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Integrals associated with Gauss's hypergeometric series and Aleph-functions of several variables

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Abstract : In this document , we introduce the Aleph-function of several variables. The aim of this document is to evaluate four finite double integrals involving the hypergeometric functions and the multivariable Aleph-functions. At the end of this paper, we evaluate s everal particular cases and remarks are given.

Keywords : Gauss's hypergeometric function , integral , Aleph-function of several variables , Mellin-Barnes contour integral, multivariable I-function

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1. Introduction and preliminaries.

The object of this document is to evaluate several finite double integrals involving the hypergeometric function and the multivariable aleph-functions. These function generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [2] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $\aleph(z_1,\cdots,z_r) = \aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i(1)},q_{i(1)},\tau_{i(1)};R^{(1)};\cdots;p_{i(r)},q_{i(r)};\tau_{i(r)};R^{(r)}}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r \end{array}\right|$

$$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}] ,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}] :$$
$$\cdots\cdots\cdots\cdots ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}] :$$

$$\left.\begin{array}{l} [(\mathrm{c}_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;\ ;[(\mathrm{c}_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}] \\ [(\mathrm{d}_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;;[(\mathrm{d}_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}] \end{array}\right)$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r) = \frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{\sum_{i=1}^{R}[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k)]} \tag{1.2}$$

and $$\theta_k(s_k) = \frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

where $j=1$ to r and $k=1$ to r. Suppose , as usual , that the parameters

$a_j, j=1,\cdots,p; b_j, j=1,\cdots,q;$

$c_j^{(k)}, j=1,\cdots,n_k; c_{ji^{(k)}}^{(k)}, j=n_k+1,\cdots,p_{i^{(k)}};$

$d_j^{(k)}, j=1,\cdots,m_k; d_{ji^{(k)}}^{(k)}, j=m_k+1,\cdots,q_{i^{(k)}}$ with $k=1\cdots,r, i=1,\cdots,R\ , i^{(k)}=1,\cdots,R^{(k)}$

are complex numbers , and the $\alpha's, \beta's, \gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} + \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} - \sum_{j=1}^{m_k} \delta_j^{(k)}$$

$$-\tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} \leqslant 0 \tag{1.4}$$

The reals numbers τ_i are positives for $i=1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)}=1$ to $R^{(k)}$

The contour L_k is in the s_k-p lane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)} - \delta_j^{(k)} s_k)$ with $j=1$ to m_k are separated from those of $\Gamma(1-a_j+\sum_{i=1}^{r}\alpha_j^{(k)} s_k)$ with $j=1$ to n and $\Gamma(1-c_j^{(k)}+\gamma_j^{(k)} s_k)$ with $j=1$ to n_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|arg z_k| < \frac{1}{2} A_i^{(k)} \pi$, where

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} - \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} > 0, \text{ with } k=1\cdots,r, i=1,\cdots,R\ , i^{(k)}=1,\cdots,R^{(k)} \tag{1.5}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1,\cdots,z_r) = 0(\ |z_1|^{\alpha_1}\ldots|z_r|^{\alpha_r}\ ), max(\ |z_1|\ldots|z_r|\ ) \rightarrow 0$$

$$\aleph(z_1,\cdots,z_r) = 0(\ |z_1|^{\beta_1}\ldots|z_r|^{\beta_r}\ ), min(\ |z_1|\ldots|z_r|\ ) \rightarrow \infty$$

where, with $k=1,\cdots,r: \alpha_k = min[Re(d_j^{(k)}/\delta_j^{(k)})], j=1,\cdots,m_k$ and

$$\beta_k = max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})], j=1,\cdots,n_k$$

We will use these following notations in this paper

$$U = p_i, q_i, \tau_i; R\ ;\ V = m_1, n_1; \cdots; m_r, n_r \tag{1.6}$$

$$\mathrm{W} = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}, \cdots, p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)} \tag{1.7}$$

$$A = \{(a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)})_{1,n}\}, \{\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \cdots, \alpha_{ji}^{(r)})_{n+1,p_i}\} \tag{1.8}$$

$$B = \{\tau_i(b_{ji}; \beta_{ji}^{(1)}, \cdots, \beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.9}$$

$$C = \{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\}, \tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\}, \cdots ,\{(c_j^{(r)};\gamma_j^{(r)})_{1,n_r}\}, \tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \quad (1.10)$$

$$D = \{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\}, \tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\}, \cdots, \{(d_j^{(r)};\delta_j^{(r)})_{1,m_r}\}, \tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \quad (1.11)$$

The multivariable Aleph-function write :

$$\aleph(z_1,\cdots,z_r) = \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c|c} z_1 & \mathrm{A : C} \\ \cdot & \\ \cdot & \cdots \\ \cdot & \\ z_r & \mathrm{B : D} \end{array}\right) \quad (1.12)$$

2. Hypergeometric function

We have the following results , see Rathie et al [3]

$$\int_0^1 x^{\rho-1}(1-x)^{\rho}[1+ax+(1-b)]^{-2\rho-1}{}_2F_1\Big[\alpha,\beta;\frac{\alpha+\beta+2}{2};\frac{x(1+a)}{1+ax+b(1-x)}\Big]\mathrm{d}x$$

$$= 2^{\alpha+\beta-2\rho}\ \frac{\Gamma(\rho-\frac{\alpha}{2}-\frac{\beta}{2})\Gamma(\frac{\alpha+\beta+2}{2})\Gamma(\rho)}{(\alpha-\beta)(1+a)^{\rho}(1+b)^{\rho}\Gamma(\alpha)\Gamma(\beta)}$$

$$\times\left[\frac{2\rho-\alpha+\beta)\Gamma(\frac{\alpha}{2}+\frac{1}{2})\Gamma(\frac{\beta}{2})}{\Gamma(\rho-\frac{\alpha}{2}-1)\Gamma(\rho-\frac{\beta}{2}+\frac{1}{2})}-\frac{(2\rho-\alpha+\beta)\Gamma(\frac{\alpha}{2}+\frac{1}{2})\Gamma(\frac{\beta}{2})}{\Gamma(\rho-\frac{\alpha}{2}+1)\Gamma(\rho-\frac{\beta}{2}+\frac{1}{2})}\right] \quad (2.1)$$

Where $Re(\rho)>0, Re(2\rho-\alpha-\beta)>0$, a and b are constants , such the expression and $1+ax+b(1-x)$ is not zero.

$$\int_0^1 x^{\rho-1}(1-x)^{\rho}\ [1+ax+b(1-x)]^{-2\rho+1}\ {}_2F_1\left[\alpha,\beta;\frac{\alpha+\beta}{2};\frac{x(1+a)}{1+ax+b(1-x)}\right]\mathrm{d}x$$

$$= 2^{\alpha+\beta-2\rho-1}\frac{\Gamma(\rho-\frac{\alpha}{2}-\frac{\beta}{2}-1)\Gamma(\frac{\alpha+\beta}{2})\Gamma(\rho-1)}{(1+a)^{\rho}(1+b)^{\rho}\Gamma(\alpha)\Gamma(\beta)}$$

$$\times\left[\frac{(2\rho-\alpha+\beta-2)\Gamma(\frac{\alpha}{2}+\frac{1}{2})\Gamma(\frac{\beta}{2})}{\Gamma(\rho-\frac{\alpha}{2})\Gamma(\rho-\frac{\beta}{2}-\frac{1}{2})}+\frac{(2\rho+\alpha-\beta)\Gamma(\frac{\alpha}{2})\Gamma(\frac{\beta+1}{2})}{\Gamma(\rho-\frac{\beta}{2})\Gamma(\rho-\frac{\alpha}{2}-\frac{1}{2})}\right] \quad (2.2)$$

Where $Re(\rho)>0, Re(2\rho-\alpha-\beta)>0$, a and b are constants , such the expression

In this document the quantity $1+ax+b(1-x)$ is not zero.

$$\int_0^{\pi/2} e^{i(2w+1)\pi\theta}(sin\theta)^{w-1}(cos\theta)^{w-1}\ {}_2F_1\left[\alpha',\beta';\frac{\alpha'+\beta'+2}{2};e^{i\theta}cos\theta\right]\mathrm{d}\theta$$

$$= \frac{e^{i\pi(w+1)/2}\Gamma(w)\Gamma(w-\frac{\alpha'-\beta'}{2})\Gamma(\frac{\alpha'-\beta'}{2}+1)}{2^{2w-\alpha'-\beta'+2}\Gamma(\alpha'-\beta')\Gamma(\alpha')\Gamma(\beta')}$$

$$\times\left[\frac{(2w-\alpha'-\beta')\Gamma(\frac{\alpha'+1}{2})\Gamma(\frac{\beta'}{2})}{\Gamma(w-\frac{\alpha'}{2}+1)\Gamma(w-\frac{\beta'-1}{2})}-\frac{(2w+\alpha'-\beta')\Gamma(\frac{\alpha'}{2})\Gamma(\frac{\beta'+1}{2})}{\Gamma(w-\frac{\beta'}{2}+1)\Gamma(w-\frac{\alpha'-1}{2})}\right] \quad (2.3)$$

where $Re(w) > 0$ and $Re(2w - \alpha' - \beta') > 0$

$$\int_0^{\pi/2} e^{i(2w+1)\pi\theta}(sin\theta)^{w-1}(cos\theta)^{w-1}\ {}_2F_1\Big[\ \alpha', \beta'; \frac{\alpha'+\beta'}{2}; e^{i\theta}cos\theta\ \Big]\,\mathrm{d}\theta$$

$$= \frac{e^{i\pi(w+1)/2}\Gamma(w-1)\Gamma(w-\frac{\alpha'-\beta'}{2}-1)\Gamma(\frac{\alpha'+\beta'}{2})}{2^{2w-\alpha'-\beta'}\Gamma(\alpha')\Gamma(\beta')}$$

$$\times\Big[\frac{(2w-\alpha'-\beta'-2)\Gamma(\frac{\alpha'+1}{2})\Gamma(\frac{\beta'}{2})}{\Gamma(w-\frac{\alpha'}{2})\Gamma(w-\frac{\beta'+1}{2})} - \frac{(2w+\alpha'-\beta'-2)\Gamma(\frac{\alpha'}{2})\Gamma(\frac{\beta'+1}{2})}{\Gamma(w-\frac{\beta'}{2})\Gamma(w-\frac{\alpha'+1}{2})}\Big] \tag{2.4}$$

where $Re(w) > 0$ and $Re(2w - \alpha' - \beta') > 0$

3 Finite double integrals

We evaluate the following four finite double integrals involving hypergeometric functions and multivariable Aleph-functions.

a) $$\int_0^1\int_0^{\pi/2} x^{\rho-1}(1-x)^{\rho}[1+ax+(1-b)]^{-2\rho-1}{}_2F_1\Big[\ \alpha, \beta; \frac{\alpha+\beta+2}{2}; \frac{x(1+a)}{1+ax+b(1-x)}\Big]$$

$$e^{i(2w+1)\pi\theta}(sin\theta)^{w-1}(cos\theta)^{w-1}\ {}_2F_1\Big[\ \alpha', \beta'; \frac{\alpha'+\beta'+2}{2}; e^{i\theta}cos\theta\ \Big]$$

$$\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1 x^{\rho_1}(1-x)^{\rho_1}[1+ax+b(1-x)]^{-2\rho_1} \\ \cdot \\ \cdot \\ \mathrm{z}_r x^{\rho_r}(1-x)^{\rho_r}[1+ax+b(1-x)]^{-2\rho_r} \end{array}\right)\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1 e^{2i\theta w_1}(sin\theta)^{w_1}(cos\theta)^{w_1} \\ \cdot \\ \cdot \\ \mathrm{z}_r e^{2i\theta w_r}(sin\theta)^{w_r}(cos\theta)^{w_r} \end{array}\right)\mathrm{d}\theta\ \mathrm{d}x$$

$$= \frac{2^{\alpha+\beta-2\rho-2}\Gamma(\frac{\alpha+\beta+2}{2})}{\Gamma(\alpha)\Gamma(\beta)(\alpha-\beta)(1+a)^{\rho}(1+b)^{\rho}}\Big[\Gamma(\frac{\alpha+1}{2})\Gamma(\frac{\beta}{2})\aleph_1(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha}{2})\Gamma(\frac{\beta+1}{2})\aleph_2(z_1,\cdots,z_r)\Big]$$

$$\times\frac{e^{i\pi(w+1)/2}\Gamma(\frac{\alpha'+\beta'+2}{2})}{2^{2w-\alpha'-\beta'}\Gamma(\alpha'-\beta')\Gamma(\alpha')\Gamma(\beta')}\Big[\Gamma(\frac{\alpha'+1}{2})\Gamma(\frac{\beta'}{2})\aleph_3(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha'}{2})\Gamma(\frac{\beta'+1}{2})\aleph_4(z_1,\cdots,z_r)\Big]$$

with the validity conditions : $Re(\rho) > 0, Re(w) > 0, |argz_k| < \frac{1}{2}A_i^{(k)}\pi$; $A_i^{(k)}$is defined by (1.5) and

$$Re(2w-\alpha'-\beta'+2\sum_{i=1}^r w_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}) > 0\,;\, Re(2\rho-\alpha-\beta+2\sum_{i=1}^r \rho_i \min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}) > 0 \tag{3.1}$$

where

$$\aleph_1(z_1,\cdots,z_r) = \aleph_{p_i+3,q_i+3,\tau_i;R:W}^{m,\mathfrak{n}+3:V}\left(\begin{array}{c} \mathrm{z}_1 A_1 \\ \cdot \\ \cdot \\ \mathrm{z}_r A_r \end{array}\left|\begin{array}{c} (\alpha-\beta-2\rho;2\rho_1,\cdots,2\rho_r),(1-\rho;\rho_1,\cdots,\rho_r), \\ \cdots \\ (1\text{ - }2\rho+\alpha+\beta;2\rho_1,\cdots,2\rho_r),(\frac{\alpha}{2}-\rho;\rho_1,\cdots,\rho_r), \end{array}\right.\right.$$

$$\left.\begin{matrix} (1+\frac{\alpha+\beta}{2} - \rho;\rho_1,\cdots,\rho_r),A:C \\ \cdots \\ (\frac{\beta+1}{2} - \rho;\rho_1,\cdots,\rho_r),B:D \end{matrix}\right) \tag{3.2}$$

$$\aleph_2(z_1,\cdots,z_r)=\aleph^{m,\mathfrak{n}+3:V}_{p_i+3,q_i+3,\tau_i;R:W}\left(\begin{matrix} z_1A_1 \\ \cdot \\ \cdot \\ z_rA_r \end{matrix}\ \middle|\ \begin{matrix} (-\alpha+\beta-2\rho;2\rho_1,\cdots,2\rho_r),(1-\rho;\rho_1,\cdots,\rho_r), \\ \cdots \\ (1 - 2\rho-\alpha+\beta;2\rho_1,\cdots,2\rho_r),(\frac{\alpha+1}{2}-\rho;\rho_1,\cdots,\rho_r), \end{matrix}\right.$$

$$\left.\begin{matrix} (1+\frac{\alpha+\beta}{2} - \rho;\rho_1,\cdots,\rho_r),A:C \\ \cdots \\ (\frac{\beta}{2} - \rho;\rho_1,\cdots,\rho_r),B:D \end{matrix}\right) \tag{3.3}$$

where $A_j = \dfrac{2^{-2\rho_j-2}}{(1+a)^{\rho_j}(1+b)^{\rho_j}}$; $j=1,\cdots,r$ (3.4)

$$\aleph_3(z_1,\cdots,z_r)= \aleph^{m,\mathfrak{n}+3:V}_{p_i+3,q_i+3,\tau_i;R:W}\left(\begin{matrix} z_1B_1 \\ \cdot \\ \cdot \\ z_rB_r \end{matrix}\ \middle|\ \begin{matrix} (\alpha'-\beta'-2w;w_1,\cdots,w_r),(1-w;w_1,\cdots,w_r) \\ \cdots \\ (1\text{-}2\mathrm{w}+\alpha'+\beta';w_1,\cdots,w_r),(\frac{\alpha'}{2}-w;w_1,\cdots,w_r) \end{matrix}\right.$$

$$\left.\begin{matrix} ,(1+\frac{\alpha'+\beta'}{2} - \mathrm{w}\ ;\mathrm{w}_1,\cdots,w_r),A:C \\ \cdots \\ ,(\frac{\beta'+1}{2} - \rho;w_1,\cdots,w_r),B:D \end{matrix}\right) \tag{3.5}$$

$$\aleph_4(z_1,\cdots,z_r)=\aleph^{m,\mathfrak{n}+3:V}_{p_i+3,q_i+3,\tau_i;R:W}\left(\begin{matrix} z_1B_1 \\ \cdot \\ \cdot \\ z_rB_r \end{matrix}\ \middle|\ \begin{matrix} (- \alpha'+\beta'-2w;w_1,\cdots,w_r),(1-w;w_1,\cdots,w_r) \\ \cdots \\ (1\text{-}2\mathrm{w}\ \text{-}\alpha'+\beta';w_1,\cdots,w_r),(\frac{\alpha'+1}{2}-w;w_1,\cdots,w_r) \end{matrix}\right.$$

$$\left.\begin{matrix} ,(1+\frac{\alpha'+\beta'}{2} - \mathrm{w}\ ;\mathrm{w}_1,\cdots,w_r),A:C \\ \cdots \\ ,(\frac{\beta'}{2} - \mathrm{w}\ ;\mathrm{w}_1,\cdots,w_r),B:D \end{matrix}\right) \tag{3.6}$$

where $B_j = \dfrac{e^{i\pi w_j}}{4^{w_j}}$; $j=1,\cdots,r$ (3.7)

b) $\displaystyle\int_0^1\int_0^{\pi/2} x^{\rho-1}(1-x)^{\rho}[1+ax+(1-b)]^{-2\rho+1}\Big[\alpha,\beta;\frac{\alpha+\beta+2}{2};\frac{x(1+a)}{1+ax+b(1-x)}\Big]$

$e^{i(2w+1)\pi\theta}(sin\theta)^{w-1}(cos\theta)^{w-1}\ {}_2F_1\left[\ \alpha',\beta';\dfrac{\alpha'+\beta'}{2};e^{i\theta}cos\theta\right]$

$$\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1x^{\rho_1}(1-x)^{\rho_1}[1+ax+b(1-x)]^{-2\rho_1}\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}(1-x)^{\rho_r}[1+ax+b(1-x)]^{-2\rho_r}\end{array}\right)\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1e^{2i\theta w_1}(sin\theta)^{w_1}(cos\theta)^{w_1}\\ \cdot\\ \cdot\\ \mathrm{z}_re^{2i\theta w_r}(sin\theta)^{w_r}(cos\theta)^{w_r}\end{array}\right)\mathrm{d}\theta\mathrm{d}x$$

$$=\frac{2^{\alpha+\beta-2\rho-1}\Gamma(\frac{\alpha+\beta+2}{2})}{\Gamma(\alpha)\Gamma(\beta)(1+a)^{\rho}(1+b)^{\rho}}\left[\Gamma(\frac{\alpha+\beta}{2})\Gamma(\frac{\beta}{2})\aleph_5(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha+\beta}{2})\Gamma(\frac{\alpha}{2})\aleph_6(z_1,\cdots,z_r)\right]$$

$$\times\frac{e^{i\pi(w+1)/2}\Gamma(\frac{\alpha'+\beta'+2}{2})}{2^{2w-\alpha'+1}\Gamma(\alpha'-\beta')\Gamma(\alpha')\Gamma(\beta')}\left[\Gamma(\frac{\alpha'+1}{2})\Gamma(\frac{\beta'}{2})\aleph_7(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha'+1}{2})\Gamma(\frac{\beta'+2}{2})\,\aleph_8(z_1,\cdots,z_r)\right]$$

with the validity conditions : $Re(\rho)>0, Re(w)>0, |argz_k|<\frac{1}{2}A_i^{(k)}\pi$; $A_i^{(k)}$ is defined by (1.5) and

$$Re(2w-\alpha'-\beta'+2\sum_{i=1}^r w_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>2\,;\,Re(2\rho-\alpha-\beta+2\sum_{i=1}^r\rho_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>2 \quad (3.8)$$

where

$$\aleph_5(z_1,\cdots,z_r)=\aleph_{p_i+3,q_i+3,\tau_i;R:W}^{m,\mathfrak{n}+3:V}\left(\begin{array}{c}\mathrm{z}_1A_1\\ \cdot\\ \cdot\\ \mathrm{z}_rA_r\end{array}\middle|\begin{array}{c}(2+\alpha-\beta-2\rho;2\rho_1,\cdots,2\rho_r),(2-\rho;\rho_1,\cdots,\rho_r),\\ \cdots\\ (3-2\rho+\alpha+\beta;2\rho_1,\cdots,2\rho_r),(1+\frac{\alpha}{2}-\rho;\rho_1,\cdots,\rho_r),\end{array}\right.$$

$$\left.\begin{array}{c}(2+\frac{\alpha+\beta}{2}-\rho;\rho_1,\cdots,\rho_r),A:C\\ \cdots\\ (\frac{\beta+3}{2}-\rho;\rho_1,\cdots,\rho_r),B:D\end{array}\right) \quad (3.9)$$

$$\aleph_6(z_1,\cdots,z_r)=\aleph_{p_i+3,q_i+3,\tau_i;R:W}^{m,\mathfrak{n}+3:V}\left(\begin{array}{c}\mathrm{z}_1A_1\\ \cdot\\ \cdot\\ \mathrm{z}_rA_r\end{array}\middle|\begin{array}{c}(2-\alpha+\beta-2\rho;2\rho_1,\cdots,2\rho_r),(2-\rho;\rho_1,\cdots,\rho_r),\\ \cdots\\ (3-2\rho-\alpha+\beta;2\rho_1,\cdots,2\rho_r),(1+\frac{\beta}{2}-\rho;\rho_1,\cdots,\rho_r),\end{array}\right.$$

$$\left.\begin{array}{c}(2+\frac{\alpha+\beta}{2}-\rho;\rho_1,\cdots,\rho_r),A:C\\ \cdots\\ (\frac{\alpha+3}{2}-\rho;\rho_1,\cdots,\rho_r),B:D\end{array}\right) \quad (3.10)$$

where $A_j=\dfrac{2^{-2\rho_j}}{(1+a)^{\rho_j}(1+b)^{\rho_j}}\,;\,j=1,\cdots,r$ (3.11)

$$\aleph_7(z_1,\cdots,z_r)=\aleph_{p_i+3,q_i+3,\tau_i;R:W}^{m,\mathfrak{n}+3:V}\left(\begin{array}{c}\mathrm{z}_1A_1\\ \cdot\\ \cdot\\ \mathrm{z}_rA_r\end{array}\middle|\begin{array}{c}(2+\alpha'-\beta'-2w,w_1,\cdots,w_r),(2-w,w_1,\cdots,w_r)\\ \cdots\\ (3-2\mathrm{w}-\alpha'+\beta',2w_1,\cdots,2w_r),(1+\frac{\alpha}{2}-w,w_1,\cdots,w_r)\end{array}\right.$$

$$, \left(2+\frac{\alpha'+\beta'}{2} - \mathrm{w} , \mathrm{w}_1,\cdots,w_r), A:C \atop \cdots \atop , (\frac{\beta'+3}{2} - \mathrm{w} , \mathrm{w}_1,\cdots,w_r), B:D \right) \tag{3.12}$$

$$\aleph_8(z_1,\cdots,z_r)= \aleph^{m,\mathfrak{n}+3:V}_{p_i+3,q_i+3,\tau_i;R:W}\left(\begin{array}{c} \mathrm{z}_1B_1 \\ \cdot \\ \cdot \\ \mathrm{z}_rB_r \end{array} \middle| \begin{array}{c} (2\text{-}\ \alpha'+\beta'-2w;w_1,\cdots,w_r),(2-w;w_1,\cdots,w_r) \\ \cdots \\ (3\text{-}2\mathrm{w}\ \text{-}\alpha'+\beta';w_1,\cdots,w_r),(1+\frac{\beta'}{2}-w;w_1,\cdots,w_r) \end{array} \right.$$

$$, \left(2+\frac{\alpha'+\beta'}{2} - \mathrm{w}\ ;\mathrm{w}_1,\cdots,w_r), A:C \atop \cdots \atop , (\frac{3+\alpha'}{2} - \mathrm{w}\ ;\mathrm{w}_1,\cdots,w_r), B:D \right) \tag{3.13}$$

where $B_j=\dfrac{e^{i\pi w_j/2}}{4^{w_j}}\ ;j=1,\cdots,r$ (3.14)

c) $$\int_0^1\int_0^{\pi/2} x^{\rho-1}(1-x)^{\rho}[1+ax+(1-b)]^{-2\rho+1}\left[\alpha,\beta;\frac{\alpha+\beta+2}{2};\frac{x(1+a)}{1+ax+b(1-x)}\right]$$

$$e^{i(2w+1)\pi\theta}(sin\theta)^{w-1}(cos\theta)^{w-1}\ {}_2F_1\left[\alpha',\beta';\frac{\alpha'+\beta'}{2};e^{i\theta}cos\theta\right]$$

$$\aleph^{0,\mathfrak{n}:V}_{U:W}\left(\begin{array}{c} \mathrm{z}_1x^{\rho_1}(1-x)^{\rho_1}[1+ax+b(1-x)]^{-2\rho_1} \\ \cdot \\ \cdot \\ \mathrm{z}_rx^{\rho_r}(1-x)^{\rho_r}[1+ax+b(1-x)]^{-2\rho_r} \end{array}\right)\aleph^{0,\mathfrak{n}:V}_{U:W}\left(\begin{array}{c} \mathrm{z}_1e^{2i\theta w_1}(sin\theta)^{w_1}(cos\theta)^{w_1} \\ \cdot \\ \cdot \\ \mathrm{z}_re^{2i\theta w_r}(sin\theta)^{w_r}(cos\theta)^{w_r} \end{array}\right)\mathrm{d}\theta\mathrm{d}x$$

$$=\frac{2^{\alpha+\beta-2\rho-1}}{\Gamma(\alpha)\Gamma(\beta)(\alpha-\beta)(1+a)^{\rho}(1+b)^{\rho}}\left[\Gamma(\frac{\alpha+1}{2})\Gamma(\frac{\beta}{2})\aleph_1(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha}{2})\Gamma(\frac{\beta+1}{2})\aleph_2(z_1,\cdots,z_r)\right]$$

$$\times\frac{e^{i\pi(w-1)/2}\Gamma(\frac{\alpha'+\beta'}{2})}{2^{2w+\alpha'-\beta'+1}\Gamma(\alpha')\Gamma(\beta')}\left[\Gamma(\frac{\alpha'+1}{2})\Gamma(\frac{\beta'}{2})\aleph_7(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha'}{2})\Gamma(\frac{\beta'+1}{2})\ \aleph_8(z_1,\cdots,z_r)\right]$$

with the validity conditions : $Re(\rho)>0, Re(w)>0,|argz_k|\ <\dfrac{1}{2}A_i^{(k)}\pi$; $A_i^{(k)}$ is defined by (1.5) and

$$Re(2w-\alpha'-\beta'+2\sum_{i=1}^r w_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>0\,;Re(2\rho-\alpha-\beta+2\sum_{i=1}^r\rho_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>0 \tag{3.15}$$

where $\aleph_1(z_1,\cdots,z_r)$, $\aleph_2(z_1,\cdots,z_r)$, $\aleph_7(z_1,\cdots,z_r)$ and $\aleph_8(z_1,\cdots,z_r)$ are defined by (3.2), (3.3), (3.12) and (3.13) respectively.

d) $$\int_0^1\int_0^{\pi/2} x^{\rho}(1-x)^{\rho-2}\,[1+ax+b(1-x)]^{-2\rho+1}\ \left[\alpha,\beta;\frac{\alpha+\beta+2}{2};\frac{x(1+a)}{1+ax+b(1-x)}\right]$$

$$e^{i\pi(2w+1)\theta}(sin\theta)^{w}(cos\theta)^{w-1}\ {}_2F_1\left[\alpha',\beta';\frac{\alpha'+\beta'+2}{2};e^{i\theta}cos\theta\right]$$

$$\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1x^{\rho_1}(1-x)^{\rho_1}[1+ax+b(1-x)]^{-2\rho_1}\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}(1-x)^{\rho_r}[1+ax+b(1-x)]^{-2\rho_r}\end{array}\right)\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1e^{2i\theta w_1}(sin\theta)^{w_1}(cos\theta)^{w_1}\\ \cdot\\ \cdot\\ \mathrm{z}_re^{2i\theta w_r}(sin\theta)^{w_r}(cos\theta)^{w_r}\end{array}\right)\mathrm{d}\theta\,\mathrm{d}x$$

$$=\frac{2^{\alpha+\beta-2\rho-1}\Gamma(\frac{\alpha+\beta}{2})\Gamma(\frac{\alpha+\beta+2}{2})}{\Gamma(\alpha)\Gamma(\beta)(1+a)^{\rho}(1+b)^{\rho}}\left[\Gamma(\frac{\alpha+1}{2})\Gamma(\frac{\beta}{2})\aleph_5(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha}{2})\Gamma(\frac{\beta+1}{2})\aleph_6(z_1,\cdots,z_r)\right]$$

$$\times\frac{e^{i\pi(w+1)/2}\Gamma(\frac{\alpha'+\beta'+2}{2})}{2^{2w+\alpha'-\beta'+1}\Gamma(\alpha')\Gamma(\beta')\Gamma(\alpha'-\beta')}\left[\Gamma(\frac{\alpha'+1}{2})\Gamma(\frac{\beta'}{2})\aleph_3(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha'}{2})\Gamma(\frac{\beta'+1}{2})\ \aleph_4(z_1,\cdots,z_r)\right]$$

with the validity conditions : $Re(\rho)>0, Re(w)>0,|argz_k|\ <\frac{1}{2}A_i^{(k)}\pi$; $A_i^{(k)}$ is defined by (1.5) and

$$Re(2w-\alpha'-\beta'+2\sum_{i=1}^{r}w_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>2\,;Re(2\rho-\alpha-\beta+2\sum_{i=1}^{r}\rho_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>0 \quad (3.16)$$

where $\aleph_5(z_1,\cdots,z_r)$, $\aleph_6(z_1,\cdots,z_r)$, $\aleph_3(z_1,\cdots,z_r)$ and $\aleph_4(z_1,\cdots,z_r)$ are defined by (3.9), (3.10), (3.5) and (3.6) respectively.

Proof : To etablish (3.1),we express the multivariable Aleph-functions on the left hande side using (1.1) in Mellin-Barnes contour multiple integral and interchanging the order of integration which is justifiable due to absolute convergence of the integrals , we have :

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)\Bigg(\int_0^1x^{\rho+\sum_{j=1}^r\rho_js_j-1}(1-x)^{\rho+\sum_{j=1}^r\rho_js_j}$$

$$[1+ax+b(1-x)]^{-2\rho-2\sum_{j=1}^r\rho_js_j-1}\times\,{}_2F_1\left[\ \alpha,\beta;\frac{\alpha+\beta+2}{2};\frac{x(1+a)}{1+ax+b(1-x)}\right]\mathrm{d}x\Bigg)$$

$$\prod_{k=1}^{r}z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r\times\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)\Bigg(\int_0^{\pi/2}e^{i(2w+2\sum_{j=1}^rw_js_j+1)}$$

$$(cos\theta)^{w+\sum_{j=1}^rw_js_j}(sin\theta)^{w+\sum_{j=1}^rw_js_j}{}_2F_1\left[\alpha',\beta';\frac{\alpha'+\beta'+2}{2};e^{i\theta}cos\theta\right]\mathrm{d}\theta\Bigg)\prod_{k=1}^{r}z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r$$

We evaluate the inner integrals with the help of (2.1) and (2.3) and applying (1.1) , we get the R.H.S of (3.1) in terms of product of multivariable Aleph-functions. The other integrals calculate in the similar method. We obtain the similar formulas for the Aleph-functions of two variables defined by K. Sharma [4].

4. Multivariable I-function

If $\tau_i=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=1$the Aleph-function of several variables degenere to the I-function of several variables. The following finite double integrals have been derived in this section for multivariable I-functions defined by Sharma et al [2]. In these section, we note

$$B_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} - \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} - \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)}$$

$+\sum_{j=1}^{m_k} \delta_j^{(k)} - \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} > 0,$ with $k = 1 \cdots, r, i = 1, \cdots, R$, $i^{(k)} = 1, \cdots, R^{(k)}$ and we have

a) $$\int_0^1 \int_0^{\pi/2} x^{\rho-1}(1-x)^{\rho}[1+ax+(1-b)]^{-2\rho-1} {}_2F_1\Big[\alpha, \beta; \frac{\alpha+\beta+2}{2}; \frac{x(1+a)}{1+ax+b(1-x)}\Big]$$

$$e^{i(2w+1)\pi\theta}(sin\theta)^{w-1}(cos\theta)^{w-1} {}_2F_1\left[\alpha', \beta'; \frac{\alpha'+\beta'+2}{2}; e^{i\theta}cos\theta\right]$$

$$I_{U:W}^{0,\mathfrak{n}:V}\begin{pmatrix} \mathrm{z}_1 x^{\rho_1}(1-x)^{\rho_1}[1+ax+b(1-x)]^{-2\rho_1} \\ \cdot \\ \cdot \\ \mathrm{z}_r x^{\rho_r}(1-x)^{\rho_r}[1+ax+b(1-x)]^{-2\rho_r} \end{pmatrix} I_{U:W}^{0,\mathfrak{n}:V}\begin{pmatrix} \mathrm{z}_1 e^{2i\theta w_1}(sin\theta)^{w_1}(cos\theta)^{w_1} \\ \cdot \\ \cdot \\ \mathrm{z}_r e^{2i\theta w_r}(sin\theta)^{w_r}(cos\theta)^{w_r} \end{pmatrix} \mathrm{d}\theta\, \mathrm{d}x$$

$$= \frac{2^{\alpha+\beta-2\rho-2}\Gamma(\frac{\alpha+\beta+2}{2})}{\Gamma(\alpha)\Gamma(\beta)(\alpha-\beta)(1+a)^{\rho}(1+b)^{\rho}}\Big[\Gamma(\frac{\alpha+1}{2})\Gamma(\frac{\beta}{2})I_1(z_1,\cdots,z_r) - \Gamma(\frac{\alpha}{2})\Gamma(\frac{\beta+1}{2})I_2(z_1,\cdots,z_r)\Big]$$

$$\times \frac{e^{i\pi(w+1)/2}\Gamma(\frac{\alpha'+\beta'+2}{2})}{2^{2w-\alpha'-\beta'}\Gamma(\alpha'-\beta')\Gamma(\alpha')\Gamma(\beta')}\Big[\Gamma(\frac{\alpha'+1}{2})\Gamma(\frac{\beta'}{2})I_3(z_1,\cdots,z_r) - \Gamma(\frac{\alpha'}{2})\Gamma(\frac{\beta'+1}{2})I_4(z_1,\cdots,z_r)\Big]$$

with the validity conditions : $Re(\rho) > 0, Re(w) > 0, |argz_k| < \frac{1}{2}B_i^{(k)}\pi$

$$Re(2w-\alpha'-\beta'+2\sum_{i=1}^{r} w_i \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}) > 0\,; Re(2\rho-\alpha-\beta+2\sum_{i=1}^{r} \rho_i \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}) > 0 \quad (4.1)$$

where $I_1(z_1,\cdots,z_r)$, $I_2(z_1,\cdots,z_r), I_3(z_1,\cdots,z_r)$ and $I_4(z_1,\cdots,z_r)$ are defined by the similar formulas that $\aleph_1(z_1,\cdots,z_r), \aleph_2(z_1,\cdots,z_r), \aleph_3(z_1,\cdots,z_r)$ and $\aleph_4(z_1,\cdots,z_r)$ respectively

b) $$\int_0^1 \int_0^{\pi/2} x^{\rho-1}(1-x)^{\rho}[1+ax+(1-b)]^{-2\rho+1}\Big[\alpha, \beta; \frac{\alpha+\beta+2}{2}; \frac{x(1+a)}{1+ax+b(1-x)}\Big]$$

$$e^{i(2w+1)\pi\theta}(sin\theta)^{w-1}(cos\theta)^{w-1} {}_2F_1\left[\alpha', \beta'; \frac{\alpha'+\beta'}{2}; e^{i\theta}cos\theta\right]$$

$$I_{U:W}^{0,\mathfrak{n}:V}\begin{pmatrix} \mathrm{z}_1 x^{\rho_1}(1-x)^{\rho_1}[1+ax+b(1-x)]^{-2\rho_1} \\ \cdot \\ \cdot \\ \mathrm{z}_r x^{\rho_r}(1-x)^{\rho_r}[1+ax+b(1-x)]^{-2\rho_r} \end{pmatrix} I_{U:W}^{0,\mathfrak{n}:V}\begin{pmatrix} \mathrm{z}_1 e^{2i\theta w_1}(sin\theta)^{w_1}(cos\theta)^{w_1} \\ \cdot \\ \cdot \\ \mathrm{z}_r e^{2i\theta w_r}(sin\theta)^{w_r}(cos\theta)^{w_r} \end{pmatrix} \mathrm{d}\theta\, \mathrm{d}x$$

$$= \frac{2^{\alpha+\beta-2\rho-1}\Gamma(\frac{\alpha+\beta+2}{2})}{\Gamma(\alpha)\Gamma(\beta)(1+a)^{\rho}(1+b)^{\rho}}\Big[\Gamma(\frac{\alpha+\beta}{2})\Gamma(\frac{\beta}{2})I_5(z_1,\cdots,z_r) - \Gamma(\frac{\alpha+\beta}{2})\Gamma(\frac{\alpha}{2})I_6(z_1,\cdots,z_r)\Big]$$

$$\times\frac{e^{i\pi(w+1)/2}\Gamma(\frac{\alpha'+\beta'+2}{2})}{2^{2w-\alpha'+1}\Gamma(\alpha'-\beta')\Gamma(\alpha')\Gamma(\beta')}\left[\Gamma(\frac{\alpha'+1}{2})\Gamma(\frac{\beta'}{2})I_7(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha'+1}{2})\Gamma(\frac{\beta'+2}{2})\ I_8(z_1,\cdots,z_r)\right]$$

with the validity conditions : $Re(\rho)>0, Re(w)>0, |argz_k|<\frac{1}{2}B_i^{(k)}\pi$ and

$$Re(2w-\alpha'-\beta'+2\sum_{i=1}^{r}w_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>2\,;Re(2\rho-\alpha-\beta+2\sum_{i=1}^{r}\rho_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>2 \quad (4.2)$$

where $I_5(z_1,\cdots,z_r)$, $I_6(z_1,\cdots,z_r)$, $I_7(z_1,\cdots,z_r)$ and $I_8(z_1,\cdots,z_r)$ are defined by the similar formulas that $\aleph_5(z_1,\cdots,z_r)$, $\aleph_6(z_1,\cdots,z_r)$, $\aleph_7(z_1,\cdots,z_r)$ and $\aleph_8(z_1,\cdots,z_r)$ respectively

c) $$\int_0^1\int_0^{\pi/2}x^{\rho-1}(1-x)^{\rho}[1+ax+(1-b)]^{-2\rho+1}\left[\alpha,\beta;\frac{\alpha+\beta+2}{2};\frac{x(1+a)}{1+ax+b(1-x)}\right]$$

$$e^{i(2w+1)\pi\theta}(sin\theta)^{w-1}(cos\theta)^{w-1}\ {}_2F_1\left[\alpha',\beta';\frac{\alpha'+\beta'}{2};e^{i\theta}cos\theta\right]$$

$$I_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1x^{\rho_1}(1-x)^{\rho_1}[1+ax+b(1-x)]^{-2\rho_1}\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}(1-x)^{\rho_r}[1+ax+b(1-x)]^{-2\rho_r}\end{array}\right)I_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1e^{2i\theta w_1}(sin\theta)^{w_1}(cos\theta)^{w_1}\\ \cdot\\ \cdot\\ \mathrm{z}_re^{2i\theta w_r}(sin\theta)^{w_r}(cos\theta)^{w_r}\end{array}\right)\mathrm{d}\theta\,\mathrm{d}x$$

$$=\frac{2^{\alpha+\beta-2\rho-1}}{\Gamma(\alpha)\Gamma(\beta)(\alpha-\beta)(1+a)^{\rho}(1+b)^{\rho}}\left[\Gamma(\frac{\alpha+1}{2})\Gamma(\frac{\beta}{2})I_1(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha}{2})\Gamma(\frac{\beta+1}{2})I_2(z_1,\cdots,z_r)\right]$$

$$\times\frac{e^{i\pi(w-1)/2}\Gamma(\frac{\alpha'+\beta'}{2})}{2^{2w+\alpha'-\beta'+1}\Gamma(\alpha')\Gamma(\beta')}\left[\Gamma(\frac{\alpha'+1}{2})\Gamma(\frac{\beta'}{2})I_7(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha'}{2})\Gamma(\frac{\beta'+1}{2})\ I_8(z_1,\cdots,z_r)\right]$$

with the validity conditions : $Re(\rho)>0, Re(w)>0,|argz_k|<\frac{1}{2}B_i^{(k)}\pi$;

$$Re(2w-\alpha'-\beta'+2\sum_{i=1}^{r}w_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>0\,;Re(2\rho-\alpha-\beta+2\sum_{i=1}^{r}\rho_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>0 \quad (4.3)$$

d) $$\int_0^1\int_0^{\pi/2}x^{\rho}(1-x)^{\rho-2}\,[1+ax+b(1-x)]^{-2\rho+1}\ \left[\alpha,\beta;\frac{\alpha+\beta+2}{2};\frac{x(1+a)}{1+ax+b(1-x)}\right]$$

$$e^{i\pi(2w+1)\theta}(sin\theta)^{w}(cos\theta)^{w-1}\ {}_2F_1\left[\alpha',\beta';\frac{\alpha'+\beta'+2}{2};e^{i\theta}cos\theta\right]$$

$$I_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1x^{\rho_1}(1-x)^{\rho_1}[1+ax+b(1-x)]^{-2\rho_1}\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}(1-x)^{\rho_r}[1+ax+b(1-x)]^{-2\rho_r}\end{array}\right)I_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1e^{2i\theta w_1}(sin\theta)^{w_1}(cos\theta)^{w_1}\\ \cdot\\ \cdot\\ \mathrm{z}_re^{2i\theta w_r}(sin\theta)^{w_r}(cos\theta)^{w_r}\end{array}\right)\mathrm{d}\theta\,\mathrm{d}x$$

$$=\frac{2^{\alpha+\beta-2\rho-1}\Gamma(\frac{\alpha+\beta}{2})\Gamma(\frac{\alpha+\beta+2}{2})}{\Gamma(\alpha)\Gamma(\beta)(1+a)^{\rho}(1+b)^{\rho}}\left[\Gamma(\frac{\alpha+1}{2})\Gamma(\frac{\beta}{2})I_5(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha}{2})\Gamma(\frac{\beta+1}{2})I_6(z_1,\cdots,z_r)\right]$$

$$\times\frac{e^{i\pi(w+1)/2}\Gamma(\frac{\alpha'+\beta'+2}{2})}{2^{2w+\alpha'-\beta'+1}\Gamma(\alpha')\Gamma(\beta')\Gamma(\alpha'-\beta')}\left[\Gamma(\frac{\alpha'+1}{2})\Gamma(\frac{\beta'}{2})I_3(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha'}{2})\Gamma(\frac{\beta'+1}{2})\,I_4(z_1,\cdots,z_r)\right]$$

with the validity conditions : $Re(\rho)>0, Re(w)>0, |argz_k|<\frac{1}{2}B_i^{(k)}\pi$ and

$$Re(2w-\alpha'-\beta'+2\sum_{i=1}^{r}w_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>2\,;Re(2\rho-\alpha-\beta+2\sum_{i=1}^{r}\rho_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>0 \quad (3.16)$$

5. Multivariable H-function

If $\tau_i=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=1$and $R=R^{(1)}=\cdots=R^{(r)}=1$the Aleph-function of several variables degenere to the H-function of several variables. The following finite double integrals have been derived in this section for multivariable H-functions defined by Srivastava et al [5]. In these section, we note

$$A_i=\sum_{j=1}^{\mathfrak{n}}\alpha_j^{(i)}-\sum_{j=\mathfrak{n}+1}^{p}\alpha_j^{(i)}-\sum_{j=1}^{q}\beta_j^{(i)}+\sum_{j=1}^{n_i}\gamma_j^{(i)}-\sum_{j=n_i+1}^{p_i}\gamma_j^{(i)}+\sum_{j=1}^{m_i}\delta_j^{(i)}-\sum_{j=m_i+1}^{q_i}\delta_j^{(i)}>0\,,$$

a) $$\int_0^1\int_0^{\pi/2}x^{\rho-1}(1-x)^{\rho}[1+ax+(1-b)]^{-2\rho-1}{}_2F_1\left[\alpha,\beta;\frac{\alpha+\beta+2}{2};\frac{x(1+a)}{1+ax+b(1-x)}\right]$$

$$e^{i(2w+1)\pi\theta}(sin\theta)^{w-1}(cos\theta)^{w-1}\,{}_2F_1\left[\alpha',\beta';\frac{\alpha'+\beta'+2}{2};e^{i\theta}cos\theta\right]$$

$$H_{p,q:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1x^{\rho_1}(1-x)^{\rho_1}[1+ax+b(1-x)]^{-2\rho_1}\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}(1-x)^{\rho_r}[1+ax+b(1-x)]^{-2\rho_r}\end{array}\right)H_{p,q:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1e^{2i\theta w_1}(sin\theta)^{w_1}(cos\theta)^{w_1}\\ \cdot\\ \cdot\\ \mathrm{z}_re^{2i\theta w_r}(sin\theta)^{w_r}(cos\theta)^{w_r}\end{array}\right)\mathrm{d}\theta\mathrm{d}x$$

$$=\frac{2^{\alpha+\beta-2\rho-2}\Gamma(\frac{\alpha+\beta+2}{2})}{\Gamma(\alpha)\Gamma(\beta)(\alpha-\beta)(1+a)^{\rho}(1+b)^{\rho}}\left[\Gamma(\frac{\alpha+1}{2})\Gamma(\frac{\beta}{2})H_1(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha}{2})\Gamma(\frac{\beta+1}{2})H_2(z_1,\cdots,z_r)\right]$$

$$\times\frac{e^{i\pi(w+1)/2}\Gamma(\frac{\alpha'+\beta'+2}{2})}{2^{2w-\alpha'-\beta'}\Gamma(\alpha'-\beta')\Gamma(\alpha')\Gamma(\beta')}\left[\Gamma(\frac{\alpha'+1}{2})\Gamma(\frac{\beta'}{2})H_3(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha'}{2})\Gamma(\frac{\beta'+1}{2})H_4(z_1,\cdots,z_r)\right]$$

with the validity conditions : $Re(\rho)>0, Re(w)>0, |argz_k|<\frac{1}{2}A_i\pi$

$$Re(2w-\alpha'-\beta'+2\sum_{i=1}^{r}w_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>0\,;Re(2\rho-\alpha-\beta+2\sum_{i=1}^{r}\rho_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>0 \quad (5.1)$$

where $H_1(z_1,\cdots,z_r)$, $H_2(z_1,\cdots,z_r)$, $H_3(z_1,\cdots,z_r)$ and $H_4(z_1,\cdots,z_r)$ are defined by the similar formulas that $\aleph_1(z_1,\cdots,z_r)$, $\aleph_2(z_1,\cdots,z_r)$, $\aleph_3(z_1,\cdots,z_r)$ and $\aleph_4(z_1,\cdots,z_r)$ respectively

b) $$\int_0^1\int_0^{\pi/2} x^{\rho-1}(1-x)^{\rho}[1+ax+(1-b)]^{-2\rho+1}\Big[\alpha,\beta;\frac{\alpha+\beta+2}{2};\frac{x(1+a)}{1+ax+b(1-x)}\Big]$$

$$e^{i(2w+1)\pi\theta}(sin\theta)^{w-1}(cos\theta)^{w-1}\,{}_2F_1\left[\alpha',\beta';\frac{\alpha'+\beta'}{2};e^{i\theta}cos\theta\right]$$

$$H^{0,\mathfrak{n}:V}_{p,q:W}\begin{pmatrix}\mathrm{z}_1x^{\rho_1}(1-x)^{\rho_1}[1+ax+b(1-x)]^{-2\rho_1}\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}(1-x)^{\rho_r}[1+ax+b(1-x)]^{-2\rho_r}\end{pmatrix}H^{0,\mathfrak{n}:V}_{p,q:W}\begin{pmatrix}\mathrm{z}_1e^{2i\theta w_1}(sin\theta)^{w_1}(cos\theta)^{w_1}\\ \cdot\\ \cdot\\ \mathrm{z}_re^{2i\theta w_r}(sin\theta)^{w_r}(cos\theta)^{w_r}\end{pmatrix}\mathrm{d}\theta\mathrm{d}x$$

$$=\frac{2^{\alpha+\beta-2\rho-1}\Gamma(\frac{\alpha+\beta+2}{2})}{\Gamma(\alpha)\Gamma(\beta)(1+a)^{\rho}(1+b)^{\rho}}\Big[\Gamma(\frac{\alpha+\beta}{2})\Gamma(\frac{\beta}{2})H_5(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha+\beta}{2})\Gamma(\frac{\alpha}{2})H_6(z_1,\cdots,z_r)\Big]$$

$$\times\frac{e^{i\pi(w+1)/2}\Gamma(\frac{\alpha'+\beta'+2}{2})}{2^{2w-\alpha'+1}\Gamma(\alpha'-\beta')\Gamma(\alpha')\Gamma(\beta')}\Big[\Gamma(\frac{\alpha'+1}{2})\Gamma(\frac{\beta'}{2})H_7(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha'+1}{2})\Gamma(\frac{\beta'+2}{2})\,H_8(z_1,\cdots,z_r)\Big]$$

with the validity conditions : $Re(\rho)>0, Re(w)>0, |argz_k|<\frac{1}{2}A_i\pi$ and

$$Re(2w-\alpha'-\beta'+2\sum_{i=1}^r w_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>2\,;Re(2\rho-\alpha-\beta+2\sum_{i=1}^r\rho_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>2 \quad (5.2)$$

where $H_5(z_1,\cdots,z_r)$, $H_6(z_1,\cdots,z_r)$, $H_7(z_1,\cdots,z_r)$ and $H_8(z_1,\cdots,z_r)$ are defined by the similar

formulas that $\aleph_5(z_1,\cdots,z_r)$, $\aleph_6(z_1,\cdots,z_r)$, $\aleph_7(z_1,\cdots,z_r)$ and $\aleph_8(z_1,\cdots,z_r)$ respectively

c) $$\int_0^1\int_0^{\pi/2} x^{\rho-1}(1-x)^{\rho}[1+ax+(1-b)]^{-2\rho+1}\Big[\alpha,\beta;\frac{\alpha+\beta+2}{2};\frac{x(1+a)}{1+ax+b(1-x)}\Big]$$

$$e^{i(2w+1)\pi\theta}(sin\theta)^{w-1}(cos\theta)^{w-1}\,{}_2F_1\left[\alpha',\beta';\frac{\alpha'+\beta'}{2};e^{i\theta}cos\theta\right]$$

$$H^{0,\mathfrak{n}:V}_{p,q:W}\begin{pmatrix}\mathrm{z}_1x^{\rho_1}(1-x)^{\rho_1}[1+ax+b(1-x)]^{-2\rho_1}\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}(1-x)^{\rho_r}[1+ax+b(1-x)]^{-2\rho_r}\end{pmatrix}H^{0,\mathfrak{n}:V}_{p,q:W}\begin{pmatrix}\mathrm{z}_1e^{2i\theta w_1}(sin\theta)^{w_1}(cos\theta)^{w_1}\\ \cdot\\ \cdot\\ \mathrm{z}_re^{2i\theta w_r}(sin\theta)^{w_r}(cos\theta)^{w_r}\end{pmatrix}\mathrm{d}\theta\mathrm{d}x$$

$$=\frac{2^{\alpha+\beta-2\rho-1}}{\Gamma(\alpha)\Gamma(\beta)(\alpha-\beta)(1+a)^{\rho}(1+b)^{\rho}}\Big[\Gamma(\frac{\alpha+1}{2})\Gamma(\frac{\beta}{2})h_1(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha}{2})\Gamma(\frac{\beta+1}{2})H_2(z_1,\cdots,z_r)\Big]$$

$$\times\frac{e^{i\pi(w-1)/2}\Gamma(\frac{\alpha'+\beta'}{2})}{2^{2w+\alpha'-\beta'+1}\Gamma(\alpha')\Gamma(\beta')}\left[\Gamma(\frac{\alpha'+1}{2})\Gamma(\frac{\beta'}{2})H_7(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha'}{2})\Gamma(\frac{\beta'+1}{2})\ H_8(z_1,\cdots,z_r)\right]$$

with the validity conditions : $Re(\rho)>0, Re(w)>0,|argz_k|<\frac{1}{2}A_i\pi$;

$$Re(2w-\alpha'-\beta'+2\sum_{i=1}^{r}w_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>0\,;Re(2\rho-\alpha-\beta+2\sum_{i=1}^{r}\rho_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>0 \quad (5.3)$$

d) $$\int_0^1\int_0^{\pi/2}x^{\rho}(1-x)^{\rho-2}\left[1+ax+b(1-x)\right]^{-2\rho+1}\left[\alpha,\beta;\frac{\alpha+\beta+2}{2};\frac{x(1+a)}{1+ax+b(1-x)}\right]$$

$$e^{i\pi(2w+1)\theta}(sin\theta)^w(cos\theta)^{w-1}\ {}_2F_1\left[\alpha',\beta';\frac{\alpha'+\beta'+2}{2};e^{i\theta}cos\theta\right]$$

$$H_{p,q:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1x^{\rho_1}(1-x)^{\rho_1}[1+ax+b(1-x)]^{-2\rho_1}\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}(1-x)^{\rho_r}[1+ax+b(1-x)]^{-2\rho_r}\end{array}\right)H_{p,q:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1e^{2i\theta w_1}(sin\theta)^{w_1}(cos\theta)^{w_1}\\ \cdot\\ \cdot\\ \mathrm{z}_re^{2i\theta w_r}(sin\theta)^{w_r}(cos\theta)^{w_r}\end{array}\right)\mathrm{d}\theta\mathrm{d}x$$

$$=\frac{2^{\alpha+\beta-2\rho-1}\Gamma(\frac{\alpha+\beta}{2})\Gamma(\frac{\alpha+\beta+2}{2})}{\Gamma(\alpha)\Gamma(\beta)(1+a)^{\rho}(1+b)^{\rho}}\left[\Gamma(\frac{\alpha+1}{2})\Gamma(\frac{\beta}{2})H_5(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha}{2})\Gamma(\frac{\beta+1}{2})H_6(z_1,\cdots,z_r)\right]$$

$$\times\frac{e^{i\pi(w+1)/2}\Gamma(\frac{\alpha'+\beta'+2}{2})}{2^{2w+\alpha'-\beta'+1}\Gamma(\alpha')\Gamma(\beta')\Gamma(\alpha'-\beta')}\left[\Gamma(\frac{\alpha'+1}{2})\Gamma(\frac{\beta'}{2})H_3(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha'}{2})\Gamma(\frac{\beta'+1}{2})\ H_4(z_1,\cdots,z_r)\right]$$

with the validity conditions : $Re(\rho)>0, Re(w)>0, |argz_k|<\frac{1}{2}A_i\pi$ and

$$Re(2w-\alpha'-\beta'+2\sum_{i=1}^{r}w_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>2\,;Re(2\rho-\alpha-\beta+2\sum_{i=1}^{r}\rho_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>0 \quad (5.4)$$

6. Particular case

If $a=b$ in (3.8) , we obtain :

$$\int_0^1\int_0^{\pi/2}x^{\rho-1}(1-x)^{\rho}(1+b)^{-2\rho+1}\ {}_2F_1\left[\alpha,\beta;\frac{\alpha+\beta+2}{2};x\right]e^{i(2w+1)\pi\theta}(sin\theta)^{w-2}(cos\theta)^{w-1}$$

$${}_2F_1\left[\alpha',\beta';\frac{\alpha'+\beta'}{2};e^{i\theta}cos\theta\right]$$

$$\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1x^{\rho_1}(1-x)^{\rho_1}(1+a)^{-2\rho_1}\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\rho_r}(1-x)^{\rho_r}(1+a)^{-2\rho_r}\end{array}\right)\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1e^{2i\theta w_1}(sin\theta)^{w_1}(cos\theta)^{w_1}\\ \cdot\\ \cdot\\ \mathrm{z}_re^{2i\theta w_r}(sin\theta)^{w_r}(cos\theta)^{w_r}\end{array}\right)\mathrm{d}\theta\mathrm{d}x$$

$$=\frac{2^{\alpha+\beta-2\rho_1-1}\Gamma(\frac{\alpha+\beta}{2})}{\Gamma(\alpha)\Gamma(\beta)(1+a)^{2\rho}}\left[\Gamma(\frac{\alpha+\beta}{2})\Gamma(\frac{\beta}{2})\aleph_5(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha+\beta}{2})\Gamma(\frac{\alpha}{2})\aleph_6(z_1,\cdots,z_r)\right]$$

$$\times\frac{e^{i\pi(w_1-1)/2}\Gamma(\frac{\alpha'+\beta'+2}{2})}{2^{2w+\alpha'-\beta'+1}\Gamma(\alpha')\Gamma(\beta')}\left[\Gamma(\frac{\alpha'+1}{2})\Gamma(\frac{\beta'}{2})\aleph_7(z_1,\cdots,z_r)\text{-}\Gamma(\frac{\alpha'+1}{2})\Gamma(\frac{\beta'+2}{2})\aleph_8(z_1,\cdots,z_r)\right]$$

with the validity conditions : $Re(\rho)>0, Re(w)>0, |argz_k|<\frac{1}{2}A_i^{(k)}\pi$; $A_i^{(k)}$ is defined by (1.5) and

$$Re(2w-\alpha'-\beta'+2\sum_{i=1}^{r}w_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>2\,;Re(2\rho-\alpha-\beta+2\sum_{i=1}^{r}\rho_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}})>2 \quad (6.1)$$

7. Conclusion

The aleph-function of several variables presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions o several variables such as multivariable I-function ,multivariable Fox's H-function, Fox's H-function, see [1] , Meijer's G-function, Wright's generalized Bessel function, Wright's generalized hypergeometric function, MacRobert's E-function, generalized hypergeometric function, Bessel function of first kind, modied Bessel function, Whittaker function, exponential function , binomial function etc. as its special cases, and therefore, various unified integral presentations can be obtained as special cases of our results.

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