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ASSESSMENT OF INTERPOLATION METHODS FOR SOLVING THE REAL LIFE PROBLEM

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Abstract: In the present paper, four different interpolation methods, namely Newton-Gregory Forward, Newton-Gregory Backward, Lagrange and Newton divided difference, are used for solving the real life problem. These methods are used to solve the following problem:

The following table represents the number of students obtained marks in the specified ranges. From the table, our aim is to estimate the number of students who obtained marks between 40 and 45.

Marks	30-40	40-50	50-60	60-70	70-80
Number of students	31	42	51	35	31

The estimated result shows that first two interpolation methods (Newton-Gregory Forward and Newton-Gregory Backward) give the same values. Since the range (40-45) is near to the starting point therefore Newton-forward is suitable for this case however Newton backward method is found computationally expensive for this case. The last two methods (Lagrange and Newton divided difference) are generally used for unequal intervals. All four methods are gives same results but computational point of view, the last three are more expensive because of the point of interest and equation complexity.

Keywords: Interpolation methods, Real life problem, equal and unequal intervals.

1. INTRODUCTION

Interpolation techniques are used to find the value of function at any point using given data points. It is very useful for interpolating the experimental data because during the experimentation it is not always possible to take all the readings. There are several methods for interpolating the data but difficult to solves all the problems because of their limitations. In the present work only four methods are used to solve a particular problem therefore the following paragraphs discuss only these methods:

Newton: Gregory forward and backward methods are used to solve those problems where point of interest is near to starting and ending point respectively. Also, these two methods are applies for equal intervals. However, Lagrange and Newton's-divided interpolation techniques are used for both equal and unequal intervals. Lagrange and Newton's divided difference methods needs more calculations than other two methods [1, 2].

Generally, the real life problems have large data points at different locations. Therefore in those cases Newton-Gregory forward and backward fails to solve because of its limitation of equal intervals.

The mathematical formulations of all these methods are discussed in the following section:

2. PROBLEM DEFINITION

In this section, the specified problem is solved using the four methods.

2.1 Newton's Forward Difference Formula

Newton's forward difference formula is

$$f(a+hu) = f(a) + u\Delta f(a) + \frac{u(u-1)}{\lfloor 2}\Delta^2 f(a) + \cdots + \frac{u(u-1)(u-2)\cdots(u-n+1)}{\lfloor n}\Delta^n f(a) \qquad (1)$$

The following Table-1 is forward difference table and used for finding the value of $f(45)$:

Table 1: Forward Difference Table

x	$y = \mathrm{f}(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
40	**31**				
50	73	**42**			
60	124	51	**9**		
70	159	35	–16	**–25**	
80	190	31	**–4**	12	**37**

First Term, $a = 40$, interval $h = 10$ and $a + hu = 45$, therefore $u = 0.5$. Putting the values in the above mentioned formula (1):

$$\begin{aligned} f(45) &= f(40) + u\Delta f(40) + \frac{u(u-1)}{\lfloor 2}\Delta^2 f(40) \\ &\quad + \frac{u(u-1)(u-2)}{\lfloor 3}\Delta^3 f(40) + \frac{u(u-1)(u-2)(u-3)}{\lfloor 4}\Delta^4 f(40) \\ &= 31 + 0.5\times 42 + \frac{0.5\times(0.5-1)}{\lfloor 2}\times 9 \\ &\quad + \frac{0.5\times(0.5-1)(0.5-2)}{\lfloor 3}\times(-25) + \frac{0.5\times(0.5-1)(0.5-2)(0.5-3)}{\lfloor 4}\times 37 \\ &= 47.8672 \approx 48 \end{aligned}$$

Hence, the number of students getting marks less than 45 is 48, but the number of students getting marks less than 40 is 31. Therefore the number of students getting marks between 40 and 45 is 48 – 31 = 17.

2.2 Newton's Backward Difference Formula

Newton's backward difference formula is

$$\begin{aligned} f(a+nh+uh) &= f(a+nh) + u\nabla f(a+nh) \\ &\quad + \frac{u(u+1)}{\lfloor 2}\nabla^2 f(a+nh) + \cdots + \frac{u(u+1)\cdots(u+\overline{n-1})}{\lfloor n}\nabla^n f(a+nh) \qquad (2) \end{aligned}$$

The following Table-2 is backward difference table and used for finding the value of f (45):

Table 2: Backward Difference Table

x	$y = f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
40	31				
50	73	42			
60	124	51	9		
70	159	35	–16	–25	**<u>37</u>**
80	**190**	**<u>31</u>**	**<u>–4</u>**	**<u>12</u>**	

First Term, $a = 40$, interval $h = 10$ and $a + hu = 80$ and also $n = 4$, therefore $u = -3.5$. Putting the values in the above mentioned formula (2):

$$f(45) = y(80) + u\nabla y(80) + \frac{u(u+1)}{\underline{|2}}\nabla^2 y(80) + \frac{u(u+1)(u+2)}{\underline{|3}}\nabla^3 y(80) + \frac{u(u+1)(u+2)(u+3)}{\underline{|4}}\nabla^4 y(80)$$

$$= 190 + (-3.5)\times 31 + \frac{(-3.5)(-3.5+1)}{\underline{|2}}\times(-4) + \frac{(-3.5)(-3.5+1)(-3.5+2)}{\underline{|3}}\times 12 + \frac{(-3.5)(-3.5+1)(-3.5+2)(-3.5+3)}{\underline{|4}}\times 37$$

$$= 47.8672 \approx 48$$

Hence, number of students getting marks less than 45 is 48. Therefore as mentioned in above method, the number of students getting marks between 40 and 45 is 17.

2.3 Lagrange Interpolation Formula

The Lagrange interpolation formula for this problem is

$$\begin{aligned} f(x) = {} & \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} f(x_1) \\ & + \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} f(x_3) \\ & + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} f(x_4) \end{aligned} \tag{3}$$

Putting all these values of $x_0 = 40$, $x_1 = 50$, $x_2 = 60$, $x_3 = 70$ and $x_4 = 80$ and corresponding functions are $f(x_0) = 31$, $f(x_1) = 42$, $f(x_2) = 51$, $f(x_3) = 35$ and $f(x_4) = 31$ in formula (3). We have to find the value of $f(x)$ at $x = 45$. Now

$$
\begin{aligned}
f(45) &= \frac{(45-50)(45-60)(45-70)(45-80)}{(40-50)(40-60)(40-70)(40-80)}\times 31+\frac{(45-40)(45-60)(45-70)(45-80)}{(50-40)(50-60)(50-70)(50-80)}\times 73 \\
&+\frac{(45-40)(45-50)(45-70)(45-80)}{(60-40)(60-50)(60-70)(60-80)}\times 124+\frac{(45-40)(45-50)(45-60)(45-80)}{(70-40)(70-50)(70-60)(70-80)}\times 159 \\
&+\frac{(45-40)(45-50)(45-60)(45-70)}{(80-40)(80-50)(80-60)(80-70)}\times 190 \\
&= 8.4765625+79.84375+(-67.8125)+34.78125+(-7.421875) \\
&= 47.8671875 \approx 48
\end{aligned}
$$

Therefore, Number of students getting marks between 40 and 45 = 48 – 31 = 17

2.4 Newton-Divided Interpolation Formula

The Newton-Divided interpolation formula for this problem is

$$
\begin{aligned}
f(x) = f(0)+(x-x_0)\blacktriangle f(0)+(x-x_0)(x-x_1)\blacktriangle^2 f(0)+(x-x_0)(x-x_1)(x-x_2)\blacktriangle^3 f(0) \\
+(x-x_0)(x-x_1)(x-x_2)(x-x_3)\blacktriangle^4 f(0)
\end{aligned} \tag{4}
$$

Below in the mentioned table, we have given the calculated values which will be used for calculating $f(45)$:

x	*y=f(x)*	*?f*	*?2 f*	*?3f*	*?4f*
40	**31**				
50	73	**4.2**			
60	124	5.1	**0.045**		
70	159	3.5	–0.08	**4.17*10^{-3}**	**1.54*10–4**
80	190	3.1	–0.02	– 2*10^{-3}	

Putting all these values in the above mentioned formula (4):

$$
\begin{aligned}
f(45) &= 31+(45-40)4.2+(45-40)(x-50)0.045 \\
&+(45-40)(45-50)(45-60)(-0.00417) \\
&+(45-40)(45-50)(45-60)(45-70)(0.0001542)
\end{aligned}
$$

$$= 47.86718790625 \approx 48$$

Therefore, Number of students getting marks between 40 and 45 is 17.

It is found from all the methods that the number of students getting marks between 40 and 45 are 17.

3. RESULTS AND DISCUSSION

The solutions are obtained by all the methods i.e., Newton-Gregory Forward and Backward formula, Lagrange's formula and Newton-Divided formula and then it is compared analytically. The result obtained by all the methods is found to be approximately equal. Hence this result gives us an expanding opportunity to find the solution by any of these methods. Since the range (40-45) is near the starting of the given data therefore Newton-forward is suitable for this case however Newton backward method is found computationally expensive for this case. Rests of the two methods (Lagrange and Newton-Divided difference method) are also useful but analytically more expensive from calculation point of view.

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