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Some integrals involving generalized polynomials sets, Aleph-function of two variables and multivariable Aleph-function with applications

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ABSTRACT
In this document, we first evaluate certain unified integrals formulas involving various products of a generalized polynomial set, a general class of polynomials and the multivariable Aleph-function. With the help of these integral formulas, we establish some expansion formulas for the product of several classes of polynomials of several variables and the multivariable Aleph-function in series of several polynomials. The results established here are quite general in character and special number of results which follow as special cases of our results are discussed brieftly.

KEYWORDS : Aleph-function of several variables, Aleph-function of two variables, finite integral, special function, general class of polynomials,Fourier-serie

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1.Introduction and preliminaries.

The function Aleph of several variables generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [7] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We define : $\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c|} \mathrm{z}_1\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\end{array}\right.$

$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}]\ ,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:$
$\ldots\ldots\ldots\ldots\ ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:$

$\left.\begin{array}{l}[(\mathrm{c}_j^{(1)});\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;[(\mathrm{c}_j^{(r)});\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]\\ [(\mathrm{d}_j^{(1)});\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;[(\mathrm{d}_j^{(r)});\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\end{array}\right)$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^r\theta_k(s_k)z_k^{t_k}\,\mathrm{d}t_1\cdots\mathrm{d}t_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^r\alpha_j^{(k)}s_k)}{\sum_{i=1}^R[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^r\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^r\beta_{ji}^{(k)}s_k)]} \tag{1.2}$$

and $$\theta_k(s_k)=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

Suppose , as usual , that the parameters

$a_j,j=1,\cdots,p;b_j,j=1,\cdots,q;$

$c_j^{(k)}, j = 1, \cdots, n_k; c_{ji^{(k)}}^{(k)}, j = n_k + 1, \cdots, p_{i^{(k)}};$

$d_j^{(k)}, j = 1, \cdots, m_k; d_{ji^{(k)}}^{(k)}, j = m_k + 1, \cdots, q_{i^{(k)}};$

with $k = 1 \cdots, r, i = 1, \cdots, R\ , i^{(k)} = 1, \cdots, R^{(k)}$

are complex numbers , and the $\alpha' s, \beta' s, \gamma' s$ and $\delta' s$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} + \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} - \sum_{j=1}^{m_k} \delta_j^{(k)}$$

$$-\tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} \leqslant 0 \tag{1.4}$$

The reals numbers τ_i are positives for $i = 1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)} = 1$ to $R^{(k)}$

The contour L_k is in the s_k-p lane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)} - \delta_j^{(k)} s_k)$ with $j = 1$ to m_k are separated from those of $\Gamma(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(k)} s_k)$ with $j = 1$ to n and $\Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)$ with $j = 1$ to n_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|arg z_k| < \frac{1}{2} A_i^{(k)} \pi$, where

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} - \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)}$$

$$+ \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} > 0, \quad \text{with } k = 1 \cdots, r, i = 1, \cdots, R\ , i^{(k)} = 1, \cdots, R^{(k)} \tag{1.5}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1, \cdots, z_r) = 0(\ |z_1|^{\alpha_1} \ldots |z_r|^{\alpha_r}\ ), max(\ |z_1| \ldots |z_r|\ ) \to 0$

$\aleph(z_1, \cdots, z_r) = 0(\ |z_1|^{\beta_1} \ldots |z_r|^{\beta_r}\ ), min(\ |z_1| \ldots |z_r|\ ) \to \infty$

where, with $k = 1, \cdots, r : \alpha_k = min[Re(d_j^{(k)} / \delta_j^{(k)})], j = 1, \cdots, m_k$ and

$\beta_k = max[Re((c_j^{(k)} - 1) / \gamma_j^{(k)})], j = 1, \cdots, n_k$

We will use these following notations in this paper

$$U = p_i, q_i, \tau_i; R\ ;\ V = m_1, n_1; \cdots; m_r, n_r \tag{1.6}$$

$$\text{W} = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}, \cdots, p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)} \tag{1.7}$$

$$A = \{(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,n}\},\{\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{n+1,p_i}\}$$
$$B = \{\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.9}$$

$$C = \{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\},\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\},\cdots,\{(c_j^{(r)};\gamma_j^{(r)})_{1,n_r}\},\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.10}$$

$$D = \{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\},\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\},\cdots,\{(d_j^{(r)};\delta_j^{(r)})_{1,m_r}\},\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.11}$$

The multivariable Aleph-function write :

$$\aleph(z_1,\cdots,z_r) = \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c|c} z_1 & \mathrm{A : C} \\ \cdot & \\ \cdot & \cdots \\ \cdot & \\ z_r & \mathrm{B : D} \end{array}\right) \tag{1.12}$$

The generalized Aleph-function of two variables is defined by K. Sharma [9], it's an extension of the I-function defined by C.K. Sharma and P.L. Mishra [8] , A. Goyal et al [1 and 2] , wich itself is a generalisation of G and H-function of two variables. The double Mellin-Barnes integral occuring in this paper will be referred to as the Aleph-function of two variables throughout our present study and will be defined and represented as follows.

$$\aleph(y_1,y_2) = \aleph^{M_1,N_1:M_2,N_2;M_3,N_3}_{P_i^{(1)},Q_i^{(1)},\iota_i;r:P_{i'}^{(2)},Q_{i'}^{(2)},\iota_{i'};r':P_{i''}^{(3)},Q_{i''}^{(3)},\iota_{i''};r''}\left(\begin{array}{c|l} \mathrm{y}_1 & [(\mathrm{u}_j;\mu_j^{(1)},\mu_j^{(2)}]_{N_1},[\iota_i(u_{ji};\mu_{ji}^{(1)},\mu_{ji}^{(2)})]_{N_1+1,P_i^{(1)}} \\ \mathrm{y}_2 & \quad\cdot\ \cdot\ [\iota_i(v_{ji};\upsilon_{ji}^{(1)},\upsilon_{ji}^{(2)})]_{M_1+1,Q_i^{(1)}} \end{array}\right.$$

$$\left.\begin{array}{l} :\ (\mathrm{a}_j;\alpha_j)_{1,N_2},[\iota_{i'}(a_{ji'};\alpha_{ji'})]_{N_2+1,P_{i'}^{(2)}};(c_j;\gamma_j)_{1,N_3},[\iota_{i''}(c_{ji''};\gamma_{ji''})]_{N_3+1,P_{i''}^{(3)}} \\ :\ (\mathrm{b}_j;\beta_j)_{1,M_2},[\iota_{i'}(b_{ji'};\beta_{ji'})]_{M_2+1,Q_{i'}^{(2)}};(\mathfrak{d}_\mathrm{j};\delta_\mathrm{j})_{1,M_3},[\iota_{i''}(\mathfrak{d}_{\mathrm{j}i'};\delta_{\mathrm{j}i''})]_{M_3+1,Q_{i''}^{(3)}} \end{array}\right)$$

$$= \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\zeta(s,t)\ \phi_1(s)\phi_2(s)y_1^t y_2^t\,\mathrm{d}s\mathrm{d}t \tag{1.13}$$

where : $\omega = \sqrt{-1}$

$$\zeta(s,t) = \frac{\prod_{j=1}^{N_1}\Gamma(1-u_j+\mu_j^{(1)}s+\mu_j^{(2)}t)}{\sum_{i=1}^{r}\iota_i[\prod_{j=M_1+1}^{Q_i^{(1)}}\Gamma(1-v_{ji}+\upsilon_{ji}^{(1)}s+\upsilon_{ji}^{(2)}t)\prod_{j=N_1+1}^{P_i^{(1)}}\Gamma(u_{ji}-\mu_{ji}^{(1)}s-\mu_{ji}^{(2)}t)]} \tag{1.14}$$

$$\phi_1(s) = \frac{\prod_{j=1}^{M_2}\Gamma(b_j-\beta_j s)\prod_{j=1}^{N_2}\Gamma(1-a_j+\alpha_j s)}{\sum_{i=1}^{r'}\iota_{i'}[\prod_{j=M_2+1}^{Q_{i'}^{(2)}}\Gamma(1-b_{ji'}+\beta_{ji'}s)\prod_{j=N_2+1}^{P_{i'}^{(2)}}\Gamma(a_{ji'}-\alpha_{ji'}s)]} \tag{1.15}$$

$$\phi_2(t) = \frac{\prod_{j=1}^{M_3}\Gamma(\mathfrak{d}_\mathrm{j}-\delta_\mathrm{j}t)\prod_{j=1}^{N_3}\Gamma(1-c_j+\gamma_j s)}{\sum_{i=1}^{r''}\iota_{i''}[\prod_{j=M_3+1}^{Q_{i''}^{(3)}}\Gamma(1-\mathfrak{d}_{\mathrm{j}i''}+\delta_{\mathrm{j}i''}t)\prod_{j=N_3+1}^{P_{i''}^{(3)}}\Gamma(c_{ji''}-\gamma_{ji''}t)]} \tag{1.16}$$

where x and y are not equal to zero. $P_1,P_{i'}^{(1)},P_{i''}^{(2)},Q_1,Q_{i'}^{(1)},Q_{i''}^{(2)},M_1,N_1,M_2,N_2,M_3,N_3$ are non-negative integers such that $0\leqslant N_1\leqslant P_1, 0\leqslant N_2\leqslant P_{i'}^{(1)}, 0\leqslant N_3\leqslant P_{i''}^{(2)}$,

$Q_i>0, Q_{i'}^{(1)}>0, Q_{i''}^{(2)}>0, ; \iota_i,\iota_{i'},\iota_{i''}>0.$

All the $\mu_1 s, \mu_2 s$ $\alpha' s, \beta' s, \delta' s, \gamma' s$ are assumed to be positive quantities ; the definition and Aleph-function of two variables given above will however , have a meaning even if some these quantities are zero. The contour L_1 is in the s -plane and run from $-\omega\infty$ to $+\omega\infty$ with loops , if necessary , to ensure that the pole of $\Gamma(b_j - \beta_j s)$, $j = 1, \cdots, M_2$ lies to the right and the poles of $\Gamma(1 - a_j + \alpha_j s)$, $j = 1, \cdots, N_2$; $\Gamma(1 - u_j + \mu^{(1)} s + \mu_j^{(2)} t)$; $j = 1, \cdots, N_1$ to the left contour.

The contour L_2 is in the t-plane and run from $-\omega\infty$ to $+\omega\infty$ with loops , if necessary , to ensure that the pole of

$\Gamma(\mathfrak{d}_j - \delta_j t)$;$j = 1, \cdots, M_3$ lies to the right and the poles of $\Gamma(1 - c_j + \gamma_j t); j = 1, \cdots, N_3$;
$\Gamma(1 - u_j + \mu^{(1)} s + \mu_j^{(2)} t)$; $j = 1, \cdots, N_1$ to the left contour.

The existence conditions of (1.13) are given below .

$$U_1 = \iota_{i'} \sum_{j=1}^{P_{i'}^{(2)}} \alpha_{ji'} + \iota_i \sum_{j=1}^{P_i^{(1)}} \alpha_{ji}^{(1)} - \iota_i \sum_{j=1}^{Q_i^{(1)}} \beta_{ji}^{(1)} - \iota_{i'} \sum_{j=1}^{Q_{i'}^{(2)}} \beta_{ji'} < 0 \tag{1.17}$$

$$U_2 = \iota_i \sum_{j=1}^{P_i^{(1)}} \alpha_{ji}^{(2)} + \iota_{i''} \sum_{j=1}^{P_{i''}^{(3)}} \gamma_{ji''} - \iota_i \sum_{j=1}^{Q_i^{(1)}} \beta_{ji}^{(2)} - \iota_{i''} \sum_{j=1}^{Q_{i''}^{(3)}} \delta_{ji''} < 0 \tag{1.18}$$

where $i = 1, \cdots, r$;$i' = 1, \cdots r'$; $i'' = 1, \cdots, r''$

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.13) can be obtained by extension of the corresponding conditions for H-function of two variables given by as :

$|arg(y_1)| < A_1 \frac{\pi}{2}$ and $|arg(y_2)| < A_2 \frac{\pi}{2}$; $i = 1, \cdots, r$;$i' = 1, \cdots r'$; $i'' = 1, \cdots, r''$, where :

$$A_1 = \iota_i \sum_{j=N_1+1}^{P_i^{(1)}} \alpha_{ji}^{(1)} - \iota_i \sum_{j=1}^{Q_i^{(1)}} \beta_{ji}^{(1)} + \sum_{j=1}^{m_2} \beta_j - \iota_{i'} \sum_{j=M_2+1}^{Q_{i'}^{(2)}} \beta_{ji'} + \sum_{j=1}^{N_2} \alpha_j - \iota_{i'} \sum_{j=N_2+1}^{P_{i''}^{(2)}} \alpha_{ji'} > 0 \tag{1.19}$$

$$+ \sum_{j=1}^{N_2} \gamma_j - \iota_{i''} \sum_{j=N_3+1}^{P_{i''}^{(3)}} \gamma_{ji''} > 0 \quad (1.20)$$

$$A_2 = \iota_i \sum_{j=N_1+1}^{P_i^{(1)}} \alpha_{ji}^{(2)} - \iota_i \sum_{j=1}^{Q_i^{(1)}} \beta_{ji}^{(2)} + \sum_{j=1}^{M_2} \delta_j - \iota_{i''} \sum_{j=M_3+1}^{Q_{i''}^{(3)}} \delta_{ji''} + \sum_{j=1}^{N_2} \gamma_j - \iota_{i''} \sum_{j=N_3+1}^{P_{i''}^{(3)}} \gamma_{ji''} > 0 \quad (1.20)$$

We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(y_1, y_2) = 0(\,|y_1|^{\alpha_1}, |y_2|^{\alpha_r}\,)\,, max(\,|y_1|, |y_2|\,) \rightarrow 0$$

$$\aleph(y_1, y_2) \; = 0(\,|y_1|^{\beta_1}, |z_r|^{\beta_r}\,)\,, min(\,|y_1|,|y_2|\,) \rightarrow \infty$$

where : $\alpha_1 = min[Re(b_j/\beta_j)], j = 1, \cdots, M_2$ and $\alpha_2 = min[Re(\mathfrak{d}_j/\delta_j)], j = 1, \cdots, M_3$

$\beta_1 = max[Re((a_j - 1)/\alpha_j)], j = 1, \cdots, N_2$ and $\beta_2 = max[Re((c_j - 1)/\gamma_j)], j = 1, \cdots, N_3$

Serie representation of Aleph-function of two variables is

$$\aleph_{P_i^{(1)},Q_i^{(1)},\iota_i;r:P_{i'}^{(2)},Q_{i'}^{(2)},\iota_{i'};r';P_{i''}^{(3)},Q_{i''}^{(3)},\iota_{i''};r''}^{M_1,N_1:M_2,N_2;M_3,N_3} \begin{pmatrix} \mathrm{y}_1 \\ \mathrm{y}_2 \end{pmatrix} = \sum_{G_1,G_2=0}^{\infty} \sum_{g_1=0}^{M_2} \sum_{g_2=0}^{M_3} \frac{(-)^{G_1+G_2}}{\beta_{g_1} G_1! \delta_{\mathfrak{g}_2} G_2!}$$

$$\times\ \zeta(\eta_{G_1,g_1},\eta_{G_2,g_2})\phi_1(\eta_{G_1,g_1})\,\phi_2(\eta_{G_2,g_2})y_1^{\eta_{G_1,g_1}}y_2^{\eta_{G_2,g_2}} \tag{1.21}$$

Where $\zeta(.,.),\phi_1(.),\phi_2(.)$ are given respectively in (1.14), (1.15) and (1.16) and

$$\eta_{G_1,g_1}=\frac{b_{g_1}+G_1}{\beta_{g_1}},\quad \eta_{G_2,g_2}=\frac{\mathfrak{d}_{\mathfrak{g}_2}+G_2}{\delta_{\mathfrak{g}_2}}$$

The generalized polynomials defined by Srivastava [10], is given in the following manner :

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1,\cdots,y_s]\ =\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}$$

$$A[N_1,K_1;\cdots;N_s,K_s]y_1^{K_1}\cdots y_s^{K_s} \tag{1.22}$$

Where $M_1,\cdots,M_s$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_s,K_s]$ are arbitrary constants, real or complex.

The generalized polynomial set defined by Raizada [6 ,p.64, eq.(2.1.2)] in the following Rodrigues type formula :

$$S_n^{\alpha,\beta,\tau}[x:\gamma,s,q,A,B,m,\zeta,l]=(Ax+B)^{-\alpha}(1-\tau x^{\gamma})^{\beta/\gamma} \tag{1.23}$$

$$\times T_{\zeta,l}^{m+n}[(Ax+B)^{\alpha+qn}(1-\tau x^{\gamma})^{(\beta/\tau)+sn}]$$

with the differential operator $T_{k,l}$ is defined by $T_{k,l}=x^l(k+x\dfrac{d}{dx})$ (1.24)

Moreover it can be expressed in the following serie :

$$S_n^{\alpha,\beta,\tau}[x:r,s,q,A,B,m,k,l]=B^{qn}x^{l(m+n)}(1-\tau x^r)^{sn}l^{m+n}\sum_{v=0}^{m+n}\sum_{u=0}^{v}\sum_{p=0}^{m+n}\sum_{c=0}^{p}\frac{(-)^p(-v)_u(-p)_e(\alpha)_p}{u!v!e!p!}$$

$$\frac{(-\alpha-qn)_e}{(1-\alpha-p)_e}\ \Big(-\frac{\beta}{\tau}-sn\Big)_v\Big(\frac{e+k+ru}{l}\Big)_{m+n}\Big(\frac{-\tau x^r}{1-\tau x^r}\Big)^{\sigma}\Big(\frac{Ax}{B}\Big)^p \tag{1.25}$$

$$\text{Let}\ \sum_{e,p,u,v}=\sum_{v=0}^{m+n}\sum_{u=0}^{v}\sum_{p=0}^{m+n}\sum_{c=0}^{p} \tag{1.26}$$

Taking $\tau\rightarrow 0$, in(1.23) and (1.25), one arrives at the following polynomial set :

$$S_n^{\alpha,\beta,0}(x)=S_n^{\alpha,\beta,0}[x:r,q,A,B,m,k,l]=\sum_{e,p,u,v}\phi(e,p,u,v)x^L \tag{1.27}$$

$\Big(\dfrac{e+k+ru}{l}\Big)_{m+n}$Where $\phi(e,p,u,v)=B^{qn-p}t^{m+n}\dfrac{(-)^p(-v)_u(-p)_e(\alpha)_p}{u!v!e!p!}\dfrac{(-\alpha-qn)_e}{(1-\alpha-p)_e}$

$$[(e+k+ru)/l]_{m+n}\,A^p\beta^v \tag{1.28}$$

and $L=l(m+n)+p+rv,(p,v=0,\cdots,m+n)$ (1.29)

Konhauser[4,p.303,304] has considered following pair of biorthogonal polynomials.

$$Z_n^{\alpha}(x;k)=\frac{\Gamma(\alpha+kn+1)}{n!}\sum_{j=0}^{n}(-)^j\binom{n}{j}\frac{x^{kj}}{\Gamma(\alpha+kj+1)} \tag{1.30}$$

and $\gamma_n^{\alpha}(x;k) = (1/n!) \sum_{\omega=0}^{n} \left(x^{\omega}/\omega!\right) \sum_{j=0}^{\omega} (-)^j \binom{\omega}{j} \left((a+j+1)/k\right)_n$ (1.31)

and the binomial expansion

$$(ax^v+b)^{\lambda} = b^{\lambda} \sum_{h=0}^{\infty} \binom{\lambda}{h} (ax^v/b)^h \text{ where } |ax^v/b|<1 \text{ and } |arg(ax^v/b)|<\pi \tag{1.32}$$

2. Main integrals

In the document , we note :

$$G(\eta_{G_1,g_1},\eta_{G_2,g_2}) = \phi(\eta_{G_1,g_1},\eta_{G_2,g_2})\theta_1(\eta_{G_1,g_1})\theta_r(\eta_{G_2,g_2}) \tag{2.1}$$

$$A = \frac{(-N_1)_{M_1K_1}}{K_1!} \cdots \frac{(-N_u)_{M_uK_u}}{K_u!} A[N_1,K_1;\cdots;N_u,K_u] \tag{2.2}$$

$g(x) = (ax^v+b)$ and $U_{21} = p_i+2, q_i+1, \tau_i; R$ (2.3)

and $A' = AG(\eta_{G_1,g_1},\eta_{G_2,g_2})(-)^{g+f}$ (2.4)

First integral

$$\int_0^{\infty} x^{\rho-1}\big(g(x)\big)^{\lambda} e^{-cx^{\eta}} S_n^{\alpha,\beta,0}[y'x^{\zeta_1}\big(g(x)\big)^{\zeta_2}: r,q,A,B,m,k,l] J_{\mu}\left[zx^{\zeta}\big(g(x)\big)^{\psi}\right]$$

$$y_{\mu}\left[1,k,x^{\rho}\big(g(x)\big)^{\zeta}\right] S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1x^{u_1}\big(g(x)\big)^{n_1},\cdots,y_sx^{u_s}\big(g(x)\big)^{n_s}]$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1x^{u_1}\big(g(x)\big)^{n_1},\cdots,y_sx^{u_s}\big(g(x)\big)^{n_s}] \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1x^{\rho_1}\big(g(x)\big)^{\sigma_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_rx^{\rho_r}\big(g(x)\big)^{\sigma_r} \end{array}\right) \mathrm{d}x$$

$$= (1/\eta)(y')^L b^{\lambda+\zeta_2L}(1/c)^{(\rho+\zeta_1L)/\eta} \sum_{g,h=0}^{\infty}\sum_{f=0}^{u}\sum_{i,\delta,t,p}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]} \phi(i,\delta,t,p)A(-)^{g+f}$$

$$\frac{(z/2)^{\mu+2g}(-u)_f(k+u-1)_f}{f!g!h!\Gamma(\mu+g+1)} y_1^{K_1}\cdots y_s^{K_s} b^{\psi(\mu+2g)-\zeta f+\sum_{i=1}^s n_iK_i}(a/b)^h(1/c)^{(\zeta(\mu+2g)+\sigma f+\sum_{i=1}^s K_iu_i+vh)\eta}$$

$$\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c} \mathrm{z}_1b^{\sigma_1}c^{\eta\sigma_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_rb^{\sigma_r}c^{\eta\sigma_r} \end{array}\middle| \begin{array}{c} ((1\text{-}\rho-\zeta_1L-\zeta(\mu+2g)+\sigma f-\sum_{i=1}^s K_iu_i-vh)/\eta;\rho_1/\eta,\cdots,\rho_r/\eta), \\ \cdots \\ \cdots \\ \cdots\cdots \end{array}\right.$$

$$\left.\begin{array}{c} (\text{-}\lambda-\zeta_2L-\psi(\mu+2g)+\zeta f-\sum_{i=1}^s K_in_i;\sigma_1,\cdots,\sigma_r),A:C \\ \cdots \\ \cdots \\ (\text{-}\lambda+h-\zeta_2L-\psi(\mu+2g)+\zeta f-\sum_{i=1}^s K_in_i;\sigma_1,\cdots,\sigma_r),B:D \end{array}\right) \tag{2.5}$$

where $\phi(i,\delta,t,p)$ and L are defined in (1.28) and (1.29) respectively, $y_\mu\left[1,k,zx^\rho\left(g(x)\right)^\zeta\right]$ is Bessel polynomial.

Provided that

a)$\zeta_1,\zeta_2,u_j,n_j,j=1,\cdots,s;\rho_i,\sigma_i,i=1,\cdots,r$ are positive real numbers (not all zero simultaneously)

b)$Re[\rho+\zeta_1 L+\zeta(\mu+2g)-\sigma f+vh+\sum_{i=1}^{s}K_iu_i+\sum_{i=1}^{r}\rho_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

c)$Re[\lambda+\zeta_2 L+\psi(\mu+2g)-\zeta f+\sum_{i=1}^{s}K_iu_i+\sum_{i=1}^{r}\sigma_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>-1$

d) $Re[\rho+\zeta_1 L+\zeta(\mu+2g)-\sigma f+vh+\sum_{i=1}^{s}K_iu_i+\sum_{i=1}^{r}\rho_i t_i]>0$; $a,b\neq 0,\eta>0,Re(c)>0$

e) $|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.5) ; $k=1,\cdots,r$

Second integral

$$\int_0^\infty x^{\rho-1}\left(g(x)\right)^\lambda e^{-cx^\eta}S_n^{\alpha,\beta,0}[y'x^{\zeta_1}\left(g(x)\right)^{\zeta_2}:r,q,A,B,m,k,l]J_\mu\left[zx^\zeta\left(g(x)\right)^\psi\right]\aleph\begin{pmatrix}\mathrm{D}x^\omega\left(g(x)\right)^v\\\mathrm{E}x^\omega\left(g(x)\right)^v\end{pmatrix}$$

$$y_\mu\left[1,k,x^\rho\left(g(x)\right)^\zeta\right]S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1x^{u_1}\left(g(x)\right)^{n_1},\cdots,y_sx^{u_s}\left(g(x)\right)^{n_s}]\aleph_{U:W}^{0,\mathfrak{n}:V}\begin{pmatrix}\mathrm{z}_1x^{\rho_1}\left(g(x)\right)^{\sigma_1}\\\cdot\\\cdot\\\cdot\\\mathrm{z}_rx^{\rho_r}\left(g(x)\right)^{\sigma_r}\end{pmatrix}\mathrm{d}x$$

$$=(1/\eta)(y')^Lb^{\lambda+\zeta_2L+\psi\mu}(1/c)^{(\rho+\zeta_1L+\zeta\mu)}\sum_{g,h=0}^{\infty}\sum_{f=0}^{u}\sum_{i,\delta,t,p}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{G_1,G_2=0}^{\infty}\sum_{g_1=0}^{M_2}\sum_{g_2=0}^{M_3}$$

$$\phi(i,\delta,t,p)A'\frac{(z/2)^{\mu+2g}(-u)_f(k+u-1)_f}{f!g!h!\Gamma(\mu+g+1)}\frac{(-)^{G_1+G_2}}{\beta_{g_1}G_1!\delta_{\mathfrak{g}_2}G_2!}y_1^{K_1}\cdots y_s^{K_s}b^{2g\psi-\zeta f+\sum_{i=1}^{s}n_iK_i+v\sum_{i=1}^{2}\eta_{Gi,gi}}$$

$$(a/b)^h(1/c)^{(2g\zeta-\sigma f+\sum_{i=1}^{s}K_iu_i+\omega\sum_{i=1}^{2}\eta_{G_i,g_i}+vh)\eta}D^{\eta_{G_1,g_1}}E^{\eta_{G_2,g_2}}\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{matrix}\mathrm{z}_1b^{\sigma_1}c^{\eta\sigma_1}\\\cdot\\\cdot\\\cdot\\\mathrm{z}_rb^{\sigma_r}c^{\eta\sigma_r}\end{matrix}\right|$$

$$((1\text{-}\rho-\zeta_1L-\zeta(\mu+2g)+\sigma f-\omega\sum_{i=1}^{2}\eta_{G_i,g_i}-\sum_{i=1}^{s}K_iu_i-vh)/\eta;\rho_1/\eta,\cdots,\rho_r/\eta),$$
$$\cdots$$
$$\cdots$$
$$\cdots\cdots$$

$$\left.\begin{matrix}(-\lambda-\zeta_2 L-\psi(\mu+2g)+\zeta f-v\sum_{i=1}^{2}\eta_{G_i,g_i}-\sum_{i=1}^{s}K_in_i;\sigma_1,\cdots,\sigma_r),A:C\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ (-\lambda+h-\zeta_2 L-\psi(\mu+2g)+\zeta f-v\sum_{i=1}^{2}\eta_{G_i,g_i}-\sum_{i=1}^{s}K_in_i;\sigma_1,\cdots,\sigma_r),B:D\end{matrix}\right) \tag{2.6}$$

where A' is defined by (2.4)

Provided that

a)$\zeta_1,\zeta_2,u_j,n_j,j=1,\cdots,s;\rho_i,\sigma_i,i=1,\cdots,r$ are positive real numbers (not all zero simultaneously)

b)$Re[\rho+\zeta_1 L+\zeta(\mu+2g)-\sigma f+vh+\omega(\min\limits_{1\leqslant j\leqslant M_2}\dfrac{b_j}{\beta_j}+\min\limits_{1\leqslant j\leqslant M_3}\dfrac{\mathfrak{d}_j}{\delta_j})+\sum\limits_{i=1}^{s}K_iu_i+\sum\limits_{i=1}^{r}\rho_i\min\limits_{1\leqslant j\leqslant m_i}\dfrac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

c)$Re[\lambda+\zeta_2 L+\psi(\mu+2g)-\zeta f+v(\min\limits_{1\leqslant j\leqslant M_2}\dfrac{b_j}{\beta_j}+\min\limits_{1\leqslant j\leqslant M_3}\dfrac{\mathfrak{d}_j}{\delta_j})+\sum\limits_{i=1}^{s}K_iu_i+\sum\limits_{i=1}^{r}\sigma_i\min\limits_{1\leqslant j\leqslant m_i}\dfrac{d_j^{(i)}}{\delta_j^{(i)}}]>-1$

d) $a,b\neq 0,\eta,\omega,v>0,Re(c)>0$

e)$Re[\rho+\zeta_1 L+\zeta(\mu+2g)-\sigma f+vh+\omega(\min\limits_{1\leqslant j\leqslant M_2}\dfrac{b_j}{\beta_j}+\min\limits_{1\leqslant j\leqslant M_3}\dfrac{\mathfrak{d}_j}{\delta_j})+\sum\limits_{i=1}^{s}K_iu_i+\sum\limits_{i=1}^{r}\rho_i\min\limits_{1\leqslant j\leqslant m_i}\dfrac{d_j^{(i)}}{\delta_j^{(i)}}+\sum\limits_{i=1}^{r}\rho_it_i]>0$

f) $|argz_k|<\dfrac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.5)

g) $|arg(D)|<A_1\dfrac{\pi}{2}$ and $|arg(E)|<A_2\dfrac{\pi}{2}$ where A_1 and A_2 are given respectively in (1.19) and (1.20)

Third integral

$$\int_0^\infty x^{\rho-1}\big(g(x)\big)^{\lambda}e^{-cx^\eta}S_n^{\alpha,\beta,0}[y'x^{\zeta_1}\big(g(x)\big)^{\zeta_2}:r,q,A,B,m,k,l]J_\mu\big[zx^\zeta\big(g(x)\big)^{\psi}\big]$$

$$\gamma_u^N\big(y\big(g(x)\big)^{\zeta};k'\big)S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1x^{u_1}\big(g(x)\big)^{n_1},\cdots,y_sx^{u_s}\big(g(x)\big)^{n_s}]\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{matrix}\mathrm{z}_1x^{\rho_1}\big(g(x)\big)^{\sigma_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}\big(g(x)\big)^{\sigma_r}\end{matrix}\right)\mathrm{d}x$$

$$=(1/\eta)(y')^Lb^{\lambda+\zeta_2L+\psi u}(1/c)^{(\rho+\zeta_1L+\zeta u)/\eta}\sum_{g,h=0}^{\infty}\sum_{f=0}^{u}\sum_{i,\delta,t,p}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{\omega=0}^{n}\sum_{j=0}^{\omega}\binom{\omega}{j}\big((N+j+1)/k'\big)_u$$

$$\phi(i,\delta,t,p)A(-1)^{g-j}\frac{(z/2)^{\mu+2g}y^\omega}{f!g!h!\Gamma(\mu+g+1)}\ \ y_1^{K_1}\cdots y_s^{K_s}b^{\omega\zeta+2\psi g+\sum_{i=1}^{s}n_iK_i}$$

$$(a/b)^h(1/c)^{(vh-\sigma\omega+2\zeta g+\sum_{i=1}^{s}K_iu_i)\eta}\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{matrix}\mathrm{z}_1b^{\sigma_1}c^{-\rho_1/\eta}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rb^{\sigma_r}c^{-\rho_r/\eta}\end{matrix}\right|$$

$$((1\text{-}\rho-\zeta_1 L-vh-\zeta(\mu+2g)-\sigma\omega-\sum_{i=1}^{s}K_iu_i)/\eta;\rho_1/\eta,\cdots,\rho_r/\eta),$$

$$\cdots$$

$$\cdots$$

$$\cdots\cdots$$

$$\left.\begin{array}{c}(\text{-}\lambda-\zeta_2 L-\psi(\mu+2g)-\zeta\omega-\sum_{i=1}^{s}K_in_i;\sigma_1,\cdots,\sigma_r),A:C\\ \cdots\\ \cdots\\ (\text{-}\lambda+h-\zeta_2 L-\psi(\mu+2g)-\zeta\omega-\sum_{i=1}^{s}K_in_i;\sigma_1,\cdots,\sigma_r),B:D\end{array}\right) \quad (2.7)$$

where $\gamma_u^N(y\big(g(x)\big)^{\zeta};k')$ is defined by (1.31)

Provided that

a)$\zeta_1,\zeta_2,u_j,n_j,j=1,\cdots,s;\rho_i,\sigma_i,i=1,\cdots,r$ are positive real numbers (not all zero simultaneously)

b)$Re[\rho+\zeta_1 L+\zeta(\mu+2g)-\sigma\omega+\sum_{i=1}^{s}K_iu_i+\sum_{i=1}^{r}\rho_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

c)$Re[\lambda+\zeta_2 L+\psi(\mu+2g)+\zeta\omega+\sum_{i=1}^{s}K_iu_i+\sum_{i=1}^{r}\sigma_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>-1$

d) $a,b\neq 0,\eta,\omega,\upsilon>0,Re(c)>0$

e)$Re[\rho+\zeta_1 L+\zeta(\mu+2g)+\sigma\omega+vh+\sum_{i=1}^{s}K_iu_i+\sum_{i=1}^{r}\rho_it_i]>0$

f) $|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.5)

Fourth integral

$$\int_0^{\infty}x^{\rho-1}\big(g(x)\big)^{\lambda}e^{-cx^{\eta}}S_n^{\alpha,\beta,0}[y'x^{\zeta_1}\big(g(x)\big)^{\zeta_2}:r,q,A,B,m,k,l]J_{\mu}\big[zx^{\zeta}\big(g(x)\big)^{\psi}\big]\aleph\left(\begin{array}{c}\mathrm{D}x^{\omega}\big(g(x)\big)^{v}\\ \mathrm{E}x^{\omega}\big(g(x)\big)^{v}\end{array}\right)$$

$$\gamma_u^N(y\big(g(x)\big)^{\zeta};k')S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1x^{u_1}\big(g(x)\big)^{n_1},\cdots,y_sx^{u_s}\big(g(x)\big)^{n_s}]\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1x^{\rho_1}\big(g(x)\big)^{\sigma_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}\big(g(x)\big)^{\sigma_r}\end{array}\right)\mathrm{d}x$$

$$=(1/\eta)(y')^Ly^{\omega}b^{\lambda+\zeta_2 L+\psi u}(1/c)^{(\rho+\zeta_1 L+\zeta u)/\eta}\sum_{g,h=0}^{\infty}\sum_{f=0}^{u}\sum_{i,\delta,t,p}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{G_1,G_2=0}^{\infty}\sum_{g_1=0}^{M_2}\sum_{g_2=0}^{M_3}\sum_{\omega=0}^{n}\sum_{j=0}^{\omega}$$

$$\binom{\omega}{j}\big((N+j+1)/k'\big)_u\phi(i,\delta,t,p)A'\frac{(z/2)^{\mu+2g}}{f!g!h!\Gamma(\mu+g+1)}\frac{(-)^{G_1+G_2}}{\beta_{g_1}G_1!\delta_{\mathfrak{g}_2}G_2!}y_1^{K_1}\cdots y_s^{K_s}D^{\eta_{G_1,g_1}}E^{\eta_{G_2,g_2}}$$

$$(a/b)^h(1/c)^{(-2\zeta g+vh-\sigma\omega+\sum_{i=1}^{s}K_iu_i-\omega\sum_{i=1}^{2}\eta_{G_i,g_i}+\upsilon h)\eta}b^{2g\psi+\omega\zeta+\sum_{i=1}^{s}n_iK_i+\upsilon\sum_{i=1}^{2}\eta_{Gi,gi}}$$

$$\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c} z_1 b^{\sigma_1} c^{-\rho_1/\eta} & (\text{-}\lambda-\zeta_2 L-\psi(\mu+2g)-v\sum_{i=1}^{2}\eta_{G_i,g_i}-\sum_{i=1}^{s}K_i n_i;\sigma_1,\cdots,\sigma_r),A \\ \cdot & \cdots \\ \cdot & \cdots \\ \cdot & \\ z_r b^{\sigma_r} c^{-\rho_r/\eta} & (\text{-}\lambda+h-\zeta_2 L-\psi(\mu+2g)-v\sum_{i=1}^{2}\eta_{G_i,g_i}-\sum_{i=1}^{s}K_i n_i;\sigma_1,\cdots,\sigma_r),B \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-}\rho-\zeta_1 L-\alpha-\zeta(\mu+2g)+\sigma\omega-\omega\sum_{i=1}^{2}\eta_{G_i,g_i}-\sum_{i=1}^{s}K_i u_i+vh)/\eta;\rho_1/\eta,\cdots,\rho_r/\eta):C \\ \cdots \\ \cdots \\ \text{D} \end{array}\right) \tag{2.8}$$

where A' is defined by (2.4)

Provided that

a)$\zeta_1,\zeta_2,u_j,n_j,j=1,\cdots,s;\rho_i,\sigma_i,i=1,\cdots,r$ are positive real numbers (not all zero simultaneously)

b)$Re[\rho+\zeta_1 L+\zeta(\mu+2g)+\sigma\omega+\omega(\min\limits_{1\leqslant j\leqslant M_2}\dfrac{b_j}{\beta_j}+\min\limits_{1\leqslant j\leqslant M_3}\dfrac{\mathfrak{d}_j}{\delta_j})+\sum\limits_{i=1}^{s}K_i u_i+\sum\limits_{i=1}^{r}\rho_i\min\limits_{1\leqslant j\leqslant m_i}\dfrac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

c)$Re[\lambda+\zeta_2 L+\psi(\mu+2g)+\zeta\omega+v(\min\limits_{1\leqslant j\leqslant M_2}\dfrac{b_j}{\beta_j}+\min\limits_{1\leqslant j\leqslant M_3}\dfrac{\mathfrak{d}_j}{\delta_j})+\sum\limits_{i=1}^{s}K_i u_i+\sum\limits_{i=1}^{r}\sigma_i\min\limits_{1\leqslant j\leqslant m_i}\dfrac{d_j^{(i)}}{\delta_j^{(i)}}]>-1$

d) $a,b\neq 0,\eta,\omega,v>0,Re(c)>0$

e)$Re[\rho+\zeta_1 L+\zeta(\mu+2g)+\sigma\omega+\omega(\min\limits_{1\leqslant j\leqslant M_2}\dfrac{b_j}{\beta_j}+\min\limits_{1\leqslant j\leqslant M_3}\dfrac{\mathfrak{d}_j}{\delta_j})+\sum\limits_{i=1}^{s}K_i u_i+\sum\limits_{i=1}^{r}\rho_i\min\limits_{1\leqslant j\leqslant m_i}\dfrac{d_j^{(i)}}{\delta_j^{(i)}}+\sum\limits_{i=1}^{r}\rho_i t_i]>0$

f) $|arg z_k|<\dfrac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.5)

g) $|arg(D)|<A_1\dfrac{\pi}{2}$ and $|arg(E)|<A_2\dfrac{\pi}{2}$ where A_1 and A_2 are given respectively in (1.19) and (1.20)

Proof of (2.5)

Let $M=\dfrac{1}{(2\pi\omega)^r}\displaystyle\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)$

Using (1.23), (1.22), (1.32) and (1.9) in (2.5) and interchanging the orders of summation and integrations which permissible under the conditions stated with (2.5), we get

L.H.S of (2.5)$=\displaystyle\sum_{g,h=0}^{\infty}\sum_{f=0}^{u}\sum_{i,\delta,t,p}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\phi(i,\delta,t,p)A(-)^{g+f}\frac{(z/2)^{\mu+2g}(-u)_f(k+u-1)_f}{f!g!h!\Gamma(\mu+g+1)}y'^L$

$$y_1^{K_1}\cdots y_s^{K_s}b^{\lambda+\zeta_2 L+\psi(\mu+2g)-\zeta f+\sum_{i=1}^{s}n_i K_i}(a/b)^h(1/c)^{(\zeta(\mu+2g)+\sigma f+\sum_{i=1}^{s}K_i u_i+vh)\eta}(a/b)^h M$$

$$\left(z_1 b^{\sigma_1}\right)^{t_1}\cdots\left(z_r b^{\sigma_r}\right)^{t_r}\frac{\Gamma(1+\lambda+\zeta_2 L+\psi(\mu+2g)-\zeta f+\sum_{i=1}^{s}K_i n_i+\sum_{i=1}^{r}\sigma_i t_i)}{\Gamma(1+\lambda-h+\zeta_2 L+\psi(\mu+2g)-\zeta f+\sum_{i=1}^{s}K_i n_i+\sum_{i=1}^{r}\sigma_i t_i)}$$

$$\int_0^{\infty}e^{-cx^{\eta}}x^{\rho-1+\zeta_1 L+\zeta(\mu+2g)-\sigma f+\sum_{i=1}^{s}K_i u_i+\sum_{i=1}^{r}\rho_i t_i}\mathrm{d}x\,\mathrm{d}t_1\cdots\mathrm{d}t_r \tag{2.9}$$

On evaluating the inner x-integral occuring on the R.H.S. of (2.9) and on reinterpreting the Mellin-Barnes contour integral in the R.H.S. of (2.9) in term of the multivariable Aleph-function given by (1.1), we arrive at the desired result.

The proofs of (2.6), (2.7) and (2.8) can be developped by similar methods.

3. Fourier series

a) First Fourier-Legendre series

$$x^{\rho-1}\left(g(x)\right)^{\lambda}e^{-cx^{\eta}}\, S_n^{\alpha,\beta,0}[y'x^{\zeta_1}\left(g(x)\right)^{\zeta_2}:r,q,A,B,m,k,l]J_{\mu}\left[zx^{\zeta}\left(g(x)\right)^{\psi}\right]$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1x^{u_1}\left(g(x)\right)^{n_1},\cdots,y_sx^{u_s}\left(g(x)\right)^{n_s}]\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1x^{\rho_1}\left(g(x)\right)^{\sigma_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}\left(g(x)\right)^{\sigma_r}\end{array}\right)$$

$$=\sum_{u,g,h=0}^{\infty}\sum_{i,\delta,t,p}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\phi(i,\delta,t,p)A(-)^g\frac{(z/2)^{\mu-2g}c^d(y')^t}{g!h!\Gamma(\mu+g+1)\Gamma(d)}y_1^{K_1}\cdots y_s^{K_s}$$

$$b^{\lambda+\psi\mu+2g\eta-\sum_{i=1}^s n_iK_i}(a/b)^h(1/c)^{(\rho+d+\zeta(\mu+2g)+l\zeta_1+\sum_{i=1}^s K_iu_i+vh-1)}/\left({}_3F_1[-u,u+1,d;1;x^{\eta}]\right)$$

$$\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c} \mathrm{z}_1b^{\sigma_1}c^{-\rho_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rb^{\sigma_r}c^{-\rho_r}\end{array}\middle|\begin{array}{c}(\text{-}\lambda-\zeta_2L-\psi(\mu+2g)-\sum_{i=1}^s K_in_i;\sigma_1,\cdots,\sigma_r),\\ \cdots\\ \cdots\\ (\text{-}\lambda+h-\zeta_2L-\psi(\mu+2g)-\sum_{i=1}^s K_in_i;\sigma_1,\cdots,\sigma_r),\end{array}\right.$$

$$\left.\begin{array}{c}(2\text{-}\rho-d-\zeta_1L-\zeta(\mu+2g)-vh-\sum_{i=1}^s K_iu_i;\rho_1,\cdots,\rho_r),A:C\\ \cdots\\ \cdots\\ \mathrm{B}:\mathrm{D}\end{array}\right)P_u(1-2xT) \tag{3.1}$$

The equation (3.5) is valid under the same conditions mentioned in (2.5) and $Re(\rho)>0$

b) Second Fourier-Legendre series

$$x^{\rho-1}\left(g(x)\right)^{\lambda}e^{-cx^{\eta}}S_n^{\alpha,\beta,0}[y'x^{\zeta_1}\left(g(x)\right)^{\zeta_2}:r,q,A,B,m,k,l]J_{\mu}\left[zx^{\zeta}\left(g(x)\right)^{\psi}\right]\aleph\left(\begin{array}{c}\mathrm{D}x^{\omega}\left(g(x)\right)^{\upsilon}\\ \mathrm{E}x^{\omega}\left(g(x)\right)^{\upsilon}\end{array}\right)$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1x^{u_1}\left(g(x)\right)^{n_1},\cdots,y_sx^{u_s}\left(g(x)\right)^{n_s}]\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1x^{\rho_1}\left(g(x)\right)^{\sigma_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}\left(g(x)\right)^{\sigma_r}\end{array}\right)$$

$$= (1/\eta)(y')^L b^{\lambda+\zeta_2 L}(1/c)^{(\rho+\zeta_1 L)/\eta}\sum_{u,g,h=0}^{\infty}\sum_{i,\delta,t,p}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{G_1,G_2=0}^{\infty}\sum_{g_1=0}^{M_2}\sum_{g_2=0}^{M_3}D^{\eta_{G_1,g_1}}E^{\eta_{G_2,g_2}}$$

$$\phi(i,\delta,t,p)A'\frac{(-1)^d(z/2)^{\mu-2g}c^d(y')^L}{g!h!\Gamma(\mu+g+1)}\frac{(-)^{G_1+G_2}}{\beta_{g_1}G_1!\delta_{\mathfrak{g}_2}G_2!}y_1^{K_1}\cdots y_s^{K_s}b^{\psi(\mu+2g)-\sum_{i=1}^s n_iK_i+\upsilon\sum_{i=1}^2\eta_{Gi,gi}}$$

$$y^L(a/b)^h(1/c)^{(\rho+d-1+\zeta_1 L+\zeta(\mu+2g)+\sum_{i=1}^s K_iu_i+\omega\sum_{i=1}^2\eta_{G_i,g_i}+vh)}/\left(\Gamma(d)_3F_1(-u,u+1,d;1;x^\eta)\right)$$

$$\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c}\mathrm{z}_1b^{\sigma_1}c^{-\rho_1} & (2\text{-}\rho-d-\zeta_1L+\alpha+\zeta(\mu+2g)-\omega\sum_{i=1}^2\eta_{G_i,g_i}+\sum_{i=1}^s K_iu_i-vh;\rho_1,\cdots,\rho_r),\\ \cdot & \cdots\\ \cdot & \cdots\\ \cdot & \cdots\cdots\\ \mathrm{z}_rb^{\sigma_r}c^{-\rho_r} & \end{array}\right.$$

$$\left.\begin{array}{c}(\text{-}\lambda-\zeta_2L-\psi(\mu+2g)-\upsilon\sum_{i=1}^2\eta_{G_i,g_i}-\sum_{i=1}^s K_in_i;\sigma_1,\cdots,\sigma_r),A:C\\ \cdots\\ \cdots\\ (\text{-}\lambda+h-\zeta_2L-\psi(\mu+2g)-\upsilon\sum_{i=1}^2\eta_{G_i,g_i}-\sum_{i=1}^s K_in_i;\sigma_1,\cdots,\sigma_r),B:D\end{array}\right)P_u(1-2xT) \quad (3.2)$$

where A' is defined by (2.4), the equation (3.6) is valid under the same conditions mentioned in (2.6) and $Re(\rho) > 0$

c)Fourier-Bessel series

$$x^{\rho-1}\big(g(x)\big)^{\lambda}e^{-cx^\eta}S_n^{\alpha,\beta,0}[y'x^{\zeta_1}\big(g(x)\big)^{\zeta_2}:r,q,A,B,m,k,l]J_\mu\big[zx^\zeta\big(g(x)\big)^{\psi}\big]\aleph\begin{pmatrix}\mathrm{D}x^\omega\big(g(x)\big)^\upsilon\\ \mathrm{E}x^\omega\big(g(x)\big)^\upsilon\end{pmatrix}$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1x^{u_1}\big(g(x)\big)^{n_1},\cdots,y_sx^{u_s}\big(g(x)\big)^{n_s}]\aleph_{U:W}^{0,\mathfrak{n}:V}\begin{pmatrix}\mathrm{z}_1x^{\rho_1}\big(g(x)\big)^{\sigma_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}\big(g(x)\big)^{\sigma_r}\end{pmatrix}$$

$$= (1/\eta)(y')^L b^{\lambda+\zeta_2 L}\sum_{u,g,h=0}^{\infty}\sum_{f=0}^{u}\sum_{i,\delta,t,p}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{G_1,G_2=0}^{\infty}\sum_{g_1=0}^{M_2}\sum_{g_2=0}^{M_3}D^{\eta_{G_1,g_1}}E^{\eta_{G_2,g_2}}(a/b)^h$$

$$\phi(i,\delta,t,p)A'\frac{(z/2)^{\mu+2g}(-u)_f(k+u-1)_f}{f!g!h!\Gamma(\mu+g+1)}\frac{(-)^{G_1+G_2}}{\beta_{g_1}G_1!\delta_{\mathfrak{g}_2}G_2!}y_1^{K_1}\cdots y_s^{K_s}b^{\psi(\mu+2g)-\sum_{i=1}^s n_iK_i+\upsilon\sum_{i=1}^2\eta_{Gi,gi}}$$

$$\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c}\mathrm{z}_1b^{\sigma_1} & (\omega'-\rho-\zeta_1L+f-\zeta(\mu+2g)-\omega\sum_{i=1}^2\eta_{G_i,g_i}+\sum_{i=1}^s K_iu_i-vh;\rho_1,\cdots,\rho_r),\\ \cdot & \cdots\\ \cdot & \cdots\\ \cdot & \cdots\cdots\\ \mathrm{z}_rb^{\sigma_r} & \end{array}\right.$$

$$\left.\begin{matrix} (\text{-}\lambda-\zeta_2 L-\psi(\mu+2g)-\upsilon\sum_{i=1}^2\eta_{G_i,g_i}-\sum_{i=1}^s K_i n_i;\sigma_1,\cdots,\sigma_r),A:C \\ \cdots \\ \cdots \\ (\text{-}\lambda+h-\zeta_2 L-\psi(\mu+2g)-\upsilon\sum_{i=1}^2\eta_{G_i,g_i}-\sum_{i=1}^s K_i n_i;\sigma_1,\cdots,\sigma_r),B:D \end{matrix}\right) Y_u(1,\omega,x) \quad (3.3)$$

where A' is defined by (2.4), the equation (3.7) is valid under the same conditions mentioned in (2.7).

d)Fourier-Konhauser series

$$x^{\rho-1}\big(g(x)\big)^{\lambda}e^{-cx^{\eta}}S_n^{\alpha,\beta,0}[y'x^{\zeta_1}\big(g(x)\big)^{\zeta_2}:r,q,A,B,m,k,l]J_{\mu}\big[zx^{\zeta}\big(g(x)\big)^{\psi}\big]\aleph\begin{pmatrix}\mathrm{Dx}^{\omega}\big(g(x)\big)^{\upsilon}\\ \mathrm{Ex}^{\omega}\big(g(x)\big)^{\upsilon}\end{pmatrix}$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1x^{u_1}\big(g(x)\big)^{n_1},\cdots,y_sx^{u_s}\big(g(x)\big)^{n_s}]\aleph_{U:W}^{0,\mathfrak{n}:V}\begin{pmatrix}\mathrm{z}_1x^{\rho_1}\big(g(x)\big)^{\sigma_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}\big(g(x)\big)^{\sigma_r}\end{pmatrix}=(1/\eta)(y')^L b^{\lambda+\zeta_2 L}$$

$$\sum_{F,g,h=0}^{\infty}\sum_{f=0}^{u}\sum_{i,\delta,t,p}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{G_1,G_2=0}^{\infty}\sum_{g_1=0}^{M_2}\sum_{g_2=0}^{M_3}\sum_{j=0}^{\omega}D^{\eta_{G_1,g_1}}E^{\eta_{G_2,g_2}}(a/b)^h\binom{\omega}{j}\frac{\big((N+j+1)/k'\big)_u}{\Gamma(N+kF+1)}$$

$$\phi(i,\delta,t,p)A'\frac{(z/2)^{\mu+2g}(a/b)^h}{\omega!f!g!h!\Gamma(\mu+g+1)}\frac{(-)^{G_1+G_2}}{\beta_{g_1}G_1!\delta_{\mathfrak{g}_2}G_2!}y_1^{K_1}\cdots y_s^{K_s}b^{\psi(\mu+2g)+\sum_{i=1}^s n_iK_i+\upsilon\sum_{i=1}^2\eta_{Gi,gi}}Z_F^N(x;k)$$

$$\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{matrix}\mathrm{z}_1b^{\sigma_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rb^{\sigma_r}\end{matrix}\,\middle|\,\begin{matrix}(\text{-}\lambda-\zeta_2 L-\psi(\mu+2g)-\upsilon\sum_{i=1}^2\eta_{G_i,g_i}-\sum_{i=1}^s K_in_i;\sigma_1,\cdots,\sigma_r),A,\\ \cdots\\ \cdots\\ (\text{-}\lambda+h-\zeta_2 L-\psi(\mu+2g)-\upsilon\sum_{i=1}^2\eta_{G_i,g_i}-\sum_{i=1}^s K_in_i;\sigma_1,\cdots,\sigma_r),B:\end{matrix}\right)$$

$$\left.\begin{matrix}(1\text{-}\rho-N-\zeta_1 L-\omega-\zeta(\mu+2g)+f-\omega\sum_{i=1}^2\eta_{G_i,g_i}+\sum_{i=1}^s K_iu_i-v;\rho_1,\cdots,\rho_r):C\\ \cdots\\ \cdots\\ \mathrm{D}\end{matrix}\right) \quad (3.4)$$

where A' is defined by (2.4), the equation (3.8) is valid under the same conditions mentioned in (2.8).

Proof of (3.5)

Let L.H.S of (3.5) $=\sum_{u=0}^{\infty}E_uP_u(1-2xt)$ (3.5)

Multiplying both sides by $e^{-cx}x^{d-1}$ and integrating with respect to x from 0 to ∞, we get

$$\int_0^\infty x^{\rho+d-2}\big(g(x)\big)^{\lambda} e^{-cx^\eta}\, S_n^{\alpha,\beta,0}[y'x^{\zeta_1}\big(g(x)\big)^{\zeta_2} : r,q,A,B,m,k,l] J_\mu\left[zx^\zeta\big(g(x)\big)^{\psi}\right]$$

$$y_\mu\left[1,k,zx^\rho\big(g(x)\big)^{\zeta}\right] S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1x^{u_1}\big(g(x)\big)^{n_1},\cdots,y_sx^{u_s}\big(g(x)\big)^{n_s}]\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1x^{\rho_1}\big(g(x)\big)^{\sigma_1} \\ . \\ . \\ . \\ \mathrm{z}_rx^{\rho_r}\big(g(x)\big)^{\sigma_r}\end{array}\right)\mathrm{d}x$$

$$=\sum_{i=0}^\infty E_u \int_0^\infty e^{-cx}x^{d-1}P_u(1-2xT)\mathrm{d}x \tag{3.6}$$

using the integral formula (2.5) and the following orthogonal property of Legendre polynomial [3.p.97,Eq.(5.2.4.17)

$$\int_0^\infty e^{-st}t^{a-1}P_n(1-2xT)\mathrm{d}t = \big(\Gamma(a)/s^a\big)\big({}_3F_1(-n,n+1,a;1;x/s)\big) \tag{3.7}$$

where $Re(a)>0, Re(s)>0$

In (3.5), we find the value E_u and substituting the value of E_u in (3.10), we get the desired formula (3.5). To prove (3.6) to (3.8), we use the similar methods and using respectively the integral formulas [3.p.97,Eq.(5.2.4.17)], [5.p.135] and [4,p.303], we can establish the expansion formulas (3.6) to (3.8).

4. Multivariable I-function

If $\tau_i=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=1$ then the multivariable Aleph-functions degenere to the multivariable I-functions defined by Sharma et al [7]. And we have the following results concerning the Fourier series .

a) First Fourier-Legendre series

$$x^{\rho-1}\big(g(x)\big)^{\lambda} e^{-cx^\eta}\, S_n^{\alpha,\beta,0}[y'x^{\zeta_1}\big(g(x)\big)^{\zeta_2} : r,q,A,B,m,k,l] J_\mu\left[zx^\zeta\big(g(x)\big)^{\psi}\right]$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1x^{u_1}\big(g(x)\big)^{n_1},\cdots,y_sx^{u_s}\big(g(x)\big)^{n_s}]I_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1x^{\rho_1}\big(g(x)\big)^{\sigma_1} \\ . \\ . \\ . \\ \mathrm{z}_rx^{\rho_r}\big(g(x)\big)^{\sigma_r}\end{array}\right)$$

$$=\sum_{u,g,h=0}^\infty\sum_{i,\delta,t,p}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\phi(i,\delta,t,p)A(-)^g\frac{(z/2)^{\mu-2g}c^dy'^t}{g!h!\Gamma(\mu+g+1)\Gamma(d)}y_1^{K_1}\cdots y_s^{K_s}b^{\lambda+\psi\mu+2g\eta-\sum_{i=1}^s n_iK_i}$$

$$(a/b)^h(1/c)^{(\rho+d-1+\zeta_1L+\zeta(\mu+2g)+\sum_{i=1}^s K_iu_i+\omega\sum_{i=1}^2\eta_{G_i,g_i}+vh)}/\big({}_3F_1(-u,u+1,d;1;x^\eta]\big)$$

$$I_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c} \mathrm{z}_1b^{\sigma_1}c^{-\rho_1} \\ . \\ . \\ . \\ \mathrm{z}_rb^{\sigma_r}c^{-\rho_r}\end{array}\left|\begin{array}{c} (\text{-}\lambda-\zeta_2L-\psi(\mu+2g)+\zeta f-\sum_{i=1}^s K_in_i;\sigma_1,\cdots,\sigma_r), \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ (\text{-}\lambda+h-\zeta_2L-\psi(\mu+2g)+\zeta f-\sum_{i=1}^s K_in_i;\sigma_1,\cdots,\sigma_r), \end{array}\right.\right.$$

$$(2\text{-}\rho - d - \zeta_1 L - \zeta(\mu+2g) - vh - \textstyle\sum_{i=1}^{s} K_i u_i; \rho_1, \cdots, \rho_r), A : C \quad \cdots \quad \cdots \quad \mathrm{B : D} \Bigg) P_u(1-2xT) \tag{4.1}$$

The equation (3.5) is valid under the same conditions mentioned in (2.5) and $Re(\rho) > 0$

b) Second Fourier-Legendre series

$$x^{\rho-1}\big(g(x)\big)^{\lambda} e^{-cx^{\eta}} S_n^{\alpha,\beta,0}[y'x^{\zeta_1}\big(g(x)\big)^{\zeta_2} : r, q, A, B, m, k, l] J_\mu\big[zx^{\zeta}\big(g(x)\big)^{\psi}\big] \aleph\begin{pmatrix} \mathrm{Dx}^{\omega}\big(g(x)\big)^{\upsilon} \\ \mathrm{Ex}^{\omega}\big(g(x)\big)^{\upsilon} \end{pmatrix}$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1 x^{u_1}\big(g(x)\big)^{n_1}, \cdots, y_s x^{u_s}\big(g(x)\big)^{n_s}] I\begin{pmatrix} \mathrm{z}_1 x^{\rho_1}\big(g(x)\big)^{\sigma_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r x^{\rho_r}\big(g(x)\big)^{\sigma_r} \end{pmatrix}$$

$$= (1/\eta)(y')^L b^{\lambda+\zeta_2 L}(1/c)^{\rho+\zeta_1 L)/\eta} \sum_{u,g,h=0}^{\infty} \sum_{i,\delta,t,p} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_s=0}^{[N_s/M_s]} \sum_{G_1,G_2=0}^{\infty} \sum_{g_1=0}^{M_2} \sum_{g_2=0}^{M_3} D^{\eta_{G_1,g_1}} E^{\eta_{G_2,g_2}}$$

$$\phi(i,\delta,t,p) A' \frac{(-1)^d (z/2)^{\mu-2g} c^d y'^t}{g!h!\Gamma(\mu+g+1)\Gamma(d)} \frac{(-)^{G_1+G_2}}{\beta_{g_1} G_1! \delta_{\mathfrak{g}_2} G_2!} y_1^{K_1} \cdots y_s^{K_s} b^{\psi(\mu+2g) - \sum_{i=1}^{s} n_i K_i + \upsilon \sum_{i=1}^{2} \eta_{Gi,gi}}$$

$$y^L (a/b)^h (1/c)^{(\rho+d-1+\zeta_1 L + \zeta(\mu+2g) + \sum_{i=1}^{s} K_i u_i + \omega \sum_{i=1}^{2} \eta_{G_i,g_i} + vh)} / \big(\Gamma(d)_3 F_1(-u, u+1, d; 1; x^{\eta})\big)$$

$$I_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{matrix} \mathrm{z}_1 b^{\sigma_1} c^{-\rho_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r b^{\sigma_r} c^{-\rho_r} \end{matrix}\,\middle|\, \begin{matrix} (2\text{-}\rho - d - \zeta_1 L + \alpha + \zeta(\mu+2g) - \omega \sum_{i=1}^{2} \eta_{G_i,g_i} + \sum_{i=1}^{s} K_i u_i - vh; \rho_1, \cdots, \rho_r), \\ \cdots \\ \cdots \\ \cdots\cdots \end{matrix}\right.$$

$$\left.\begin{matrix} (\text{-}\lambda - \zeta_2 L - \psi(\mu+2g) - \upsilon \sum_{i=1}^{2} \eta_{G_i,g_i} - \sum_{i=1}^{s} K_i n_i; \sigma_1, \cdots, \sigma_r), A : C \\ \cdots \\ \cdots \\ (\text{-}\lambda + h - \zeta_2 L - \psi(\mu+2g) - \upsilon \sum_{i=1}^{2} \eta_{G_i,g_i} - \sum_{i=1}^{s} K_i n_i; \sigma_1, \cdots, \sigma_r), B : D \end{matrix}\right) P_u(1-2xT) \tag{4.2}$$

where A' is defined by (2.4), the equation (3.6) is valid under the same conditions mentioned in (2.6) and $Re(\rho) > 0$

c)Fourier-Bessel series

$$x^{\rho-1}\big(g(x)\big)^{\lambda} e^{-cx^{\eta}} S_n^{\alpha,\beta,0}[y'x^{\zeta_1}\big(g(x)\big)^{\zeta_2} : r, q, A, B, m, k, l] J_\mu\big[zx^{\zeta}\big(g(x)\big)^{\psi}\big] \aleph\begin{pmatrix} \mathrm{Dx}^{\omega}\big(g(x)\big)^{\upsilon} \\ \mathrm{Ex}^{\omega}\big(g(x)\big)^{\upsilon} \end{pmatrix}$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1x^{u_1}\left(g(x)\right)^{n_1},\cdots,y_sx^{u_s}\left(g(x)\right)^{n_s}]I\left(\begin{array}{c} \mathrm{z}_1x^{\rho_1}\left(g(x)\right)^{\sigma_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}\left(g(x)\right)^{\sigma_r}\end{array}\right)$$

$$=(1/\eta)(y')^Lb^{\lambda+\zeta_2L}\sum_{u,g,h=0}^{\infty}\sum_{f=0}^{u}\sum_{i,\delta,t,p}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{G_1,G_2=0}^{\infty}\sum_{g_1=0}^{M_2}\sum_{g_2=0}^{M_3}D^{\eta_{G_1,g_1}}E^{\eta_{G_2,g_2}}(a/b)^h$$

$$\phi(i,\delta,t,p)A'\frac{(z/2)^{\mu+2g}(-u)_f(k+u-1)_f}{f!g!h!\Gamma(\mu+g+1)}\frac{(-)^{G_1+G_2}}{\beta_{g_1}G_1!\delta_{\mathfrak{g}_2}G_2!}y_1^{K_1}\cdots y_s^{K_s}b^{\psi(\mu+2g)-\sum_{i=1}^s n_iK_i+\upsilon\sum_{i=1}^2\eta_{Gi,gi}}$$

$$I_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c} \mathrm{z}_1b^{\sigma_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rb^{\sigma_r}\end{array}\middle|\begin{array}{c}(\omega'-\rho-\zeta_1L+f-\zeta(\mu+2g)-\omega\sum_{i=1}^2\eta_{G_i,g_i}+\sum_{i=1}^sK_iu_i-\upsilon h;\rho_1,\cdots,\rho_r),\\ \cdots\\ \cdots\\ \cdots\cdots\end{array}\right.$$

$$\left.\begin{array}{c}(\text{-}\lambda-\zeta_2L-\psi(\mu+2g)-\upsilon\sum_{i=1}^2\eta_{G_i,g_i}-\sum_{i=1}^sK_in_i;\sigma_1,\cdots,\sigma_r),A:C\\ \cdots\\ \cdots\\ (\text{-}\lambda+h-\zeta_2L-\psi(\mu+2g)-\upsilon\sum_{i=1}^2\eta_{G_i,g_i}-\sum_{i=1}^sK_in_i;\sigma_1,\cdots,\sigma_r),B:D\end{array}\right)Y_u(1,\omega,x)\quad(4.3)$$

where A' is defined by (2.4), the equation (3.7) is valid under the same conditions mentioned in (2.7).

c)Fourier-Konhauser series

$$x^{\rho-1}\left(g(x)\right)^{\lambda}e^{-cx^\eta}S_n^{\alpha,\beta,0}[y'x^{\zeta_1}\left(g(x)\right)^{\zeta_2}:r,q,A,B,m,k,l]J_\mu\left[zx^\zeta\left(g(x)\right)^\psi\right]\aleph\left(\begin{array}{c}\mathrm{D}x^\omega\left(g(x)\right)^\upsilon\\ \mathrm{E}x^\omega\left(g(x)\right)^\upsilon\end{array}\right)$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1x^{u_1}\left(g(x)\right)^{n_1},\cdots,y_sx^{u_s}\left(g(x)\right)^{n_s}]I\left(\begin{array}{c} \mathrm{z}_1x^{\rho_1}\left(g(x)\right)^{\sigma_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}\left(g(x)\right)^{\sigma_r}\end{array}\right)=(1/\eta)(y')^Lb^{\lambda+\zeta_2L}$$

$$\sum_{F,g,h=0}^{\infty}\sum_{f=0}^{u}\sum_{i,\delta,t,p}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{G_1,G_2=0}^{\infty}\sum_{g_1=0}^{M_2}\sum_{g_2=0}^{M_3}\sum_{j=0}^{\omega}D^{\eta_{G_1,g_1}}E^{\eta_{G_2,g_2}}(a/b)^h\binom{\omega}{j}\frac{\left((N+j+1)/k'\right)_u}{\Gamma(N+kF+1)}$$

$$\phi(i,\delta,t,p)A'\frac{(z/2)^{\mu+2g}(a/b)^h}{\omega!f!g!h!\Gamma(\mu+g+1)}\frac{(-)^{G_1+G_2}}{\beta_{g_1}G_1!\delta_{\mathfrak{g}_2}G_2!}y_1^{K_1}\cdots y_s^{K_s}b^{\psi(\mu+2g)+\sum_{i=1}^s n_iK_i+\upsilon\sum_{i=1}^2\eta_{Gi,gi}}Z_F^N(x;k)$$

$$I_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c} \mathrm{z}_1 b^{\sigma_1} & (\text{-}\lambda-\zeta_2 L-\psi(\mu+2g)-\upsilon\sum_{i=1}^{2}\eta_{G_i,g_i}-\sum_{i=1}^{s}K_i n_i;\sigma_1,\cdots,\sigma_r), A, \\ \cdot & \cdots \\ \cdot & \cdots \\ \cdot & \\ \mathrm{z}_r b^{\sigma_r} & (\text{-}\lambda+h-\zeta_2 L-\psi(\mu+2g)-\upsilon\sum_{i=1}^{2}\eta_{G_i,g_i}-\sum_{i=1}^{s}K_i n_i;\sigma_1,\cdots,\sigma_r), B: \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-}\rho-N-\zeta_1 L-\omega-\zeta(\mu+2g)+f-\omega\sum_{i=1}^{2}\eta_{G_i,g_i}+\sum_{i=1}^{s}K_i u_i-v;\rho_1,\cdots,\rho_r):C \\ \cdots \\ \cdots \\ \mathrm{D} \end{array}\right) \quad (4.4)$$

where A' is defined by (2.4), the equation (3.8) is valid under the same conditions mentioned in (2.8).

5. Multivariable H-function

If$\tau_i=\tau_{i^{(1)}}=\cdots=\tau_{i^{(s)}}=1$ and $r=r^{(1)}=\cdots=r^{(s)}=1$, then the multivariable Aleph-function degenere to the multivariable H-function defined by Srivastava et al [11]. And we have the following result.

a) First Fourier-Legendre series

$$x^{\rho-1}\big(g(x)\big)^{\lambda}e^{-cx^{\eta}}S_n^{\alpha,\beta,0}[y'x^{\zeta_1}\big(g(x)\big)^{\zeta_2}:r,q,A,B,m,k,l]J_{\mu}\big[zx^{\zeta}\big(g(x)\big)^{\psi}\big]$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1x^{u_1}\big(g(x)\big)^{n_1},\cdots,y_sx^{u_s}\big(g(x)\big)^{n_s}]H\left(\begin{array}{c} \mathrm{z}_1 x^{\rho_1}\big(g(x)\big)^{\sigma_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r x^{\rho_r}\big(g(x)\big)^{\sigma_r} \end{array}\right)$$

$$=\sum_{u,g,h=0}^{\infty}\sum_{i,\delta,t,p}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\phi(i,\delta,t,p)A(-)^g\frac{(z/2)^{\mu-2g}c^d y'^t}{g!h!\Gamma(\mu+g+1)\Gamma(d)}y_1^{K_1}\cdots y_s^{K_s}b^{\lambda+\psi\mu+2g\eta-\sum_{i=1}^{s}n_iK_i}$$

$$(a/b)^h(1/c)^{(\rho+d-1+\zeta_1L+\zeta(\mu+2g)+\sum_{i=1}^{s}K_iu_i+\omega\sum_{i=1}^{2}\eta_{G_i,g_i}+vh)}/\big({}_3F_1(-u,u+1,d;1;x^{\eta}]\big)$$

$$H_{p+2,q+1:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c} \mathrm{z}_1 b^{\sigma_1}c^{-\rho_1} & (\text{-}\lambda-\zeta_2 L-\psi(\mu+2g)+\zeta f-\sum_{i=1}^{s}K_i n_i;\sigma_1,\cdots,\sigma_r), \\ \cdot & \cdots \\ \cdot & \cdots \\ \cdot & \\ \mathrm{z}_r b^{\sigma_r}c^{-\rho_r} & (\text{-}\lambda+h-\zeta_2 L-\psi(\mu+2g)+\zeta f-\sum_{i=1}^{s}K_i n_i;\sigma_1,\cdots,\sigma_r), \end{array}\right.$$

$$\left.\begin{array}{c} (2\text{-}\rho-d-\zeta_1 L-\zeta(\mu+2g)-vh-\sum_{i=1}^{s}K_i u_i;\rho_1,\cdots,\rho_r), A:C \\ \cdots \\ \cdots \\ \mathrm{B}:\mathrm{D} \end{array}\right)P_u(1-2xT) \quad (5.1)$$

The equation (3.5) is valid under the same conditions mentioned in (2.5) and $Re(\rho)>0$

b) Second Fourier-Legendre series

$$x^{\rho-1}\big(g(x)\big)^{\lambda}e^{-cx^{\eta}}S_n^{\alpha,\beta,0}[y'x^{\zeta_1}\big(g(x)\big)^{\zeta_2}:r,q,A,B,m,k,l]J_{\mu}\big[zx^{\zeta}\big(g(x)\big)^{\psi}\big]\aleph\begin{pmatrix}\mathrm{Dx}^{\omega}\big(g(x)\big)^{\upsilon}\\ \mathrm{Ex}^{\omega}\big(g(x)\big)^{\upsilon}\end{pmatrix}$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1x^{u_1}\big(g(x)\big)^{n_1},\cdots,y_sx^{u_s}\big(g(x)\big)^{n_s}]H\begin{pmatrix}\mathrm{z}_1x^{\rho_1}\big(g(x)\big)^{\sigma_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}\big(g(x)\big)^{\sigma_r}\end{pmatrix}$$

$$=(1/\eta)(y')^Lb^{\lambda+\zeta_2L}(1/c)^{\rho+\zeta_1L)/\eta}\sum_{u,g,h=0}^{\infty}\sum_{i,\delta,t,p}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{G_1,G_2=0}^{\infty}\sum_{g_1=0}^{M_2}\sum_{g_2=0}^{M_3}D^{\eta_{G_1,g_1}}E^{\eta_{G_2,g_2}}$$

$$\phi(i,\delta,t,p)A'\frac{(-1)^d(z/2)^{\mu-2g}c^dy'^t\quad(-)^{G_1+G_2}}{g!h!\Gamma(\mu+g+1)\Gamma(d)\beta_{g_1}G_1!\delta_{\mathfrak{g}_2}G_2!}y_1^{K_1}\cdots y_s^{K_s}b^{\psi(\mu+2g)-\sum_{i=1}^{s}n_iK_i+\upsilon\sum_{i=1}^{2}\eta_{Gi,gi}}$$

$$y^L(a/b)^h(1/c)^{(\rho+d-1+\zeta_1L+\zeta(\mu+2g)+\sum_{i=1}^{s}K_iu_i+\omega\sum_{i=1}^{2}\eta_{G_i,g_i}+vh)}/\big(\Gamma(d)_3F_1(-u,u+1,d;1;x^{\eta})\big)$$

$$H_{p+2,q+1:W}^{0,\mathfrak{n}+2:V}\left(\begin{matrix}\mathrm{z}_1b^{\sigma_1}c^{-\rho_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rb^{\sigma_r}c^{-\rho_r}\end{matrix}\middle|\begin{matrix}(2\text{-}\rho-d-\zeta_1L+\alpha+\zeta(\mu+2g)-\omega\sum_{i=1}^{2}\eta_{G_i,g_i}+\sum_{i=1}^{s}K_iu_i-vh;\rho_1,\cdots,\rho_r),\\ \cdots\\ \cdots\\ \cdots\cdots\end{matrix}\right.$$

$$\left.\begin{matrix}(\text{-}\lambda-\zeta_2L-\psi(\mu+2g)-\upsilon\sum_{i=1}^{2}\eta_{G_i,g_i}-\sum_{i=1}^{s}K_in_i;\sigma_1,\cdots,\sigma_r),A:C\\ \cdots\\ \cdots\\ (\text{-}\lambda+h-\zeta_2L-\psi(\mu+2g)-\upsilon\sum_{i=1}^{2}\eta_{G_i,g_i}-\sum_{i=1}^{s}K_in_i;\sigma_1,\cdots,\sigma_r),B:D\end{matrix}\right)P_u(1-2xT)\quad(5.2)$$

where A' is defined by (2.4), the equation (3.6) is valid under the same conditions mentioned in (2.6) and $Re(\rho)>0$

c)Fourier-Bessel series

$$x^{\rho-1}\big(g(x)\big)^{\lambda}e^{-cx^{\eta}}S_n^{\alpha,\beta,0}[y'x^{\zeta_1}\big(g(x)\big)^{\zeta_2}:r,q,A,B,m,k,l]J_{\mu}\big[zx^{\zeta}\big(g(x)\big)^{\psi}\big]\aleph\begin{pmatrix}\mathrm{Dx}^{\omega}\big(g(x)\big)^{\upsilon}\\ \mathrm{Ex}^{\omega}\big(g(x)\big)^{\upsilon}\end{pmatrix}$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1x^{u_1}\big(g(x)\big)^{n_1},\cdots,y_sx^{u_s}\big(g(x)\big)^{n_s}]H\begin{pmatrix}\mathrm{z}_1x^{\rho_1}\big(g(x)\big)^{\sigma_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}\big(g(x)\big)^{\sigma_r}\end{pmatrix}$$

$$=(1/\eta)(y')^Lb^{\lambda+\zeta_2L}\sum_{u,g,h=0}^{\infty}\sum_{f=0}^{u}\sum_{i,\delta,t,p}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{G_1,G_2=0}^{\infty}\sum_{g_1=0}^{M_2}\sum_{g_2=0}^{M_3}D^{\eta_{G_1,g_1}}E^{\eta_{G_2,g_2}}(a/b)^h$$

$$\phi(i,\delta,t,p)A'\frac{(z/2)^{\mu+2g}(-u)_f(k+u-1)_f}{f!g!h!\Gamma(\mu+g+1)}\frac{(-)^{G_1+G_2}}{\beta_{g_1}G_1!\delta_{\mathfrak{g}_2}G_2!}y_1^{K_1}\cdots y_s^{K_s}b^{\psi(\mu+2g)-\sum_{i=1}^{s}n_iK_i+\upsilon\sum_{i=1}^{2}\eta_{Gi,gi}}$$

$$H^{0,\mathfrak{n}+2:V}_{p+2,q+1:W}\left(\begin{array}{c|c} \mathrm{z}_1b^{\sigma_1} & (\omega'-\rho-\zeta_1L+f-\zeta(\mu+2g)-\omega\sum_{i=1}^{2}\eta_{G_i,g_i}+\sum_{i=1}^{s}K_iu_i-vh;\rho_1,\cdots,\rho_r), \\ . & \cdots \\ . & \cdots \\ . & \cdots\cdots \\ \mathrm{z}_rb^{\sigma_r} & \end{array}\right.$$

$$\left.\begin{array}{c} (\text{-}\lambda-\zeta_2L-\psi(\mu+2g)-\upsilon\sum_{i=1}^{2}\eta_{G_i,g_i}-\sum_{i=1}^{s}K_in_i;\sigma_1,\cdots,\sigma_r),A:C \\ \cdots \\ \cdots \\ (\text{-}\lambda+h-\zeta_2L-\psi(\mu+2g)-\upsilon\sum_{i=1}^{2}\eta_{G_i,g_i}-\sum_{i=1}^{s}K_in_i;\sigma_1,\cdots,\sigma_r),B:D \end{array}\right)Y_u(1,\omega,x) \quad (5.3)$$

where A' is defined by (2.4), the equation (3.7) is valid under the same conditions mentioned in (2.7).

c)Fourier-Konhauser series

$$x^{\rho-1}\big(g(x)\big)^{\lambda}e^{-cx^{\eta}}S_n^{\alpha,\beta,0}[y'x^{\zeta_1}\big(g(x)\big)^{\zeta_2}:r,q,A,B,m,k,l]J_\mu\big[zx^{\zeta}\big(g(x)\big)^{\psi}\big]\aleph\begin{pmatrix}\mathrm{D}x^{\omega}\big(g(x)\big)^{\upsilon}\\ \mathrm{E}x^{\omega}\big(g(x)\big)^{\upsilon}\end{pmatrix}$$

$$S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}[y_1x^{u_1}\big(g(x)\big)^{n_1},\cdots,y_sx^{u_s}\big(g(x)\big)^{n_s}]H\begin{pmatrix}\mathrm{z}_1x^{\rho_1}\big(g(x)\big)^{\sigma_1}\\ .\\ .\\ .\\ \mathrm{z}_rx^{\rho_r}\big(g(x)\big)^{\sigma_r}\end{pmatrix}=(1/\eta)(y')^Lb^{\lambda+\zeta_2L}$$

$$\sum_{F,g,h=0}^{\infty}\sum_{f=0}^{u}\sum_{i,\delta,t,p}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{G_1,G_2=0}^{\infty}\sum_{g_1=0}^{M_2}\sum_{g_2=0}^{M_3}\sum_{j=0}^{\omega}D^{\eta_{G_1,g_1}}E^{\eta_{G_2,g_2}}(a/b)^h\binom{\omega}{j}\frac{\big((N+j+1)/k'\big)_u}{\Gamma(N+kF+1)}$$

$$\phi(i,\delta,t,p)A'\frac{(z/2)^{\mu+2g}(a/b)^h}{\omega!f!g!h!\Gamma(\mu+g+1)}\frac{(-)^{G_1+G_2}}{\beta_{g_1}G_1!\delta_{\mathfrak{g}_2}G_2!}y_1^{K_1}\cdots y_s^{K_s}b^{\psi(\mu+2g)+\sum_{i=1}^{s}n_iK_i+\upsilon\sum_{i=1}^{2}\eta_{Gi,gi}}Z_F^N(x;k)$$

$$H^{0,\mathfrak{n}+2:V}_{p+2,q+1:W}\left(\begin{array}{c|c} \mathrm{z}_1b^{\sigma_1} & (\text{-}\lambda-\zeta_2L-\psi(\mu+2g)-\upsilon\sum_{i=1}^{2}\eta_{G_i,g_i}-\sum_{i=1}^{s}K_in_i;\sigma_1,\cdots,\sigma_r),A, \\ . & \cdots \\ . & \cdots \\ . & \\ \mathrm{z}_rb^{\sigma_r} & (\text{-}\lambda+h-\zeta_2L-\psi(\mu+2g)-\upsilon\sum_{i=1}^{2}\eta_{G_i,g_i}-\sum_{i=1}^{s}K_in_i;\sigma_1,\cdots,\sigma_r),B: \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-}\rho-N-\zeta_1L-\omega-\zeta(\mu+2g)+f-\omega\sum_{i=1}^{2}\eta_{G_i,g_i}+\sum_{i=1}^{s}K_iu_i-v;\rho_1,\cdots,\rho_r):C \\ \cdots \\ \cdots \\ \mathrm{D} \end{array}\right) \quad (5.4)$$

where A' is defined by (2.4), the equation (3.8) is valid under the same conditions mentioned in (2.8).

6. Conclusion

Due to the nature of the multivariable Aleph-function and the general class of polynomials $S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}$, we can get general product of Laguerre, Legendre, Jacobi and other polynomials, the special functions of one and several variables.

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