**τ₁ τ₂* Boundary set on Bigeneralized Topological Space**

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Abstract

In this paper we define τ₁ τ₂* boundary set in a bigeneralized topological space we study the properties of the set.

Key words: τ₁ τ₂* boundary point,

I. Introduction:


II. Preliminaries

Let X be a non-empty set and τ be a collection of subsets of X. Then τ is called a generalized topology on X if φ ∈ τ and arbitrary union of elements of τ in τ. A space with a generalized topology is called a generalized topological space.

In a generalized topological space (X, τ) the following are true

i) \( \text{Cl} (A) = X - \text{int} (X - A) \)

ii) \( \text{Int} (A) = X - \text{Cl} (X - A) \)

iii) \( \text{Cl} (X - A) = X - \text{int} (A) \) and \( \text{int} (X - A) = X - \text{Cl} (A) \)

iv) If \( X - A \in τ \) then \( \text{Cl} (A) = A \) and if \( A \in τ \) then \( \text{int} (A) = A \).

v) If \( A \subseteq B \) then \( \text{Cl} (A) \subseteq \text{Cl} (B) \) and \( \text{int} (A) \subseteq \text{int} (B) \)

vi) \( A \subseteq \text{Cl} (A) \) and \( \text{int} (A) \subseteq A \)

Let X be a non-empty set and τ₁, τ₂ be generalized topologies on X. The triple \( (X, τ₁, τ₂) \) is called a bigeneralized topological space (briefly BGTS).

III. τ₁ τ₂* Boundary set

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Definition: 3.1
A subset $A$ of a bigeneralized topological space $(X, \tau_1, \tau_2)$ is called $\tau_1 \tau_2$ closed if $A = \tau_1 \text{Cl}(A) \cap \tau_2 \text{Cl}(A)$. The complement of $\tau_1 \tau_2$ closed set is called $\tau_1 \tau_2$ open. The $\tau_1 \tau_2$ closure of a set $A$ is defined in the usual way.

Result: 3.2
Let $(X, \tau_1, \tau_2)$ be a bigeneralized topological space and let $A$ be a subset of $X$. Then $\tau_1 \tau_2 \text{ Cl}(A) = \tau_1 \text{ Cl}(A) \cap \tau_2 \text{ Cl}(A)$.

In this section, we introduce the concept $\tau_1 \tau_2$ boundary set and study their properties in bigeneralized topological space.

Definition: 3.3
Let $(X, \tau_1, \tau_2)$ be a BGTS. Let $A$ be a subset of $X$ and $x \in X$. Then $x$ is called $\tau_1 \tau_2$ boundary point of $A$ if $x \in (\tau_1 \tau_2 \text{ Cl}(A) \cap \tau_1 \tau_2 \text{ Cl}(X - A))$. We denote the set of all $\tau_1 \tau_2$ boundary point of $A$ by $\tau_1 \tau_2 \text{ Bd}(A)$. From definition we have $\tau_1 \tau_2 \text{ Bd}(A) = (\tau_1 \text{ Cl}(A) \cap \tau_2 \text{ Cl}(A)) \cap (\tau_1 \text{ Cl}(X - A) \cap \tau_2 \text{ Cl}(X - A))$

Example: 3.2
Let $X = \{a, b, c, d\}$
$\tau_1 = \{\emptyset, X, \{a, b\}, \{b, c\}\}$ and
$\tau_2 = \{\emptyset, X, \{a, c\}, \{a, b\}\}$
Now, $\tau_1$ closed sets are $\{\emptyset, X, \{c\}, \{a\}\}$ and
$\tau_2$ closed sets are $\{\emptyset, X, \{b\}, \{c\}\}$
$\tau_1 \tau_2 \text{ Bd}(\{a, b\}) = \{c\}$ and
$\tau_1 \tau_2 \text{ Bd}(\{b, c\}) = \{a\}$

Theorem: 3.3
Let $(X, \tau_1, \tau_2)$ be a bigeneralized topological space. Let $A$ be a subset of $X$. Then $\tau_1 \tau_2 \text{ Bd}(A) = \tau_1 \tau_2 \text{ Bd}(X - A)$

Proof:
$\tau_1 \tau_2 \text{ Bd}(A) = (\tau_1 \text{ Cl}(A) \cap \tau_2 \text{ Cl}(A)) \cap (\tau_1 \text{ Cl}(X - A) \cap \tau_2 \text{ Cl}(X - A))$
$= (\tau_1 \text{ Cl}(X - (X - A)) \cap \tau_2 \text{ Cl}(X - (X - A))) \cap (\tau_1 \text{ Cl}(X - A) \cap \tau_2 \text{ Cl}(X - A))$
$= \tau_1 \tau_2 \text{ Bd}(X - A)$

Theorem: 3.4
Let $(X, \tau_1, \tau_2)$ be a bigeneralized topological space and $A, B$ be a subset of $X$. Then the following are true.

\begin{align*}
\text{i) } &\tau_1 \tau_2 \text{ Bd}(A) = (\tau_1 \text{ Cl}(A) \cap \tau_2 \text{ Cl}(A)) \setminus (\tau_1 \text{ int}(A) \cup \tau_2 \text{ int}(A)) \\
\text{ii) } &\tau_1 \tau_2 \text{ Bd}(A) \cap (\tau_1 \text{ int}(A) \cup \tau_2 \text{ int}(A)) = \emptyset \\
\text{iii) } &\tau_1 \tau_2 \text{ Bd}(A) \cap (\tau_1 \text{ int}(X - A) \cup \tau_2 \text{ int}(X - A)) = \emptyset \\
\text{iv) } &\tau_1 \text{ Cl}(A) \cap \tau_2 \text{ Cl}(A) = \tau_1 \tau_2 \text{ Bd}(A) \cup (\tau_1 \text{ int}(A) \cup \tau_2 \text{ int}(A)) \\
\text{v) } &X = (\tau_1 \text{ int}(X - A) \cup \tau_2 \text{ int}(X - A)) \cup \tau_1 \tau_2 \text{ Bd}(A) \cup (\tau_1 \text{ int}(A) \cup \tau_2 \text{ int}(A)) \\
\end{align*}

is a pair wise disjoint Union.

Proof:
\begin{align*}
\text{i) } &\tau_1 \tau_2 \text{ Bd}(A) = (\tau_1 \text{ Cl}(A) \cap \tau_2 \text{ Cl}(A)) \cap (\tau_1 \text{ Cl}(X - A) \cap \tau_2 \text{ Cl}(X - A)) \\
&= (\tau_1 \text{ Cl}(A) \cap \tau_2 \text{ Cl}(A)) \cap (X - (\tau_1 \text{ int}(A) \cup \tau_2 \text{ int}(A))) \\
&= (\tau_1 \text{ Cl}(A) \cap \tau_2 \text{ Cl}(A)) \setminus (\tau_1 \text{ int}(A) \cup \tau_2 \text{ int}(A)) \\
\end{align*}
\(\tau_1 \tau_2^*\) Boundary set on Bigeneralized...

ii) From (i), it’s obvious that
\[
\tau_1 \tau_2^* \text{Bd}(A) \cap (\tau_1 \text{int}(A) \cup \tau_2 \text{int}(A)) = \phi
\]
iii) \[
\tau_1 \tau_2^* \text{Bd}(A) \cap (\tau_1 \text{int}(X - A) \cup \tau_2 \text{int}(X - A)) = \tau_1 \tau_2^* \text{Bd}(X - A) \cap (\tau_1 \text{int}(X - A) \cup \tau_2 \text{int}(X - A)) = \phi
\]
iv) \[
\tau_1 \text{Cl} A \cap \tau_2 \text{Cl} A
\]
\[
= [\tau_1 \text{Cl}(A) \cap \tau_2 \text{Cl}(A) \setminus \tau_1 \text{int}(A) \cup \tau_2 \text{int}(A)] \cup (\tau_1 \text{int}(A) \cup \tau_2 \text{int}(A)) = \tau_1 \tau_2^* \text{Bd}(A) \cup (\tau_1 \text{int}(A) \cup \tau_2 \text{int}(A))
\]
v) \[
(\tau_1 \text{int}(X - A) \cup \tau_2 \text{int}(X - A)) \cup \tau_1 \tau_2^* \text{Bd}(A) \cup (\tau_1 \text{int}(A) \cup \tau_2 \text{int}(A))
\]
\[
= [X - ((\tau_1 \text{Cl}(A) \cap \tau_2 \text{Cl}(A))] \cup (\tau_1 \text{Cl}(A) \cap \tau_2 \text{Cl}(A))
\]
\[
X.
\]
By ii) and (iii) \(\tau_1 \tau_2^* \text{Bd}(A) \cap (\tau_1 \text{int}(A) \cup \tau_2 \text{int}(A)) = \phi\) and
\[
\tau_1 \tau_2^* \text{Bd}(A) \cap (\tau_1 \text{int}(X - A) \cup \tau_2 \text{int}(X - A)) = \phi
\]
Now, \((\tau_1 \text{int}(A) \cup \tau_2 \text{int}(A)) \cap (\tau_1 \text{int}(X - A) \cup \tau_2 \text{int}(X - A)) \subseteq A \cap (X - A) = \phi
\]
Therefore, \(X = (\tau_1 \text{int}(A) \cup \tau_2 \text{int}(A)) \cup \tau_1 \tau_2^* \text{Bd}(A) \cup (\tau_1 \text{int}(X - A) \cup \tau_2 \text{int}(X - A))\) is a pair wise disjoint union.

**Theorem 3.5**

Let \((X, \tau_1, \tau_2)\) be a BGTS and \(A\) be a subset of \(X\). Then

i) \(A\) is \(\tau_1 \tau_2^*\) closed if and only if \(\tau_1 \tau_2^* \text{Bd}(A) \subseteq A\).

ii) \(A\) is \(\tau_1 \tau_2^*\) open if and only if \(\tau_1 \tau_2^* \text{Bd}(A) \subseteq X - A\).

**Proof:**

i) Assume that \(A\) is \(\tau_1 \tau_2^*\) closed,
Thus \(\tau_1 \text{Cl} A \cap \tau_2 \text{Cl} A = A\)
Let \(x \in \tau_1 \tau_2^* \text{Bd}(A)\). Then
\[
x \in (\tau_1 \text{Cl} A \cap \tau_2 \text{Cl} A) \cap (\tau_1 \text{Cl}(X - A) \cap \tau_2 \text{Cl}(X - A))
\]
\[
\Rightarrow x \in A \cap (\tau_1 \text{Cl}(X - A) \cap \tau_2 \text{Cl}(X - A))
\]
\[
\Rightarrow x \in A
\]
Therefore, \(\tau_1 \tau_2^* \text{Bd}(A) \subseteq A\).
Conversely let \(\tau_1 \tau_2^* \text{Bd}(A) \subseteq A\)
Then \(\tau_1 \tau_2^* \text{Bd}(A) \cap (X - A) = \phi\)
Now, \((\tau_1 \text{Cl} A \cap \tau_2 \text{Cl} A) \cap (X - A)\)
\[
= (\tau_1 \text{Cl} A \cap \tau_2 \text{Cl} A) \cap [(\tau_1 \text{Cl}(X - A) \cap \tau_2 \text{Cl}(X - A)) \cap (X - A)]
\]
\[
= \tau_1 \tau_2^* \text{Bd}(A) \cap (X - A)
\]
\[
= \phi
\]
Therefore, \(\tau_1 \text{Cl} A \cap \tau_2 \text{Cl} A \subseteq A\)
Also \(A \subseteq \tau_1 \text{Cl} A \cap \tau_2 \text{Cl} A\)
That implies \(\tau_1 \text{Cl} A \cap \tau_2 \text{Cl} A = A\)
Hence \(A\) is \(\tau_1 \tau_2^*\) closed.

ii) Assume that \(A\) is \(\tau_1 \tau_2^*\) open,
Thus \(\tau_1 \text{int}(A) \cup \tau_2 \text{int}(A) = A\)
Now \(\tau_1 \tau_2^* \text{Bd}(A) \cap A = [(\tau_1 \text{Cl}(A) \cap \tau_2 \text{Cl}(A)) \cap (\tau_1 \text{int}(A) \cup \tau_2 \text{int}(A))] \cap A\)
\[
= (\tau_1 \text{Cl}(A) \cap \tau_2 \text{Cl}(A)) \cap (\tau_1 \text{int}(A) \cup \tau_2 \text{int}(A))\cap A
\]
\[
= \phi
\]
Hence \(\tau_1 \tau_2^* \text{Bd}(A) \subseteq X - A\)
Conversely, Assume that $\tau_1 \tau_2^* \text{Bd}(A) \subseteq X - A$
Thus $\tau_1 \tau_2^* \text{Bd}(A) \cap A = \phi$.
This implies, $[(\tau_1 \text{Cl}(A) \cap \tau_2 \text{Cl}(A)) \setminus (\tau_1 \text{int}(A) \cup \tau_2 \text{int}(A))] \cap A = \phi$
Since $A \subseteq (\tau_1 \text{Cl}(A) \cap \tau_2 \text{Cl}(A))$, $A - (\tau_1 \text{int}(A) \cup \tau_2 \text{int}(A)) = \phi$
But $\tau_1 \text{int}(A) \cup \tau_2 \text{int}(A) \subseteq A$. Hence $A = \tau_1 \text{int}(A) \cup \tau_2 \text{int}(A)$.
A is $\tau_1 \tau_2^*$ open.

**Theorem: 3.6**
Let $(X, \tau_1, \tau_2)$ be a bigeneralized topological space and $A$ be a subset of $X$. Then $\tau_1 \tau_2^* \text{Bd}(A) = \phi$ if and only if $A$ is $\tau_1 \tau_2^*$ closed and $\tau_1 \tau_2^*$ open.

**Proof:**
Let $\tau_1 \tau_2^* \text{Bd}(A) = \phi$
Then by theorem 3.5 $A$ is $\tau_1 \tau_2^*$ closed and $\tau_1 \tau_2^*$ open.
Conversely, let $\tau_1 \tau_2^*$ closed and $\tau_1 \tau_2^*$ open.
Then $\tau_1 \text{Cl}(A) \cap \tau_2 \text{Cl}(A) = A$ and
$\tau_1 \text{Cl}(X - A) \cap \tau_2 \text{Cl}(X - A) = X - A$
$\tau_1 \tau_2^* \text{Bd}(A) = (\tau_1 \text{Cl}(A) \cap \tau_2 \text{Cl}(A)) \cap (\tau_1 \text{Cl}(X - A) \cap \tau_2 \text{Cl}(X - A))$
$= A \cap (X - A)$
$= \phi$

**Definition: 3.7**
Let $(X, \tau_1, \tau_2)$ be a bigeneralized topological space. A subset $A$ of $X$ is said $\tau_1 \tau_2^*$ dense if $\tau_1 \tau_2^* \text{Cl}(A) = X$.

**Theorem: 3.8**
Let $(X, \tau_1, \tau_2)$ be a bigeneralized topological space. If $A$ is $\tau_1 \tau_2^*$ closed and $X - A$ is $\tau_1 \tau_2^*$ dense then $\tau_1 \tau_2^* \text{Bd}(A) = A$

**Proof:**
Since $A$ is $\tau_1 \tau_2^*$ closed $\tau_1 \tau_2^* \text{Bd}(A) \subseteq A$.
Since $X - A$ is $\tau_1 \tau_2^*$ dense, $\tau_1 \text{Cl}(X - A) \cap \tau_2 \text{Cl}(X - A) = X$
Clearly $A \subseteq \tau_1 \text{Cl}(A) \cap \tau_2 \text{Cl}(A)$. Hence $A \subseteq (\tau_1 \text{Cl}(A) \cap \tau_2 \text{Cl}(A)) \cap X$
This implies, $A \subseteq (\tau_1 \text{Cl}(A) \cap \tau_2 \text{Cl}(A)) \cap (\tau_1 \text{Cl}(X - A) \cap \tau_2 \text{Cl}(X - A))$
This implies, $A \subseteq \tau_1 \tau_2^* \text{Bd}(A)$. Hence $A = \tau_1 \tau_2^* \text{Bd}(A)$

**References**