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Cone Metric Spaces and Common Fixed Point Results of Generalized Contractive Mappings

S. K. Tiwari[1], Dr. R. P. Dubey[2]

[1]Department of Mathematics Dr. C. V. Raman University Bilaspur, Chhattisgarh, India

E-mail: sk10tiwari@gmail.com

[2]Department of Mathematics, Dr. C. V. Raman University Bilaspur, Chhattisgarh, India

E- mail: **ravidubey_1963@yahoo.co.in**

Abstract: The purpose of this paper is to obtain sufficient condition for the existence of a unique common fixed point of generalized contractive mappings in the setting of complete cone metric space depending on another function.

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***Keywords*:** Fixed Point, common fixed point, contractive mapping, Complete cone metric Space, Sequentially convergent,Normal cone.

I INTRODUCTION

The well known Banach contraction principle and its several generalizations in the setting of metric spaces have appeared. The classical contraction mapping principle in 1922, Banach proved the following famous fixed point theorem [1].

Let (X, d) is a complete metric space and $T: X \to X$ be a self map, satisfies

$$d(Tx, Ty) \leq \alpha d(x, y), \text{ for all} x, y \in X, i.e\ T \text{is a contractive maspping.}$$

Then T has a unique fixed point. This principle has been generalized in different directions in different spaces by mathematicians over the year. Also in contemporary research, it is still seriously investigated.

Recently, the real number's as the co-domain of a metric, by an ordered Banach space obtain a generalized metric space, called a cone metric space, was introduced by Huang and Zhang [4]. The described the convergence in cone metric space, introduced their completeness, and proved some fixed point theorem for contractive mapping on cone metric space. The initial work of Huang and Zhang[4] inspired many authors to prove fixed point theorems as well as common fixed point theorems for two or more mappings on cone metric space see for instance[13,14]. After wards, many authors have generalized the results of [4] and studied the existence of common fixed point of a pair of self mapping in the frame work of normal cone metric space, see for instance[2], [3], [5], [6] [7] [8], [9] to [16].

Recently, Morals and Rojas [17], [18] have extended the concept T-contraction mappings to cone metric space by proving fixed point theorems for T- Kannan – Zamfirescu. T-weak contraction mappings.
S. Moradi in [19] introduced the T- Kannan contractive mapping which extends the well known Kannan fixed point theorem given in [20, 21, and 22]. The results [3] and [19] very recently, generalized by [23], [24] and [26].In view of these facts, the purpose of this paper is to study sufficient condition for the existence of common fixed point of generalized contractive type mappings on complete cone metric space(X, d).Our results generalize and extend the respective theorem 2.1 of [25].

2. PRELIMINARY NOTES:

First, we recall some standard notations and definitions in cone metric spaces with some of their properties [4]. Let E be a real Banach space and P be a subset of E. P is called a cone if and only if:

(i) P is closed, non – empty and P $\neq \{0\}$,

(ii) ax + by $\in$ P for all x, $y \in$ P and non – negative real number a, b,

(iii) $x \in$ P and – $x \in$ P => x = 0 <=> P ∩ (-P) = {0}.

Given a cone $P \subset E$, we define a partial ordering $\leq$ on E with respect to P by $x \leq y$ if and only if $y - x \in P$. We shall write $x \ll$ y if $y - x \in$ int P, int P denotes the interior of P. The cone P is called normal if there is a number $K > 0$ such that for all $x, y \in E$,

$$0 \leq x \leq y \text{ implies } \| x \| \leq K \| y \|.$$

The least positive number K satisfying the above is called the normal constant of P.

Definition 2.1 [4] Let X be a non – empty set. Suppose the mapping $d: X\, x\, X \rightarrow E$ satisfies

(d_1) $0 \leq d\,(x, y)$ for all x, $y \in X$ and $d\,(x, \mathrm{y}) = 0$ iff $x = y$;

(d_2) $d\,(x, y) = d\,(y, x)$ for all $x, y \in X$;

(d_3) $d\,(x,y) \leq d\,(x,z) + d\,(z,y)$ $for\ all\ x,y,z \in X$.

Then d is called a cone metric on X, and (X, d) is called a cone metric space [1]. It is obvious that cone metric spaces generalize metric space.

Example 2.2 [4] Let $E = R^2$, $P = \{(x, y) \in E : x, y \geq 0\}$, $X = R$ and $d : X \times X \rightarrow E$ defined by

$d\,(x, y) = (\,|\,x - y\,|, \alpha\,|\,x- y\,|\,)$, where $\alpha \geq 0$ is a constant. Then (X, d) is a cone metric space.

Definition 2.3 [4] Let (X, d) be a cone metric space, $x \in X$ and $\{x_n\}_{n \geq 1}$ a sequence in X.

Then,

(i) $\{x_n\}_{n \geq 1}$ converges to x whenever for every $c \in E$ with o $\ll$ c , there is a natural Number

(ii) N such that d $(x_n, x) \ll$c for all $n \geq N$. We denote this by $\lim_{n\rightarrow\infty} x_n = x$ or $x_n \rightarrow x$, $(n\rightarrow \infty)$.

(iii) $\{x_n\}_{n \geq 1}$ is said to be a Cauchy sequence if for every $c \in E$ with $o \ll c$, there is a natural number N such that d $(x_n, x_m) \ll$ c for all $n, m \geq$ N.

(iv) (X, d) is called a complete cone metric space if every Cauchy sequence in X is converge.

Lemma 2.4 [4] Let (X, d) be a cone metric space, $P \subset E$ a normal cone with normal constant K. Let $\{x_n\}$, $\{y_n\}$ be a sequences in X and $x, y \in X$. Then,

(i) $\{x_n\}$ converges to x if and only if $\lim_{n\to\infty} d(x_n, x) = 0$;

(ii) If $\{x_n\}$ converges to x and $\{x_n\}$ converges to y then $x = y$. That is the limit of $\{x_n\}$

(iii) If $\{x_n\}$ converges to x, then $\{x_n\}$ is Cauchy sequence.

(iv) $\{x_n\}$ is a Cauchy sequence if and only if $\lim_{n,m\to\infty} d(x_n, x_m) = 0$;

(iv) If $x_n \to x$ and $\{y_n\}$ is another sequence in X such that $(n \to \infty)$.

then $d(x_n, y_n) \to d(x, y)$.

Definition 2.5 Let (X, d) be a cone metric space. If for any sequence $\{x_n\}$ in X, there is a Subsequence $\{x_{ni}\}$ of $\{x_n\}$ such that $\{x_n\}$ is convergent in X then X is called a Sequentially compact cone metric space.

Definition 2.6[8] Let (X, d) be a cone metric space, P be a normal cone with normal constant K and $T : X \to X$ then

(i) T is said to be continuous if $\lim_{n\to\infty} x_n = x$ implies that $\lim_{n\to\infty} Tx_n = Tx$ for all $\{x_n\}$ in X;

(ii) T is said to be sub sequentially convergent, if for every sequence $\{y_n\}$ that $\{Ty_n\}$ is Convergent, implies $\{y_n\}$ has a convergent subsequence.

(iii) T is said to be sequentially convergent if for every sequence $\{y_n\}$, if $\{Ty_n\}$ is Convergent, then $\{y_n\}$ is also convergent

K. P. R. Sastry, et al [25] proved the following theorem.

Theorem 2.7(K.P.R. Sastry, at al [25], Theorem 2.1): Let (X, d) be a complete cone metric space, P be a normal cone with normal constant K $(K \geq 1)$. Suppose the mapping $T: X \to X$ satisfies the following contractive condition

$d(Tx, Ty) \leq A_1(x, y)d(x, y) + A_2(x, y)d(x, Tx) + A_3(x, y)d(y, Ty) + A_4(x, y)d(x, Ty) + A_5(x, y)d(y, Tx)$ for all $x, y \in X$ where a_i = 1, 2, 3, 4, 5 are all nonnegative constant such that $a_1 + a_2 + a_3 + a_4 + a_5 < 1$. Then T has a unique fixed point.

Main Results

In this section, we improved and generalize our main result of above theorem, following as :

Theorem 3.1: Let (X,d) is a complete cone metric space, P be a normal cone with normal constant K, and let $T: X \to X$ be an one to one and continuous function. Moreover, let $A, B: X \to X$ be any two mappings of satisfying X such that

$d(TAx, TBy) \leq a_1(x,y)d(Tx,Ty)+a_2(x,y)d(Tx,TAx)+ a_3(x,y)d(Ty,TBy)$

$+ \ (x,y)d(Tx,T\text{By}) + a_5(Ty,TAx)$ For all $x, y \in X$, where a_i = 1, 2, 3, 4, 5 are all nonnegative constant such that $a_1 + a_2 + a_3 + a_4 + a_5 < 1$. Then

(1) For every $x_0 \in X$,

$$\lim_{n\to\infty} d(TA^{2n+1}x_0, TA^{2n+2}x_0) = 0 \text{ and } \lim_{n\to\infty} d(TB^{2n+1}x_0, TB^{2n+2}x_0) = 0.$$

(2) There is $v \in X$ such that

$$\lim_{n\to\infty} TA^{2n+1}x_0, \ = v = \lim_{n\to\infty} TB^{2n+2}x_0,$$

(3) If T is sub sequentially convergent, then $(A^{2n+1}x_0)$ and $(B^{2n+2}x_0)$ have a convergent subsequences.

(4) There is common fixed point $u \in X$ such that $Au = u = Bu$.

(5) If T is sequentially convergent, then for each $x_0 \in X$ the iterate sequences$(A^{2n+1}x_0)$ and $(B^{2n+2}x_0)$ converge to u

Proof: Let $x_0 \in X$ is an arbitrary point and the Picard iteration associated to A (x_{2n+1})given by

$$x_{2n+2} = Ax_{2n+1} = A^{2n+1}x_0, n = 0, 1, 2. 3\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Similarly, associated to $B(x_{2n+2})$ given by

$$x_{2n+3} = Bx_{2n+2} = B^{2n+2}x_0, n = 0, 1, 2. 3\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Now consider

$$d(Tx_{2n+1}, Tx_{2n+2}) = d(TAx_{2n}, TAx_{2n+1})$$

$$\leq a_1(x_{2n}, x_{2n+1})d(Tx_{2n}, Tx_{2n+1})$$

$$+ a_2(x_{2n}, x_{2n+1})d(Tx_{2n}, TAx_{2n})$$

$$+ a_3(x_{2n}, x_{2n+1})d(Tx_{2n+1}, TAx_{2n+1})$$

$$+ a_4(x_{2n}, x_{2n+1})d(Tx_{2n}, TAx_{2n+1})$$

$$+ a_5(x_{2n}, x_{2n+1})d(Tx_{2n+1}, TAx_{2n})$$

$$= a_1(x_{2n}, x_{2n+1})d(Tx_{2n}, Tx_{2n+1})$$

$$+ a_2(x_{2n}, x_{2n+1})d(Tx_{2n}, Tx_{2n+1})$$

$$+ a_3(x_{2n}, x_{2n+1}) d(Tx_{2n+1}, Tx_{2n+2})$$

$$+ a_4(x_{2n}, x_{2n+1}) d(Tx_{2n}, Tx_{2n+2})$$

$$+ a_5(x_{2n}, x_{2n+1}) d(Tx_{2n+1}, Tx_{2n+1})$$

$$\leq a_1(x_{2n}, x_{2n+1}) d(Tx_{2n}, Tx_{2n+1})$$

$$+ a_2(x_{2n}, x_{2n+1}) d(Tx_{2n}, Tx_{2n+1})$$

$$+ a_3(x_{2n}, x_{2n+1}) d(Tx_{2n+1}, Tx_{2n+2})$$

$$+ a_4(x_{2n}, x_{2n+1}) [d(Tx_{2n}, Tx_{2n+2}) + d(Tx_{2n+1}, Tx_{2n+2})\ ]$$

$$d(Tx_{2n+1}, Tx_{2n+2}) \ \leq \ (a_1 + a_2 + a_4)\,(x_{2n}, x_{2n+1}) d(Tx_{2n}, Tx_{2n+1})$$

$$+ (a_3 + a_4)(x_{2n}, x_{2n+1}) d(Tx_{2n+1}, Tx_{2n+2})$$

Consequently

$$d(Tx_{2n+1}, Tx_{2n+2}) \leq \frac{(a_1+a_2+a_4)(x_{2n}, x_{2n+1})}{1-(a_3+a_4)(x_{2n}, x_{2n+1})}\ d(Tx_{2n}, Tx_{2n+1})$$

$$\leq \ \frac{(a_1+a_2+a_4)}{1-(a_3+a_4)}\ d(Tx_{2n}, Tx_{2n+1})$$

$$\leq \ \ h\, d(Tx_{2n}, Tx_{2n+1})$$

Where $\quad \frac{(a_1+a_2+a_4)}{1-(a_3+a_4)} = h \ < 1.$

Similarly, we can show

$$d(Tx_{2n+2}, Tx_{2n+3}) \leq \frac{(a_1+a_3+a_5)}{1-(a_2+a_5)}\ d(Tx_{2n+1}, Tx_{2n+2})$$

$$\leq h'\, d(Tx_{2n+1}, Tx_{2n+2})$$

Where $\frac{(a_1+a_3+a_5)}{1-(a_2+a_5)} = h' \ < 1.$

We conclude, by repeating the same argument, that

$$d\big(TA^{2n+1}x_{0,}\, TA^{2n+2}x_0\big) \leq \ h^{2n+1} d(Tx_0, TAx_0) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \text{(3.1)}$$

and

$$d\big(TB^{2n+2}x_{0,}\, TB^{2n+3}x_0\big) \leq \ h'^{2n+2} d(Tx_0, TBx_0) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ \text{(3.2)}$$

From (3.1) and the fact the cone P is a normal cone with normal constant, we get

$$\big\|d\big(TA^{2n+1}x_{0,}\, TA^{2n+2}x_0\big)\big\| \leq K\, h^{2n+1} \|d(Tx_0, TAx_0)\|$$

Taking limit in the above inequality, we get

$$\lim_{n\to\infty} \big\|d\big(TA^{2n+1}x_{0,}\, TA^{2n+2}x_0\big)\big\| = 0.$$

Hence $\lim_{n\to\infty} d\big(TA^{2n+1}x_{0,}\, TA^{2n+2}x_0\big) = 0$ ……………………………………………(3.3)

Similarly, from (3.2) we have

$$\lim_{n\to\infty} d\left(TB^{2n+2}x_{0,}\, TA^{2n+3}x_0\right) = 0 .$$

Now, for $m, n \in N$ with $m > n$, we get

$$d(TA^{2n+1}x_0, TA^{2m+1}x_0) \leq (\, h^{2n+1} + h^{2n+2} + \cdots \dots \dots \dots \dots \dots \dots \dots + h^{2n})\, d(Tx_0, TAx_0)$$

$$\leq \frac{h^{2n+1}}{1-h}\, d(Tx_0, TAx_0)$$

Taking norm to inequality above, we obtain that

$$\|\, d(TA^{2n+1}x_0, TA^{2m+1}x_0)\| \leq \frac{h^{2n+1}}{1-h}\, K\|d(Tx_0, TAx_0)\| \dots\dots\dots\dots\dots\dots\dots\dots(3.5)$$

Again taking limit, we obtain

$\lim_{n,m\to\infty} d\, \left\|d\left(TA^{2n+1}x_{0,}\, TA^{2m+1}x_0\right)\right\| = 0$. In this way, we have

$\lim_{n\to\infty} d\left(TA^{2n+1}x_{0,}\, TA^{2m+1}x_0\right) = 0$ which implies that $(A^{2n+1}x_0)$ is a Cauchy sequence in X. Since (X, d) is a complete cone metric space, then there is $v \in X$ such that

$$\lim_{n\to\infty} TA^{2n+1}x_{0,} = v \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (3.6)$$

Similarly, one can show easily that $\lim_{n\to\infty} TB^{2n+2}x_{0,} = v$………………………… (3.6)’

So, $\lim_{n\to\infty} TA^{2n+1}x_{0,} = v = \lim_{n\to\infty} TB^{2n+2}x_{0,}$.Proving in this way assertion (2).

Now, if T is sub sequentially convergent, then $(A^{2n+1}x_0)$ has a convergent subsequence. So, there exist $u \in X$ and $\{x_{(2n+1)i}\}$ such that

$$\lim_{i\to\infty} A^{(2n+1)i}x_{0,} = u \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (3.7)$$

Since T is continuous and by (3.7) we have

$$\lim_{i\to\infty} TA^{(2n+1)i}x_{0,} = Tu \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots .(3.8)$$

From inequality (3.6) we conclude that

$$Tu = v \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (3.9)$$

Since R is continuous (and also using (3.7)). Then

$$\lim_{i\to\infty} A^{(2n+1)i}x_{0,} = Au$$

As well as

$$\lim_{i\to\infty} TA^{(2n+1)i+1}x_{0,} = TAu \dots\dots\dots\dots\dots\dots\dots\dots\dots (3.10)$$

Therefore $Tu = v = TAu$ .Since T is one to one, then $Au = u$. Thus u is the fixed point of A.

Now, If v is another fixed point of A. Then,

$$d(TAu, TAv) = a_1(u, v)d(Tu, Tv) + a_2\, (u, v)d(Tu, TAu) + a_3 d(Tv, TAv)$$

$$+ a_4(u,v)d(Tu,TAv)+ a_5 d(Tv,TAu)$$

$$\leq (a_1 + a_4 + a_5)\ d(Tu,Tv)$$

$$\leq (a_1 + a_2 + a_3 + a_4 + a_5)\ d(Tu,Tv)$$

$$<\ d(Tu,Tv) \text{ a contradiction.}$$

Hence $\|d(TAu,TAv)\|$ = 0 which implies TAu = TAv.

Since T is one to one. So, $Au = Av \Rightarrow u = v$ is the unique fixed point of A.

It is clear that if T is sequentially convergent, then for each $x_0 \in X$, the iterate $(A^{2n+1}x_0)$ is convergent to u , $i.e. \lim_{n\to\infty} A^{(2n+1)}x_{0,}$=$u$.

Proving in this way conclusion (5).Similarly, we can prove that $(B^{2n+2}x_0)$ convergent to the fixed point of B. $i.e. \lim_{n\to\infty} A^{(2n+1)}x_{0,}= u\ = . \lim_{n\to\infty} B^{(2n+2)}x_{0,}$.

Thus u is common fixed point of A and B. This completes the prove the theorem.

The following results obtained from theorem3.1.

Corollary 3.2 Let (X,d) is a complete cone metric space, P be a normal cone with normal constant K, and let $T: X \to X$ be an one to one and continuous function. Moreover, let $A, B: X \to X$ be any two mappings of satisfying X such that

$d(TAx,TBy) \leq kd(Tx,Ty)$, for all $x,y \in X$, for some < 1 . Then

(1) For every $x_0 \in X$,

$$\lim_{n\to\infty} d(TA^{2n+1}x_0, TB^{2n+2}x_0) = 0 \text{ and } \lim_{n\to\infty} d(TA^{2n+1}x_0, TB^{2n+2}x_0) = 0.$$

(2) There is $v \in X$ such that

$$\lim_{n\to\infty} TA^{2n+1}x_0,\ = v = \lim_{n\to\infty} TB^{2n+2}x_0,$$

(3) If T is sub sequentially convergent, then $(A^{2n+1}x_0)$ and $(B^{2n+2}x_0)$ have a convergent subsequences.

(4) There is common fixed point $u \in X$ such that $Au = u = Bu$.

(5) If T is sequentially convergent, then for each $x_0 \in X$ the iterate sequences$(A^{2n+1}x_0)$ and $(B^{2n+2}x_0)$ converge to u

Proof: The proof of the corollary immediately follows by putting k = $a_1 \leq a_1 + a_2 + a_3 + a_4 + a_5 <$ 1 and k = $a_1, = a_2 = a_3 = a_4 = a_5 = 0$. In theorem 3.1.

Corollary 3.3 Let (X,d) is a complete cone metric space, P be a normal cone with normal constant K, and let $T: X \to X$ be an one to one and continuous function. Moreover, let $A, B: X \to X$ be any two mappings of satisfying X such that

$d(TAx, TBy) \leq a_1 d(Tx, Ty) + a_2 d(Tx, TAx) + a_3 d(Ty, TBy) + a_4 d(Tx, TBy) + a_5 (Ty, TAx)$ For all $x, y \in X$, where $a_i = 1$, 2, 3, 4, 5 are all nonnegative constant and $a_1, = a_2, a_3 = a_4 = a_5$. Then (1), (2), (3), (4), (5) of theorem 3.1 hold.

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