Int. Jr. of Mathematical Sciences & Applications
Vol. 8, No. 1, (January-June, 2018)
Copyright: Mind Reader Publications
ISSN No: 2230-9888
www.ijmsa.yolasite.com

New Class Of Contra-Continuous Functions in Soft Topology

January 29, 2018

*C.Santhini **M.Santhiyaa
*Associate Professor of Mathematics,
V.V.Vanniaperumal College For Women, Virudhunagar-626001,
Tamil Nadu, INDIA.
E-mail : santhinimdu@gmail.com
**Department of Mathematics,
V.V.Vanniaperumal College For Women, Virudhunagar-626001,
Tamil Nadu, INDIA.
E-mail :santhiyaamuthukrishnan@yahoo.in

Abstract:The aim of this paper is to introduce the concepts of soft contra $^{\#\#}g$-continuity and soft almost contra $^{\#\#}g$-continuity.Also a new class of set namely soft $^{\#\#}glc^{**}$ set is introduced along with its continuity and irresoluteness.Some of their characterizations are also studied.

Keywords and Phrases : Soft contra $^{\#\#}g$-continuous function, soft almost contra $^{\#\#}g$-continuous function, soft $^{\#\#}glc^{**}$ set, soft $^{\#\#}glc^{**}$-continuous function, soft $^{\#\#}glc^{**}$-irresolute function.

1 INTRODUCTION

Soft set theory was initiated by Molodtsov[11] as a new method for vagueness.Muhammad Shabir and Munazza naz [13] introduced soft topological spaces and studied some concepts of soft set such as soft interior,soft closure and soft separation axioms.In 1996 , Dontchev[5] introduced the notion of contra continuity.In 1968, M.K.Singal et al[15] introduced the concept of almost continuous mappings.The first step of locally closedness was done by Bourbaki[4].He defined a set A to be locally closed if it is the intersection of an open set and closed set.Recently soft locally closed sets in soft topological spaces was introduced by Ali Haydar Kocaman[1] .In this paper, the notion of soft $^{##}g$-closed sets in soft topological space is applied to present and study a new class of functions called soft contra$^{##}g$-continuous and soft almost contra $^{##}g$-continuous functions. Also new class of soft locally closed set namely soft $^{##}glc^{**}$ -set is introduced and their properties,characterizations are obtained.

2 PRELIMINARIES

Throughout the study (X, τ, E),(Y, σ, E) and (Z, τ, E)(or simply X, Y and Z) represent soft topological spaces on which no separation axioms are assumed unless otherwise mentioned.For a subset (A,E) of X, $\tilde{s}$cl(A,E), $\tilde{s}$ int(A,E) and $(A, E)^c$ denote the soft closure of (A,E),the soft interior of (A,E) and the soft complement of (A,E) respectively.Let us recall the following definitions,which are useful in the sequel

Definition 2.1. *[11] Let X be an initial universe set and E be the set of parameters.Let $P(X)$ denotes the power set of X.For A $\subseteq$E,the pair (F, A) is called a soft set over X where F is a mapping given by F:A $\rightarrow P(X)$.*

Definition 2.2. *[11] The Union of two soft sets (F,A) and (G,B) over X is the soft set (H,C),where C=A$\cup$B,and for all eϵC, H(e)=F(e),if eϵA-B,H(e)=G(e) if eϵB-A and H(e)=F(e)$\cup$G(e)if e ϵA$\cap$B and is denoted as (F,A)$\cup$(G,B)=(H,C).*

Definition 2.3. *[11] The intersection of two soft sets (F,A) and (G,B) over X is the soft set (H,C),where C=A $\cap$ B,and H(e)=F(e) $\cap$ G(e) for all e $\in$ C and is denoted as (F,A) $\cap$ (G,B)=(H,C).*

Definition 2.4. *[11] Let τ be the collection of soft sets over X then τ is said to be a soft topology on X if*

1. *$\emptyset$ and X ϵ τ*
2. *The union of any number of soft sets in $\tau\epsilon$ τ.*
3. *The intersection of any two soft sets in $\tau\epsilon$ τ.*

The triplet (X,τ,E) is called a soft topological space over X. Then the members of τ are called soft open sets in X and complements of them are called soft closed sets in X..

Definition 2.5. *[11] Let (X,τ,E) be a soft topological space over X.The soft Interior of (F,E) denoted by $\tilde{s}$int(F,E),is the largest soft open set over X which contained in (F,E).*

Definition 2.6. *[11] The soft closure of the soft set (F,E) is denoted by $\tilde{s}$cl(F,E),is the smallest soft closed set containing (F,E).*

Definition 2.7. *A soft topological space (X, τ, E) is called soft*

(i) *regular-open[2] if (A,E)= $\tilde{s}$int($\tilde{s}$cl(A,E))and its complement is soft regular closed.*

(ii) *g-closed[9] if $\tilde{s}$cl(A,E)$\tilde{\subseteq}$(U,E) whenever(A,E) $\tilde{\subseteq}$(U,E) and (U,E) is soft open in X.*

(iii) *gs-closed [10]if $\tilde{s}$scl(A,E)$\tilde{\subseteq}$(U,E) whenever(A,E) $\tilde{\subseteq}$(U,E) and (U,E) is soft open in X.*

(iv) *rg-closed[2] if $\tilde{s}$cl(A,E)$\tilde{\subseteq}$(U,E) whenever(A,E) $\tilde{\subseteq}$(U,E) and (U,E) is soft regular open in X.*

(v) *gβ-closed [2]if $\tilde{s}\beta$cl(A,E)$\tilde{\subseteq}$(U,E) whenever(A,E) $\tilde{\subseteq}$(U,E) and (U,E) is soft open in X.*

(vi) *$g^{\#}\alpha$-closed [10]if $\tilde{s}\alpha$cl(A,E)$\tilde{\subseteq}$(U,E) whenever(A,E) $\tilde{\subseteq}$(U,E) and (U,E) is soft g- open in X.*

(vii) *$^{\#}g\alpha$-closed[10] if $\tilde{s}\alpha$cl(A,E)$\tilde{\subseteq}$(U,E) whenever(A,E) $\tilde{\subseteq}$(U,E) and (U,E) is soft $g^{\#}\alpha$ open in X.*

(viii) *$^{\#\#}g$-closed[14] if $\tilde{s}$cl(A,E)$\tilde{\subseteq}$ (U,E) whenever (A,E)$\tilde{\subseteq}$ (U,E) and (U,E) is soft $^{\#}g\alpha$-open in X.*

The complement of soft g-closed(rep.soft gs-closed,soft rg-closed,soft $g\beta$-closed,soft $g^{\#}\alpha$-closed,soft $^{\#}g\alpha$-closed,soft $^{\#\#}g$-closed)is called soft g-open(rep.soft gs-open,soft rg-open,soft $g\beta$-open,soft $g^{\#}\alpha$-open,soft $^{\#}g\alpha$-open,soft $^{\#\#}g$-open).Also soft $^{\#\#}g$-open is denoted as $^{\#\#}g$O.

Definition 2.8. *A soft mapping f :X$\rightarrow$Y is said to be*

(i) *soft $^{\#\#}g$-continuous[14] if the inverse image of each soft closed set in Y is soft $^{\#\#}g$-closed in X.*

(ii) *soft $^{\#\#}g$-irresolute [14] if the inverse image of each soft $^{\#\#}g$-closed set in Y is soft $^{\#\#}g$-closed in X.*

Definition 2.9. *A soft mapping f :X→Y is said to be*

(i) *soft contra continuous[7] if the inverse image of each soft closed(open) set in Y is soft open(closed) in X.*

(ii) *soft almost continuous [7] if the inverse image of each soft regular closed(open) set in Y is soft closed(open) in X.*

(iii) *soft almost contra continuous[7] if the inverse image of each soft regular closed(open) set in Y is soft open(closed) in X.*

Definition 2.10. *A soft mapping f :X→Y is called*

(i) *soft contra g-continuous [3] if f^{-1}(A,E) is soft g-closed in X for every soft open set (A,E) in Y.*

(ii) *soft contra gs-continuous[8] if f^{-1}(A,E) is soft gs-closed in X for every soft open set (A,E) in Y.*

(iii) *soft contra rg-continuous[7] if f^{-1}(A,E) is soft rg-closed in X for every soft open set (A,E) in Y.*

(iv) *soft contra gβ-continuous[8] if f^{-1}(A,E) is soft gβ-closed in X for every soft open set (A,E) in Y.*

Definition 2.11. *[14] A soft topological space (X, τ, E) is called a soft $T^{\#\#g}$-space if for every soft $^{\#\#}g$-closed set is soft closed in X.*

Definition 2.12. *[14] A soft subset (A,E) of a space (X, τ, E) is called a soft $^{\#\#}glc^*$-set if (A,E)=(F,E) ∩ (G,E) where (F,E) is soft $^{\#}g\alpha$-open and (G,E) is soft closed in X.*

Definition 2.13. *[14] A soft function f :X→ Y is said to be a soft $^{\#\#}glc^*$-continuous if the inverse image of every soft closed set in Y is a soft $^{\#\#}glc^*$-set in X.*

3 SOFT CONTRA $^{\#\#}g$-CONTINUOUS FUNCTION

In this section , soft contra $^{\#\#}g$-continuous function is introduced and some of its basic properties are derived.

Definition 3.1. *A function f :X → Y is called soft contra $^{\#\#}g$-continuous if f^{-1}(A,E) is soft $^{\#\#}g$-closed(open) in X for every soft open (closed) set (A,E) in X.*

Example 3.2. *Let X=Y=$\{a,b\}$ E=$\{e_1, e_2\}$.Then the soft subsets over X are defined as follows:*
$(F,E)_1 = \{(e_1, \emptyset), (e_2, \emptyset)\}$
$(F,E)_2 = \{(e_1, \emptyset), (e_2, \{a\})\}$
$(F,E)_3 = \{(e_1, \emptyset), (e_2, \{b\})\}$
$(F,E)_4 = \{(e_1, \emptyset), (e_2, \{a, b\})\}$
$(F,E)_5 = \{(e_1, \{a\}), (e_2, \emptyset)\}$
$(F,E)_6 = \{(e_1, \{a\}), (e_2, \{a\})\}$
$(F,E)_7 = \{(e_1, \{a\}), (e_2, \{b\})\}$
$(F,E)_8 = \{(e_1, \{a\}), (e_2, \{a, b\})\}$
$(F,E)_9 = \{(e_1, \{b\}), (e_2, \emptyset)\}$
$(F,E)_{10} = \{(e_1, \{b\}), (e_2, \{a\})\}$
$(F,E)_{11} = \{(e_1, \{b\}), (e_2, \{b\})\}$
$(F,E)_{12} = \{(e_1, \{b\}), (e_2, \{a, b\})\}$
$(F,E)_{13} = \{(e_1, \{a, b\}), (e_2, \emptyset)\}$
$(F,E)_{14} = \{(e_1, \{a, b\}), (e_2, \{a\})\}$
$(F,E)_{15} = \{(e_1, \{a, b\}), (e_2, \{b\})\}$
$(F,E)_{16} = \{(e_1, \{a, b\}), (e_2, \{a, b\})\}$

Let $\tau = \{\tilde{\emptyset}, \tilde{X}, (F,E)_5, (F,E)_7, (F,E)_8, \}$; $\sigma = \{\tilde{\emptyset}, \tilde{X}, (F,E)_6\}$.Then the function $f : X \to Y$ is defined by f(a)=b,f(b)=a is soft contra $^{\#\#}g$-continuous.

Theorem 3.3. *Every soft contra continuous function is soft contra $^{\#\#}g$-continuous.*

Proof. Let(A,E) be a soft open set in Y.By hypothesis , f^{-1}(A,E) is soft closed set in X.By theorem 3.2 [14], f^{-1}(A,E) is soft $^{\#\#}g$-closed set in X.Hence the result. □

Remark 3.4. *The converse of the above theorem is not true as shown in the following example.*

Example 3.5. *Let X=Y=$\{a,b\}$; E=$\{e_1, e_2\}$.Then the soft sets over X are defined as in example 3.2 ,$\tau = \{\tilde{\emptyset}, \tilde{X}, (F,E)_5, (F,E)_7, (F,E)_8, \}$; $\sigma = \{\tilde{\emptyset}, \tilde{X}, (F,E)_6\}$. Then the function $f : X \to Y$ is defined by f(a)=b,f(b)=a is soft contra $^{\#\#}g$-continuous function. but not soft contra continuous.*

Theorem 3.6.

- **(i)** *Every soft contra $^{\#\#}g$-continuous is soft contra g-continuous.*
- **(ii)** *Every soft contra $^{\#\#}g$-continuous is soft contra gs-continuous.*
- **(iii)** *Every soft contra $^{\#\#}g$-continuous is soft contra rg-continuous.*

(iv) *Every soft contra $^{\#\#}g$-continuous is soft contra $g\beta$-continuous.*

Proof. (i) Let f :X→ Y be a soft contra $^{\#\#}g$-continuous function and (U,E) be a soft open set in Y.Then $f^{-1}(U,E)$ is soft $^{\#\#}g$-closed in X.By theorem 3.5(i) [14] , $f^{-1}(U,E)$ is soft g-closed.Consequently f is contra g-continuous.

(ii)Let f :X→ Y is a soft contra $^{\#\#}g$-continuous function and (U,E) be a soft open set in Y.Then $f^{-1}(U,E)$ is soft $^{\#\#}g$-closed in X.By theorem 3.5(ii)[14],$f^{-1}(U,E)$ is soft gs-closed and so f is contra gs-continuous.

(iii)Let f :X→ Y is a soft contra $^{\#\#}g$-continuous function and (U,E) be a soft open set in Y.Then $f^{-1}(U,E)$ is soft $^{\#\#}g$-closed in X.By theorem 3.5(iv) [14] , $f^{-1}(U,E)$ is soft rg-closed.Hence the result.

(iv)Let f :X→ Y is a soft contra $^{\#\#}g$-continuous function and (U,E) be a soft open set in Y.Then $f^{-1}(U,E)$ is soft $^{\#\#}g$-closed in X.By theorem 3.5(v)[14], $f^{-1}(U,E)$ is soft $g\beta$-closed.Hence the result. □

However none of these implications are reversible as shown in the following examples.

Example 3.7. *Let X=Y=$\{a,b\}$; E=$\{e_1,e_2\}$ .Then the soft sets over X are defined as in example 3.2 , τ=$\{\tilde{\emptyset}, \tilde{X}$,(F,E)$_6\}$ and σ=$\{\tilde{\emptyset}$,$\tilde{X}$,(F,E)$_7$, $(F,E)_{10}\}$.Then the function f :X → Y defined by f(a)=b,f(b)=a is both soft contra g-continuous and soft contra gs-continuous but not soft $^{\#\#}g$-continuous.*

Example 3.8. *Let X=Y=$\{a,b\}$; E=$\{e_1,e_2\}$ .Then the soft sets over X are defined as in example 3.2 , τ=$\{\tilde{\emptyset}, \tilde{X}$,(F,E)$_{11}\}$ and σ=$\{\tilde{\emptyset}$,$\tilde{X}$,(F,E)$_6\}$.Then the function f :X → Y defined by f(a)=b,f(b)=a is soft contra rg-continuous but not soft contra $^{\#\#}g$-continuous.*

Example 3.9. *Let X=Y=$\{a,b\}$; E=$\{e_1,e_2\}$ as soft sets defined in example 3.2 , τ=$\{\tilde{\emptyset}, \tilde{X}$,(F,E)$_6\}$ and σ=$\{\tilde{\emptyset}$,$\tilde{X}$,(F,E)$_{11}\}$.Then the function f :X → Y defined by f(a)=b,f(b)=a is soft contra $g\beta$-continuous but not soft contra $^{\#\#}g$-continuous.*

Remark 3.10. *The following table shows the relationships between soft contra $^{\#\#}g$-closed continuous function and other known existing contra-continuous functions.The symbol"1" in a cell means that a map on the corresponding row implies a map on the corresponding column and the symbol"0" means that a map on the corresponding row does not imply a map on the corresponding column.*

soft contra mappings	A	B	C	D	E	F
A	**1**	**1**	**1**	**1**	**1**	**1**
B	**0**	**1**	**1**	**1**	**1**	**1**
C	**0**	**0**	**1**	**0**	**0**	**0**
D	**0**	**1**	**1**	**1**	**1**	**1**
E	**0**	**0**	**1**	**0**	**1**	**1**
F	**0**	**0**	**1**	**0**	**0**	**1**

Table-1

*Here, A-soft contra continuous.B-soft contra g-continuous.C-soft contra rg-continuous.D-soft contra $^{\#\#}g$-continuous.E-soft contra $g\beta$-continuous.
F-soft contra gs-continuous.*

Theorem 3.11. *Let $f:\ X \rightarrow Y$ be a mapping where (X, τ, E) and (Y, σ, E) are soft topological spaces and soft $^{\#\#}g$-O(X,τ, E) is closed under arbitrary union.Then the following conditions are equivalent.*

(i) *f : $X \rightarrow Y$ is soft contra $^{\#\#}g$ continuous.*

(ii) *The inverse image of each soft closed set in Y is soft $^{\#\#}g$-open in X.*

(iii) *For each $x \in X$ and for each soft closed set (F,E) containing f(x), there exist a soft $^{\#\#}g$-open set (G,E) in X such that f((G,E))$\tilde{\subseteq}$ (F,E)*

Proof. (i) $\Rightarrow$ (ii) Let (F,E) is soft closed in Y.Then Y$\backslash$(F,E) is soft open in Y.By assumption , $f^{-1}(Y\backslash$(F,E))=X$\backslash$f$^{-1}((F,E))$ is soft $^{\#\#}g$-closed in X which implies $f^{-1}(F,E)$ is soft $^{\#\#}g$-open set in X.

(ii) $\Rightarrow$ (i) Let (G,E) be a soft open set in Y.Then Y$\backslash$(G,E) is soft closed in X.By (ii) , $f^{-1}(Y\backslash$(G,E))=X$\backslash$f$^{-1}((G,E))$ is soft $^{\#\#}g$-open in X which implies $f^{-1}(G,E)$ is soft $^{\#\#}g$ -closed in X and so f is soft contra $^{\#\#}g$-continuous.

(ii) $\Rightarrow$ (iii) Let (F,E) be a soft closed set in Y containing f(x) and hence $x \in$ f$^{-1}(F,E)$. By (ii) , $f^{-1}(F,E)$ is soft $^{\#\#}g$-open in X . Take (G,E)=$f^{-1}(F,E)$. Then f((G,E)) $\tilde{\subseteq}$ (F,E).

(iii) $\Rightarrow$ (ii) Let (F,E) be any soft closed set in Y containing f(x) and $x \in$ f$^{-1}(F,E)$.By (iii) ,there exists a soft $^{\#\#}g$-open set (G_x, E) containing x such that f((G$_x, E)$)$\tilde{\subseteq}$(F,E).Therefore $f^{-1}(F,E)$=$\tilde{\cup}\{($G$_x, E), x \epsilon f^{-1}(F,E)\}$ and $f^{-1}(F,E)$ is soft $^{\#\#}g$-open in X. □

Theorem 3.12. *Let f :X→ Y be a function where X is a soft $T^{\#\#g}$-space.Then the following are equivalent:*

(i) *f is soft contra $^{\#\#}g$- continuous.*

(ii) *f is soft contra-continuous.*

Proof. (i) $\Rightarrow$(ii) Let (U,E) be a soft open set in Y. By hypothesis , $f^{-1}(U,E)$ is soft $^{\#\#}g$-closed in X. Since X is a soft $T^{\#\#g}$-space , $f^{-1}(U,E)$ is soft closed in X and thus f is soft contra continuous.
(ii) $\Rightarrow$ (i) By theorem 3.3, proof follows. □

Definition 3.13. *A space (X,τ,E) is called soft $^{\#\#}g$-locally indiscrete if every soft $^{\#\#}g$-open (closed) set of (X,τ,E) is soft closed (open) in X.*

Theorem 3.14. *Let f :X$\rightarrow$ Y be a soft mapping then*

(i) *if f is soft $^{\#\#}g$-continuous and X is soft $^{\#\#}g$-locally indiscrete then f is soft contra-continuous.*

(ii) *if f is soft $^{\#\#}g$-irresolute and X is soft $^{\#\#}g$-locally indiscrete , then f is a soft contra-continuous function.*

Proof. (i) Let (V,E) be a soft closed set in X.By theorem 3.3[14],(V,E) is soft $^{\#\#}g$-closed in X. Since f is soft $^{\#\#}g$-irresolute, $f^{-1}(V,E)$ is soft $^{\#\#}g$-closed in X. As X is soft $^{\#\#}g$-locally indiscrete, $f^{-1}(V,E)$ is soft open in X . Thus the proof follows.

(ii) By theorem 6.6 [14] and and by (i) the result follows. □

Remark 3.15. *Composition of two soft contra $^{\#\#}g$-continuous functions is not a soft contra $^{\#\#}g$-continuous function.*

Example 3.16. *Consider $X=\{a,b\}, E=\{e_1,e_2\}$.Then the soft sets are defined over X,Y,Z is as in example 3.2 , τ=$\{\tilde{X}$,$\tilde{\emptyset}$,$(F,E)_5$, $(F,E)_7$, $(F,E)_8\}$ $\sigma=\{\tilde{X}$,$\tilde{\emptyset}$,$(F,E)_6\}$ $\gamma=\{\tilde{X}$,$\tilde{\emptyset}$,$(F,E)_{11}\}$.Then f :X $\rightarrow$Y is defined by f(a)=b,f(b)=a and g :Y$\rightarrow$Z is defined by g(a)=a,g(b)=b are soft contra $^{\#\#}g$-continuous functions.Observe that $(g\circ f\ )^{-1}((F,E)_{11}) = f^{-1}(g^{-1}(F,E)_{11}) = f^{-1}((F,E)_{11}) = (F,E)_6$ which is not soft $^{\#\#}g$-closed set in X where $(F,E)_{11}$ is soft open in Z.*

COMPOSITION OF SOFT CONTRA $^{\#\#}g$ FUNCTIONS

Theorem 3.17. *Let f: X $\rightarrow$ Y be a soft $^{\#\#}g$-irresolute function and g :Y$\rightarrow$Z be a soft contra $^{\#\#}g$-continuous function.Then g$\circ$f : X$\rightarrow$ Y is a soft contra $^{\#\#}g$-continuous function.*

Proof. Let (G,E) be a soft open set in Z. Since f is soft contra $^{\#\#}g$-continuous, $g^{-1}(G,E)$ is soft $^{\#\#}g$-closed in Y. Now f is soft $^{\#\#}g$-irresolute which implies $f^{-1}(g^{-1}(G,E)) = (g\circ\text{f})^{-1}$(G,E) is soft $^{\#\#}g$-closed in X.Hence the result. □

Theorem 3.18. *Let f : X→ Y be a soft $^{\#\#}g$-continuous function and g : Y→Z be a soft contra continuous function.Then g∘f : X →Z is a soft contra $^{\#\#}g$-continuous function.*

Proof. By theorem 6.6[14] and 3.3 the result follows.

Corollary 3.19. *Let f : X→ Y be a soft $^{\#\#}g$-irresolute function and g:Y→Z be a soft contra-continuous functions . Then g∘f : X →Y is a soft contra $^{\#\#}g$-continuous function.*

Proof. The proof follows from the theorem 6.6[14] and 3.18 □

Theorem 3.20. *Let f :X→ Y and g : Y→ Z be two soft functions and Y is a soft $T^{\#\#g}$-space then the following conditions holds:*

- **(i)** *If f is soft contra continuous and g is soft contra $^{\#\#}g$-continuous then g∘f is soft continuous.*
- **(ii)** *If f is soft contra continuous and g is soft $^{\#\#}g$-irresolute,then g∘f is soft contra-continuous.*

Proof. (i) Let (B,E) be soft open in Z. Since g is soft contra $^{\#\#}g$-continuous , then $g^{-1}(B,E)$ is soft $^{\#\#}g$-closed in Y. But Y is a soft $T^{\#\#g}$-space , $g^{-1}(B,E)$ is soft closed in Y . Now f is soft contra continuous implies $f^{-1}(g^{-1}(B,E))$=$(g\circ\text{f})^{-1}(B,E)$ is soft open in X .Consequently $g\circ$f is soft continuous.

(ii)Let (B,E) be soft closed in Z. By theorem 3.2[14],(B,E) is soft $^{\#\#}g$-closed in Z. Since g is soft $^{\#\#}g$-irresolute , then $g^{-1}(B,E)$ is soft $^{\#\#}g$-closed in Y. But Y is a soft $T^{\#\#g}$-space $g^{-1}(B,E)$ is soft closed in Y . Now f is soft contra continuous , then $f^{-1}(g^{-1}(B,E))$=$(g\circ\text{f})^{-1}(B,E)$ is soft open in X and thus $g\circ$f is soft contra continuous.

4 SOFT ALMOST CONTRA $^{\#\#}g$-CONTINUOUS

Definition 4.1. *A function f : X→Y is said to be soft almost contra continuous function if $f^{-1}(V,E)$ is soft $^{\#\#}g$-closed (open) in X for every soft regular open (closed)(V,E) in Y.*

Example 4.2. *Let X=$\{a,b,c\}$,E=$\{e\}$, then the soft subsets over X is defined as follows*

$(G,E)_1 = \{(e, \emptyset)\}$
$(G,E)_2 = \{(e, \{a\})\}$
$(G,E)_3 = \{(e, \{b\})\}$
$(G,E)_4 = \{(e, \{c\})\}$
$(G,E)_5 = \{(e, \{a,b\})\}$
$(G,E)_6 = \{(e, \{b,c\})\}$
$(G,E)_7 = \{(e, \{a,c\})\}$
$(G,E)_8 = \{(e, \{a,b,c\})\}$
$\tau=\{\tilde{\emptyset},\tilde{X},(F,E)_3,(F,E)_4,(F,E)_5,(F,E)_6\}$; $\sigma=\{\tilde{\emptyset},\tilde{X},(F,E)_2,(F,E)_3,(F,E)_5\}$
Then the function $f : X \to Y$ defined by f(a)=a , f(b)= c , f(c)= b is soft almost contra $^{\#\#}g$-continuous .

Theorem 4.3. *Every contra $^{\#\#}g$-continuous is soft almost contra $^{\#\#}g$-continuous.*

Proof. Let (U,E) be a soft regular open in Y. By remark 3.2[12], (U,E) is soft open in Y. By hypothesis , $f^{-1}(U,E)$ is soft $^{\#\#}g$-closed in X which implies f is soft almost contra $^{\#\#}g$-continuous. □

Theorem 4.4. *Let f :X→ Y be a function . Then the following are equivalent:*

- **(i)** *f is soft almost contra $^{\#\#}g$-continuous.*
- **(ii)** *The inverse image of each soft closed set in Y is soft $^{\#\#}g$-open in X.*

Proof. follows from the theorem 3.4[6].

Theorem 4.5. *Let f : X→ Y be a soft function . Then the following are equivalent :*

- **(i)** *f is soft almost contra $^{\#\#}g$-continuous.*
- **(ii)** *$f^{-1}(\tilde{s}int(\tilde{s}cl(A,E)))$ is a soft $^{\#\#}g$-closed set in X for every soft open set (A,E) of Y.*
- **(iii)** *$f^{-1}(\tilde{s}\ cl(\tilde{s}\ int(A,E)))$ is a soft $^{\#\#}g$-open set in X for every soft closed subset (B,E) of Y.*

Proof. (i) ⇒ (ii) Let (A,E) be a soft open set in X . By the definition of soft regular open set ,(A,E) = ($\tilde{s}$int($\tilde{s}$cl(A,E))) is regular open in Y. Since f is soft almost contra $^{\#\#}g$-continuous which impliesf^{-1} ($\tilde{s}$int($\tilde{s}$cl(A,E))) is soft $^{\#\#}g$-closed in X.

(ii) ⇒ (i) Let (A,E)be regular open in Y. By remark 3.2[12], (A,E) is soft open in Y. By (ii)$f^{-1}((\tilde{s}$ int($\tilde{s}$ cl(A,E))) is soft $^{\#\#}g$-closed in X. And so f$^{-1}(A,E)$ is soft $^{\#\#}g$-closed in X. Hence f is soft almost contra $^{\#\#}g$-continuous.

(i) $\Rightarrow$ (iii) Proof is similar to (i) $\Rightarrow$ (ii)

(iii) $\Rightarrow$ (i) Proof is similar to (ii) $\Rightarrow$ (i) □

Theorem 4.6. *If $f : X \to Y$ is soft almost contra $^{\#\#}g$-continuous and X is a soft $^{\#\#}g$ locally-indiscrete space. Then f is soft almost continuous.*

Proof. Let (U,E) be soft regular open set in Y. Since f is soft almost contra $^{\#\#}g$-continuous , $f^{-1}(U,E)$ is soft $^{\#\#}g$-closed set in X. As X is a soft $^{\#\#}g$-locally indiscrete space ,$f^{-1}(U,E)$ is soft open in X. Hence f is soft almost continuous. □

5 SOFT $^{\#\#}glc^{**}$-SET

In this section, we introduce a new soft locally closed set namely $^{\#\#}glc^{**}$-set and its properties are studied.

Definition 5.1. *A soft subset (A,E) of a soft topological space $(X, \tau\, E)$ is said to be a soft $^{\#\#}glc^{**}$-set if $(A,E) = (F,E)\tilde{\cap}$(H,E) where (F,E) is soft $^{\#\#}g$-closed and (H,E) is soft $^{\#\#}g$-open.*

Example 5.2. *Let $X = \{a,b\}; E = \{e_1, e_2\}$; as soft dets defined in example 3.1 , $\tau = \{\tilde{X},\tilde{\emptyset},(F,E)_7,(F,E)_{10}\}$.Here $(F,E)_7 = \{(e_1,\{a\}),(e_2,\{b\})\}$ and $(F,E)_{10} = \{(e_1,\{b\}),(e_2,\{a\})\}$ are soft $^{\#\#}glc^{**}$-sets in X.*

Theorem 5.3. *For a soft subset (A,E) of a space (X,τ,E) ,the following statements are equivalent:*

(i) *(A,E) $\in$ $^{\#\#}gcl^{**}(X,\tau,E)$*

(ii) *$(A,E) = (U,E) \cap^{\#\#} gcl$(A,E) for some $^{\#\#}g$-open set (U,E) in (X,τ, E)*

Proof. (i)$\Rightarrow$(ii) Let (A,E) $\in$ $^{\#\#}gcl^{**}(X,\tau,E)$.Then by definition 5.1 ,
(A,E)=(F,E)$\tilde{\cap}$(G,E) where(F,E) is soft $^{\#\#}g$-open set and (G,E) is soft $^{\#\#}g$-closed set which implies (A,E)$\tilde{\subseteq}$(U,E) .Since (A,E)$\tilde{\subseteq}$ $^{\#\#}g$cl(A,E)
then $(A,E)\tilde{\subseteq}$(U,E) $\tilde{\cap}$ $^{\#\#}g$cl(A,E). Since (G,E) is soft$^{\#\#}g$-closed set which implies $^{\#\#}g$cl(A,E)$\tilde{\subseteq}$ $^{\#\#}g$cl(G,E)=(G,E).Then (U,E)$\tilde{\cap}^{\#\#}g$cl(A,E)$\tilde{\subseteq}$(U,E)$\tilde{\cap}$(G,E) =(A,E). Hence the result.
(ii)$\Rightarrow$(i) Since $^{\#\#}g$cl(A,E) is soft $^{\#\#}g$-closed set By hypothesis,
(A,E)=(U,E)$\tilde{\cap}^{\#\#}g$cl(A,E). Hence (A,E) is $^{\#\#}glc^{**}$-set . □

Theorem 5.4. *Let (X,τ,E) be a $T^{\#\#g}$-space. For a soft subset (A,E)of the space (X,τ,E) the following are equivalent:*

(i) ${}^{\#\#}gcl(A,E)\backslash(A,E)$ *is soft* ${}^{\#\#}g$*-closed*

(ii) $(A,E)\tilde{\cap}$ $({}^{\#\#}gcl(A,E))^{\complement}$

Proof. (i) $\Rightarrow$ (ii) Let $(A,E)\tilde{\cup}({}^{\#\#}gcl(A,E))^{\complement}$ which implies $({}^{\#\#}g\mathrm{cl(A,E)}\backslash\mathrm{(A,E)})^{\complement}$.By assumption $({}^{\#\#}g\mathrm{cl(A,E)}\backslash\mathrm{(A,E)})$ is soft ${}^{\#\#}g$-closed.$({}^{\#\#}gcl(A,E)\backslash\mathrm{(A,E)})^{\complement}$ is soft ${}^{\#\#}g$-open implies $(A,E)\tilde{\cup}({}^{\#\#}g\mathrm{cl(A,E)})^{\complement}$ is soft ${}^{\#\#}g$-open.

(ii)$\Rightarrow$(i) Let $(U,E) = (A,E)\tilde{\cup}$ $({}^{\#\#}g\mathrm{cl(A,E)})^{\complement}$. By (i) (U,E) is soft ${}^{\#\#}g$-closed.Then $(U,E)^{\complement}$ is soft ${}^{\#\#}g$-closed which implies $((A,E)\tilde{\cup}({}^{\#\#}gcl(A,E))^{\complement})^{\complement}$ is soft ${}^{\#\#}g$-closed set.Then $(({}^{\#\#}gcl(A,E))^{\complement}\tilde{\cap}\,\mathrm{(A,E)}^{\complement})^{\complement} ={}^{\#\#} gcl(A,E)\tilde{\cap}\,\mathrm{(A,E)}$ is soft ${}^{\#\#}g$-closed set.Hence ${}^{\#\#}g\mathrm{cl(A,E)}\backslash\mathrm{(A,E)}$ is soft ${}^{\#\#}g$-closed. □

Definition 5.5. *Let f : $X \to Y$ be a soft map then f is called a soft ${}^{\#\#}glc^{**}$-continuous function if for every soft closed(V,E) in Y, f^{-1}(V,E) is soft ${}^{\#\#}glc^{**}$-set in X*

Example 5.6. *Let X=$\{a,b\}$ E=$\{e_1,e_2\}$ as soft sets defined in example 3.2 $\tau = \{\tilde{X},\tilde{\emptyset},(F,E)_6,(F,E)_{11}\}.\sigma = \{\tilde{X},\tilde{\emptyset},(F,E)_6\}.f : X \to Y$ is a identity function then f is a soft ${}^{\#\#}glc^{**}$-continuous function.*

Definition 5.7. *A soft function f : X$\to$ Y is called a soft ${}^{\#\#}glc^{**}$-irresolute function if for every soft ${}^{\#\#}glc^{**}$-set (V,E) in Y , f^{-1}(V,E) is a soft ${}^{\#\#}glc^{**}$-set in X.*

Example 5.8. *Let X=$\{a,b\}$ E=$\{e\}$ as soft sets defined in example 4.2 , $\tau = \{\tilde{X},\tilde{\emptyset},(F,E)_6\}.\sigma = \{\tilde{X},\tilde{\emptyset},(F,E)_7\}.f :$ X$\to$ Y is a function defined by f(a)=b,f(b)=a,f(c)=c .Then f is a soft ${}^{\#\#}glc^{**}$-irresolute function.*

Theorem 5.9. *If f :X$\to$ Y is a soft ${}^{\#\#}glc^{**}$-continuous and X is a soft $T^{\#\#}g$-space then f is soft ${}^{\#\#}glc^{*}$-continuous.*

Proof. Let (V,E) be soft closed in Y. By hypothesis , f^{-1}(V,E) is soft ${}^{\#\#}glc^{**}$-set in X.By definition 5.1, f^{-1}(V,E)=f^{-1}(U,E) $\tilde{\cap}$ f^{-1}(H,E) where$f^{-1}(U,E)$ is soft ${}^{\#\#}g$-closed and $f^{-1}(H,E)$ is soft ${}^{\#\#}g$-open . Since X is soft T ${}^{\#\#}g$-space which implies $f^{-1}(U,E)$ is soft open and $f^{-1}(H,E)$ is soft closed. By theorem 7[10],$f^{-1}(U,E)$ is soft ${}^{\#}g\alpha$-open .And so $f^{-1}(V,E)$ is a soft ${}^{\#\#}glc^{*}$-set Then f is soft ${}^{\#\#}glc^{*}$-continuous. □

6 APPLICATION

Theorem 6.1. *Let f : X$\to$ Y be a soft mapping.If f is soft ${}^{\#\#}glc^{**}$-continuous and soft contra ${}^{\#\#}g$-continuous and Y is a soft T ${}^{\#\#}g$-space, then f is a soft ${}^{\#\#}glc^{**}$-irresolute function.*

Proof. Let (U,E) be a soft $^{\#\#}glc^{**}$-set in Y which implies (U,E)=(F,E) $\tilde{\cap}$(G,E) where (G,E) is soft $^{\#\#}g$-open and (F,E) soft $^{\#\#}g$-closed. Since Y is soft T $^{\#\#}g$-space, (G,E) is soft closed and (F,E) is soft open . As f is soft contra $^{\#\#}g$-continuous , then $f^{-1}(G,E)$ is soft$^{\#\#}g$-open and $f^{-1}(F,E)$ is soft $^{\#\#}g$-closed and hence$f^{-1}(U,E)$=$f^{-1}(G,E)\tilde{\cap}f^{-1}(F,E)$ and so $f^{-1}(U,E)$ is soft $^{\#\#}glc^{**}$-set in X.Hence f is soft $^{\#\#}glc^{**}$-continuous. □

Corollary 6.2. *Let f :X→ Y be a soft mapping.If f is soft $^{\#\#}glc^{**}$-continuous and soft almost contra continuous and Y is soft $^{\#\#}g$-locally indiscrete.Then f is soft $^{\#\#}glc^{**}$-irresolute.*

Proof. By theorem 4.3 and 6.4 the result follows. □

7 CONCLUSION

We have introduced soft contra $^{\#\#}g$-continuous function and soft contra $^{\#\#}g$-irresolute function.Also we have introduced soft almost contra $^{\#\#}g$-continuous function and soft almost contra $^{\#\#}g$-irresolute function in soft topological spaces and investigated their properties.Finally a new soft locally closed set namely soft $^{\#\#}glc^{**}$-set is also introduced and some of the basic characterizations of soft $^{\#\#}glc^{**}$-set is also examined.

References

[1] ALI HAYDAR KOCAMAN , NAIME TOZLU :"Soft locally closed sets and decompositions of soft continuity", *Annals of Fuzzy Mathematics and Informatics .,(2015), 1-9.*

[2] AROCKIARANI.I,AROKIALANCY.A*:* "Generalized gβ-closed sets and gsβ-closed sets in soft topological space"*., International Journal of Mathematical Archive.,4(2).,(2013).,17-23.*

[3] AROCKIARANI.I AND SELVI.A*:*"On soft contra Πg-continuous functions in soft topological spaces "*,International Journal of Mathematics Trends and Technology.,Vol 19(1).,(2015).,1-11.*

[4] BOURBAKI.N*:*"General Topology, Part I "*, Addison-Wesley, Reading, Mass, (1966).*

[5] DONTCHEV.J *:* "Contra-continuous functions and strongly S-closed spaces" *, International Journal of Mathematics and Mathematical Sciences.,Vol.19(2)(1996),303-310.*

[6] HAI LONG YANG;XIUWU LIAO AND SHENG-GANG LI:"On soft mappings and connectedness of soft topological spaces"., *Hacettepe Journal of Mathematics and Statistics.,Vol-44.,(2015).,385-398.*

[7] INDIRANI.K AND SMITHA .M.G:"On soft contra rgβ-continuous function and soft almost rgβ-continuous function in soft topological spaces".,*Journal of Global Research in Mathematical Archives., Vol-2(3).,(2014).,1-14.*

[8] JEYANTHI.V AND JANAKI.C:"πgr-closed sets in topological spaces"*Asian Journal of Current Engginerring and Mathematics., Vol-1(5).,(2015).,241-246.*

[9] KANNAN.K:"Soft generalized closed sets in soft topological spaces".,*Journal of Theoretical and Applied Information Technology.,Vol-37.,(2012).,17-20.*

[10] KOKILAVANI.V,SINDU.D AND VIVEK PRABU.M:"Soft $^{\#}g\alpha$-closed sets and soft $^{\#}g\alpha$-continuous functions".,*International Journal of Advanced and Innovative research.,(2014),Vol-3.,1-7.*

[11] MOLODTSOV.D:"Soft set theory-First results"., *Computers and Mathematics with Applications.,Vol-37.,(1999).,19-31.*

[12] SAZIYE YUKSEL;NAIME TOZLU AND ZEHRA GUZEL ERGUL:"Soft regular generalized closed sets in soft topological spaces"., *International Journal of Mathematical Analysis.,Vol-8(8).,(2014).,355-367.*

[13] SHABIR.M AND NAZ.M:"On soft topological spaces"., *Computers and Mathematics with applications.,Vol-61.,(2011).,Issue 7.,1786-1799.*

[14] SANTHINI.C AND SANTHIYAA.M:"On soft $^{\#\#}g$-closed set in soft topological spaces".,*Global Journal of Pure and Applied Mathematics ,Accepted for publication.*

[15] SINGAL.M.K AND SINGAL A.R :"Almost continuous mappings" , *Yokohama Mathematical Journal., 16 (1968), 63-73.*