International Journal of Mathematical Sciences and Applications
Vol. 1 No. 1 (January, 2011)

www.ijmsa.yolasite.com

HOW TO TASTE MATHEMATICS

Fayez Fok Al Adeh
President of the Syrian Cosmological Society

P.O. Box 13187
Damascus, Syria
Phone: 00963-11-2776729
00963-11-2713005
E-mail: hayfa@scs-net.org

Abstract

We are facing a real danger as mathematics continues to be driven further outside the realm of modern culture. The danger lies in the fact that mathematical structures reflect all possible patterns of mental activity, which are creative and unlimited. This proves itself especially in mathematical breakthroughs. Mathematics is integral to culture. In fact, mathematics merits as legitimate a place in modern culture as music.
The fundamental sequence of doing mathematics is as follows. In the first phase, the mind formulates some simple or rather involved problem. Soon the problem transforms into a sort of obsession that pushes the mind into a lengthy search for a solution. In the case where a solution is found, the mind is satisfied at least temporarily – until it creates a new problem.

Mathematics is not a pragmatic science in any sense. Hence, there is a rarity of great mathematicians in our epoch. Another reason for a dearth of modern, brilliant mathematicians is that mathematics is, unfortunately, being relegated primarily to the economic sector. Music is moving towards the same fate. No one discipline can attest to having a monopoly on how to appeal to our deepest aesthetic sensibilities – not music or any other of the arts. And, in fact,

mathematics has more in common with music than students of either are led to believe.

Unfortunately, the prevailing classical methods of teaching mathematics, are partly responsible for the decline in this great art. We have to modify these methods so as to achieve a deeper understanding of mathematics. Our incomprehension of a thing often leads to our criticism of it as being too involved or complicated, followed by our tendency to thus render it unimportant, and then to our conveniently neglecting it altogether. To experience the beauty and wonder – the exaltation – of mathematics, just like that of music, requires only that we attempt to truly understand it.

Keywords: General Mathematics, Geometry, Topology, Number Theory.

AMS Mathematics Classification: 00Axx, 00Bxx.

1. Introduction

Searching for knowledge in the field of mathematics is akin to excavating for the aesthetic elements in their purest most abstract formulae. The simplest theory is the most beautiful theory is the most correct theory. It is not difficult, therefore, to see how mathematics qualifies as one of the arts.

Nevertheless, mathematics is consistently classified as one of the sciences. Some scholars go so far as to say that it is the central science. This incorrect delineation still prevails as the principle view in most educational milieu. Imagine if mathematical theses were merely a system of relations to be memorized with proofs, and confined to using them on the necessary prerequisite??. Hence, those relations change to useful tools that are put aside when consuming the pragmatic utilitarian benefit from them??.

In this research I present quite a contrary point of view to this. My premise is that, not only is mathematics an art, it is the most sublime art of all. An d mathematics and music have more in common then things which separate them. Mathematics is the music of intuition, and recognition, while what we familiarly refer to as music is the music of the senses.

This can be better understood if we consider how humans sense the world around them. Humans have the capacity to sense or interpret

one, unified music. However, there is no reason that this music cannot be notated using different systems: one being musical notation, and the other being mathematical principles.

Musical notation includes the notes of the musical scale: Do, Re, Mi, Fa, So, La, Te, Do; and musical marks and symbols such as clefs, time signatures, note value, rests, repetitions, codas, and so on. Mathematical modalities include a variety of proofs: direct proof, reversal, contradiction proof, proof by choice, proof by extrapolation, proof by allocation, direct oneness proof, indirect oneness proof, proof by deletion, proof by the cases, proof by resorting to the greatest and smallest values. These two types of notation, while different, can be converted to form a unified wholeness, which is, in essence, an eternal cosmic music.

Here, there emerges the following pressing question: Could we not then integrate musical education with mathematics education? If mathematics can be approached step by step to reach an absolute, then music could as well. If mathematics is a scale that elevates mankind above the stars, could not musical notation provide the scaffolding for the steps of that scale?

Though of different origins, both mathematics and music emanate from similarly inexplicable intuition and indiscernible vision. If this is so, what justifies the continued isolation of mathematics? Why do we continue to regard mathematics merely as lifeless tables and data? Is it not, indeed, the same inspirational brainwave that is responsible for creating, generating and framing a musical melody that is required to recognize a mathematical definition, and form and prove a mathematical system? In the interest of aesthetics and excellence, it is therefore prudent that we begin immediately to integrate musical lessons with mathematical lessons at all levels of education.

My research consists of thirty pairs each consisting of a musical phrase and a mathematical clause selected from a variety of recorded musical works, and mathematical clauses. I intentionally chose pairs

that do not, at first glance, resemble one another. I then attempt to present them in a way that illustrates their similarities. The goal is to help readers conceive of the possibility of mathematics as being capable of achieving a harmony similar, if not identical, to that achieved in the musical realm. While each discipline has its own distinct harmony, there are nevertheless, common denominators.

In terms of methodology, I have identified certain criteria that allow the commonalities or symmetry in the two elements within each pair – mathematical and musical –to be highlighted.

2. **Criteria for Analysis of Musical Phrases**

characteristics of the composer
description of the musical phrase: preface, introduction, interval, movement, bound, light musical piece, pondering, dance, enthusiastic song, special work, choir, the machine and /or dominant machines,
the psychological impact of the phrase/the search for a result such as: expectation of salvation, sensation of simplicity, ease or difficulty of adaptability, adding adobes/building blocks? to the imagination, the experience of fear, awe, joy, sorrow, openness, giving credibility?, closing and possession?,
phrase rhythm: intermittent, separate, mixed
harmony pattern caused by the phrase: deep and entire, partial, superficial
musical enumeration: apparent lyrical tune, tuneless phrase
musical composition: direct display, gradual creation
instant evaluation of the phrase?: beautiful and could always be heard, medial beauty heard from time to time, lack of aesthetic elements and could be heard in favor of separate periods.

3. Criteria for Analysis of the Mathematical Obverse

The characteristics of the author
description of the mathematical clause in accordance with subjective intention, undefined elements, result, theory, definition, preliminary,

mathematical codifying, adoption of activist term produce expected result, particular work, multiplying structures, method of proof and / or the dominant methods of proof,
the psychological impact of the phrase/the search for a result such as: expectation of salvation, sensation of simplicity, ease or difficulty of adaptability, adding adobes to imagination?, the experience of fear, awe, joy, sorrow, openness, giving credibility?, closing and possession?,
clause rhythm: deep and entire, partial, superficial
mathematical enumeration: apparent target, mysterious context
mathematical composition: direct display, gradually creation
instant evaluation of the clause: beautiful and could always be heard, medial beauty heard from time to time, lack of aesthetic elements and could be heard in favor of separate periods.

Before presenting the selected pairs of musical phrases and their mathematical obverses for discussion, I will draw on some aspects of well-known psychological theory which support my research.

4. Mathematics and Music from the Psychological Perspective

Here, rather than trying to interpret psychology from a mathematical perspective, I am going to address the psychological effects which occur as a result of the impact of mathematical activities. To do this, and in continuing with this paper's assertion of deep similarities between the disciplines, it will be helpful to compare such effects to those which occur as a result of music.

From the physical point of view, music is made up of merely sound waves, not unlike other sound waves evident in various other material conglomerations These physical sound waves cause different harmonic conditions in listeners.
It is very difficult to classify the conditions of harmony, or bind these conditions by the rules of a law or a few laws, and so it is with mathematics. Nonetheless, the deep psychological phenomena, is attributable to the harmony of those conditions in both mathematics and music.

Physics, may provide a counter explanation, and one that is simpler and easier to understand. It is the phenomenon of resonance whereby the physical response within a certain physical formation will reach the maximum climactic level if the natural, recurring frequency of natural vibration of formation is equal to the influential? or near enough.

Full resonance leads to the extinction of influential and related? factors, as at the confluence of electron and positron. This is equivalent to the life cycle of some species, where one or both of the parents die upon completion of reproduction. Is not deep grief capable of killing the author, creator, composer? Cannot overwhelming joy become a leading cause of death?

The phenomenon of resonance is narrower in scope than the state of harmony and perhaps it occupies a separate site. If music is a harmony achieved on the sidelines??/as a byproduct of very complicated feeling combinations, then without any doubt, the same can be said for mathematics. For, according to Albert Einstein, the father of the *Theory of Relativity*, the purpose of mathematics and physics is not the discovery of relationships between physical objects, but relationships between feelings. Reality is comprised of many combinations of feelings. For great scientists, who use mathematics and physics to explain reality, psychology is a key component.

We note here that the physics stands out as the most important academic course when it comes to the applications of mathematics.

The most correct mathematical case? is the simplest case?. It is an interior system and achieves the highest harmony within the internal world of the creator/originator/mathematics scholar?/mathematican?. The mathematical case is the internal projection of harmony, a harmony which is born of the complexities of internal feelings – feelings to which the world's very existence can almost be attributed.

It is true that the world is filled with waves and particles, but these remain neither interpreted nor classified until impacted with consciousness. Thus, the phenomenon of harmony and the phenomenon of resonance can be reduced to a pure creative act of consciousness. Creative acts can be categorized and they have degrees, but they all share in the fact that each creative act is more or less harmonious.

The originator translates the creative act according to a certain alphabet. Mathematical symbols represent the alphabet of mathematics; musical notation is the alphabet of musical pieces. Since everything is joined by frequencies, and each symbol expresses a frequency, then the frequency released by a star and the frequency of a musical phrase are alike in that they are no more or less than mere frequencies. We can, therefore, justify the repeated attempts of mathematician/philosopher, Pythagoras, mathematician/astronomer, Johannes Kepler, and others to translate the innovations of mathematics, physics and the cosmos into mathematical symbols and figures.

No wonder that they each ventured forth in their respective attempts to blend the various translations. There is, however, a difference. The mathematician can present an extensive, original, mathematical formula and interpret it into clear rhetorical phrases, however, it may remain difficult for others to understand. The difference between mathematics and music is in the rhetorical presentation when it comes to the origins of internal harmony. In the mathematical conceptualization something is lost in terms of the internal harmony – that which created it, its central component and that which is considered its real vision – when it comes to the process of imitation by others.

The composer, in the field of music, similarly faces a great, but different difficulty whenever he/she tries to transfer creative innovations to an explicit rhetorical text. Despite its very difficulty, it is also the major advantageous quality inherent in the creation of music. The listener can personally experience directly the sensation of

the agitation of internal harmony, which led to the musical phrase. What if we probed deeper into the notion of the internal harmony responsible for the formation of mathematical innovations and attempted to find commonalities with the internal harmony responsible for the creation of musical works?

The psychological impact of tones resulting from musical notation seems to be deeper and stronger than that of mathematical symbols and of language alphabets. This is why it is more relevant to the core of cosmic structure and responsible even for the existence of such infrastructure as pointed out by Einstein in regard to mathematics and physical laws.

Let me use an analogy here: because red lipstick is such a collectively known thing, it is impossible for one of us to experience the taste of red lipstick according to taste of the lipstick, or the internal harmony of it, for others. The taste – the internal harmony – is thus subjective. Both musical phrases and mathematical clauses have internal harmony translated into alphabets, and while one can be explained in words of rhetoric, it is impossible to do so in the case of the other.

Because of the core similarity we can examine mathematical clauses or innovations to try to get at the origins of their internal harmony. This, though, demands that mathematics curricula be transformed from direct, neutral elocution to a different method. The new method must attempt to expose the hidden emotions so as to ensure the student's assimilation of mathematics is through 'taste' or experience and not memorization or invocation.

If harmony were at its zenith, and every superior accomplishment were unique and genuine, then the innovations in mathematics would all be correct, and musical creations all awe-inspiring. What some believe to be a lack of harmony in some cases is no more than merely a wrong mathematical construct or just bad music. Music is integral within the discipline of psychology, and the proposed methods for teaching mathematics in order to make the case for harmony in mathematics demands urgent attention by psychologists.

A discussion about a single, common structure for mathematics and music requires a little more discussion about psychology. We can use to our advantage the tenets of this science for the formulation of psychologically based mathematical and musical structures. The successful formulation of such structures using psychology would affirm the idea of the single structure and would justify the comparative approaches used in the body of this research.

5. Psychological Theory: The Collective Unconscious

At the beginning of the 20th century, the great psychologist, Carl Gustav Jung, formulated an idea he referred to as the *collective unconscious*. He proposed that the collective unconscious is a consciousness shared by and inherited by all humans. He believed that in each person's psyche exists a shared consciousness that contains all of life's latent and inherited functions from the ancestral strain, where every child imbibed with pre-awareness and suitable psychological functional readiness (1).

According to Jung, though it is sub-conscious, this instinctive, concealed function continually asserts its presence and activity in the midst of a conscious adult's life. The sub-conscious is transient and attuned to the immediate surroundings in the present. Additionally, the sub-conscious contains an unspecified mass of observational perceptions which are a huge treasure consisting of the accumulated contributions of all the generations of humankind.

In short, the entirety of past cosmic experience, which has accumulated since the *"Big Bang"* said to have created the universe, lies in that collective unconsciousness. Can we, by using language, any language, sculpt enough idioms to describe what lies waiting to be revealed in its concealed contents?

The collective unconscious is indeed the reservoir of all knowledge. We return to our question by saying that the answer can only be negative. The proof of this is simple in principle. Language based on the alphabet is limited to a number of symbols which build words.

Now, if we are trying to assess the highest number of words that can be produced by a particular language, we have to assume a maximum number of characters in each word from the words of the language in the research. Let us accept that this limit is equal to fifty characters. No person will be able to pronounce any word which consists of more than fifty characters. This is what justifies our acceptance of the mentioned limit.

This highest limit of the number of words that can be produced by a given alphabet is equivalent to the number of characters in the alphabet brought] to the equivalent of fifty which is the highest limit to the number of letters in each word from the words of the language.

Life, in return, has its own alphabet. This alphabet consists of twenty amino acids. Life can install proteins as much as it needs, starting from these acids. We can say that the mentioned amino acids serve as the characters in the alphabet of life, while the proteins are the words in that language. In this case, there is no limit to the number of characters needed to create each of the constituent 'words' of life. For example, scientists have discovered a protein consisting of more than one hundred amino acids (2).

The absence of a limit opens the way for life to construct new words with increasing lengths. In short, the number of words in the language of life will always surpass the number of words in any language spoken by members of the human race, and all languages based on alphabets will remain unable to formulate words corresponding to the 'words' built by the continuation of life.

We conclude from this simple comparison that human language is not an adequate medium by which to access the wealth of all knowledge and awareness present in the collective unconscious. This holds true despite the fact that all new knowledge continues to be expressed by humans in the form of language-specific phrases, however inadequate they may be. Still, humans have always hoped to discover some means, perhaps a talisman, that would allow them a window into all of the progress and knowledge achieved and

accumulated through the ages thus far.

Since ancient times, man has invented alphabets to achieve this aim. In addition to basic alphabets with letters man has also employed an alphanumeric system. In this system, characters from a certain alphabet, as well as certain symbols like points, commas and so on can be transferred into positive integers according to specific rules.

Thus, we can translate *the Diwan* by Al-Mutanabbi, books by Dostoevsky, mathematical theses, *the Theory of Relativity*, the cosmic sciences and so on into 'syllables' consisting of numbers.

In the first part of this paper the aim of the research has been to discuss how both mathematics and music can be assimilated – how they can, in fact, be unified. Through an analysis of harmonic pairs in the body of this paper, a reasonable pattern emerges that integrates not only mathematics and music, but perhaps all human knowledge. This pattern is characterized as a metamathematical pattern like the type formulated by Kurt Gödel to explain his *Incompleteness Theroem* (3).

As shown by Jung, mathematics and music can both be discussed from the same perspective. And because it is a metamathematical pattern, he reduces mathematics to its harmonic roots in the collective unconscious and classifies music as one of its results.

We must turn to the decimal fractions for the construction of our desired pattern, namely, those fractions that have numbers after the decimal point only.

The numerical phrase/clause delineating any unit of human knowledge will, without doubt, come in an endless number of decimal fractions. However, the closest decimal fraction to knowledge is that which is made of a numerical phrase/clause of this knowledge that has been repeated an endless number of times after the decimal point. This numerical phrase/clause is the integer, which we can get by the appropriate use of the numerical alphabet

we have produced. The decimal fraction, in this case, is described as a non-random. For example, we can translate Gauss's *Law of Quadratic Reciprocity* to an integer, and build a repeating fraction by reiterating this integer an endless number of times after the decimal point. From the mathematical point of view, there is precise relationship between the figures in this repeated numerical phrase to allow some prediction of the usage of others.

In general, we describe the decimal fraction as non-random if we can define it by useful phrases/clauses. Now we imagine a decimal fraction including one or more numerical phrases/clauses, but repeating only a finite number of times. We say that such a fraction is non-random fraction of the second type. A random fraction is that which lacks an explicit definition. If we resort to defining such a fraction by mentioning its constituents, we will face an impossible task.

Numerical alphabets provide us with great potential for translating the language versions of numbers and vice versa. Thus, if we look at a non-random decimal fraction of the first type, as a kind of linguistic random text, we can conclude that a fraction group of this kind certainly forms an endless milieu of knowledge. It may be a milieu which includes knowledge already discovered, and knowledge not yet revealed.

What if we tried to isolate one of these fractions and translate it into the original alphabet? By doing so, could we not then move from a fraction to a theory or a poem or lyrical tune? The number of decimal fractions is much more than the number of non-random fractions of the second type and the number of latest fractions is more than the number of non-random fractions of the first type. The conversion of any random fraction to the alphabet will also provide a linguistic text free of any meaning. The corresponding conversion of a non-random fraction of the second type, however, will produce a text that is linguistically sparse and thin on content. The non-random fractions of the first type have a unique nature, as they result in linguistic texts with great depth of content.

If the mentioned process of isolation were not described in clear terms, that is, if it were random, it would provide only a random fraction, and at best would only highlight the non-random fraction of the second type. The high system and the high-symmetry fully implied in the non-random fractions of the first type compel the mathematical scientist or the musical composer to make a major effort in his or her persistent pursuit of a desired object or goal whether it be a scientific theory or a lyrical masterpiece.

Finding the knowledge-equivalent of the decimal fraction, without doubt, is a kind of talent. There is no particular system to giving a language equivalent for a decimal fraction of the mentioned type starting from the repeated numerical component to it. If we knew some numbers of this phrase, we could predict the other terms, since the phrase numbers in this case are linked to each other in specific relations. But if we knew some numbers from the numerical phrase of the repeated non-random decimal fractions of the second type, the predictability of other terms of the phrase would be very difficult and the matter would appear to us as if we were dealing with a random decimal fraction.

The decimal fractions representing the activities of mathematics and the musical pieces are non-random decimal fractions of the first type. This group of decimal fractions is a non-closed group and as long as we are able to convert any of them to a linguistic text, it means that we will find many meaningless theses and fractures resulting from the transfer of random and non-random fractions of the second type to the original language.

Due to the number of decimal fractions and non-random and random fractions of the second type compared with the random fractions of the first type, the above-mentioned processes take place in isolation from talent and without any effort. There is evidence that parties engaged in scientific dialogues with others, on topics about which they knew very little, could even draft many phrases and use scientific terminology, without any logical basis. Are there not, for

that matter, many people trying to compose pieces of music with artificially carved frames and empty contents. We find the explanation for this in the abundance of the corresponding decimal fractions – both random and non-random decimal fractions of the second type.

6. Mathematical and Musical Harmonic Types

I have tried, through selected types to confirm the idea, that mathematics is a major internal source of harmony, in the same way that music is. The selection process was not spontaneous or random, as I carried it out in accordance with a specific standard. Each selected musical phrase puts the listener in a certain harmonic condition. For the new idea that I propose to be effective, the respective mathematical obverse must push the student to a debate of harmonic case, and must, shortly after, lead to a little effort to represent the content corresponding to the student in question. This is achieved on simple subjective grounds.

That the emotional elements inherent in the mathematical concepts contain similar elements to those within the musical phrase means, depending on the quality of the student, the mathematical concept has the capacity to move the student in as profound ways as a musical phrase does. As the musical phrase renders passion in the student, the mathematical opposite permeates his mind through a common means, in this case, harmony. The musical phrase and mathematical obverse are linked in their capacity to create two approximate experiences of harmony whereby the rhythm of the musical phrase and the essence of the mathematical obverse are stabilized as well as contrasted against the background of the conscious and subconscious elements of the mind. Because of this, I was desirous that selected pairs of a musical phrase and its mathematical obverse have a minimum /variance of harmony between them.

Following the presentation of each pair, through a brief analysis, I attempt to prove that the harmony created by both the musical

phrase and its mathematical obverse is closely aligned. It should then become clear that a close study of the musical phrase can be used to help the student of mathematics understand its mathematical obverse.

I propose that the examples of harmonic pairs outlined in the remainder of this research paper, can be used effectively in selected classes. I hope that the educational community accepts my research and my resulting proposal. Furthermore, I hope those responsible for future curriculum development in the disciplines of both Mathematics and Music, take into account the basic idea underlying my research. There is no doubt that it would be beneficial to unify Music and Mathematics courses and possibly even other courses in order to enhance learning.

7. Types of Musical and Mathematical Harmonic Pairs

7.1 Type 1

Musical Phrase: *Hungarian Dance No. 5*

Composer: Johannes Brahms. Brahms worked humbly as a musician for a long time before turning to composition. He used to say, Beethoven stands on my shoulders. Latterly, his prime focus was upon composition. His musical phrases are strict and non-bending especially in the violin concertos, but that does not apply to the Hungarian Dances.

Description of the musical phrase: a folk dance.

Dominant musical instrument(s): orchestral performance.

Psychological impact of the phrase: adding adobes/building blocks to the imagination.

Rhythm of the phrase: separate.

Harmonic pattern generated by the phrase: deep and holistic.

Musical enumeration/delineation?: apparent lyrical.
Musical composition: direct presentation.

Real-time evaluation of the phrase: beautiful and can always be heard .
Mathematical Obverse: *The Betweenness Axioms* (4).

Originator(s): David Hilbert. Hilbert was a great German Mathematician known for very abstract mathematics. He reformulated mathematics based on the concept of consistency. He also formulated the theory of general relativity a few days before Einstein started from purely mathematical elements, but the theory was attributed to Einstein for several reasons, including the physical assets Einstein set out.

Description of mathematical clause: thematic structure.

Method of proof and/or dominant methods of proof: thematic and does not require a proof.

Psychological impact of the clause: sense of simplicity.

Rhythm of the clause: separate.

Mathematical enumeration/delineation?: apparent target.

Mathematical formation: direct presentation.

Real-time evaluation of the clause: attractive and can always be referred to.

Analysis: The Hungarian Dance No. 5 consists of two separate tunes. The repetition of listening to it no doubt will leave a profound impact about the concept of "Betweenness," because the first tune is

repeated in the introduction and conclusion while the second tune highlights a clear and opposite between the introduction and conclusion. It is a musical "betweenness" like the Betweenness of Hilbert, the mathematician. The passionate situation created by the Dance could be the necessary deprivation type in the context of Hilbert's concept of Betweenness.

7.2 Type 2

Musical Phrase: Mendelssohn's *Violin Concerto*.

Composer: Felix Mendelssohn. Mendelssohn was a German musician and composer. His creativity in music began at an early age and approached peak glory at the age of seven. His heightened aesthetic feelings enabled him to reflect his impressions faithfully in his music. We remember in particular, in this context, his remarkable work: the Italian Symphony. Mendelssohn's *Violin Concerto* is considered one of the most beautiful concertos ever played on this instrument.

Description of the musical phrase: movement.

Dominant musical instrument(s): violin.

Psychological impact of the phrase: sense of joy.

Rhythm of the phrase: mixed.

Harmonic pattern generated by the phrase: deep and holistic.

Musical enumeration: apparent lyrical.

Musical composition: direct presentation.

Real-time evaluation of the phrase: beautiful and can always be heard.

Mathematical Obverse: The Perfect Numbers (5).

Originator(s): Pythagorean School. Pythagoras had established this school, which settled moral and aesthetic problems in all aspects of life. The school inspired particular universal patterns from the aesthetics of numbers.

Description of mathematical clause:
Definition: The total number is equal to the total divisors.
For example: 6 = 1 +2 +3
28 = 1 +2 +4 +7 +14
The perfect number is distinguished with a spectacularly beautiful advantage found through the following process:

We consider the total number: 28.
28 = 1 +2 +4 +7 +14
Divide both sides of this equation by 28:

$$1 = \frac{1}{28} + \frac{1}{14} + \frac{1}{7} + \frac{1}{4} + \frac{1}{2}$$

Re-write the last equality in the formula:

$$1 = \frac{1}{7} + \frac{1}{14} + \frac{1}{28} + \frac{1}{4} + \frac{1}{2}$$

We convert to the binary system of numbers:

$$\frac{1}{7} = 0.001001001\ 001 \ldots \ldots$$

$$\frac{1}{14} = 0.0001001001\ 001 \ldots \ldots$$

$$\frac{1}{24} = 0.00001001001\ 001 \ldots \ldots$$

$$\frac{1}{4} = 0.010000000 \ldots \ldots$$

$$\frac{1}{2} = 0.010000000 \ldots \ldots$$

Total = 1 = 0.111 111 1111 11.....
Note that the collection of binary fractions described does not only lead to an integer, but that, in addition, it does not include any carrying.

Method of proof and/or dominant methods of proof: a definition has no need of proof.

Psychological impact of the clause: sense of joy.

Rhythm of the clause: mixed.

Harmonic pattern generated by the clause: deep and holistic.

Mathematical enumeration/delineation?: apparent target.

Mathematical formation: direct presentation.

Real-time evaluation of the clause: attractive and can always be referred to.

Analysis: Mendelssohn's *Violin Concerto* and the definition of the numbers are replete with aesthetic elements. Both are full of beauty. They are like a very sweet drink. This creates the possibility of hearing this Concerto when experiencing the definition of the full perfect numbers.

7.3 Type 3

Musical Phrase: *La Cumparsita*.

Composer: Gerardo Matos Rodríguez: Rodríguez was a popular emotional musician and composer from Uruguay in Latin America.

Description of the musical phrase: dance.

Dominant musical instrument(s): Spanish guitar.

Psychological impact of the phrase: sense of simplicity of adaptability.

Rhythm of the phrase: sporadic.

Harmonic pattern generated by the phrase: partial.

Musical enumeration/delineation?: apparent lyrical.

The musical composition: direct presentation.

Real-time evaluation of the phrase: beautiful and can always be heard.

Mathematical Obverse: Mathematical Relation: $[6]\, d\bar{N} + kd\,\bar{X} = 0$ (6)

Where $\bar{N}$ is the organizer on a surface, k is the arc in the direction of the curvature line d x. In the context of Gaspard Monge's theory, the importance of this relation highlights the notion that it is a necessary and sufficient condition for there to be a bend on a certain surface in order for there to be a curved line, the established organizers lines along the curved surface to shape a spread able surface.

Originator(s): Olend Rodrigo. Rodrigo was one of Monge's outstanding, distinguished students. His name emerged in particular with regards to differential Geometry and in the functions of Legendre.

Description of mathematical clause: theory.

Method of proof and/or dominant methods of proof: direct proof.

Let us assume a given curve: X = X (s)
Thus we will have:

$$\left(\frac{d\bar{x}}{ds}\right).\bar{N} = \bar{1}.\bar{N} = 0$$

The point on a ruled surface created by the organizers on that surface is determined by the Relation:
$\bar{Y} = \bar{X}(S) + u\bar{N}(S)$
Where u is the distance between points $\bar{Y}$ and $\bar{X}$.

We conclude from the above to arrive at the following relations:

$\bar{Y} = \bar{X}(S) + u\bar{N}(S)$
$\bar{Y}s = \bar{t} + u\,\bar{N}s$
$\bar{Y}u = \bar{N}$
$\bar{Y}su = \bar{N}s$
$\bar{Y}uu = \bar{N}s$
$eg - f2 = o$
$(\bar{T}\bar{N}\bar{N}) = O$

If 0 = g the extension condition changed to eg-f2 = o
Thus f = 0 or $(\bar{T}\bar{N}\bar{N}) = O$
Where:
$e \propto (\overline{XuuXuXv})$
$g \propto (\overline{Xvv\ Xu\ Xv})$
$f \propto (\overline{XuvXuXv})$
In other words, the $\bar{N}$ is vertical with both T and $\bar{N}$s either the $\bar{N}$ s = 0 or that t is on the straightness of the $\bar{N}$s.
Thus we get the relation of Rodrigo.

Psychological impact of the clause: sense of simplicity of adaptability.

Rhythm of the clause: separate.

Harmonic pattern generated by the clause: partial.

Mathematical enumeration: apparent target.
Mathematical formation: direct presentation.

Real-time Evaluation of the clause: attractive and can always be consulted.

Analysis: As Rodrigo's relations survey closed extension able surfaces for tripartite solids, *La Cumparsita* also sways in return. which is for another Rodrigo, on the surrounding surfaces of our internal worlds? The teacher must address this idea, and the fact that the ripples caused by Rodrigo's relations survey, are of the same quality and perhaps have the same ripples formed after hearing the phrase of Rodriguez.

7.4 Type 4

Musical Phrase: Beethoven's *Fifth Symphony*.

Composer: Ludwig van Beethoven. Beethoven was a great musician who revealed the nature of the deep intellectual, mental impact of music.

Description of the musical phrase: the first movement, a part of that movement.

Dominant musical instrument(s): stringed and brass instruments.

Psychological impact of the phrase: expectation of salvation, sense of the difficulty of adaptation, awe

Rhythm of the phrase: sporadic.

Harmonic pattern generated by the phrase: deep and holistic

Musical Enumeration: tuneless phrase.

Musical composition: gradual synthesis.

Real-time evaluation of the phrase: beautiful and can always be heard.

The Mathematical Obverse: *The Law of Quadratic Reciprocity* (7)

$(q/p) = (-1)^{(p-1)(q.1)/4} (p/q)$ Where p and q are prime numbers and (p/q) (q/p) are Legender symbols.

Originator(s): Carl Friedrich Gauss. Gauss was a great, multi-talented mathematician who excelled in all branches of mathematics.

Description of mathematical clause: theory.

Method of proof and/or dominant methods of proof: direct proof; proof by cases. It is one of sixteen proofs produced by Gauss to the same theory.

Psychological impact of the clause: sense of difficulty of adaptation; awe.

Rhythm of the clause: sporadic.

Harmonic pattern generated by the clause: deep and holistic.

Mathematical enumeration: ambiguous context.

Mathematical formation: gradual synthesis.

Real-time evaluation of the clause: attractive and can always be consulted.

Analysis: Beethoven's *Fifth Symphony* raises to the surface the myth of the eternal conflict of man with Fate. The Symphony provides what the listener does not expect at all. Both opponents of the conflict are equal. Each opponent is imitated by a set of instruments. Here, we cannot differentiate between the machinery embodied by each of

the opponents, but enough is implied for us from the symphony in the wonderful analog format, which includes information about the two strange opponents. The elements, which form each opposing side, are not related to any of the known earlier forms However, the great Beethoven surprises us with the mentioned great form.

Gauss, in turn, does something similar. If not in conformity, while the prime numbers have a different structure. Kummer's Optimum Analysis confirms this (8). Once again, it took the genius of Gauss to explore a similar pattern which emerged from the genius of Beethoven. A symmetry joins the two opponents who are impossible to annex. Here, the teacher must clearly show the discrepancy between the fatalistic, destiny currents and the voluntary structure of the human being. On the other hand, in a corresponding form, he/she must present the differences between prime numbers based on the analysis of Kummer's Optimum Analysis and uneven distribution among the Natural Numbers. The latter task of the teacher is reduced to presenting the two corresponding formulae by including fate and human beings in one, and the two prime numbers in the other.

7.5 Type 5

Musical Phrase: Édouard Lalo's *Violin Symphony*

Composer: Édouard Lalo. Lalo was a French composer who was able to imitate the Spanish spirit through self-research though he did not act like those who produced the instant Spanish tunes.

Description of the musical phrase: the third movement of the symphony.

Dominant musical instrument(s): violin.

Psychological impact of the phrase: a sense of questing for refuge, and the expectation of salvation.

Rhythm of the phrase: mixed.

Harmonic pattern generated by the phrase: deep and holistic

Musical enumeration: tuneless phrase.

Musical composition: gradual synthesis.

Real-time evaluation of the phrase: beautiful and can always be heard.

The Mathematical Obverse: There is a polynomial whose coefficients are integers. If we replace its variable by any positive integer, the result will be a prime number. (8)

Originator(s): Davis Putnam Robinson and Mantegna Civic . A group of experienced mathematicians weaving mathematical ideas one by one, as a tailor does when he sews clothes by hand.

Description of mathematical clause: theory.

Method of proof and/or dominant methods of proof: indirect unity proof.

Psychological impact of the clause: sense of the expectation of salvation and the difficulty of adaptation.

Rhythm of the clause: sporadic.

Harmonic pattern generated by the clause: deep and holistic.

Mathematical enumeration: ambiguous context.

Mathematical formation: gradual synthesis.

Real-time evaluation of the clause: medium , because of beauty, and

the movement from one period to another.

Analysis: The Lalo phrase starts with a large question proposed by a crowd of orchestral instruments, in a mysterious, melodic context. This is followed by the violin, which tries to answer. It is just as generations of mathematicians wondered about mathematical formulae to generate prime numbers. Thus mathematicians weaved a law because one after another hoped to achieve this goal. Perhaps the laws:

$n^2 - n + 41$

$n^2 - 79n + 1601$

are useful examples in this context. The first law fails to generate a prime number in the prime n = 41 while the second law fails when n = 1601.

In this example, the violin moves from one tune to another hoping to provide the answer to the big question. In the midst of the ebb and tide, the violin turns carefully to the first entry to answer. The movement ends with a single, short, critical, sharp phrase. It is the confluence of the answer. Yes, there are many limits generating all prime numbers. All prime numbers are a pattern of overall Diaphontain numbers. At this point, Lalo closes the third movement, stressing the significant positive question.

7.6 Type 6

Musical Phrase: Bruckner's *Ninth Symphony*.

Composer: Anton Bruckner. Bruckner's music was characterized by a mixture of expressive rise zone. He intended not to write a ninth symphony afraid that, if he did, he may die. This is because he believed that all composers died after writing their ninth works. When he finally attempted to compose his own ninth symphony, he died before he could complete the last movement.

Description of the musical phrase: Part II of the second movement of the symphony.

Dominant musical instrument(s): stringed instruments with an orchestral background.

Psychological impact of the phrase: adding adobes to the imagination.

Rhythm of the phrase: separate.

Harmonic pattern generated by the phrase: partial.

Musical enumeration: apparent target.

Musical composition: direct presentation.

Real-time evaluation of the phrase: medium due to the beauty because it moves from one period to another.

Mathematical Obverse: *The Four-Color Theorem* (9).

Originator(s): Kenneth Appel and Wolfgang Haken. Mathematicians, Appel and Haken, both used computers to prove theories.
The theory was proved by the author of this paper under the title"a Pure Mathematical Proof of The Four Color Problem" and was published in GJPAM Vol.4 No.3.
Description of mathematical clause: theory.

Method of proof and/or dominant methods of proof: proof by cases.

Psychological impact of the clause: adding adobes to the imagination.

Rhythm of the clause: sporadic.

Harmonic pattern generated by the clause: partial.

Mathematical enumeration: apparent target.

Mathematical formation: direct presentation.

Real-time evaluation of the clause: medium due to the beauty, because of moving from one period to another.

Analysis: Bruckner reveals the melodic potential taking place in his heart by crafting a simple dialogue between the stringed instruments and the other instruments in the orchestra. Haken and Appel shall turn to survey a limited number of cases, covering the proof of the *Four-Color Theorem* inventoried by computer.

7.7 Type 7

Musical Phrase: *the Matanzas*.

Composer: unknown.

Description of the musical phrase: a light piece of music.

Dominant musical instrument(s): guitar.

Psychological impact of the phrase: joy.

Rhythm of the phrase: separate.

Harmonic pattern generated by the phrase: superficial.

Musical enumeration: apparent target.

Musical composition: direct presentation.

Real-time evaluation of the phrase: medium due to the beauty, to be due to from a period to another.

Mathematical Obverse: Palindromes (10).

With a palindrome, the positive integer can be read the same way from the right side as read from the left.

Originator(s): unknown.

Description of mathematical clause: definition.

Method of proof and/or dominant methods of proof: guesswork . **Psychological impact of the clause**: joy.

Rhythm of the clause: sporadic.

Harmonic pattern generated by the clause: superficial.

Mathematical enumeration: apparent target.

Mathematical formation: direct presentation.

Real-time evaluation of the clause: medium due to the beauty, to be due to from a period to another.

Analysis: The search for a numerical palindrome involves repetition and seeking without obtaining an answer in most cases. Similarly, in *the Matanzas*, the theme is repeated, while formulating a limited and short, lackluster answer.

7.8 Type 8

Musical Phrase: /*The Moldau*?

Composer: Bedřich Smetana. Smetana was a Czech sensation and sensitive musician known for his poetic music.

Description of the musical phrase: impressionistic music, reaching up to a spectacular poetic limit.

Dominant musical instrument(s): orchestral work.

Psychological impact of the phrase: a sense of the simplicity of adaptability.

Rhythm of the phrase: mixed.

Harmonic pattern generated by the phrase: deep and holistic.

Musical enumeration: apparent lyrical.

Musical composition: direct presentation.

Real-time evaluation of the phrase: beautiful and can always be heard.

Mathematical Obverse: *Möbius Transformations* or the Geometry of Complex Numbers (11)

In particular, the conversions which transfer circles into circles. **Originator(s):** a number of mathematicians, including August Möbius in particular.

Description of mathematical clause: a set of definitions, theories, and results.

Method of proof and/or dominant methods of proof: many methods of proof with the direct proof and proof by cases being the most important.

Psychological impact of the clause: a sense of the simplicity of adaptability.

Rhythm of the clause: mixed.

Harmonic pattern generated by the clause: deep and holistic.

Mathematical enumeration: apparent target.

Mathematical formation: direct presentation.

Real-time evaluation of the clause: attractive and can always be consulted.

Analysis: the circles which are transferred by Möbius' transformation to other circles look like ripples as they spread on water. They are just like the wave circuits in Smetana's piece which describes the great Bohemian river, Vltava, which flows toward Prague.

7.9 Type 9

Musical Phrase: *The Pearl Fishers*.

Composer: Georges Bizet. Bizet was a French musician known for his strong, sweet music, in particular, the opera, *Carmen*.

Description of the musical phrase: meditative.

Dominant musical instrument(s): orchestral work.

Psychological impact of the phrase: causes a quest for refuge; inspires awe.

Rhythm of the phrase: mixed

Harmonic pattern generated by the phrase: deep and holistic.

Musical enumeration: apparent lyrical.

Musical composition: direct presentation.

Real-time evaluation of the phrase: beautiful and can always be heard.

Mathematical Obverse: The use of only the ruler, without measuring, in order to expand the given straight piece beyond a barrier making use of the harmonic division. (12)

Originator(s): A number of adherents to Abstract Mathematics.

Description of mathematical clause: a set of definitions and theories which produced unexpected results.

Method of proof and/or dominant methods of proof: direct proof

Psychological impact of the clause: the expectation of salvation, and sense of the difficulty of adaptation.

Rhythm of the clause: mixed.

Harmonic pattern generated by the clause: mixed.

Mathematical enumeration: apparent target and ambiguous context simultaneously.

Mathematical formation: direct presentation and gradual synthesis.

The real-time evaluation of the paragraph states: attractive and can always be consulted.

Analysis: George Bizet starts from his exciting apparent melody involving fear and vague Sufi meditation. As the orchestra ends at the same melody in the performance of such repetition based therefore to find unexpected result which is a familiar rhythms to the ears, as well as the application of consensual division of the mysterious context affect to achieve the natural requirement to extend the straight piece.

7.10 Type10

Musical Phrase: *Toccata and Fugue in D Minor*,
Composer: Johann Sebastian Bach. Bach, the founder and father of classical music is known for his prolific body of work. There is little doubt that Bach was the first musical architect to show the world the best way to build a classical musical phrase using his trademark, systemic approach.

Description of the musical phrase: entrance and meditation.

Dominant musical instrument(s): keyboard/organ.

Psychological impact of the phrase: adding adobes to the imagination; inspiring awe, dread; openness and the granting of credibility?.

Rhythm of the phrase: sporadic.

Harmonic pattern generated by the phrase: deep and holistic.

Musical enumeration: apparent lyrical.

Musical composition: gradual synthesis.

Real-time evaluation of the phrase: beautiful and can always be heard.

Mathematical Obverse: *Set Theory* (13)

Originator(s): Zorn and Georg Cantor and other adherents of abstract mathematics.

Description of mathematical clause: objective structure, definitions, theories.

Method of proof and/or dominant methods of proof: direct proof in most cases.

Psychological impact of the clause: adding adobes to the imagination, inspiring awe, dread, openness and the granting of credibility.

Rhythm of the clause: sporadic.

Harmonic pattern generated by the clause: deep and holistic.

Mathematical enumeration: ambiguous context.

Mathematical formation: gradual synthesis.

Real-time evaluation of the clause: attractive and can always be consulted.

Analysis: The supplication starts with a preliminary and questioning introduction with a quick intervention of the organ to gradually build the mental substantive rationality structure, on the one hand, and the romantic mysticism on the other hand. The rational is used because it can answer and the romantic mysticism can open the door to the creation of anomalies and to prevent access to an integrated response. This is the general plan and the precise context of *Set Theory*.

7.11 Type 11

Musical Phrase: *Csardas*: Hungarian folk music.

Composer: Vittorio Monti. Monti was a famous, Italian musician, composer, and conductor with a good reputation for writing popular music in a glossy, serious format.

Description of the musical phrase: a dance.

Dominant musical instrument(s): violin.

Psychological impact of the phrase: joy.

Rhythm of the phrase: mixed.

Harmonic pattern generated by the phrase: superficial.

Musical enumeration: apparent target.

Musical composition: direct presentation.

Real-time evaluation of the phrase: medium due to the beauty, to be due to from a period to another.

Mathematical Obverse: The number of stars that can be drawn through a specified number of points without raising the pen or passing on the previous points (14) is equal to the number

$$\frac{\phi(n)-2}{2}$$

Originator(s): Leonhard Euler. Euler was a Swiss genius who wrote in all branches of mathematics, pure and applied sciences.

Description of mathematical clause: theory.

Method of proof and/or dominant methods of proof: direct proof.

Psychological impact of the clause: sense of simplicity and joy.

Rhythm of the clause: mixed.

Harmonic pattern generated by the clause: superficial.

Mathematical enumeration: apparent target.

Mathematical formation: direct presentation.

Real-time evaluation of the clause: medium due to the beauty, to be due to from a period to another.

Analysis: the musical phrase consists of two parts. The first part presents the problem in a quiet/quick distribution, while the solution drives to a noisy violin, which leaves a quick, dynamic impression. Very soon it becomes extinct, which is the second part. On the other hand, the leveling in the mathematical obverse is reduced by the question of the possibility of drawing such stars. The answer is formulated in order to transfer a central definition of mathematics which is a function of Euler. In spite of the clamor of the issue, it still holds only a limited significance.

7.12 Type 12

Musical Phrase: *Recuerdos de la Alhambra/Memories of the Alhambra*.

Composer: Francisco Tárrega. Tárrega was an important Spanish composer and guitarist who composed, in particular, from the spirit of the Iberian Peninsula.

Description of the musical phrase: separation

Dominant musical instrument(s): harp/classical guitar?

Psychological impact of the phrase: a sense of joy, awe, openness and the granting of credibility.
Rhythm of the phrase: mixed.

Harmonic pattern generated by the phrase: partial.

Musical enumeration: apparent target.

Musical composition: direct presentation.

Real-time evaluation of the phrase: medium due to the beauty, to be due to from a period to another.

Mathematical Obverse: The theory: There is a set of plain **2n** points, no three of which are collinear. Assume that **n** of these points has been colored red, while the other points are colored blue. Demonstrate that there are **n** straight pieces that any two of them do not intersect and that each piece is characterized by two ends different in color and belonging to the mentioned set (15).

Originator(s): Karl Theodor Wilhelm Weierstrass. Weierstrass put mathematical and scientific analysis on solid foundations.

Description of mathematical clause: theory.

Method of proof and/or dominant methods of proof: proof of the theory, extrapolation proof, direct proof by using the average value, proof of the cases.

Psychological impact of the clause: sense of joy, openness, fear, and the granting of credibility.

Rhythm of the clause: mixed.

Harmonic pattern generated by the clause: partial.

Mathematical enumeration: apparent target.

Mathematical formation: direct presentation.

Real-time evaluation of the clause: medium due to the beauty, to be due to from a period to another.

Analysis: As Tárrega's melodies swing easily and slowly, and the musical phrases talk to each other continuously without stopping to pick up the guitar's breath, the listener already thinks that some separate component of [guitar] strings has entered the world in a

continuous manner in spite of the divergence and differentiation of its elements. If we turn to the world of mathematics, in particular mathematical analysis using the theory of medium value, a central theory of the continuity theories, for sentencing on a sporadic points, we find, in a sense, the strings of Tárrega's guitar .

7.13 Type 13

Musical Phrase: Romance.

Composer: P. M. Gomez. Gomez composed in the context of light music.

Description of the musical phrase: light piece of music.

Dominant musical instrument(s): harp.

Psychological impact of the phrase: openness and the granting of credibility.

Rhythm of the phrase: mixed.

Harmonic pattern generated by the phrase: partial.

Musical composition: direct presentation.

Real-time evaluation of the phrase: beautiful and can always be heard.

Mathematical Obverse: *Kummer's Criterion*. Kummer did an analysis of prime numbers and the cyclotomic field and was thus able to prove Fermat's last theorem for certain primes(16).

Originator(s): Ernst Eduard Kummer. Kummer was an outstanding German mathematician and a veritable human computer.

Description of mathematical clause: the integrated theory of ideal

analysis.

Method of proof and/or dominant methods of proof: direct proof.

Psychological impact of the clause: adding adobes to the imagination, creating a sense of openness and the granting of credibility.

Rhythm of the clause: mixed.

Harmonic pattern generated by the clause: deep and holistic.

Mathematical enumeration: ambiguous context.

Real-time evaluation of the clause: medium due to the beauty, to be due to from a period to another.

Analysis: We will not pay attention in this context to a detailed analysis of the musical phrase, as the combined effect it creates is enough. That effect is to put us in a romantic scrutiny? Similarly, it is a staggering fact that the analysis to a prime Cyclostomes number/field? remains single as long as the Cyclostome's/ic??? exponent is less than 23, but the unity fails when it equals 23 according to Kummer's conclusions. Why 23 in particular?

7.14 Type 14

Musical Phrase: *Saltarello*

Composer: unknown.

Description of the musical phrase: light piece of music/folk dance.

Dominant musical instrument(s): harp.

Psychological impact of the phrase: sense of simplicity of adaptability.

Rhythm of the phrase: separate.

Harmonic pattern generated by the phrase: superficial.

Musical enumeration: apparent target.

Musical composition: direct presentation.

Real-time evaluation of the phrase medium due to the beauty, accessible to people in various periods/eras.
Mathematical Obverse: *Sieve of Eratosthenes* (17)

Originator(s): Eratosthenes of Cyrene. Eratosthenes was one of the great Greek mathematicians and scholars at Alexandria.

Description of mathematical clause: adopting an activist term.

Method of proof and/or dominant methods of proof: direct proof.

Psychological impact of the clause: sense of simplicity.

Rhythm of the clause: sporadic.

Harmonic pattern generated by the clause: superficial.

Mathematical enumeration: apparent target.

Mathematical formation: direct presentation.

Real-time evaluation of the clause: has a lack of aesthetic elements and may refer to it in favor of separate periods.

Analysis: The sieve of prime numbers, in the same way as the *Saltarello* does, repeats frequently and deliberately, with no creativity. It is just an expected direct move that will not allow us to reach very large prime numbers and the formulation of normative controlling

the distribution.

7.15 Type 15

Musical Phrase: the Score for the film *the Godfather*.
Composer: Nino Rota. Rota was an award-winning composer especially of film scores. He composed in the context of light music.

Description of the musical phrase: an introduction.

Dominant musical instrument(s): general orchestral form.

Psychological impact of the phrase: the expectation of salvation, and sense of simplicity.

Rhythm of the phrase: mixed.

Harmonic pattern generated by the phrase: partial.

Musical enumeration: apparent target.

Musical composition: direct presentation.

Real-time evaluation of the phrase: medium due to the beauty, to be listened to from a period to another.

Mathematical Obverse: The time needed to find a common denominator for the two positive integers according to Euclid's [Euclid's what?] commensurate with the algorithm: [18] $o(\log^3(\alpha))$.

Originator(s): Neal I. Koblitz. Koblitz is a professor of mathematics at the University of Washington. He works in the computer sector.

Description of mathematical clause: theory.

Method of proof and/or dominant methods of proof: direct proof.

Psychological impact of the clause: The programmer should adopt a final algorithm requiring a minimum limit of time . Accordingly, the psychological impact here is the expectation of salvation and sense of simplicity.

Rhythm of the clause: sporadic.

Harmonic pattern generated by the clause: partial.

Mathematical enumeration: apparent target.

Mathematical formation: direct presentation.

Real-time evaluation of the clause: medium due to the beauty, to be due to from a period to another.

Analysis: Calmly, Nino Rota provides his melody and pushes it to the surface within the framework of a clear dialogue between the two parts of the tune. The listener is engulfed by a euphoria emanating from the imminence of salvation. The proof of the mathematical obverse, as well, consists of two parts. The first is concerned with the relative comparison for the size of the successive remainders in Euclid's algorithm, while the second part deals with timing based on the preliminary estimates of the times [length of time on computer?] of the computer. The expectation of the programmer is to find an algorithm which is characterized within a minimum limit of time.

7.16 Type 16

Musical Phrase: *The Song of Lara*. composer.

Composer: Maurice Bijar. Bijar is a well-known composer with a good reputation for composing movie scores.

Description of the musical phrase: an introduction.

Dominant musical instrument(s): general orchestral form.

Psychological impact of the phrase: adding adobes to the imagination, creating a sense of openness, and the granting of credibility.

Rhythm of the phrase: mixed.

Harmonic pattern generated by the phrase: partial.

Musical enumeration: apparent target.

Musical composition: direct presentation.

Real-time evaluation of the phrase: medium due to the beauty, to be listened to from a period to another.

Mathematical Obverse: *Borel Measure* - the cumulative distribution function (19)

Originator(s): Émile Borel. Borel was the first to apply Cantor's ideas on mathematical analysis and probability theory.

Description of mathematical clause: definition.

Method of proof and/or dominant methods of proof: none.

Psychological impact of the clause: a sense of simplicity.

Rhythm of the clause: sporadic.

Harmonic pattern generated by the clause: superficial.

Mathematical enumeration: apparent target.

Mathematical formation: direct presentation.

Real-time evaluation of the clause: medium due to the beauty, to be due to from a period to another.

Analysis: Bijar adds the sounds which can be emitted from the various instruments for the expression of deep lyrical simplicity, in the same way Borel combines the separate impacts/influences of the measurement/distribution? function to conclude/arrive at a holistic way of doing so for all those affected separately.

7.17 Type17

Musical Phrase: *The Butterfly*.
Composer: Jack/Jerry? Goldsmith. Goldsmith is a composer of light music.

Description of the musical phrase: light piece of music.

Dominant musical instrument(s): orchestra with accordion.

Psychological impact of the phrase: joy.

Harmonic pattern generated by the phrase: superficial.

Musical enumeration: apparent target.

Musical composition: direct presentation.

Real-time evaluation of the phrase: medium due to the beauty, to be listened to from a period to another.

Mathematical Obverse: Archimedes' Inequality of the Real Numbers (20).

Assuming that the $x > o$ and y is a real number, there is a positive integer, n, satisfying the inequality $n x > y$

Originator(s): Archimedes of Syracuse**.** Archimedes was a well-known, classical Greek scientist and mathematician.

Description of mathematical clause: theory.

Method of proof and/or dominant methods of proof: direct proof.

Psychological impact of the phrase: joy.

Harmonic pattern generated by the phrase: superficial.

Mathematical enumeration: apparent target.

Mathematical formation: direct presentation.

Real-time evaluation of the clause: medium due to the beauty, to be due to from a period to another.

Analysis: Despite the apparent aesthetic element in *The Butterfly*, the effect quickly dissipates because of the extreme simplicity of the piece, which makes it rather naïve. Similarly, Archimedes' theory, in spite of being a theory, is closer to the naïve than the obvious. That real true [Real truth?] obviously has its very serious objects.

17.18 Type 18

Musical Phrase: *Egmont*.

Composer: Ludwig van Beethoven. Beethoven was the great musician whose music reveals the nature of the deep intellectual impact that music can have on the mind.

Description of the musical phrase: an introduction.

Dominant musical instrument(s): orchestral.

Psychological impact of the phrase: a search for sanctuary and the inspiration of awe.

Rhythm of the phrase: mixed.

Harmonic pattern generated by the phrase: deep and holistic.

Musical enumeration: apparent lyrical.

Musical composition: gradual synthesis.

Real-time evaluation of the phrase: beautiful and can always be heard.

Mathematical Obverse: *Riemann Hypothesis* (21)

$$L\,(x) = \sum_{n=2}^{x} \frac{1}{logn}$$

$$\prod(x) I\,(x) \approx \frac{x}{logx}$$

Total gives L(X) almost the best for $\prod(x)$comparison with $\frac{x}{logx}$.
[wording in the above and below is not clear.]

Riemann's Hypothesis dictates that the validity of equality provides the following:

$$\prod(x) = L\,(x) + O\left(\sqrt{x}\,logx\right)$$

Originator(s): Georg Friedrich Bernhard Riemann. Riemann lived in

the nineteenth century; his brilliant ideas served as an incentive and motivation for later mathematicians for more than a century afterwards. He laid the foundations for the theory of general relativity several decades before Einstein.

Description of mathematical clause: a special work.

Method of proof and/or dominant methods of proof: hypothesis.

Psychological impact of the clause: a sense of awe, and searching for sanctuary.

Rhythm of the clause: mixed.

Harmonic pattern generated by the clause: deep and holistic.

Mathematical enumeration: apparent target.

Mathematical formation: gradual synthesis.

Real-time evaluation of the clause: attractive to stand the test of time.

Analysis: As Beethoven's melodies rise up looking for refuge, so mathematicians strive to approach the number of all prime numbers contained until a specified number, Here, the genius, Riemann, elevated us to the set L (x) as he took refuge in the fever of searching for the indicated number.

7.19 Type 19

Musical Phrase: The Third Movement of Sibelius' *Violin Concerto.*

Composer: Jean Sibelius. Sibelius was a Finnish Musician, known for his deep, sentimental, thought-provoking music. He rose at the end of his life to Sufism.

Description of the musical phrase: movement.

Dominant musical instrument(s): violin.

Psychological impact of the phrase: adding adobes to the imagination, creating a sense of openness, and the granting of credibility.

Rhythm of the phrase: mixed.

Harmonic pattern generated by the phrase: deep and holistic.

Musical enumeration: apparent lyrical.

Musical composition: gradual synthesis.

Real-time evaluation of the phrase: beautiful and can always be heard.

Mathematical Obverse: *Brouwer's Fixed Point Theorem* (22)

Originator(s): Luitzen Egbertus Jan Brouwer. Brouwer, a Dutch mathematician was one of the founders of Algebraic Topology.

Description of mathematical clause: theory.

Method of proof and/or dominant methods of proof: direct proof.

Psychological impact of the clause: adding adobes to the imagination, creating a sense of openness, and the granting of credibility.

Rhythm of the clause: mixed.

Harmonic pattern generated by the clause: deep and holistic.

Mathematical enumeration: apparent target.

Mathematical formation: gradual synthesis.

Real-time evaluation of the clause: attractive to be due to from always.

Analysis: The discourse between orchestra and violin is like a continuous transition between the surface and the same, while the fixed point is that sweet tune, where the orchestra alternates with the violin to focus upon it. Sibelius' *Violin Concerto* is an auditory parallel which can be used as the best possible presentation to clarify the theory of fixed points.

7.20 Type 20

Musical Phrase: *Tannhäuser*.

Composer: Richard Wagner. Wagner was a great German composer and conductor who specialized in the operatic form. He was attracted by the tendency to aim for all that is great and glorious which was sweeping Germany in the mid-1800's.

Description of the musical phrase: operatic introduction and choir.

Dominant musical instrument(s): orchestral work**.**

Psychological impact of the phrase: creation of a sense of awe, fear, and the difficulty of adaptability.

Rhythm of the phrase: mixed.

Harmonic pattern generated by the phrase: deep and holistic.

Musical enumeration: apparent lyrical.

Musical composition: gradual synthesis.

Real-time evaluation of the phrase: beautiful and can always be heard.

Mathematical Obverse: *The Theory of Elasticity* using tensor analysis (23).

Originator(s): a group of applied mathematicians.

Description of mathematical clause: increased structures.

Method of proof and/or dominant methods of proof: a number of theories using direct proof most cases.

Psychological impact of the clause: a sense of awe, fear, and the difficulty of adaptability.

Rhythm of the clause: mixed.

Harmonic pattern generated by the clause: deep and holistic.

Mathematical enumeration: ambiguous context .

Mathematical formation: gradual synthesis.

Real-time evaluation of the clause: attractive to be due to from always???.

Analysis: As Wagner rises to the top in an attempt to reach for the briefing of all things, he uses all possible machinery as well as human voices, to achieve a high musical prestige and glory. The same is true for those who have given the world [the Theory of Elasticity using tensor analysis.] flexibility/elasticity? theory of tensor analysis. The originating mathematicians employed every possible perception of mathematics and considered all possibilities where traditional methods, before, had failed.

7.21 Type 21

Musical Phrase: *sprach Zarathustra* (Thus spoke Zarathustra).

Composer: Richard Strauss. Stauss was a German composer, strongly influenced by the writings of the German writer and philosopher, Friedrich Nietzsche, in particular, his work *Übermensch* (Superman). His musical compositions, therefore, have focused on great personalities and on a yearning for glory and grandeur**.**

Description of the musical phrase: introduction of a tone poem.

Dominant musical instrument(s): orchestral**.**

Psychological impact of the phrase: seeking for refuge, inspiring a sense of awe, fear, and recognition of the difficulty of adaptability.

Rhythm of the phrase: sporadic.

Harmonic pattern generated by the phrase: partial.

Musical enumeration: a tuneless phrase.

Musical composition: gradual synthesis.

Real-time evaluation of the phrase: It lacks the aesthetic elements, in favor of listening to it in interval periods of time???.

Mathematical Obverse: Lotka-Volterra Equation? The Relationship of Predators to Victims/Prey? (24).

Originator(s): a group of applied mathematicians.

Description of mathematical clause: increased structures.

Method of proof and/or dominant methods of proof: qualitative solutions of differential equations, direct proofs.

Psychological impact of the clause: inspiration of awe, fear, and a sense of the difficulty of adaptability.

Rhythm of the clause: sporadically.

Harmonic pattern generated by the clause: partial.

Mathematical enumeration: apparent target.

Mathematical formation: gradually synthesis.

Real-time evaluation of the clause: lacks the aesthetic elements, being able to be listened to in interval periods of time.

Analysis: What Strauss' introduction revealed is a form of emergence or appearance of the background of abuse. It is significant to predators and prey, and is caused when the victims' star vanishes away, the star of the predators emerges.

7.22 Type 22

Musical Phrase: A Mass for Choir and Orchestra.

Composer: George Legate. Legate is/was a composer in the world of light music.

Description of the musical phrase: special work.

Dominant musical instrument(s): choir.

Psychological impact of the phrase: sense of awe, fear, acquisition, isolation.

Rhythm of the phrase: sporadic.

Harmonic pattern generated by the phrase: deep and holistic.

Musical enumeration: tuneless phrase.

Musical composition: gradual synthesis.

Real-time evaluation of the phrase: lacks aesthetic elements, in favor of listening to it in interval periods of time.

Mathematical Obverse: *Dirichlet's Theorem* (25).
Take D to be any partial element in the Algebraic number, k of the degree of n = s +2 t
Accordingly, there are units: ϵr .....$\epsilon 1$ where r = s + t-1 as a, per unit $\delta \epsilon D$ of analysis and a single form:
$\varepsilon = \delta^{ar} \ldots \varepsilon = \delta^{ar}$
Here: ar .... A1 are integers and the root of the number **1** is valid for the content of D.

Originator(s): Johann Peter Gustav Lejeune Dirichlet. Dirichlet, a German mathematician, created the analytical theory of numbers; he was famous for his complex theories.

Description of mathematical clause: theory.

Method of proof and/or dominant methods of proof: direct proof.

Psychological impact of the clause: sense of awe, fear, acquisition, isolation.

Rhythm of the clause: sporadic.

Harmonic pattern generated by the clause: deep and holistic.

Mathematical enumeration: ambiguous context.

Mathematical formation: gradual synthesis.

Real-time evaluation of the clause: lacks aesthetic elements, in favor of listening to it in interval periods of time.

Analysis: The expectation, dread, isolation, ambiguous context, missing tune, probing the forms in unfamiliar ways, these are all fulcrum points for Legate and Dirichlet, and lead to the lack of a practical review and only when necessary.

7.23 Type 23

Musical Phrase: *The Blue Danube*.

Composer: Johann Strauss II. Strauss II was the son of the famous Strauss and was an Austrian composer, famous for his soft music in the spirit of Vienna.

Description of the musical phrase: Dance-Vales.

Dominant musical instrument(s): orchestral work.

Psychological impact of the phrase: sense of simplicity, joy.

Rhythm of the phrase: mixed.

Harmonic pattern generated by the phrase: deep and holistic.

Musical enumeration: apparent lyrical.

Musical composition: direct presentation.

Real-time Evaluation of the phrase: beautiful and can always be heard.

Mathematical Obverse: *Goldbach's Conjecture*. Each even number equals the sum of two prime numbers (26).

Originator(s): Christian Goldbach. Goldbach was a German submerged/'underground' mathematician who raised the possibility without raising the proof.

Description of mathematical clause: conjecture.

Method of proof and/or dominant methods of proof: unproven.

Psychological impact of the clause: sense of simplicity, joy.

Rhythm of the clause: sporadic.

Harmonic pattern generated by the clause: deep and holistic.

Mathematical enumeration: apparent target.

Mathematical formation: direct presentation.

Real-time evaluation of the clause: attractive to be due to from always.

Analysis: The sweetness and lyrical softness of Johann Strauss, Jr. was translated by Christian Goldbach with amazing success. It is the simplicity ripple, and horizon of joy. Unfortunately, it remains speculative – in Goldbach by an easy pattern, an impregnable form. How could a very simple formula be difficult without the capacity of the imagination? Perhaps the same was true in the imagination of Strauss.

7.24 Type 24

The Musical Phrase: Beethoven's *Romance for Violin and Orchestra No. 2.*

Composer: Ludwig van Beethoven.

Description of the musical phrase: meditation.

Dominant musical instrument(s): violin.

Psychological impact of the phrase: expectation of salvation, sense of simplicity, openness and the granting of credibility.

Rhythm of the phrase: mixed.

Harmonic pattern generated by the phrase: deep and holistic.

Musical enumeration: apparent lyrical.

Musical composition: gradually synthesis.

Real-time evaluation of the phrase: beautiful and can always be heard.

Mathematical Obverse: The Gaussian Integer? The assigning/graphing of the prime numbers of Gauss on a plane (27).

Originator(s): Carl Friedrich Gauss.

Description of mathematical clause: increased structures.

Method of proof and/or dominant methods of proof: direct proof.

Psychological impact of the clause: expectation of salvation, sense of simplicity, openness and the granting of credibility.

Rhythm of the clause: mixed.

Harmonic pattern generated by the clause: deep and holistic.

Mathematical enumeration: apparent target.

Mathematical formation: direct presentation.

Real-time evaluation of the clause: attractive to be due to from always.

Analysis: The sweetness and tenderness in *Romance 2* separates the listener from his world and allows the listener to move to the magical world of beauty. Gauss priorities/prime numbers do the same when set on the plane. There is the impact of unexpected beauty.

7.25 Type 25

Musical Phrase: *Warsaw Concerto*.

Composer: Richard Addinsell. Addinsell [1904-1977] was a British composer in the world of light music.

Description of the musical phrase: introduction.

Dominant musical instrument(s): piano with orchestra.

Psychological impact of the phrase: causing a quest for refuge, adding adobes to the imagination.

Rhythm of the phrase: mixed.

Harmonic pattern generated by the phrase: partial.

Musical enumeration: apparent lyrical.

Musical composition: direct presentation.

Real-time evaluation of the phrase: medium due to the beauty, to be listened to from a period to another.

Mathematical Obverse: The *Principle of Repetition* in the context of the

signing/graphing of points and drawing using a computer (28).

Originator(s): a number of contemporary designers of soft/ware? structures.

Description of mathematical clause: increased structures.

Method of proof and/or dominant methods of proof: proof by extrapolation.

Psychological impact of the clause: creating the expectation of salvation, and a quest for refuge.

Rhythm of the clause: mixed.

Harmonic pattern generated by the clause: partial.

Mathematical enumeration: ambiguous context.

Mathematical formation: direct presentation.

Real-time evaluation of the clause: medium due to the beauty, to be due to from a period to another.

Analysis: Addinsell was trying, by repeating the basic tune, to leave an impression for the listener of a separate and independent musical pattern. It does not, however, exceed the determination/go beyond the scope of light music [in which it is difficult to single out a particular classification of music]. As well as the repetition techniques applied on the computer, it is not a new creation, if the originators try to present these techniques as one of the new branches of mathematics.

7.26 Type 26

Musical Phrase: *Für Elise*.

Composer: Ludwig van Beethoven:

Description of the musical phrase: special work.

Dominant musical instrument(s): piano.

Psychological impact of the phrase: adding adobes to the imagination, creating a sense of joy.

Rhythm of the phrase: sporadic.

Harmonic pattern generated by the phrase: partial.

Musical enumeration: apparent target.

Musical composition: direct presentation.

Real-time evaluation of the phrase: beautiful and can always be heard.

Mathematical Obverse: The visual projection of *Lagrange's Four-Square Theorem* (29). This theorem states that every positive integer is equal to the sum of no more than four positive squares.

Originator(s): Joseph-Louis Lagrange. Lagrange was an Italian-born mathematician and astronomer. He had a good reputation for outstanding achievements in all branches of mathematics.

Description of mathematical clause: theory.

Method of proof and/or dominant methods of proof: direct proof.

Psychological impact of the clause: expectation of salvation, the quest for refuge.

Rhythm of the clause: mixed.

Harmonic pattern generated by the clause: sporadic.

Mathematical enumeration: apparent target.

Mathematical formation: direct presentation.

Real-time evaluation of the clause: attractive to be due to from always.

Analysis: With the playing of each one of the keys of the piano, Beethoven adds to the direct introduction of the integrated image, which embodies the feelings about Elise. In the same way Lagrange's theory was proved by deleting successively small squares from a bigger one.

7.27 Type 27

Musical Phrase: *La campanella*.

Composer: Niccolò Paganini [1782-1840] and Franz Liszt [1811-1886]. Paganini was an Italian composer and Liszt Hungarian. Liszt adapted Paganini's Violin Concerto No. 2 to create *La campanella* for piano.

Description of the musical phrase: light piece of music.

Dominant musical instrument(s): piano.

Psychological impact of the phrase: expectation of salvation, and ease of internalization.

Rhythm of the phrase: sporadic.

Harmonic pattern generated by the phrase: partial.

Musical enumeration: apparent target.

Musical composition: direct presentation.

Real-time evaluation of the phrase: beautiful and can always be heard.

Mathematical Obverse: Flat Lattice Networks (30).

Originator(s): János Bolyai. A Hungarian mathematician and one of the founders of non-Euclidean geometries.

Description of mathematical clause: increasing structures.

Method of proof and/or dominant methods of proof: a set of definitions, most of them using direct proof.

Psychological impact of the clause: expectation of salvation, and ease of internalization.

Rhythm of the clause: sporadic.

Harmonic pattern generated by the clause: partial.

Mathematical enumeration: apparent target.

Mathematical formation: direct presentation, gradual synthesis.

Real-time evaluation of the clause: attractive to be due to from always.

Analysis: Listeners of Paganini's melodies will note that there is no clear attempt by him to fill his own emotional frequent performer in accordance with an exact temporal symmetry rules. The Lattice Networks are characterized by similar mechanisms. A teacher could integrate the visual impact of the establishment of the lattice networks with a supporting audio recording of the musical piece of *La campanella* which affects maximum harmony in the student

7.28 Type 28

Musical Phrase: *Polonaise in A flat, Op. 53* (Polonaise No. 6).

Composer: Frédéric Chopin. Chopin [1810-1849] was a musical genius. He was a sensitive Polish composer.

Description of the musical phrase: special work.

Dominant musical instrument(s): piano.

Psychological impact of the phrase: adding adobes to the imagination, creation of a sense of openness, and the granting of credibility **.**

Rhythm of the phrase: sporadic.

Harmonic pattern generated by the phrase: deep and holistic.

Musical enumeration: apparent lyrical.

Musical composition: direct presentation.

Real-time evaluation of the phrase: beautiful and can always be heard.

Mathematical Obverse: Combinatorial Mathematics (31)

Originator(s): a number of mathematicians across several generations.

Description of mathematical clause: increasing structures.

Method of proof and/or dominant methods of proof: more than one method, most using direct proof.

Psychological impact of the clause: adding adobes to the

imagination, a sense of openness, and the granting of credibility**.**

Rhythm of the clause: sporadic.

Harmonic pattern generated by the clause: deep and holistic.
Mathematical enumeration: apparent target, ambiguous context.

Mathematical formation: direct presentation.

Real-time evaluation of the clause: attractive to be due to from always.

Analysis: Perhaps Chopin had asked himself some of the synthetic naturee questions: How could he compose a specific piece using the alignment of musical notation within the constraints of the music in advance, in order to charm the listener's heart. This is the same mechanism that Mathematicians used to establish the various structures of combinatorial mathematics. The teacher here is responsible for the selection of appropriate examples of synthetic mathematics.

7.29 Type 29

Musical Phrase: Rahmaninov's *Piano Concerto No. 2.* – first movement.

Composer: Sergei Rachmaninov. Rachmaninov was a great Russian composer .

Description of the musical phrase: movement.

Dominant musical instrument(s): orchestra with piano.

Psychological impact of the phrase: results in the search for safe haven, and realization of the difficulty of internalization.

Rhythm of the phrase: sporadic.

Harmonic pattern generated by the phrase: deep and holistic.

Musical enumeration: apparent target.

Musical composition: direct presentation.

Real-time evaluation of the phrase: medium due to the beauty, to be listened to from a period to another.
Mathematical Obverse: Smirnov's *Metric Theory* (32)

Originator(s): Vladimir Ivanovich Smirnov. Smirnov [1887-1974] was a senior Russian mathematician.

Description of mathematical clause:

Method of proof and/or dominant methods of proof: direct proof.

Psychological impact of the clause: results in the search for a safe haven, the realization of the difficulty of internalization.

Rhythm of the clause: sporadic.

Harmonic pattern generated by the clause: deep and holistic.

Mathematical enumeration: apparent target.

Mathematical formation: direct presentation.

Real-time evaluation of the clause: medium due to the beauty, to be due to from a period to another.

Analysis: As Rachmaninov went through a difficult path, so Smirnov tried to follow the same path. Both of them were looking for an apparently easy target, but in abstention/without success.

7.30 Type 30

Musical Phrase: *The Valse Triste* (Sad Waltz).

Composer: Jean Sibelius.

Description of the musical phrase: special work.

Dominant musical instrument(s): orchestral.

Psychological impact of the phrase: results in a search for shelter, and a sense of the deplorable**.**
Rhythm of the phrase: mixed.

Harmonic pattern generated by the phrase: deep and holistic.

Musical enumeration: apparent lyrical.

Musical composition: gradual synthesis.

Real-time evaluation of the phrase: beautiful and can always be heard.

Mathematical Obverse: The Cantor set of zero measure (33).

Originator(s): Georg Cantor. Cantor was a brilliant German mathematician, who addressed to notion and issues of infinity.

Description of mathematical clause: theory.

Method of proof and/or dominant methods of proof: direct proof.

Psychological impact of the clause: results in a search for shelter, and a sense of the deplorable**.**

Rhythm of the clause: mixed.

Harmonic pattern generated by the clause: deep and holistic.

Mathematical enumeration: apparent target.

Mathematical formation: gradually synthesis.

Real-time evaluation of the clause: attractive to be due to from always.

Analysis: *Valse Triste* is a sad waltz indeed, and it really is deplorable. On the other hand, it is not only sad, but also frustrating to have a zero measure in Cantor's set, although the number of elements is equal to the real numbers. The teacher here is responsible for the task of creating a state of grief and frustration at the proof/realization? of this fact and could do so using the sad waltz of Sibelius as a background, on the one hand, and analytical necessity, on the other.

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