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A Threefold Classification of the Binary Event Probability

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Abstract: This paper completes and refines previous works about the New Physical Approach to Probability (NPAP), a third-way model filling the gap between the Standard Probability Theory (SPT) and the Gambling Mathematics (GM). In this way, the whole range of binary predictive patterns would be ascribable to three approaches only. A three-fold classification of the binary event probability into SPT, GM and NPAP would have the here elucidated epistemological and heuristic reasons.

MSC(2010): 60-02, 60A05, 97K50, 97K60

Keywords: Binary Event, Probability Classification, Physical Probability, Standard Probability, Gambling Mathematics

1. Introduction

In some recent works [9-13] we analyzed a random binary test via the Standard Probability Theory (SPT for brevity) and the Gambling Mathematics (GM for short).

We found that, however motivated (by the Principle of Indifference or by symmetry), the SPT has the physical implication of a *linear time* and its probability of success, after n consecutive failures, is $p_{n+1}(\mathrm{E}) = \frac{1}{2}$.

The mainstream GM seems grounded on what we defined "Principle of Sufficient Reason" which implies a *non-linear* temporal frame not precisely identifiable for the wide range of strategies. The GM's probability of success, after n consecutive failures, was individuated in the interval $\frac{1}{2} < p_{n+1}(\mathrm{E}) \leq \frac{2^n}{1+2^n}$.

In order to complete our investigation, we introduced a New Physical Approach to Probability (acronym NPAP) based on the "Principle of Certainty" (i.e., the axiom that the success must occur after a *finite* number of trials). Such semi-empirical model assumes a *cyclical time* and its probability of success, after n consecutive failures, is $p_{n+1}(\mathrm{E}) = \frac{2m-\sqrt[k]{m^k-n^k}}{2m}$ $(k, m \in \mathbb{Z}^+, m \geq 23)$, reducible to $p_{n+1}(\mathrm{E}) = \frac{m+n}{2m}$ if $k = 1$.

We now wish to focus on the plausible classification of the whole range of binary event probabilities into SPT, GM and NPAP.

2. A random binary game

We imagined [9-13] to flip an unbiased coin n times letting us place bets on each toss (also called "trial") and on the same outcome until the first success.

We inferred the probability distributions of SPT, GM and NPAP facing also [9] the problem of the right wager size (i.e., the sum of money we should bet on the next trial if the unfavorable outcomes have always occurred since we started the binary event game) from each perspective.

Let E be the favorable event, arbitrarily "Heads" or "Tails", and let $\bar{\mathrm{E}}$ be the unfavorable event.

Let n be the number of times the coin has already been flipped and let $n(\bar{\mathrm{E}})$ be the number of unfavorable events already occurred since the first trial; according to our premise:

(1) $\quad n(\bar{\mathrm{E}}) = n \in \mathbb{N}$

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Let $p_{n+1}(\mathrm{E})$ be the probability of success in the next trial, after *n* consecutive failures.

Let $p_{n+1}(\bar{\mathrm{E}})$ be the probability of failure in the next trial, after *n* consecutive failures; obviously:

(2) $\quad p_{n+1}(\bar{\mathrm{E}}) = 1 - p_{n+1}(\mathrm{E})$

3. Standard Probability Theory

3.1 Axioms and formulas of the SPT

In the SPT a repeated coin flipping is a Bernoulli process [1] where each outcome is equally likely by the "Principle of Insufficient Reason" [18]:

(3) $\quad p_{n+1}(\mathrm{E}) = \frac{1}{2} \quad (n \in \mathbb{N})$

(4) $\quad p_{n+1}(\bar{\mathrm{E}}) = \frac{1}{2} \quad (n \in \mathbb{N})$

We inferred [9] that all the memoryless random [16] processes (i.e., successive trials independent of each other) require the *linear time*, a physical concept not free from paradoxes when applied to chances [21].

3.2 Graph of the SPT

The Fig. 1 describes the solutions of the Eqs. (3) and (4) as a set of points on the horizontal line

(5) $\quad y = \frac{1}{2}$

which never intersects the null probability axis $y = 0$.

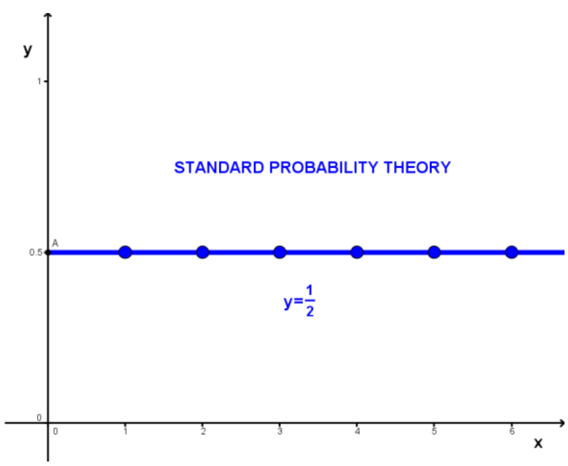

Figure 1. The Eqs. (3), (4) and (5) of the SPT.

4. Gambling Mathematics

4.1 Axioms and formulas of the GM

The GM's expectation of regularities in the event sequences obtained an *epistemological* rehabilitation [9] from the suspicion of being cognitively biased [19, 26].

In fact, we noticed that the GM's different predictions about an upcoming trial on account of the previous sequence of outcomes, known as "Gambler's fallacy" [24], are motivated by a sort of "Principle of Sufficient Reason" (opposite to the SPT's Principle of Insufficient Reason) by which there is *a reason* (not the certainty) to exclude that one out of two incompatible events can occur infinite times in a row while the other event never. We understood [9] that this axiom involves a *non-linear time*, an alternative hypothesis not to be *a priori* excluded [2-8, 14, 22, 25, 30] but indefinable when applied to the GM's contradictory strategies.

Despite the variety of dissimilar gambling strategies, denoting the GM's *heuristic* inconsistency, we chose the "Martingale" (an old-fashioned strategy where we double the wager after each losing toss) as best representative of the GM's manifold probabilities:

(6) $\quad p_{n+1}(\mathrm{E}) = \frac{2^n}{1+2^n} \quad (n \in \mathbb{N})$

(7) $\quad p_{n+1}(\bar{\mathrm{E}}) = \frac{1}{1+2^n} \quad (n \in \mathbb{N})$

The Eqs. (6) and (7) are bounds of intervals containing all the gambling behaviors based on the belief in the "fairness" of the coin [29]:

(8) $\quad \frac{1}{2} < p_{n+1}(\mathrm{E}) \leq \frac{2^n}{1+2^n}$

(9) $\quad \frac{1}{1+2^n} \leq p_{n+1}(\bar{\mathrm{E}}) < \frac{1}{2}$

4.2 Graphs of the GM

The Fig. 2 describes the solutions of the Eq. (6) as a set of points on the curve

(10) $\quad y = \frac{2^x}{1+2^x} \quad (x \in \mathbb{R}_0^+)$

which never intersects the sure event axis $y = 1$.

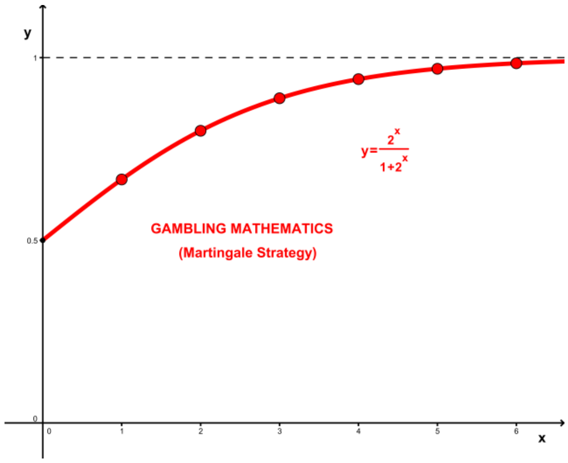

Figure 2. The Eqs. (6) and (10) of the GM.

The Fig. 3 describes the solutions of the Eq. (7) as a set of points on the curve

$$(11) \quad y = \frac{1}{1+2^x} \quad (x \in \mathbb{R}_0^+)$$

which never intersects the null probability axis $y = 0$.

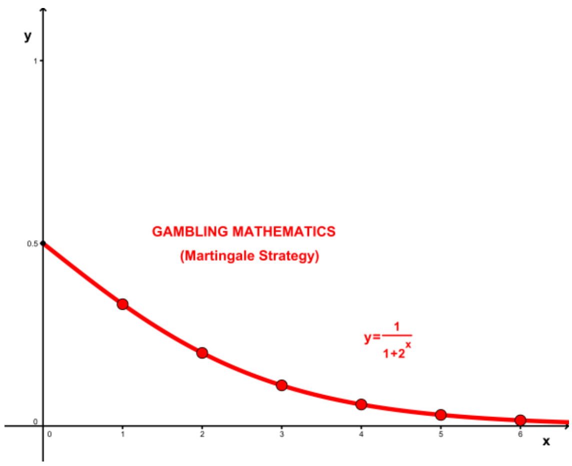

Figure 3. The Eqs. (7) and (11) of the GM.

Never intersecting $y = 1$ are also the solutions of the Eq. (8) represented by curves $y = f(x) \in \mathbb{R}^+$ in the interval

$$(12) \quad \frac{1}{2} < f(x) \leq \frac{2^x}{1+2^x}$$

some of which reported in Fig. 4.

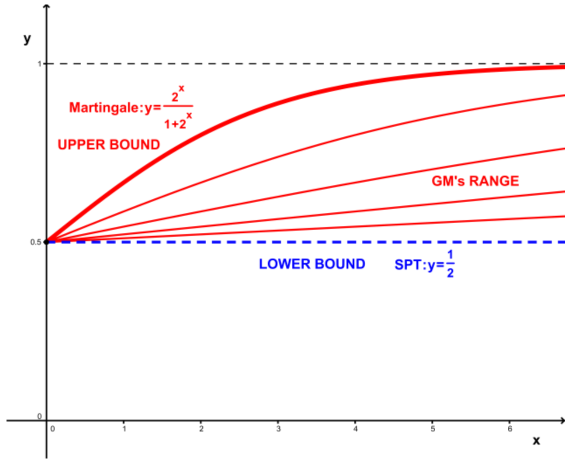

Figure 4. GM's range according to the Eq. (12).

The Fig. 5 reports some solutions of the Eq. (9) represented by the curves $y = g(x) \in \mathbb{R}^+$ in the interval

$$(13) \quad \frac{1}{1+2^x} \leq g(x) < \frac{1}{2}$$

never intersecting the null probability axis $y = 0$.

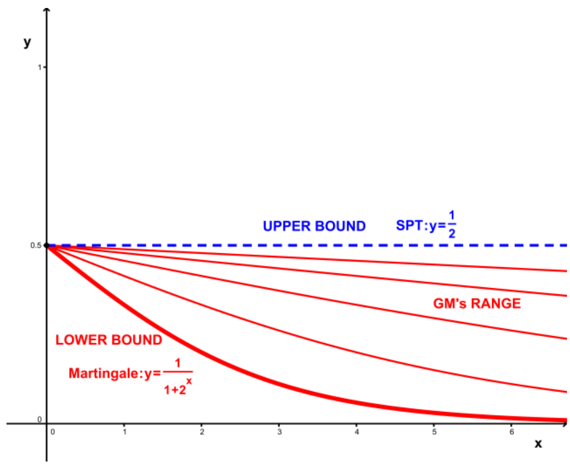

Figure 5. GM's range according to the Eq. (13).

5. New Physical Approach to Probability

5.1 Axioms and formulas of the NPAP

The NPAP's different predictions about an upcoming trial on the basis of the previous sequence of outcomes are motivated by what we called "Principle of Certainty", i.e., the *certainty* to exclude that one out of two incompatible events can occur infinite times in a row while the other event never [9].

We remarked that the NPAP is innovative because the number of failures before a success is *finite* (not potentially infinite as for the SPT and the GM) and it assumes a *non-linear time* recursively structured (cyclical).

We established the following NPAP's probabilities:

$$(14) \quad p_{n+1}(\mathrm{E}) = \frac{23+n}{46} \quad (n \in \mathbb{N})$$

$$(15) \quad p_{n+1}(\bar{\mathrm{E}}) = \frac{23-n}{46} \quad (n \in \mathbb{N})$$

We improved the Eqs. (14) and (15) as follows:

$$(16) \quad p_{n+1}(\mathrm{E}) = \frac{m+n}{2m} \quad (n \in \mathbb{N},\ m \geq 23 \text{ integer})$$

$$(17) \quad p_{n+1}(\bar{\mathrm{E}}) = \frac{m-n}{2m} \quad (n \in \mathbb{N},\ m \geq 23 \text{ integer})$$

Thus we examined the possibility of higher degree curves with the integer parameters $k \geq 1$ and $m \geq 23$:

$$(18) \quad p_{n+1}(\mathrm{E}) = \frac{2m-\sqrt[k]{m^k-n^k}}{2m} \quad (k, m \in \mathbb{Z}^+, m \geq 23)$$

$$(19) \quad p_{n+1}(\bar{\mathrm{E}}) = \frac{\sqrt[k]{m^k-n^k}}{2m} \quad (k, m \in \mathbb{Z}^+,\ m \geq 23)$$

5.2 Graphs of the NPAP

The Fig. 6 describes the solutions of the Eq. (14) as a set of points on the straight line

$$(20) \quad y = \frac{x}{46} + \frac{1}{2} \quad (x \in \mathbb{R}_0^+,\ x \leq 23)$$

which intersects $y = 1$ for $x = 23$.

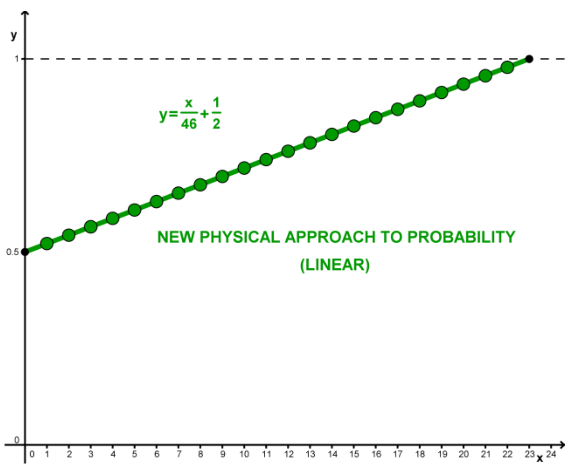

Figure 6. The Eqs. (14) and (20) of the NPAP.

Intersecting $y = 1$ for the integer value $x = m \geq 23$ are the solutions of the Eq. (16) represented by the lines

$$(21) \quad y = \frac{m+x}{2m} \quad (x \in \mathbb{R}_0^+,\ m \geq 23 \text{ integer})$$

some of which reported in Fig. 7.

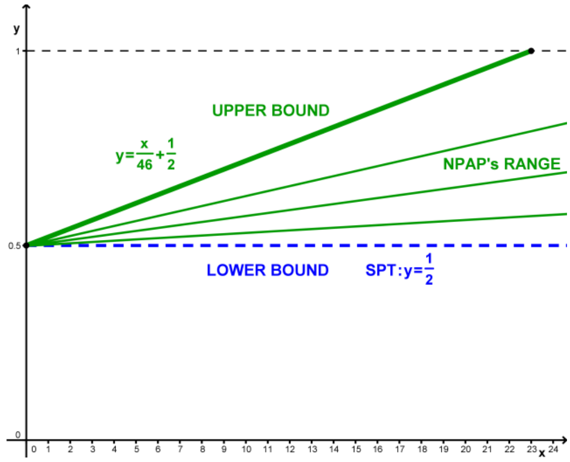

Figure 7. The Eqs. (16) and (21) of the NPAP.

The same intersections with $y = 1$ of the Eq. (21) are shared by the curves containing the solutions of the Eq. (18):

$$(22) \quad y = \frac{2m-\sqrt[k]{m^k-x^k}}{2m} \quad (k, m \in \mathbb{Z}^+,\ m \geq 23)$$

If $k = 2$ and $m = 23$, we represent the solutions of the Eqs. (18) and (22) as the arch of the ellipse

$$(23) \quad \left(\frac{x}{23}\right)^2 + 4(y-1)^2 = 1$$

reported in Fig. 8.

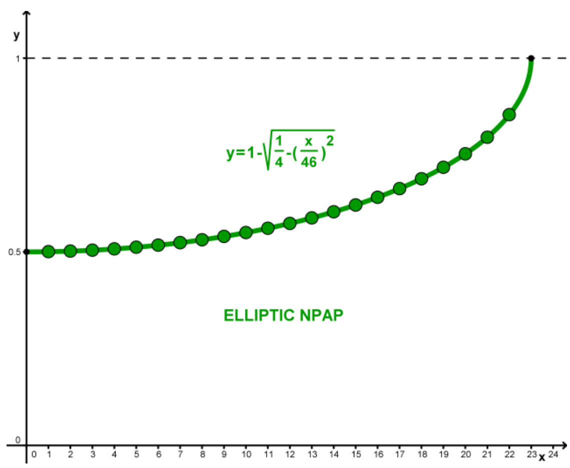

Figure 8. The NPAP according to the Eq. (23).

The Fig. 9 describes the solutions of the Eq. (15) as a set of points on the straight line

(24) $\quad y = -\frac{x}{46} + \frac{1}{2} \quad (x \in \mathbb{R}_0^+,\ x \le 23)$

which intersects the null probability axis in $x = 23$.

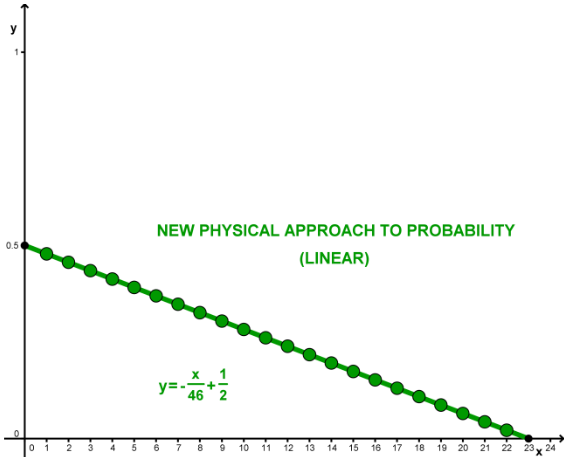

Figure 9. The Eqs. (15) and (24) of the NPAP.

Intersecting $y = 0$ in the integer roots $x = m \ge 23$ are the solutions of the Eq. (17) represented by the lines

(25) $\quad y = \frac{m-x}{2m} \quad (x \in \mathbb{R}_0^+,\ m \ge 23 \text{ integer})$

some of which reported in Fig. 10.

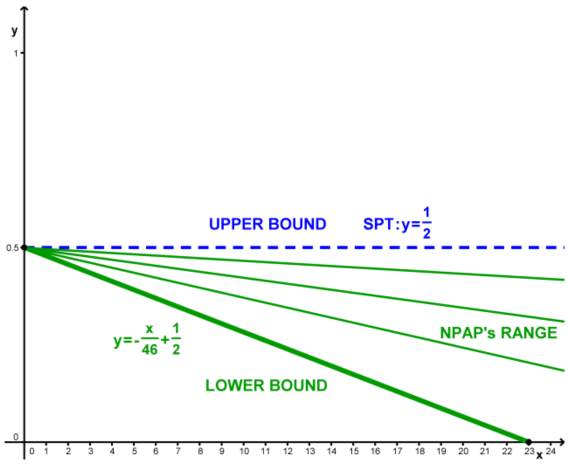

Figure 10. The Eqs. (17) and (25) of the NPAP.

The same roots of the Eq. (25) are shared by the curves containing the solutions of the Eq. (19):

(26) $\quad y = \frac{\sqrt[k]{m^k - x^k}}{2m} \quad (k, m \in \mathbb{Z}^+,\ m \ge 23)$

If $k = 2$ and $m = 23$, we represent the solutions of the Eqs. (19) and (26) as the arch of the ellipse

(27) $\quad \left(\frac{x}{23}\right)^2 + (2y)^2 = 1$

reported in Fig. 11.

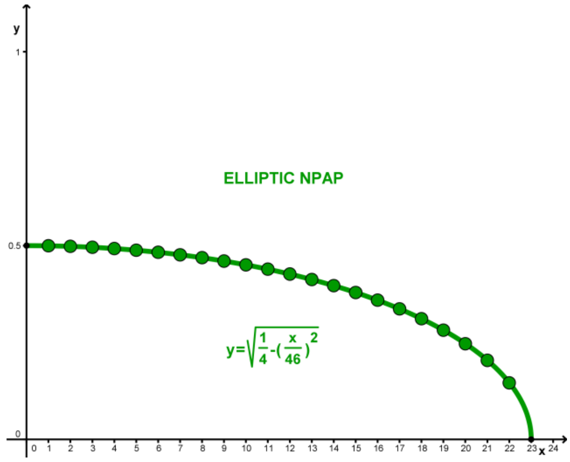

Figure 11. The NPAP according to the Eq. (27).

6. Comparison among the achievements

6.1 Comparison among the epistemological results

Let us summarize and compare all the epistemological results, i.e., the logical axioms (Table 1) and the physical hypotheses (Table 2) of each theory.

Table 1 Logical axioms of the predictive patterns

Theory	*Principle*
SPT	**Insufficient reason**: there is *no reason* to exclude that one out of two incompatible events can occur infinite times in a row while the other event never.
GM	**Sufficient reason**: there is *a reason* (not the certainty) to exclude that one out of two incompatible events can occur infinite times in a row while the other event never.
NPAP	**Certainty**: we exclude that one out of two incompatible events can occur infinite times in a row while the other event never.

Table 2 Physical hypotheses of each theory

Theory	*Nature of Time*
SPT	**Linear time**: with a potentially infinite future not affected from past results (past outcomes do

	not influence future events; e.g., in coin-flipping the probability of success in any trial is exactly the same of the first toss). The SPT implicitly assumes a *structureless time* without any scaling up allowing the Laws of Large Numbers to be applicable to short sequences of trials too; the absence of structure necessitates *scalarity* and *homogeneity*, two more features (in addition to its *infinity*, due to the existence of incommensurable samples) completing the definition of "linearity".
GM	**Non-linear indefinable time**: with observable interactions among successive outcomes from any "present" into an indefinitely far away future. The GM implicitly assumes a *structured time* whose features are not univocally defined because of the multiplicity of the developed strategies, even opposite to each other.
NPAP	**Non-linear recursive time**: with an indefinitely (but finitely) far away future linked with the past through cyclical (not necessarily identical) sequences similar, e.g., to those tracked in skilled performances. The NPAP explicitly assumes a time with some *recursion* by which the substantially same structure is scaled indefinitely up.

6.2 Comparison among the formulas

Let us summarize and compare all the heuristic results expressed in algebraic form, i.e., the probability of a favorable (Table 3) and of an unfavorable (Table 4) event, after n consecutive failures.

Table 3 Probability of the favorable event

Theory	*Formulae in the variable* $n \in \mathbb{N}$
SPT	$p_{n+1}(\mathrm{E}) = \frac{1}{2}$
GM's bound	$p_{n+1}(\mathrm{E}) = \frac{2^n}{1+2^n}$
GM's range	$\frac{1}{2} < p_{n+1}(\mathrm{E}) \le \frac{2^n}{1+2^n}$
NPAP's bound	$p_{n+1}(\mathrm{E}) = \frac{23+n}{46}$
Linear NPAP	$p_{n+1}(\mathrm{E}) = \frac{m+n}{2m}$ $(m \ge 23 \text{ integer})$
General NPAP	$p_{n+1}(\mathrm{E}) = \frac{2m - \sqrt[k]{m^k - n^k}}{2m}$ $(k \in \mathbb{Z}^+)$

Table 4 Probability of the unfavorable event

Theory	*Formulae in the variable* $n \in \mathbb{N}$
SPT	$p_{n+1}(\bar{\mathrm{E}}) = \frac{1}{2}$
GM's bound	$p_{n+1}(\bar{\mathrm{E}}) = \frac{1}{1+2^n}$
GM's range	$\frac{1}{1+2^n} \le p_{n+1}(\bar{\mathrm{E}}) < \frac{1}{2}$
NPAP's bound	$p_{n+1}(\bar{\mathrm{E}}) = \frac{23-n}{46}$
Linear NPAP	$p_{n+1}(\bar{\mathrm{E}}) = \frac{m-n}{2m}$ $(m \ge 23 \text{ integer})$
General NPAP	$p_{n+1}(\bar{\mathrm{E}}) = \frac{\sqrt[k]{m^k - n^k}}{2m}$ $(k \in \mathbb{Z}^+)$

6.3 Comparison among the graphs

Let us summarize and compare some of the heuristic results expressed in graphic form, i.e., the probability after n consecutive failures, for the linear NPAP, of a favorable (Fig. 12) or an unfavorable (Fig. 14) event and, similarly for the elliptic NPAP, of a favorable (Fig. 13) or an unfavorable (Fig. 15) event.

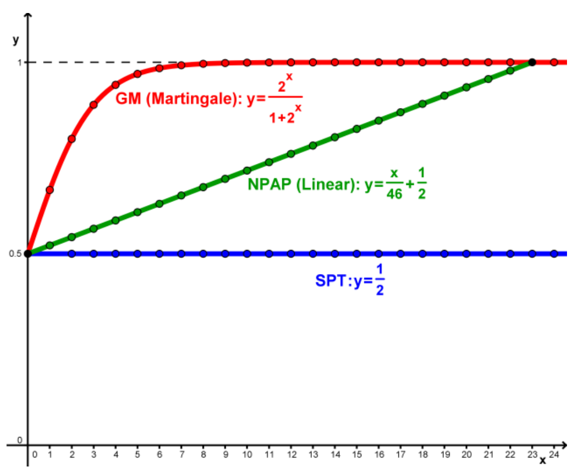

Figure 12. Comparing the Figs. 1, 2 and 6.

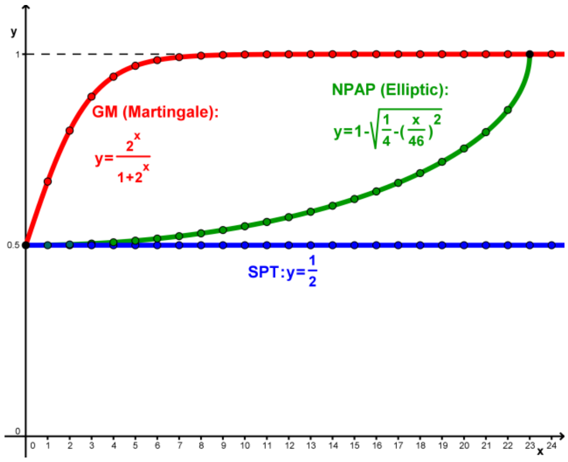

Figure 13. Comparing the Figs. 1, 2 and 8.

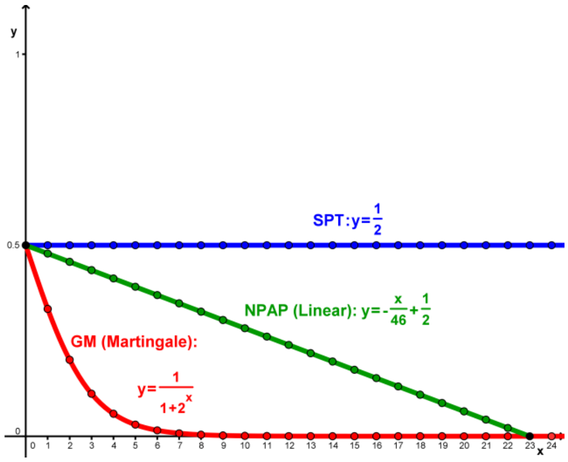

Figure 14. Comparing the Figs. 1, 3 and 9.

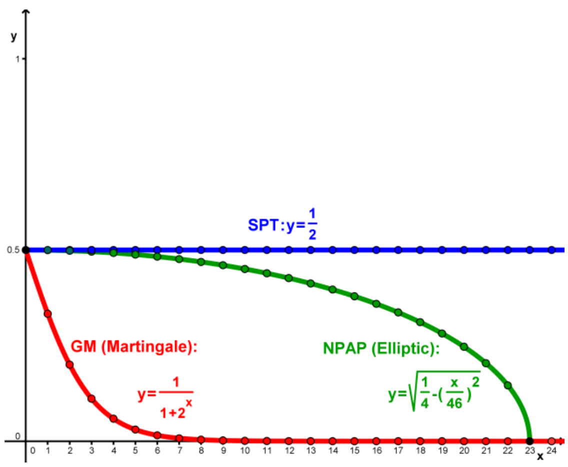

Figure 15. Comparing the Figs. 1, 3 and 11.

7. Conclusions

We have explored the tripartition of the binary event probability into SPT, GM and NPAP. It is founded on the epistemological and heuristic arguments recalled from previous works [9-13] and further expounded here, with additional tables and graphs.

Epistemologically speaking, the SPT is based on a Principle of Insufficient Reason needing the linear time, the GM is grounded on a Principle of Sufficient Reason requiring a not modelable non-linear temporal conception, the NPAP is rooted on a Principle of Certainty demanding a cyclical time.

Heuristically speaking, for a binary event:

1) the SPT employs linear equations and the probability distribution is a horizontal segment, on $y = 0.5$ line, parallel to the null probability axis (because the number of failures before a success is potentially *infinite*);

2) the GM 's mainstream strategies use fractional exponential equations and the probability distributions are arches of curves never intersecting their asymptote $y = 0$ (because the number of failures before a success is potentially *infinite*);

3) the NPAP 's predictive tools are a variety of formulas, from linear to fractional irrational equations, and the probability distribution, always intersecting the x-axis (because the number of failures before a success is *finite*), can be a segment (of course not horizontal), an arch of ellipse or of a higher degree curve.

Acknowledgments

I wish to thank all the people whose inspirational works contributed to motivate this research, in particular: Szabó [27, 28] for the Physicalist Interpretation of Probability; Diaconis et al. [15] for the physical experiment showing that the coin-tossing is fair at the second digit but not at the third; Gilden and Wilson [17], Green and Zwiebel [20] and Raab et al. [23] for their countercurrent studies about the streaks in some sports.

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