

COMPARISON/OPTIMIZATION OF DIFFERENT FITTED CURVES OF AN EXPERIMENTAL DATA

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Abstract: In the present work, curve fitting of the experimental data has been done using the three different curves ($y = a + bx$, $y = ae^{bx}$ and $y = ab^x$). Analytical solutions are obtained at the different stages. Experimental data is adopted from the open access website, in which the aim of the experiment is to measure the distance of a body moves in time 't' second when the body is dropped from the height. The aim of the present study is to compare the results using the above three curves. It is found that exponential curve fitting is the optimum fitted curve for such real life problem.

Keywords: Curve Fitting, Statistical Techniques, Experimental data.

1. INTRODUCTION:

Let there be the two variable x and y which give us a set of n pairs of numerical value $(x_1, y_1), (x_2, y_2), (\dots) (x_n, y_n)$. In order to have an approximate idea about the relationship of these variables, we plot these n paired points on a graph thus; we get a diagram showing the simultaneous variation in values of both the variables called **Scatter or Dot diagram**. From scatter diagram, we get only an approximate non mathematical relation between two variables [1-3, 5].

Curve Fitting means an exact relationship between two variables by algebraic equations; in fact this relationship is the equation of the curve[1-3]. Therefore, curve fitting means to form an equation of the curve from the given data.

Curve fitting of a Straight line: $y = a + bx$,

Normal equations are:

$$\left. \begin{aligned} \sum y &= na + b \sum x \\ \sum xy &= a \sum x + b \sum x^2 \end{aligned} \right\} \quad (1)$$

Fitting of an exponential curve: $y = ae^{bx}$,

Normal equations are:

$$\left. \begin{aligned} \sum Y &= nA + B \sum X \\ \sum XY &= A \sum X + B \sum X^2 \end{aligned} \right\} \quad (2)$$

where,

$$\begin{aligned} X &= x, Y = \log_{10} y \\ A &= \log_{10} a, B = b \log_{10} e \end{aligned}$$

Fitting of the curve: $y = ab^x$,

Normal equations are:

$$\left. \begin{aligned} \sum Y &= nA + B \sum X \\ \sum XY &= A \sum X + B \sum X^2 \end{aligned} \right\} \tag{3}$$

where, $X = x, Y = \log_{10} y$
 $A = \log_{10} a, B = \log_{10} b$

2. METHOD/PROBLEM DEFINITION:

In an experiment to measure the distance a body moves in a time (t) second when dropped from a height: the following results were obtained [4]. It is thought that the body obeys the law of gravity.

t(s)	0	0.15	0.2	0.25	0.3	0.35	0.4	0.45
S(m)	0.1	0.2	0.3	0.4	0.6	0.8	1.0	1.2

The different terms of the equations (1), (2) and (3) are calculated in the following Table-1.

Table 1: Calculation of Constants in the Normal Equations

x	y	x ²	xy	X = x	Y = log ₁₀ y	X ²	XY
0	0.1	0.0	0.0	0	-1	0.0	0
0.15	0.2	0.0225	0.03	0.15	-0.8239	0.0225	-0.1236
0.2	0.3	0.04	0.06	0.2	-0.5288	0.04	-0.1046
0.25	0.4	0.0625	0.1	0.25	-0.3979	0.0625	-0.0995
0.3	0.6	0.09	0.18	0.3	-0.2218	0.09	-0.0665
0.35	0.8	0.1225	0.28	0.35	-0.0969	0.1225	-0.0339
0.4	1.0	0.16	0.4	0.4	0	0.16	0
0.45	1.2	0.2025	0.54	0.45	0.07918	0.2025	0.0356
$\sum x = 2.1$	$\sum y = 4.6$	$\sum x^2 = 0.7$	$\sum xy = 1.59$	$\sum X = 2.1$	$\sum Y = -2.98$	$\sum X^2 = 0.7$	$\sum XY = 0.39$

Substituting all these above tabulated values in equations (1), (2) and (3) then after calculating the values of constants we obtained the required curves.

3. RESULTS AND DISCUSSION

Thus, the following three curves are obtained after the calculations:

- (i) Curve fitting by straight line: $s = 2.571t - 0.1$
- (ii) Curve fitting by exponential curve: $s = 0.0865 e^{6.0498t}$
- (iii) Curve fitting by curve : $(y = ab^x) : s = 0.0865 * 424.033^t$

On putting the different value of t in above three equations we get the different value of s and with the help the value of 't' and 's'. the following Figure 1 has been obtained by putting the values of 't' at the different levels of time.

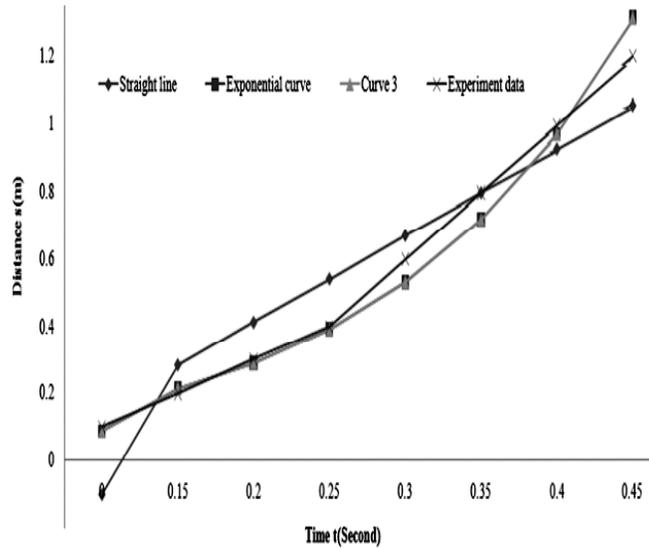


Figure 1: Comparison of Different Curve with the Experimental Data

The above result show a comparison of curve fitting of the straight line ($s = 2.57 lt - 0.1$), fitting of an exponential curve ($s = 0.0865e^{6.0498t}$), and fitting of the curve ($s = 0.0865 * 424.033t$) this shows that the exponential curve is the best method of the fitting the curve.

4. CONCLUSIONS

From the above result it is concluded that the fitting of the curve by exponential curve *i.e.*, $y = ae^{bx}$ is the one of the best method for the curve fitting for such real life problems.

REFERENCES

- [1] Goyal M., Bali N. P., 2013. "A textbook of engineering mathematics". ISBN: 978-93-80386-53-9.
- [2] Dass H. K. and Verma R., 2012. "Introduction to Engineering Mathematics". 3. ISBN: 81-219-3227-0.
- [3] Dunn D.J. "Mathematics for Engineers' Statistics Tutorial 2- Curve Fitting, Self Assessment Exercise no. 1 (2). (www.freestudy.co.uk).
- [4] www.google.com. Experimental Data or Research Question on Curve Fitting of Statistical Techniques-I.
- [5] www.asks.com. Curve Fitting of Statistical Techniques-I.

