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$\tau_1\ \tau_2$* Boundary set on Bigeneralized Topological Space

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Abstract

In this paper we define $\tau_1\ \tau_2$* boundary set in a bigeneralized topological space we study the properties of the set.

Key words: $\tau_1\tau_2$* boundary point,

I.Introduction:

In 1963, J.C. Kely [1] defined the study of Bitopological spaces. The concept of generalized topology was introduced by A. Csaszar [2] in 2002. In 2010, C. Boonpok [3] introduced the concept of bigeneralized topological spaces. In 2013, sompong and muangchan [4] introduced the concepts of boundary set on Bigeneralized topological space. In this paper, we introduce the concepts of $\tau_1\tau_2$* boundary point, and study their properties in bigeneralized topological spaces.

II. Preliminaries

Let X be a non-empty set and τ be a collection of subsets of X. Then τ is called a generalized topology on X if $\phi \in \tau$ and arbitrary union of elements of τ in τ. A space with a generalized topology is called a generalized topological space.

In a generalized topological space (X, τ) the following are true

i) Cl (A) = X – int (X – A)
ii) Int (A) = X - Cl (X – A)
iii) Cl (X – A) = X – int (A) and int (X – A) = X – Cl (A)
iv) If X – A $\in \tau$ then Cl (A) = A and if A $\in \tau$ then int (A) = A.
v) If A $\subseteq$ B then Cl (A) $\subseteq$ Cl (B) and int (A) $\subseteq$ int (B)
vi) A $\subseteq$ Cl (A) and int (A) $\subseteq$ A

Let X be a non – empty set and τ_1, τ_2 be generalized topologies on X. The triple (X, τ_1,τ_2) is called a bigeneralized topological space (briefly BGTS).

III. $\tau_1\ \tau_2$* Boundary set

Definition: 3.1

A subset A of a bigeneralized topological space (X, τ_1, τ_2) is called $\tau_1 \tau_2^*$ closed if $A = \tau_1 Cl(A) \cap \tau_2 Cl(A)$. The complement of $\tau_1 \tau_2^*$ closed set is called $\tau_1 \tau_2^*$ open. The $\tau_1 \tau_2^*$ closure of a set A is defined in the usual way.

Result: 3.2

Let (X, τ_1, τ_2) be a bigeneralized topological space and let A be a subset of X. Then $\tau_1 \tau_2^* Cl(A) = \tau_1 Cl A \cap \tau_2 Cl A$.

In this section, we introduce the concept $\tau_1 \tau_2^*$ boundary set and study their properties in bigeneralized topological space.

Definition: 3.3

Let (X, τ_1, τ_2) be a BGTS. Let A be a subset of X and $x \in X$. Then x is called $\tau_1 \tau_2^*$ boundary point of A if $x \in (\tau_1 \tau_2^* Cl A \cap \tau_1 \tau_2^* Cl(X - A))$. We denote the set of all $\tau_1 \tau_2^*$ – boundary point of A by $\tau_1 \tau_2^* Bd(A)$. From definition we have $\tau_1 \tau_2^* Bd(A) = (\tau_1 Cl A \cap \tau_2 Cl A) \cap (\tau_1 Cl(X - A) \cap \tau_2 Cl(X - A))$

Example: 3.2

Let $X = \{a, b, c, d\}$

$\tau_1 = \{\phi, X, \{a, b\}, \{b, c\}\}$ and

$\tau_2 = \{\phi, X, \{a, c\}, \{a, b\}\}$

Now, τ_1 closed sets are $\{\phi, X, \{c\}, \{a\}\}$ and

τ_2 closed sets are $\{\phi, X, \{b\}, \{c\}\}$

$\tau_1 \tau_2 Bd(\{a, b\}) = \{c\}$ and

$\tau_1 \tau_2 Bd(\{b, c\}) = \{a\}$

Theorem: 3.3

Let (X, τ_1, τ_2) be a bigeneralized topological space. Let A be a subset of X. Then $\tau_1\tau_2^* Bd(A) = \tau_1 \tau_2^* Bd(X - A)$

Proof:

$$\begin{aligned}\tau_1 \tau_2^* Bd(A) &= (\tau_1 Cl A \cap \tau_2 Cl A) \cap (\tau_1 Cl(X - A) \cap \tau_2 Cl(X - A)) \\ &= (\tau_1 Cl(X - (X - A)) \cap \tau_2 Cl(X - (X - A))) \cap (\tau_1 Cl(X - A) \cap \tau_2 Cl(X - A)) \\ &= \tau_1 \tau_2^* Bd(X - A)\end{aligned}$$

Theorem: 3.4

Let (X, τ_1, τ_2) be a bigeneralized topological space and A, B be a subset of X. Then the following are true.

i) $\tau_1 \tau_2^* Bd(A) = (\tau_1 Cl(A) \cap \tau_2 Cl(A)) \setminus (\tau_1 int(A) \cup \tau_2 int(A))$

ii) $\tau_1 \tau_2^* Bd(A) \cap (\tau_1 int(A) \cup \tau_2 int(A)) = \phi$

iii) $\tau_1 \tau_2^* Bd(A) \cap (\tau_1 int(X - A) \cup \tau_2 int(X - A)) = \phi$

iv) $\tau_1 Cl A \cap \tau_2 Cl A = \tau_1 \tau_2^* Bd(A) \cup (\tau_1 int(A) \cup \tau_2 int(A))$

v) $X = (\tau_1 int(X - A) \cup \tau_2 int(X - A)) \cup \tau_1 \tau_2^* Bd(A) \cup (\tau_1 int(A) \cup \tau_2 int(A))$

is a pair wise disjoint Union.

Proof:

i)
$$\begin{aligned}\tau_1 \tau_2^* Bd(A) &= (\tau_1 Cl(A) \cap \tau_2 Cl(A)) \cap (\tau_1 Cl(X - A) \cap \tau_2 Cl(X - A)) \\ &= (\tau_1 Cl(A) \cap \tau_2 Cl(A)) \cap (X - (\tau_1 int(A) \cup \tau_2 int(A))) \\ &= (\tau_1 Cl(A) \cap \tau_2 Cl(A)) \setminus (\tau_1 int(A) \cup \tau_2 int(A))\end{aligned}$$

ii) From (i), it's obvious that

$\tau_1\ \tau_2^*$ Bd (A) $\cap$ (τ_1 int (A) $\cup$ τ_2 int (A)) = ϕ

iii) $\tau_1\ \tau_2^*$ Bd (A) $\cap$ (τ_1 int (X – A) $\cup$ τ_2 int (X – A))

= $\tau_1\ \tau_2^*$ Bd (X – A) $\cap$ (τ_1 int (X – A) $\cup$ τ_2 int (X – A))

= ϕ

iv) τ_1 Cl A $\cap$ τ_2 Cl A

= [τ_1 Cl (A) $\cap$ τ_2 Cl (A) \ τ_1 int (A) $\cup$ τ_2 int (A)] $\cup$ (τ_1 int (A) $\cup$ τ_2 int (A))

= $\tau_1\ \tau_2^*$ Bd (A) $\cup$ (τ_1 int (A) $\cup$ τ_2 int (A))

v) (τ_1 int (X – A) $\cup$ τ_2 int (X – A)) $\cup$ $\tau_1\ \tau_2^*$ Bd (A) $\cup$ (τ_1 int (A) $\cup$ τ_2 int (A))

= [X – ((τ_1 Cl (A) $\cap$ τ_2 Cl (A))] $\cup$ (τ_1 Cl (A) $\cap$ τ_2 Cl (A))

= X.

By ii) and (iii) $\tau_1\ \tau_2^*$ Bd (A) $\cap$ (τ_1 int (A) $\cup$ τ_2 int (A)) = ϕ and

$\tau_1\ \tau_2^*$ Bd (A) $\cap$ (τ_1 int (X – A) $\cup$ τ_2 int (X – A)) = ϕ

Now, (τ_1 int (A) $\cup$ τ_2 int (A)) $\cap$ (τ_1 int (X – A) $\cup$ τ_2 int (X – A)) $\subseteq$ A $\cap$ (X – A)

= ϕ

Therefore, X = (τ_1 int (A) $\cup$ τ_2 int (A)) $\cup$ $\tau_1\ \tau_2^*$ Bd (A) $\cup$ (τ_1 int (X – A) $\cup$ τ_2 int (X – A)) is a pair wise disjoint union.

Theorem: 3.5

Let (X, τ_1, τ_2) be a BGTS and A be a subset of X. Then

i) A is $\tau_1\ \tau_2^*$ closed if and only if $\tau_1\ \tau_2^*$ Bd (A) $\subseteq$ A.

ii) A is $\tau_1\ \tau_2^*$ open if and only if $\tau_1\ \tau_2^*$ Bd (A) $\subseteq$ X – A.

Proof:

i) Assume that A is $\tau_1\ \tau_2^*$ closed,

Thus τ_1 Cl A $\cap$ τ_2 Cl A = A

Let $x \in \tau_1\ \tau_2^*$ Bd (A). Then

$x \in$ (τ_1 Cl A $\cap$ τ_2 Cl A) $\cap$ (τ_1 Cl (X – A) $\cap$ τ_2 Cl (X – A))

$\Rightarrow x \in$ A $\cap$ (τ_1 Cl (X – A) $\cap$ τ_2 Cl (X – A))

$\Rightarrow x \in$ A

Therefore, $\tau_1\ \tau_2^*$ Bd (A) $\subseteq$ A.

Conversely let $\tau_1\ \tau_2^*$ Bd (A) $\subseteq$ A

Then $\tau_1\ \tau_2^*$ Bd (A) $\cap$ (X – A) = ϕ

Now, (τ_1 Cl A $\cap$ τ_2 Cl A) $\cap$ (X – A)

= (τ_1 Cl A $\cap$ τ_2 Cl A) $\cap$ [(τ_1 Cl (X – A) $\cap$ τ_2 Cl (X – A)) $\cap$ (X – A)]

= $\tau_1\ \tau_2^*$ Bd (A) $\cap$ (X – A)

= ϕ

Therefore, τ_1 Cl A $\cap$ τ_2 Cl A $\subseteq$ A

Also A $\subseteq$ τ_1 Cl A $\cap$ τ_2 Cl A

That implies τ_1 Cl A $\cap$ τ_2 Cl A = A

Hence A is $\tau_1\ \tau_2^*$ closed.

ii) Assume that A is $\tau_1\ \tau_2^*$ open,

Thus τ_1 int (A) $\cup$ τ_2 int (A) = A

Now $\tau_1\ \tau_2^*$ Bd (A) $\cap$ A = [(τ_1 Cl (A) $\cap$ τ_2 Cl (A)) – (τ_1 int (A) $\cup$ τ_2 int (A))] $\cap$ A

= (τ_1 Cl (A) $\cap$ τ_2 Cl (A)) – A) $\cap$ A

= ϕ

Hence $\tau_1\ \tau_2^*$ Bd (A) $\subseteq$ X – A

Conversely, Assume that $\tau_1 \tau_2^*$ Bd (A) $\subseteq$ X – A

Thus $\tau_1 \tau_2^*$ Bd (A) $\cap$ A = ϕ.

This implies, $[(\tau_1 \text{ Cl } (A) \cap \tau_2 \text{ Cl } (A)) \setminus (\tau_1 \text{ int } (A) \cup \tau_2 \text{ int } (A))] \cap A = \phi$

Since $A \subseteq (\tau_1 \text{ Cl } (A) \cap \tau_2 \text{ Cl } (A))$, $A - (\tau_1 \text{ int } (A) \cup \tau_2 \text{ int } (A)) = \phi$

But τ_1 int (A) $\cup$ τ_2 int (A) $\subseteq$ A. Hence A = τ_1 int (A) $\cup$ τ_2 int (A).

A is $\tau_1 \tau_2^*$ open.

Theorem: 3.6

Let (X, τ_1, τ_2) be a bigeneralized topological space and A be a subset of X. Then $\tau_1 \tau_2^*$ Bd (A) = ϕ if and only if A is $\tau_1 \tau_2^*$ closed and $\tau_1 \tau_2^*$ open.

Proof:

Let $\tau_1 \tau_2^*$ Bd (A) = ϕ

Then by theorem 3.5 A is $\tau_1 \tau_2^*$ closed and $\tau_1 \tau_2^*$ open.

Conversely, let $\tau_1 \tau_2^*$ closed and $\tau_1 \tau_2^*$ open.

Then τ_1 Cl A $\cap$ τ_2 Cl A = A and

τ_1 Cl (X – A) $\cap$ τ_2 Cl (X – A) = X – A

$\tau_1 \tau_2^*$ Bd (A) = (τ_1 Cl A $\cap$ τ_2 Cl A) $\cap$ (τ_1 Cl (X – A) $\cap$ τ_2 Cl (X – A))

= A $\cap$ (X – A)

= ϕ

Definition: 3.7

Let (X, τ_1, τ_2) be a bigeneralized topological space. A subset A of X is said $\tau_1 \tau_2^*$ dense if $\tau_1 \tau_2^*$ Cl A = X.

Theorem: 3.8

Let (X, τ_1, τ_2) be a bigeneralized topological space. If A is $\tau_1 \tau_2^*$ closed and X – A is $\tau_1 \tau_2^*$ dense then $\tau_1 \tau_2^*$ Bd (A) = A

Proof:

Since A is $\tau_1 \tau_2^*$ closed $\tau_1 \tau_2^*$ Bd (A) $\subseteq$ A.

Since X – A is $\tau_1 \tau_2^*$ dense, τ_1 Cl (X – A) $\cap$ τ_2 Cl (X – A) = X

Clearly A $\subseteq$ τ_1 Cl A $\cap$ τ_2 Cl A. Hence A $\subseteq$ (τ_1 Cl A $\cap$ τ_2 Cl A) $\cap$ X

This implies, A $\subseteq$ (τ_1 Cl A $\cap$ τ_2 Cl A) $\cap$ (τ_1 Cl (X – A) $\cap$ τ_2 Cl (X – A))

This implies, A $\subseteq$ $\tau_1 \tau_2^*$ Bd (A). Hence A = $\tau_1 \tau_2^*$ Bd (A)

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