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A Padé-Chebyshev Resolution of the Gibbs Phenomenon in Function Approximation

(A Dissertation Abstract)

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The occurrence of the Gibbs phenomenon is an undesirable peculiarity in approximating functions with finite jumps by partial sums of an orthogonal series expansion, causing non-uniform convergence near the points of discontinuities and slow convergence elsewhere in the domain under consideration. Rational approximation, notably Padé-type approximation, is a way of reducing the adverse effect of such phenomenon for a better reconstruction of the function. Using the trigonometric definition of the Chebyshev polynomials which allows a transformation leading to the Laurent series expansion of the function, we aim to resolve the Gibbs phenomenon by constructing an amplified Padé-Chebyshev approximant based on the concept introduced by Driscoll and Fornberg in [1, 2] which incorporates into the approximation process the singularities of the function. When the jump locations are known exactly, our method practically eliminates the Gibbs phenomenon. In the absence of the knowledge of jump locations as in the case of noisy data, we invoke a Padé-Chebyshev approximation on the differentiated transformed Chebyshev expansion associated with the function to obtain jump information needed in the process. In cases when the function's expansion coefficients are difficult to determine analytically, the method adapts the Gauss-Chebyshev quadrature rule to facililate approximation. Numerical experiments reveal that even under these restrictive situations, our method can successfully generate approximants for which the Gibbs phenomenon is remarkably reduced. Furthermore, we look at the strength of the derived approximant in terms of its steepness and ability to produce a better Gibbs constant. Implementation of the method basically requires the solution of a linear system similar in structure to that of the standard Padé approximation.

[1] T. A. Driscoll and B. Fornberg, *A Padé-based algorithm for overcoming the Gibbs phenomenon*, Numerical Algorithms, 26 (2001), pp.77 – 92.

[2]T. A. Driscoll and B. Fornberg, *Padé-based interpretation and correction of the Gibbs phenomenon*, in the Advances in the Gibbs Phenomenon with Detailed Introduction, ed. A. Jerri, Sampling Publishing, Postdam, New York, 2007.

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