# A unified multiple integral transformation formula involving the product of special functions and general class of multivariable polynomials

$F.Y.\ AYANT^1$

1 Teacher in High School , France
E-mail : fredericayant@gmail.com

ABSTRACT
In the present paper we evaluate an unified multiple integral transformation whose integrand involves the products of general class of multivariable polynomials , the multivariable A-function and modified multivariable H-function of $(s+1)$-variables having general arguments. The result is believed to be new and is capable of giving a very large number of simpler integrals involving a large number of special functions and polynomials as its special cases. We shall see several corollaries and special cases at the end.

**Keywords**: Modified multivariable H-function, multiple integral transformation, multivariable A-function, A-function, class of multivariable polynomials, Fox H-function.

**2010 Mathematics Subject Classification. 33C60, 82C31**

## Introduction.

Recently Jain and Sharma [5] have studied a multiple integral transformation formula involving the multivariable H-function defined by Srivastava and Panda [10,11] and the product of general class of polynomials defined by Srivastava [7]. The aim of this paper is studied a unified multiple integral tranformation involving the product of general class of multivariable polynomials defined by Srivastava [8], the multivariable A-function defined by Gautam and Asgar [4] and the modified multivariable H-function defined by Prasad and Singh [6]. This function is an extension of the multivariable H-function defined by Srivastava and Panda [10,11]. A the end, we shall several corollaries and special cases.

The generalized polynomials of multivariables defined by Srivastava [8], is given in the following manner :

$$S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[y_1,\cdots,y_v] = \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\frac{(-N_1)_{\mathfrak{M}_1K_1}}{K_1!}\cdots\frac{(-N_v)_{\mathfrak{M}_vK_v}}{K_v!}A[N_1,K_1;\cdots;N_v,K_v]y_1^{K_1}\cdots y_v^{K_v} \quad (1.1)$$

where $\mathfrak{M}_1,\cdots,\mathfrak{M}_v$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_v,K_v]$ are arbitrary constants, real or complex.

We shall note

$$a_v' = \frac{(-N_1)_{\mathfrak{M}_1K_1}}{K_1!}\cdots\frac{(-N_v)_{\mathfrak{M}_vK_v}}{K_v!}A[N_1,K_1;\cdots;N_v,K_v] \quad (1.2)$$

The Fox H-function [2,9] is defined and represented in the following manner [9, p.10,Eq.(2.1.1)]

$$H_{P,Q}^{M,N}\left(\mathrm{x}\ \middle|\ \begin{matrix}(\mathrm{g}_j,G_j)_{1,P}\\(\mathrm{h}_j,H_j)_{1,Q}\end{matrix}\right) = \frac{1}{2\pi\omega}\int_L\theta(\xi)x^{\xi}\mathrm{d}\xi \quad (1.3)$$

where $\omega=\sqrt{-1}$ and

$$\theta(\xi) = \frac{\prod_{j=1}^{M}\Gamma(h_j-H_j\xi)\prod_{j=1}^{N}\Gamma(1-g_j+G_j\xi)}{\prod_{j=M+1}^{Q}\Gamma(1-h_j+H_j\xi)\prod_{j=N+1}^{P}\Gamma(g_j-G_j\xi)} \quad (1.4)$$

The A-function of several variables is an extension of multivariable H-function.The serie representation of the multivariable A-function is given by Gautam and Asgar [4] as

$$A[z_u,\cdots,z_u]=A^{0,\lambda:(\alpha',\beta');\cdots;(\alpha^{(u)},\beta^{(u)})}_{A,C:(M',N');\cdots;(M^{(u)},N^{(u)})}\left(\begin{array}{c|c} \mathrm{z}_u & [(\mathrm{g}_j);\gamma',\cdots,\gamma^{(u)}]_{1,A}: \\ \cdot & \cdots \\ \cdot & [(\mathrm{f}_j);\xi',\cdots,\xi^{(u)}]_{1,C}: \\ \mathrm{z}_u & \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{q}^{(1)},\eta^{(1)})_{1,M^{(1)}};\cdots;(q^{(u)},\eta^{(u)})_{1,M^{(u)}} \\ \cdots \\ \cdots \\ (\mathrm{p}^{(1)},\epsilon^{(1)})_{1,N^{(1)}};\cdots;(p^{(u)},\epsilon^{(u)})_{1,N^{(u)}} \end{array}\right)=\sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=0}^{\infty}\phi\frac{\prod_{i=1}^{u}\phi_i u_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^{u}g_i}}{\prod_{i=1}^{u}\epsilon_{G_i}^{(i)}g_i!} \tag{1.5}$$

where

$$\phi=\frac{\prod_{j=1}^{\lambda}\Gamma\left(1-g_j+\sum_{i=1}^{u}\gamma_j^{(i)}\eta_{G_i,g_i}\right)}{\prod_{j=\lambda'+1}^{A}\Gamma\left(g_j-\sum_{i=1}^{u}\gamma_j^{(i)}U_i\right)\prod_{j=1}^{C}\Gamma\left(1-f_j+\sum_{i=1}^{u}\xi_j^{(i)}\eta_{G_i,g_i}\right)} \tag{1.6}$$

$$\phi_i=\frac{\prod_{j=1,j\neq m_i}^{\alpha^{(i)}}\Gamma\left(p_j^{(i)}-\epsilon_j^{(i)}\eta_{G_i,g_i}\right)\prod_{j=1}^{\beta^{(i)}}\Gamma\left(1-q_j^{(i)}+\eta_j^{(i)}\eta_{G_i,g_i}\right)}{\prod_{j=\alpha^{(i)}+1}^{N^{(i)}}\Gamma\left(1-p_j^{(i)}+\epsilon_j^{(i)}\eta_{G_i,g_i}\right)\prod_{j=\beta^{(i)}+1}^{M^{(i)}}\Gamma\left(q_j^{(i)}-\eta_j^{(i)}\eta_{G_i,g_i}\right)},i=1,\cdots,u \tag{1.7}$$

$$\eta_{G_i,g_i}=\frac{p_{G_i}^{(i)}+g_i}{\epsilon_{G_i}^{(i)}},i=1,\cdots,u \tag{1.8}$$

$$\sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=0}^{\infty}=\sum_{G_1,\cdots,G_u=1}^{\alpha^{(1)},\cdots,\alpha^{(u)}}\sum_{g_1,\cdots,g_u=0}^{\infty} \tag{1.9}$$

and

$$u_i\neq 0,\sum_{j=1}^{A}\gamma_j^{(i)}-\sum_{j=1}^{C}\xi_j^{(i)}+\sum_{j=1}^{M^{(i)}}\eta_j^{(i)}-\sum_{j=1}^{N^{(i)}}\epsilon_j^{(i)}<0,i=1,\cdots,u \tag{1.10}$$

Here $\lambda,A,C,\alpha_i,\beta_i,M_i,N_i\in\mathbb{N}^*;i=1,\cdots,u;f_j,g_j,p_j^{(i)},q_j^{(i)},\gamma_j^{(i)},\xi_j^{(i)},\eta_j^{(i)},\epsilon_j^{(i)}\in\mathbb{C}$

The modified H-function defined by Prasad and Singh [6] generalizes the multivariable H-function defined by Srivastava and Panda [10,11]. It is defined in term of multiple Mellin-Barnes type integral :

$$H(z_1,\cdots,z_r)=H^{\mathbf{m},\mathbf{n}:|\mathbf{R}:m_1,n_1;\cdots;m_r,n_r}_{\mathbf{p},\mathbf{q}:|R:p_1,q_1;\cdots,p_r,q_r}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathbf{p}}: \\ \cdot & \\ \cdot & \\ \cdot & (\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(r)})_{1,\mathbf{q}}: \\ \mathrm{z}_r & \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{e}_j;u_j^{(1)}g_j^{(1)},\cdots,u_j^{(r)}g_j^{(r)})_{1,\mathbf{R}}:(c_j^{(1)};\gamma_j^{(1)})_{1,p_1},\cdots,(c_j^{(r)};\gamma_j^{(r)})_{1,p_r} \\ \\ (\mathrm{l}_j;U_j^{(1)}f_j^{(1)},\cdots,U_j^{(r)}f_j^{(r)})_{1,R}:(d_j^{(1)};\delta_j^{(1)})_{1,q_1},\cdots,(d_j^{(r)};\delta_j^{(r)})_{1,q_r} \end{array}\right) \tag{1.11}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\theta(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.12}$$

where $\phi(s_1,\cdots,s_r), \theta_i(s_i), i=1,\cdots,r$ are given by :

$$\theta(s_1,\cdots,s_r) = \frac{\prod_{j=1}^{\mathbf{m}}\Gamma(b_j-\sum_{i=1}^{r}\beta_j^{(i)}s_i)\prod_{j=1}^{\mathbf{n}}\Gamma\left(1-a_j+\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)}{\prod_{j=\mathbf{n}+1}^{\mathbf{p}}\Gamma\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)\prod_{j=\mathbf{m}+1}^{\mathbf{q}}\Gamma\left(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}s_j\right)}$$

$$\frac{\prod_{j=1}^{\mathbf{R}}\Gamma(e_j+\sum_{i=1}^{r}u_j^{(i)}g_j^{(i)}s_i)}{\prod_{j=1}^{R}\Gamma(l_j+\sum_{i=1}^{r}U_j^{(i)}f_j^{(i)}s_i)} \tag{1.13}$$

$$\theta_i(s_i) = \frac{\prod_{j=1}^{n_i}\Gamma\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)\prod_{j=1}^{m_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)}{\prod_{j=n_i+1}^{p_i}\Gamma\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)\prod_{j=m_i+1}^{q_i}\Gamma\left(1-d_j^{(i)}+\delta_j^{(i)}s_i\right)} \tag{1.14}$$

The multiple integral (1.12) converges absolutely if

$$|argz_i| < \frac{1}{2}U_i\pi \quad (i=1,\cdots,r) \tag{1.15}$$

$$\text{with} \qquad U_i = \sum_{j=1}^{\mathbf{m}}\beta_j^{(i)} - \sum_{j=\mathbf{m}+1}^{\mathbf{q}}\beta_j^{(i)} + \sum_{j=1}^{\mathbf{n}}\alpha_j^{(i)} - \sum_{j=\mathbf{n}+1}^{\mathbf{p}}\alpha_j^{(i)} + \sum_{j=1}^{m_i}\delta_j^{(i)} - \sum_{j=1+m_i}^{q_i}\delta_j^{(i)}$$

$$+\sum_{j=1}^{n_i}\gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i}\gamma_j^{(i)} + \sum_{j=1}^{\mathbf{R}}g_j^{(i)} - \sum_{j=1}^{R}f_j^{(i)} > 0(i=1,\cdots,r) \tag{1.16}$$

For more details ,see Prasad and Singh [6].

## 2. Required integral

The following formula that is t-dimensional analogue of a known result [1, p.172] will be required in the sequel :

**Lemma**

$$\int_0^\infty\cdots\int_0^\infty x_1^{\rho_1-1}\cdots x_t^{\rho_t-1}f\left(\sum_{k=1}^{t}A_kx_k^{\gamma_k}\right)\mathrm{d}x_1\cdots\mathrm{d}x_t = \frac{\prod_{k=1}^{t}\frac{1}{\gamma_k}\Gamma\left(\frac{\rho_k}{\gamma_k}\right)A^{-\frac{\rho_k}{\gamma_k}}}{\Gamma\left(\sum_{k=1}^{t}\frac{\rho_k}{\gamma_k}\right)}\int_0^\infty z^{\sum_{k=1}^{t}\left(\frac{\rho_k}{\gamma_k}\right)-1}f(z)\mathrm{d}z \tag{2.1}$$

where $min\{A_k,\gamma_k,Re(\rho_k)\} > 0(k=1,\cdots,t)$

## 3. Main integral

Let

$$X = m_1,n_1;\cdots;m_r,n_r;M,0 \tag{3.1}$$

$$Y = p_1,q_1;\cdots;p_r,q_r;P,Q \tag{3.2}$$

$$\mathbb{A} = (a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)},0)_{1,\mathbf{p}};(\mathrm{e}_j;u_j^{(1)}g_j^{(1)},\cdots,u_j^{(r)}g_j^{(r)},0)_{1,\mathbf{R}}:(\mathrm{c}_j^{(1)};\gamma_j^{(1)})_{1,p_1};\cdots,(c_j^{(r)};\gamma_j^{(r)})_{1,p_r};(g_j,G_j)_{1,P} \tag{3.3}$$

$$\mathbb{B} = (b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)},0)_{1,\mathbf{q}};(\mathrm{l}_j;U_j^{(1)}f_j^{(1)},\cdots,U_j^{(r)}f_j^{(r)},0)_{1,R}:(d_j^{(1)};\delta_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)};\delta_j^{(r)})_{1,q_r};(h_j,H_j)_{1,Q} \tag{3.4}$$

We have the following unified multiple integral

**Theorem**

$$\int_0^\infty \cdots \int_0^\infty x_1^{\rho_1-1}\cdots x_t^{\rho_t-1}\left\{\sum_{k=1}^t A_k x_k^{\gamma_k}\right\}^\sigma S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\begin{pmatrix} \mathrm{y}_1\prod_{k=1}^t x_k^{u_k^{(1)}}\left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right]^{w^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{y}_v\prod_{k=1}^t x_k^{u_k^{(v)}}\left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right]^{w^{(v)}} \end{pmatrix}$$

$$A\begin{pmatrix} \mathrm{z}_1\prod_{k=1}^t x_k^{v_k^{(1)}}\left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right]^{w'^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_u\prod_{k=1}^t x_k^{v_k^{(u)}}\left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right]^{w'^{(u)}} \end{pmatrix} H_{\mathbf{p},\mathbf{q}:|R:Y}^{\mathbf{m},\mathbf{n}:|\mathbf{R}:X}\left(\begin{array}{c|c} \mathrm{z}'_1\prod_{k=1}^t x_k^{\mu_k^{(1)}}\left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right]^{w''^{(1)}} & \mathbb{A} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}'_r\prod_{k=1}^t x_k^{\mu_k^{(r)}}\left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right]^{w''^{(r)}} & \cdot \\ \zeta\left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right] & \mathbb{B} \end{array}\right)$$

$$\mathrm{d}x_1\cdots\mathrm{d}x_t = \zeta^\sigma\prod_{k=1}^t\frac{(A_k\zeta)^{-\frac{\rho_k}{\gamma_k}}}{\gamma_k}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=0}^{\infty}\phi\frac{\prod_{i=1}^u\phi_k z_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^u g_i}}{\prod_{i=1}^u\delta_{G^{(i)}}^{(i)}\prod_{i=1}^u g_i!}a'_v y_1^{K_1}\cdots y_v^{K_v}$$

$$\left[\prod_{i=1}^v\zeta^{-w^{(i)}}\prod_{k=1}^t(A_k\zeta)^{-\frac{u_k^{(i)}}{\gamma_k}}\right]^{K_i}\left[\prod_{j=1}^u\zeta^{-w'^{(j)}}\prod_{k=1}^t(A_k\zeta)^{-\frac{v_k^{(j)}}{\gamma_k}}\right]^{\eta_{G_j,g_j}}$$

$$H_{\mathbf{p}+Q+t,\mathbf{q}+P+1:|R:Y'}^{\mathbf{m},\mathbf{n}+t+M:|\mathbf{R}:X'}\left(\begin{array}{c|c} \mathrm{z}'_1\prod_{k=1}^t(A_k\zeta)^{\frac{\mu^{(1)}}{\gamma_k}}\zeta^{-w''^{(1)}} & \mathrm{A'} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}'_r\prod_{k=1}^t(A_k\zeta)^{\frac{\mu^{(r)}}{\gamma_k}}\zeta^{-w''^{(r)}} & \mathrm{B'} \end{array}\right) \tag{3.5}$$

where

$$X' = m_1, n_1; \cdots; m_r, n_r \tag{3.6}$$

$$Y' = p_1, q_1; \cdots; p_r, q_r \tag{3.7}$$

$$A' = \left(1-\frac{\rho_l}{\gamma_l}-\sum_{i=1}^v\frac{u_l^{(i)}K_i}{\gamma_l}-\sum_{j=1}^u\frac{v_l^{(j)}\eta_{G_j,g_j}}{\gamma_l};\frac{\mu_l^{(1)}}{\gamma_l},\cdots,\frac{\mu_l^{(r)}}{\gamma_l}\right)_{1,r},$$

$$\left(1-h_l-H_l\left[\sum_{k=1}^t\left(\frac{\rho_k}{\gamma_k}+\sum_{i=1}^v\frac{u_k^{(i)}K_i}{\gamma_k}+\sum_{j=1}^u\frac{v_k^{(j)}\eta_{G_j,g_j}}{\gamma_k}\right)+\sum_{i=1}^v w^{(i)}K_i+\sum_{j=1}^u w'^{(j)}\eta_{G_j,g_j}+\sigma\right];\right.$$

$$H_l\left[\sum_{k=1}^{t}\left(\frac{\mu_k^{(1)}}{\gamma_k}\right)+w''^{(1)}\right],\cdots,H_l\left[\sum_{k=1}^{t}\left(\frac{\mu_k^{(r)}}{\gamma_k}\right)+w''^{(r)}\right]\Bigg)_{1,Q},$$

$$(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathbf{p}};(\mathrm{e}_j;u_j^{(1)}g_j^{(1)},\cdots,u_j^{(r)}g_j^{(r)})_{1,\mathbf{R}}:(\mathrm{c}_j^{(1)};\gamma_j^{(1)})_{1,p_1};\cdots,(c_j^{(r)};\gamma_j^{(r)})_{1,p_r} \tag{3.8}$$

$$B'=\left(1-\sum_{k=1}^{t}\left(\frac{\rho_k}{\gamma_k}+\sum_{i=1}^{v}\frac{u_k^{(i)}K_i}{\gamma_k}+\sum_{j=1}^{u}\frac{\upsilon_k^{(j)}\eta_{G_j,g_j}}{\gamma_k}\right);\sum_{k=1}^{t}\frac{\mu_k^{(1)}}{\gamma_k},\cdots,\sum_{k=1}^{t}\frac{\mu_k^{(r)}}{\gamma_k}\right),$$

$$\left(1-g_l-G_l\left[\sum_{k=1}^{t}\left(\frac{\rho_k}{\gamma_k}+\sum_{i=1}^{v}\frac{u_k^{(i)}K_i}{\gamma_k}+\sum_{j=1}^{u}\frac{\upsilon_k^{(j)}\eta_{G_j,g_j}}{\gamma_k}\right)+\sum_{i=1}^{v}w^{(i)}K_i+\sum_{j=1}^{u}w'^{(j)}\eta_{G_j,g_j}+\sigma\right];\right.$$

$$G_l\left[\sum_{k=1}^{t}\left(\frac{\mu_k^{(1)}}{\gamma_k}\right)+w''^{(1)}\right],\cdots,G_l\left[\sum_{k=1}^{t}\left(\frac{\mu_k^{(r)}}{\gamma_k}\right)+w''^{(r)}\right]\Bigg)_{1,P},$$

$$(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)})_{1,\mathbf{q}};(\mathrm{l}_j;U_j^{(1)}f_j^{(1)},\cdots,U_j^{(r)}f_j^{(r)})_{1,R}:(d_j^{(1)};\delta_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)};\delta_j^{(r)})_{1,q_r} \tag{3.9}$$

Under the following conditions

$$y_i\neq 0,\sum_{j=1}^{A}\gamma_j^{(i)}-\sum_{j=1}^{C}\xi_j^{(i)}+\sum_{j=1}^{M^{(i)}}\eta_j^{(i)}-\sum_{j=1}^{N^{(i)}}\epsilon_j^{(i)}<0,i=1,\cdots,v:Re(\rho_k)>0(k=1,\cdots,t)$$

$$max\{\sigma,A_k,\gamma_k,u_k^{(i)},\upsilon_k^{(j)},\mu_k^{(l)}\}>0;i=1,\cdots,v;j=1,\cdots,u;l=1,\cdots,r;k=1,\cdots,t$$

$$\min_{1\leqslant k\leqslant t}\left\{Re\left[\rho_k+\gamma_k\sigma+\sum_{i=1}^{u}\left(\sum_{j=1}^{\alpha^{(i)}}(\upsilon_k^{(i)}+\gamma_k w'^{(i)})\right)\eta_{G_j,g_j}\right]+\sum_{i=1}^{r}\sum_{j=1}^{m_i}(\mu_k^{(i)}+\gamma_k w''^{(i)})\min_{1\leqslant j\leqslant m_i}Re\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)+\min_{1\leqslant j\leqslant M}\gamma_k Re\left(\frac{h_j}{H_j}\right)\right\}\geqslant 0$$

$|argz_k|<\frac{1}{2}U_i\pi$, $U_i$ is defined by (1.16)

and the multiple series in the left-hand side of (3.8) converges absolutely.

**Proof**

To evaluate the main integral (3.5), first we replace the class of multivariable polynomials $S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[.]$ ,the multivariable A-function occuring in the left-hand side of the main integral in term of series with the help (1.1) and (1.5) respectively. Now we express the modified (r+1)-variables H-function in terms of the product of the Fox H-function and modified r-variables H-function in Mellin-Barnes contour integrals. Nest, we change the order of the $(s_1,\cdots,s_r)$-integrals and $(x_1,\cdots,x_t)$-integrals, ( which is justified under the conditions stated) and use the Lemma to obtain the following result (say L.H.S.) :

$$\text{L.H.S.}=\frac{1}{\gamma_k}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=0}^{\infty}\phi\frac{\prod_{i=1}^{u}\phi_k z_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^{u}g_i}}{\prod_{i=1}^{u}\delta_{G^{(i)}}^{(i)}\prod_{i=1}^{u}g_i!}a'_v y_1^{K_1}\cdots y_v^{K_v}$$

$$\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\theta(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i'^{s_i}\prod_{k=1}^{t}\left\{A_k^{-\eta_k}\Gamma(\eta_k)\right\}\Gamma(\sum_{k=1}^{t}\eta_k)$$

$$\left\{\int_0^{\infty} z^{\sum_{k=1}^{t}\eta_k+\sum_{i=1}^{v}w^{(i)}K_i+\sum_{j=1}^{u}w'^{(j)}\eta_{G_j,g_j}+\sum_{l=1}^{r}w''^{(l)}s_l+\sigma-1}\, H_{P,Q}^{M,0}\left(\zeta x \;\middle|\; \begin{matrix}(\mathrm{g}_j,G_j)_{1,P}\\(\mathrm{h}_j,H_j)_{1,Q}\end{matrix}\right)\mathrm{d}z\right\}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{3.10}$$

where

$$\eta_k=\frac{\rho_k+\sum_{i=1}^{v}u_k^{(i)}K_i+\sum_{j=1}^{u}\upsilon_k^{(j)}\eta_{G_j,g_j}+\sum_{l=1}^{r}\mu_k^{(l)}s_l}{\gamma_k} \tag{3.11}$$

Now evaluating the z-integral with the help of a result [9, p.15. Eq.(2.4.1)] and reinterpreting the result thus obtained in terms of modified multivariable H-function, we get the equation (3.5) after algebric manipulations.

## 4. Corollaries

Let $S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[.]\rightarrow S_{\mathfrak{N}}^{\mathfrak{M}}[.]$ [7], we obtain the following formula

**Corollary 1**

$$\int_0^{\infty}\cdots\int_0^{\infty}x_1^{\rho_1-1}\cdots x_t^{\rho_t-1}\left\{\sum_{k=1}^{t}A_kx_k^{\gamma_k}\right\}^{\sigma}S_{\mathfrak{N}}^{\mathfrak{M}}\left(\mathrm{y}\prod_{k=1}^{t}x_k^{u_k}\left[\sum_{k=1}^{t}A_kx_k^{\gamma_k}\right]^{w}\right)$$

$$A\left(\begin{matrix}\mathrm{z}_1\prod_{k=1}^{t}x_k^{\upsilon_k^{(1)}}\left[\sum_{k=1}^{t}A_kx_k^{\gamma_k}\right]^{w'^{(1)}}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_u\prod_{k=1}^{t}x_k^{\upsilon_k^{(u)}}\left[\sum_{k=1}^{t}A_kx_k^{\gamma_k}\right]^{w'^{(u)}}\end{matrix}\right)H_{\mathbf{p},\mathbf{q}:|R:Y}^{\mathbf{m},\mathbf{n}:|\mathbf{R}:X}\left(\begin{matrix}\mathrm{z}_1'\prod_{k=1}^{t}x_k^{\mu_k^{(1)}}\left[\sum_{k=1}^{t}A_kx_k^{\gamma_k}\right]^{w''^{(1)}}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r'\prod_{k=1}^{t}x_k^{\mu_k^{(r)}}\left[\sum_{k=1}^{t}A_kx_k^{\gamma_k}\right]^{w''^{(r)}}\\ \zeta\left[\sum_{k=1}^{t}A_kx_k^{\gamma_k}\right]\end{matrix}\;\middle|\;\begin{matrix}\mathbb{A}\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ \mathbb{B}\end{matrix}\right)$$

$$\mathrm{d}x_1\cdots\mathrm{d}x_t=\zeta^{\sigma}\prod_{k=1}^{t}\frac{(A_k\zeta)^{-\frac{\rho_k}{\gamma_k}}}{\gamma_k}\sum_{K=0}^{[\mathfrak{N}/\mathfrak{M}]}\sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=0}^{\infty}\phi\frac{\prod_{i=1}^{u}\phi_k z_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^{u}g_i}}{\prod_{i=1}^{u}\delta_{G^{(i)}}^{(i)}\prod_{i=1}^{u}g_i!}\frac{(-\mathfrak{N})_{\mathfrak{M}K}A_{\mathfrak{N},K}}{K!}y^K$$

$$\zeta^{-w}\prod_{k=1}^{t}(A_k\zeta)^{-\frac{u_k}{\gamma_k}K}\left[\prod_{j=1}^{u}\zeta^{-w'^{(j)}}\prod_{k=1}^{t}(A_k\zeta)^{-\frac{\upsilon_k^{(j)}}{\gamma_k}}\right]^{\eta_{G_j,g_j}}$$

$$H_{\mathbf{p}+Q+t,\mathbf{q}+P+1:|R:Y'}^{\mathbf{m},\mathbf{n}+t+M:|\mathbf{R}:X'}\left(\begin{matrix}\mathrm{z}_1'\prod_{k=1}^{t}(A_k\zeta)^{\frac{\mu_k^{(1)}}{\gamma_k}}\zeta^{-w''^{(1)}}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r'\prod_{k=1}^{t}(A_k\zeta)^{\frac{\mu_k^{(r)}}{\gamma_k}}\zeta^{-w''^{(r)}}\end{matrix}\;\middle|\;\begin{matrix}\mathrm{A''}\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{B''}\end{matrix}\right) \tag{4.1}$$

where

$$A''=\left(1-\frac{\rho_l}{\gamma_l}-\frac{u_lK}{\gamma_l}-\sum_{j=1}^{u}\frac{\upsilon_l^{(j)}\eta_{G_j,g_j}}{\gamma_l};\frac{\mu_l^{(1)}}{\gamma_l},\cdots,\frac{\mu_l^{(r)}}{\gamma_l}\right)_{1,r},$$

$$\left(1-h_l-H_l\left[\sum_{k=1}^{t}\left(\frac{\rho_k}{\gamma_k}+\frac{u_kK}{\gamma_k}+\sum_{j=1}^{u}\frac{\upsilon_k^{(j)}\eta_{G_j,g_j}}{\gamma_k}\right)+wK+\sum_{j=1}^{u}w'^{(j)}\eta_{G_j,g_j}+\sigma\right];\right.$$

$$\left.H_l\left[\sum_{k=1}^{t}\left(\frac{\mu_k^{(1)}}{\gamma_k}\right)+w''^{(1)}\right],\cdots,H_l\left[\sum_{k=1}^{t}\left(\frac{\mu_k^{(r)}}{\gamma_k}\right)+w''^{(r)}\right]\right)_{1,Q},$$

$$(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathbf{p}};(\mathrm{e}_j;u_j^{(1)}g_j^{(1)},\cdots,u_j^{(r)}g_j^{(r)})_{1,\mathbf{R}}:(\mathrm{c}_j^{(1)};\gamma_j^{(1)})_{1,p_1};\cdots,(c_j^{(r)};\gamma_j^{(r)})_{1,p_r} \tag{4.2}$$

$$B''=\left(1-\sum_{k=1}^{t}\left(\frac{\rho_k}{\gamma_k}+\frac{u_kK}{\gamma_k}+\sum_{j=1}^{u}\frac{\upsilon_k^{(j)}\eta_{G_j,g_j}}{\gamma_k}\right);\sum_{k=1}^{t}\frac{\mu_k^{(1)}}{\gamma_k},\cdots,\sum_{k=1}^{t}\frac{\mu_k^{(r)}}{\gamma_k}\right),$$

$$\left(1-g_l-G_l\left[\sum_{k=1}^{t}\left(\frac{\rho_k}{\gamma_k}+\frac{u_kK}{\gamma_k}+\sum_{j=1}^{u}\frac{\upsilon_k^{(j)}\eta_{G_j,g_j}}{\gamma_k}\right)+wK+\sum_{j=1}^{u}w'^{(j)}\eta_{G_j,g_j}+\sigma\right];\right.$$

$$\left.G_l\left[\sum_{k=1}^{t}\left(\frac{\mu_k^{(1)}}{\gamma_k}\right)+w''^{(1)}\right],\cdots,G_l\left[\sum_{k=1}^{t}\left(\frac{\mu_k^{(r)}}{\gamma_k}\right)+w''^{(r)}\right]\right)_{1,P},$$

$$(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)})_{1,\mathbf{q}};(\mathrm{l}_j;U_j^{(1)}f_j^{(1)},\cdots,U_j^{(r)}f_j^{(r)})_{1,R}:(d_j^{(1)};\delta_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)};\delta_j^{(r)})_{1,q_r} \tag{4.3}$$

where the same conditions that (3.5) and

$max\{\sigma,A_k,\gamma_k,u_k,\upsilon_k^{(j)},\mu_k^{(l)}\}>0;j=1,\cdots,u;l=1,\cdots,r;k=1,\cdots,t$

Consider the above corollary. If the multivariable A-function reduces to A-function defined by Gautam and Asgar [3], we obtain :

**Corollary 2**

$$\int_0^\infty\cdots\int_0^\infty x_1^{\rho_1-1}\cdots x_t^{\rho_t-1}\left\{\sum_{k=1}^{t}A_kx_k^{\gamma_k}\right\}^{\sigma}S_{\mathfrak{N}}^{\mathfrak{M}}\left(\ \mathrm{y}\prod_{k=1}^{t}x_k^{u_k}\left[\sum_{k=1}^{t}A_kx_k^{\gamma_k}\right]^{w}\ \right)$$

$$A\left(\ \mathrm{z}\prod_{k=1}^{t}x_k^{\upsilon_k}\left[\sum_{k=1}^{t}A_kx_k^{\gamma_k}\right]^{w'}\ \right)H_{\mathbf{p},\mathbf{q}:|R:Y}^{\mathbf{m},\mathbf{n}:|\mathbf{R}:X}\left(\begin{array}{c|c}\mathrm{z}'_1\prod_{k=1}^{t}x_k^{\mu_k^{(1)}}\left[\sum_{k=1}^{t}A_kx_k^{\gamma_k}\right]^{w''^{(1)}} & \mathbb{A}\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}'_r\prod_{k=1}^{t}x_k^{\mu_k^{(r)}}\left[\sum_{k=1}^{t}A_kx_k^{\gamma_k}\right]^{w''^{(r)}} & \cdot\\ \zeta\left[\sum_{k=1}^{t}A_kx_k^{\gamma_k}\right] & \mathbb{B}\end{array}\right)$$

$$\mathrm{d}x_1\cdots\mathrm{d}x_t=\zeta^{\sigma}\prod_{k=1}^{t}\frac{(A_k\zeta)^{-\frac{\rho_k}{\gamma_k}}}{\gamma_k}\sum_{K=0}^{[\mathfrak{N}/\mathfrak{M}]}\sum_{G=1}^{\alpha}\sum_{g=0}^{\infty}\phi_1\frac{z_k^{\eta_{G,g}}(-)^g}{\delta_Gg!}\frac{(-\mathfrak{N})_{\mathfrak{M}K}A_{\mathfrak{N},K}}{K!}y^K$$

$$\zeta^{-w}\prod_{k=1}^{t}(A_k\zeta)^{-\frac{u_k}{\gamma_k}\,K}\zeta^{-w'}\prod_{k=1}^{t}(A_k\zeta)^{-\frac{\upsilon_k}{\gamma_k}\,\eta_{G,g}}H^{\mathbf{m},\mathbf{n}+t+M:|\mathbf{R}:X'}_{\mathbf{p}+Q+t,\mathbf{q}+P+1:|R:Y'}\left(\begin{array}{c|c} z'_1\prod_{k=1}^{t}(A_k\zeta)^{\frac{\mu^{(1)}}{\gamma_k}}\zeta^{-w''^{(1)}} & \text{A"'} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ z'_r\prod_{k=1}^{t}(A_k\zeta)^{\frac{\mu^{(r)}}{\gamma_k}}\zeta^{-w''^{(r)}} & \text{B"'} \end{array}\right) \quad (4.4)$$

where

$$A''' = \left(1-\frac{\rho_l}{\gamma_l}-\frac{u_lK}{\gamma_l}-\frac{\upsilon_l\eta_{G,g}}{\gamma_l};\frac{\mu_l^{(1)}}{\gamma_l},\cdots,\frac{\mu_l^{(r)}}{\gamma_l}\right)_{1,r},$$

$$\left(1-h_l-H_l\left[\sum_{k=1}^{t}\left(\frac{\rho_k}{\gamma_k}+\frac{u_kK}{\gamma_k}+\frac{\upsilon_k\eta_{G,g}}{\gamma_k}\right)+wK+w'\eta_{G,g}+\sigma\right];\right.$$

$$\left.H_l\left[\sum_{k=1}^{t}\left(\frac{\mu_k^{(1)}}{\gamma_k}\right)+w''^{(1)}\right],\cdots,H_l\left[\sum_{k=1}^{t}\left(\frac{\mu_k^{(r)}}{\gamma_k}\right)+w''^{(r)}\right]\right)_{1,Q},$$

$$(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathbf{p}};(\mathrm{e}_j;u_j^{(1)}g_j^{(1)},\cdots,u_j^{(r)}g_j^{(r)})_{1,\mathbf{R}}:(\mathrm{c}_j^{(1)};\gamma_j^{(1)})_{1,p_1};\cdots,(c_j^{(r)};\gamma_j^{(r)})_{1,p_r} \quad (4.5)$$

$$B''' = \left(1-\sum_{k=1}^{t}\left(\frac{\rho_k}{\gamma_k}+\frac{u_kK}{\gamma_k}+\frac{\upsilon_k\eta_{G,g}}{\gamma_k}\right);\sum_{k=1}^{t}\frac{\mu_k^{(1)}}{\gamma_k},\cdots,\sum_{k=1}^{t}\frac{\mu_k^{(r)}}{\gamma_k}\right),$$

$$\left(1-g_l-G_l\left[\sum_{k=1}^{t}\left(\frac{\rho_k}{\gamma_k}+\frac{u_kK}{\gamma_k}+\frac{\upsilon_k\eta_{G,g}}{\gamma_k}\right)+wK+w'\eta_{G,g}+\sigma\right];\right.$$

$$\left.G_l\left[\sum_{k=1}^{t}\left(\frac{\mu_k^{(1)}}{\gamma_k}\right)+w''^{(1)}\right],\cdots,G_l\left[\sum_{k=1}^{t}\left(\frac{\mu_k^{(r)}}{\gamma_k}\right)+w''^{(r)}\right]\right)_{1,P},$$

$$(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)})_{1,\mathbf{q}};(\mathrm{l}_j;U_j^{(1)}f_j^{(1)},\cdots,U_j^{(r)}f_j^{(r)})_{1,R}:(d_j^{(1)};\delta_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)};\delta_j^{(r)})_{1,q_r} \quad (4.6)$$

where the same conditions that (3.5) and

$max\{\sigma,A_k,\gamma_k,u_k,\upsilon_k,\mu_k^{(l)}\}>0;l=1,\cdots,r;k=1,\cdots,t$ and

$$\min_{1\leqslant k\leqslant t}Re\left\{\left[\rho_k+\gamma_k\sigma+(\upsilon_k^{(i)}+\gamma_kw'^{(i)})\eta_{G,g}\right]+\sum_{i=1}^{r}\sum_{j=1}^{m_i}(\mu_k^{(i)}+\gamma_kw''^{(i)})\min_{1\leqslant j\leqslant m_i}Re\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)+\min_{1\leqslant j\leqslant M}\gamma_kRe\left(\frac{h_j}{H_j}\right)\right\}\geqslant 0$$

Consider the above corollary. If the modified multivariable H-function reduces to Multivariable H-function, we obtain :

**Corollary 3**

$$\int_0^{\infty}\cdots\int_0^{\infty}x_1^{\rho_1-1}\cdots x_t^{\rho_t-1}\left\{\sum_{k=1}^{t}A_kx_k^{\gamma_k}\right\}^{\sigma}S_{\mathfrak{N}}^{\mathfrak{M}}\left(\ \mathrm{y}\prod_{k=1}^{t}x_k^{u_k}\left[\sum_{k=1}^{t}A_kx_k^{\gamma_k}\right]^{w}\ \right)$$

$$A\left( \text{ z } \prod_{k=1}^t x_k^{\upsilon_k} \left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right]^{w'} \right) H^{0,\mathbf{n}:X}_{\mathbf{p},\mathbf{q}:Y} \left( \begin{array}{c|c} \text{z}'_1 \prod_{k=1}^t x_k^{\mu_k^{(1)}} \left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right]^{w''^{(1)}} & \text{A}_1 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \text{z}'_r \prod_{k=1}^t x_k^{\mu_k^{(r)}} \left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right]^{w''^{(r)}} & \cdot \\ \zeta \left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right] & \text{B}_1 \end{array} \right)$$

$$\mathrm{d}x_1 \cdots \mathrm{d}x_t = \zeta^{\sigma} \prod_{k=1}^t \frac{(A_k\zeta)^{-\frac{\rho_k}{\gamma_k}}}{\gamma_k} \sum_{K=0}^{[\mathfrak{N}/\mathfrak{M}]} \sum_{G=1}^{\alpha} \sum_{g=0}^{\infty} \phi_1 \frac{z^{\eta_{G,g}}(-)^g}{\delta_G g!} \frac{(-\mathfrak{N})_{\mathfrak{M}K} A_{\mathfrak{N},K}}{K!} y^K$$

$$\zeta^{-w} \prod_{k=1}^t (A_k\zeta)^{-\frac{u_k}{\gamma_k}K} \zeta^{-w'} \prod_{k=1}^t (A_k\zeta)^{-\frac{\upsilon_k}{\gamma_k}\eta_{G,g}} H^{0,\mathbf{n}+t+M:X'}_{\mathbf{p}+Q+t,\mathbf{q}+P+1:Y'} \left( \begin{array}{c|c} \text{z}'_1 \prod_{k=1}^t (A_k\zeta)^{\frac{\mu^{(1)}}{\gamma_k}} \zeta^{-w''^{(1)}} & \text{A}'_1 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \text{z}'_r \prod_{k=1}^t (A_k\zeta)^{\frac{\mu^{(r)}}{\gamma_k}} \zeta^{-w''^{(r)}} & \text{B}'_1 \end{array} \right) \tag{4.7}$$

where

$$A'_1 = \left(1 - \frac{\rho_l}{\gamma_l} - \frac{u_l K}{\gamma_l} - \frac{\upsilon_l \eta_{G,g}}{\gamma_l}; \frac{\mu_l^{(1)}}{\gamma_l}, \cdots, \frac{\mu_l^{(r)}}{\gamma_l}\right)_{1,r},$$

$$\left(1 - h_l - H_l \left[\sum_{k=1}^t \left(\frac{\rho_k}{\gamma_k} + \frac{u_k K}{\gamma_k} + \frac{\upsilon_k \eta_{G,g}}{\gamma_k}\right) + wK + w'\eta_{G,g} + \sigma\right];\right.$$

$$\left. H_l \left[\sum_{k=1}^t \left(\frac{\mu_k^{(1)}}{\gamma_k}\right) + w''^{(1)}\right], \cdots, H_l \left[\sum_{k=1}^t \left(\frac{\mu_k^{(r)}}{\gamma_k}\right) + w''^{(r)}\right]\right)_{1,Q},$$

$$(a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)})_{1,\mathbf{p}}; (\text{c}_j^{(1)}; \gamma_j^{(1)})_{1,p_1}; \cdots, (c_j^{(r)}; \gamma_j^{(r)})_{1,p_r} \tag{4.8}$$

$$B'_1 = \left(1 - \sum_{k=1}^t \left(\frac{\rho_k}{\gamma_k} + \frac{u_k K}{\gamma_k} + \frac{\upsilon_k \eta_{G,g}}{\gamma_k}\right); \sum_{k=1}^t \frac{\mu_k^{(1)}}{\gamma_k}, \cdots, \sum_{k=1}^t \frac{\mu_k^{(r)}}{\gamma_k}\right),$$

$$\left(1 - g_l - G_l \left[\sum_{k=1}^t \left(\frac{\rho_k}{\gamma_k} + \frac{u_k K}{\gamma_k} + \frac{\upsilon_k \eta_{G,g}}{\gamma_k}\right) + wK + w'\eta_{G,g} + \sigma\right];\right.$$

$$\left. G_l \left[\sum_{k=1}^t \left(\frac{\mu_k^{(1)}}{\gamma_k}\right) + w''^{(1)}\right], \cdots, G_l \left[\sum_{k=1}^t \left(\frac{\mu_k^{(r)}}{\gamma_k}\right) + w''^{(r)}\right]\right)_{1,P},$$

$$(b_j; \beta_j^{(1)}, \cdots, \beta_j^{(r)})_{1,\mathbf{q}}; (d_j^{(1)}; \delta_j^{(1)})_{1,q_1}; \cdots; (d_j^{(r)}; \delta_j^{(r)})_{1,q_r} \tag{4.9}$$

where the same conditions that (3.5) and

$max\{\sigma, A_k, \gamma_k, u_k, \upsilon_k, \mu_k^{(l)}\} > 0; l = 1, \cdots, r; k = 1, \cdots, t$

$$\min_{1\leqslant k\leqslant t}\left\{Re\left[\rho_k+\gamma_k\sigma+(\upsilon_k^{(i)}+\gamma_k w'^{(i)})\eta_{G,g}\right]+\sum_{i=1}^{r}\sum_{j=1}^{m_i}(\mu_k^{(i)}+\gamma_k w''^{(i)})\min_{1\leqslant j\leqslant m_i} Re\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)+\min_{1\leqslant j\leqslant M}\gamma_k Re\left(\frac{h_j}{H_j}\right)\right\}\geqslant 0$$

and $|argz_k| < \frac{1}{2}U_i\pi$

where $U_i \;=\sum_{j=1}^{\mathbf{n}}\alpha_j^{(i)}-\sum_{j=\mathbf{n}+1}^{\mathbf{p}}\alpha_j^{(i)}+\sum_{j=1}^{m_i}\delta_j^{(i)}-\sum_{j=1+m_i}^{q_i}\delta_j^{(i)}+\sum_{j=1}^{n_i}\gamma_j^{(i)}-\sum_{j=n_i+1}^{p_i}\gamma_j^{(i)}>0(i=1,\cdots,r)$

## 5. Particular cases

In this section, we shall the corollary 3

1) By applying our result given in (4.7) to the case of Hermite polynomials ([13], page 106, eq.(5.54)) and ([12] ,page 158) and by setting

$$S_N^2 x) \to x^{n/2} H_n\left(\frac{1}{2\sqrt{x}}\right)$$

we obtain

$$\int_0^\infty\cdots\int_0^\infty x_1^{\rho_1-1}\cdots x_t^{\rho_t-1}\left\{\sum_{k=1}^t A_k x_k^{\gamma_k}\right\}^\sigma \left[y\prod_{k=1}^t x_k^{u_k}\left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right]\right]^{n/2} H_n\left(\left[\frac{y}{2}\prod_{k=1}^t x_k^{u_k}\left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right]^{w}\right]^{-1/2}\right)$$

$$A\left(\text{ z }\prod_{k=1}^t x_k^{\upsilon_k}\left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right]^{w'}\right) H_{\mathbf{p},\mathbf{q}:Y}^{0,\mathbf{n}:X}\left(\begin{array}{c|c} \text{z}'_1\prod_{k=1}^t x_k^{\mu_k^{(1)}}\left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right]^{w''^{(1)}} & \text{A}_1 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \text{z}'_r\prod_{k=1}^t x_k^{\mu_k^{(r)}}\left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right]^{w''^{(r)}} & \cdot \\ \zeta\left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right] & \text{B}_1 \end{array}\right)$$

$$\mathrm{d}x_1\cdots\mathrm{d}x_t = \zeta^\sigma\prod_{k=1}^t\frac{(A_k\zeta)^{-\frac{\rho_k}{\gamma_k}}}{\gamma_k}\sum_{K=0}^{[n/2]}\sum_{G=1}^{\alpha}\sum_{g=0}^{\infty}\phi_1\frac{z^{\eta_{G,g}}(-)^g}{\delta_G g!}\frac{(-n)_{2K}}{K!}(-y)^K$$

$$\zeta^{-w}\prod_{k=1}^t(A_k\zeta)^{-\frac{u_k}{\gamma_k}K}\zeta^{-w'}\prod_{k=1}^t(A_k\zeta)^{-\frac{\upsilon_k}{\gamma_k}\eta_{G,g}} H_{\mathbf{p}+Q+t,\mathbf{q}+P+1:Y'}^{0,\mathbf{n}+t+M:X'}\left(\begin{array}{c|c} \text{z}'_1\prod_{k=1}^t(A_k\zeta)^{\frac{\mu^{(1)}}{\gamma_k}}\zeta^{-w''^{(1)}} & \text{A}'_1 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \text{z}'_r\prod_{k=1}^t(A_k\zeta)^{\frac{\mu^{(r)}}{\gamma_k}}\zeta^{-w''^{(r)}} & \text{B}'_1 \end{array}\right) \qquad (5.1)$$

under the same conditions that (4.7).

2) By applying our result given in (4.7) to the case the Laguerre polynomials ([13], page 101, eq.(15.1.6)) and ([12] ,page 159) and by setting

$$S_N^1(x) \to L_N^{\alpha'}(x)$$

In which case $\mathfrak{M}=1, A_{N,K}=\binom{N+\alpha'}{N}\frac{1}{(\alpha'+1)_K}$ we have the following interesting consequencies of the main results.

we obtain

$$\int_0^\infty\cdots\int_0^\infty x_1^{\rho_1-1}\cdots x_t^{\rho_t-1}\left\{\sum_{k=1}^t A_k x_k^{\gamma_k}\right\}^\sigma \quad L_N^{\alpha'}\left(\frac{y}{2}\prod_{k=1}^t x_k^{u_k}\left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right]^w\right)$$

$$A\left(\ z\prod_{k=1}^t x_k^{v_k}\left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right]^{w'}\ \right)H_{\mathbf{p},\mathbf{q}:Y}^{0,\mathbf{n}:X}\left(\begin{array}{c|c} \mathrm{z}_1'\prod_{k=1}^t x_k^{\mu_k^{(1)}}\left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right]^{w''(1)} & \mathrm{A}_1\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_r'\prod_{k=1}^t x_k^{\mu_k^{(r)}}\left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right]^{w''(r)} & \cdot\\ \zeta\left[\sum_{k=1}^t A_k x_k^{\gamma_k}\right] & \mathrm{B}_1 \end{array}\right)$$

$$\mathrm{d}x_1\cdots\mathrm{d}x_t=\zeta^\sigma\prod_{k=1}^t\frac{(A_k\zeta)^{-\frac{\rho_k}{\gamma_k}}}{\gamma_k}\sum_{K=0}^N\sum_{G=1}^\alpha\sum_{g=0}^\infty\phi_1\frac{z^{\eta_{G,g}}(-)^g}{\delta_G g!}\frac{(-N)_K}{K!}\binom{N+\alpha'}{N-K}(-y)^K$$

$$\zeta^{-w}\prod_{k=1}^t(A_k\zeta)^{-\frac{u_k}{\gamma_k}K}\zeta^{-w'}\prod_{k=1}^t(A_k\zeta)^{-\frac{v_k}{\gamma_k}\eta_{G,g}}H_{\mathbf{p}+Q+t,\mathbf{q}+P+1:Y'}^{0,\mathbf{n}+t+M:X'}\left(\begin{array}{c|c} \mathrm{z}_1'\prod_{k=1}^t(A_k\zeta)^{\frac{\mu^{(1)}}{\gamma_k}}\zeta^{-w''(1)} & \mathrm{A}_1'\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_r'\prod_{k=1}^t(A_k\zeta)^{\frac{\mu^{(r)}}{\gamma_k}}\zeta^{-w''(r)} & \mathrm{B}_1' \end{array}\right) \tag{5.2}$$

under the same conditions that (4.7).

## 6. Conclusion

In this paper we have evaluated a unified multiple integral transformation involving the product of a modified multivariable H-functions defined by Prasad and Singh [6], a expansion of multivariable A-function defined by Gautam and Asgar [4] and a class of multivariable polynomials defined by Srivastava [8] with general arguments. The formula established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

## References


[1] J. Edwards, (1922), A treatise on integral calculus, Vol 2, Chelsea Publ. Co. New York.

[2] C. Fox, The G and H-functions as symmetrical Fourier Kernels, Trans. Amer. Math. Soc. 98 (1961), 395-429.

[3] B.P. Gautam, A.S. Asgar and A.N.Goyal. The A-function. Revista Mathematica. Tucuman (1980)

[4] B.P. Gautam, A.S. Asgar and Goyal A.N, On the multivariable A-function. Vijnana Parishas Anusandhan Patrika 29(4) 1986, 67-81.

[5] R. Jain and A. Sharma, A unified multiple integral transformation formula involving the multivariable H-function

and the product of general class of polynomials, J. Rajasthan Acad. Phy. Sci, 1(2) (2002), 95-102.

[6] Y.N. Prasad and A.K.Singh, Basic properties of the transform involving and H-function of r-variables as kernel. Indian Acad Math, (2) (1982), 109-115.

[7] H.M. Srivastava, A contour integral involving Fox's H-function. Indian. J. Math, (14) (1972), 1-6.

[8] H.M. Srivastava, A multilinear generating function for the Konhauser set of biorthogonal polynomials suggested by Laguerre polynomial, Pacific. J. Math. 177(1985), 183-191.

[9] H.M. Srivastava, K.C. Gupta and S.P. Goyal, (1982), The H-function of one and two variables with applications, South Asian Publisher, New Delhi and Madras.

[10] H.M.Srivastava and R. Panda, Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24 (1975),119-137.

[11] H.M.Srivastava and R.Panda, Some expansion theorems and generating relations for the H-function of several complex variables II. Comment. Math. Univ. St. Paul. 25 (1976), 167-197.

[12] H.M. Srivastava and N.P. Singh, The integration of certain products of the multivariable H-function with a general class of polynomials. Rend. Circ. Mat. Palermo. Vol 32 (No 2) (1983), 157-187.

[13] C. Szego, (1975), Orthogonal polynomials. Amer. Math. Soc. Colloq. Publ. 23 fourth edition. Amer. Math. Soc. Providence. Rhodes Island, 1975.