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ON KAHLER MANIFOLD

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Abstract

In the present paper we study Kahler manifold with conservative conharmonic curvature tensor and kahler manifold with conservative conformal curvature tensor and conservative con-circular curvature tensor.

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1 Introduction

Let M^n be a n-dimensional (n=2m) almost Hermitian manifold with almost Hermitian structure field ϕ and the associated Riemannnian metric g. Then M^n is called a kahler manifold if

$$(\nabla_X^{\phi})Y = 0 \tag{1}$$

where ∇ denotes the Riemannian connection of g.
In such a manifold , Conharmonic curvature tensor L[3] is given by

$$L(X,Y)Z = R(X,Y)Z - \frac{1}{n-2}\{S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\} \tag{2}$$

where S(X,Y)=g(QX,Y)
where Q is the symmetric endomorphism of the tangent space at each point of the manifold and R and S be the Riemannian curvature tensor and Ricci tensor respectively.

2 Preliminaries

It is known that the following relation hold in M^n[1]

$$\phi^2 X = -X \tag{3}$$

$$g(\phi X, \phi Y) = g(X,Y) \tag{4}$$

$$'\phi(X,Y) = -'\phi(X,Y) \tag{5}$$

$$(\nabla_Z{}'\phi)(X,Y) = 0 \tag{6}$$

$$'S(X,Y) = -'S(Y,X) \tag{7}$$

$$'S(X,\phi Y) = -'S(\phi X,Y) = S(X,Y) \tag{8}$$

$$2(divL)Y = Y.r \tag{9}$$

$$(\nabla_Z{}'S)(Y,X) + (\nabla_Y{}'S)(X,Z) + (\nabla_X{}'S)(Z,Y) = 0 \tag{10}$$

Writing ϕZ for Z in (10) one gets

$$(\nabla_{\phi Z}{'}S)(Y,X) = (\nabla_Y{'}S)(\phi Z,X) - (\nabla_X{'}S)(\phi Z,Y)$$

Using (8) we obtain

$$(\nabla_{\phi^2 Z}{'}S)(Y,X) = (\nabla_X S)(Z,Y) - (\nabla_Y S)(Z,X) \tag{11}$$

Writing $\phi Z, \phi Y$ for Z, Y respectively in (11) and using (3) and (8) one gets

$$(\nabla_Z S)(Y,X) = (\nabla_X S)(\phi Z,\phi Y) - (\nabla_{\phi Y} S)(\phi Z,X) \tag{12}$$

We will use the above results in the next sections.

3 Conservative conharmonic tensor on Kahler manifold

Let L is conservative , so divL=0
So, from (2) , we have

$$(divR)(X,Y,Z) = \frac{1}{n-2}\{(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z) + g(Y,Z).(divQX) - g(X,Z).(divQY)\}$$

Using (9) we have or,

$$(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z) = \frac{1}{n-2}\{(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z) + g(Y,Z).\frac{1}{2}(X.r) - g(X,Z).\frac{1}{2}(Y.r)\}$$

or,

$$(n-3)\{(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z)\} = g(Y,Z).\frac{1}{2}(X.r) - g(X,Z).\frac{1}{2}(Y.r)$$

If the scalar curvature r is constant, then

$$(n-3)\{(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z)\} = 0$$

Then

$$(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z) = 0$$

$\Rightarrow (divR)(X,Y)Z = 0$ as $n > 3$.

Theorem 1: If a Kahler manifold ($n > 3$) of conservative curvature tensor has constant scalar curvature then the Riemannian curvature tensor is also conservative.

Conversely, let (div R)(X,Y)Z=0
so,

$$(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z) = 0$$

From (2) , we have

$$(divL)(X,Y)Z = (divR)(X,Y)Z - \frac{1}{n-2}\{(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z) + g(Y,Z).\frac{1}{n-2}(X.r) - g(X,Z).\frac{1}{n-2}(Y.r)\}$$

since (div R)(X,Y)Z=0
so,

$$(divL)(X,Y)Z = -\frac{1}{n-2}\{(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z) + g(Y,Z).\frac{1}{2}(X.r) - g(X,Z).\frac{1}{2}(Y.r)\}$$

If r is constant then (div L)(X,Y)Z=0 .

Theorem 2: If a Kahler ($n > 3$) of conservative curvature tensor has constant scalar curvature then the conharmonic curvature tensor is also conservative.
In a Kahler manifold of constant scalar curvature div R implies div L and vice-versa.

4 Conservative conformal curvature tensor in Kahler manifold

The conformal curvature tensor[5] is given by

$$\begin{aligned} C(X,Y)Z =& R(X,Y)Z - \frac{1}{n-2}\{S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\} + \\ & \frac{r}{(n-1)(n-2)}\{g(Y,Z)X - g(X,Z)Y\}. \end{aligned} \tag{13}$$

Let $divC = 0$. Then from (3) we get,

$$\begin{aligned} (divR)(X,Y)Z =& \frac{1}{n-2}\{(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z) + \\ & g(Y,Z)(divQ)X - g(X,Z)(divQ)Y\} + \\ & \frac{1}{(n-1)(n-2)}\{g(Y,Z)(X.r) - g(X,Z)(Y.r)\}. \end{aligned} \tag{14}$$

Using (9) we have

$$\{(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z)\} = \frac{n+1}{2(n-3)(n-2)(n+1)}\{g(Y,Z)(X.r) - g(X,Z)(Y.r)\}$$

If r is constant then

$$(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z) = 0 \tag{15}$$

$$(divR)(X,Y)Z = 0 \tag{16}$$

and put ϕY , ϕZ in the place of Y and Z in (3.1) we get $(\nabla_Z S)(X,Y) = 0$.

Theorem 3: If a Kahler manifold of dimension n ($n > 2$) of conservative conformal curvature then the manifold is Ricci symmetric.

5 Kahler manifold with conservative con -circular curvature tensor

The con-circular tensor[8] is given by

$$P(X,Y)Z = R(X,Y)Z - \frac{r}{n(n-1)}\{g(Y,Z)X - g(X,Z)Y\} \tag{17}$$

where r is the scalar curvature of the manifold.
Let div P=0, Then from (4), we get

$$(divR)(X,Y)Z = \frac{1}{n(n-1)}\{g(Y,Z)(X.r) - g(X,Z)(Y.r)\}$$

or,

$$(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z) = \frac{1}{n(n-1)}\{g(Y,Z)(X.r) - g(X,Z)(Y.r)\}$$

putting ϕY , ϕZ in the place of Y and Z,and using (1.9 a) we get,

$$(\nabla_Z S)(Y,X) = \frac{1}{n(n-1)}\{g(Y,Z)(X.r) - g(X,\phi Z)(\phi Y.r)\}$$

If r is constant then $(\nabla_Z S)(Y,X) = 0$
So, we can state
Theorem 4: If a Kahler manifold of conservative con -circular curvature tensor has constant scalar curvature the manifold is Ricci symmetric.

From (13)and (17) , we see that

$$\begin{aligned} C(X,Y)Z + P(X,Y)Z =& 2R(X,Y)Z - \frac{1}{n-2}\{S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\}+ \\ & \frac{r}{(n-1)(n-2)}\{g(Y,Z)X - g(X,Z)Y\} - \frac{r}{n(n-1)}\{g(Y,Z)X - g(X,Z)Y\}. \end{aligned} \tag{18}$$

Now,

$$\begin{aligned} (divC)(X,Y)Z + (divP)(X,Y)Z =& 2(divR)(X,Y)Z - \frac{1}{n-2}\{(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z)+ \\ & g(Y,Z)(divQ)X - g(X,Z)(divQ)Y\} + \frac{1}{(n-1)(n-2)}\{g(Y,Z)(X.r)- \\ & g(X,Z)(Y.r)\} - \frac{1}{n(n-1)}\{g(Y,Z)(X.r) - g(X,Z)(Y.r)\}. \end{aligned} \tag{19}$$

Using(9) we get

$$\begin{aligned} (divC)(X,Y)Z + (divP)(X,Y)Z =& \frac{2n-5}{n-2}\{(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z)\}+ \\ & \frac{-n^2+2n+4}{2n(n-1)(n-2)}\{g(Y,Z)(X.r) - g(X,Z)(Y.r)\}. \end{aligned} \tag{20}$$

Let us suppose div C=0 and r is constant and applying (3.2) we get from (4.1), div P=0
Similarly , if div P=0 and r is constant then div C=0
Hence we can state the following theorem

Theorem 5: In a Kahler manifold of dimensional n($n > 2$) of constant scalar curvature , conformal curvature property implies con-circular conservative property and vice-versa.

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