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LINEAR OSCILLATION OF AN INEXTENSIBLE CABLE CONNECTED SATELLITES SYSTEM UNDER THE INFLUENCE OF AIR RESISTANCE, MAGNETIC FORCE AND THE SHADOW OF OBLATE EARTH DUE TO SOLAR PRESSURE ABOUT THE POSITION OF STABLE EQUILIBRIUM FOR SMALL ECCENTRICITY IN ELLIPTIC ORBIT NEAR THE MAIN RESONANCE N = 1

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Abstract: *This paper is devoted to study the Linear main resonant oscillates about the stable position of equilibrium where the system oscillates about the position of equilibrium like a dumb-bell satellite in the phase plane (a,q). B.K.M. method has been exploited to get the general solution valid at and near the main resonance n=1.*

INTRODUCTION

The Russian mathematicians Beletsky; V.V. (1960) and Austin, F. (1965) made significant studies on the effect of perturbative forces on cable-connected satellites system. Similar problems have been studied by Singh; R.B.; and Das; S.K. and Bhattacharya; P.K.

Linear oscillation of the system about the stable position of equilibrium for small eccentricity near the main resonance n = 1.

The equation of the motion of cable connected satellite system in the central gravitational field of oblate earth under the influence of air resistance, magnetic force and shadow of the earth due to solar pressure in polar form is given by $-(1+e\cos v)\psi''-e\psi'\sin v+3\sin\psi\cos\psi$

$$=2e\sin v+5A(1+e\cos v)^2\sin\psi\cos\psi-Bo\rho^3(\cos\psi\sin\alpha-\sin\psi\cos\alpha)$$

$$-f\rho^2(\cos\psi-e\rho\sin\psi\sin v)+\frac{c}{\rho}(\sin\psi-e\rho\sin v\cos\psi) \qquad (1)$$

Where, $A=-\frac{3k_2}{p^2}$ = Oblations force parameter.

$B_o=\frac{B\sin\theta}{\pi}$ = Joint force parameter due to solar pressure and shadow of the earth.

$f=\frac{a_1p^3}{\sqrt{\mu p}}$ = Air resistance force parameter

C = Magnetic force parameter

$$\rho = \frac{1}{1 + e\cos v}$$

e = ecentricity of orbit

p = focal parameter

v = true anomaly of the centre of masses orbit

Here dashes denote differentiation with respect to true anomaly V

The equilibrium position is given by-

$$\phi = \phi_0 = 0 \text{ and } \psi = \psi_0 = \frac{-f - B_o \sin\alpha}{3 - 5A - c - B_0 \cos\alpha} = \mathrm{d}_0 \tag{3}$$

The equation of small oscillation about the stable position of equilibrium is obtained by putting $\psi = \psi_0 + \eta = d_o + \eta$ from (1) assuming e to be small quantity of first order infinitesimal as –

$$\eta'' + n^2\eta = e\begin{bmatrix} 2\sin v - \eta''\cos v + 2\eta'\sin v + 10A\{d_o + (1 + d_0^{\,2})\eta\}\cos v \\ + 3B_0\{1 - d_0\eta\}\sin\alpha\cos v - 3B_0\{d_0 + \eta\}\cos\alpha\cos v \\ + 2f\{1 - d_0\eta\}\cos v - f\{d_0 + \eta\}\sin v \\ + c(d_0 + \eta)\cos v - c(1 - d_0\eta)\sin v \end{bmatrix} \tag{4}$$

Where, $$n^2 = (3 - 5A)(1 - d_0^{\,2}) - B_0\cos\alpha - \mathrm{B}_0 d_0\sin\alpha - fd_0 - c \tag{5}$$

Now let us construct the general solution of the oscillatory system based on B.K.M. method which will be valid at and near the main resonance n=1. Assuming e to be small parameter, the solution in the first approximation can be sought in the form :

$$\begin{aligned} &\eta = a\cos k \\ &\textit{Where, } k = v + \theta \end{aligned} \tag{6}$$

The amplitude 'a' and phase must satisfy the system of ordinary differential equations :

$$\begin{aligned} \frac{da}{dv} &= eA_1(a,\theta) \\ \frac{d\theta}{dv} &= (n-1) + eB_1(a,\theta) \end{aligned} \tag{7}$$

Where, $A_1(a, \theta)$ and $B_1(a, \theta)$ are periodic solutions periodic with respect to of the system of partial differential equations :

$$(n-1)\frac{\partial A_1}{\partial \theta} - 2anB_1 = \frac{1}{\pi}\int_0^{2\pi} f_0(v,\eta,\eta',\eta'')\cos kdk$$

and

$$a(n-1)\frac{\partial B_1}{\partial \theta} + 2nA_1 = -\frac{1}{\pi}\int_0^{2\pi} f_0(v,\eta,\eta',\eta'')\sin kdk \tag{8}$$

Where,

$$\begin{aligned} f_0(v,\eta,\eta',\eta'') &= (2-c-fd_0)\sin v - 2an\sin k\sin v + cd_0 a\cos k\sin v + an^2\cos v\cos k \\ &- fa\cos k\sin v + (10Ad_0 + 3B_0\sin\alpha - 3B_0 d_0\cos\alpha + f + cd_0)\cos v \\ &+ \{10A(1-d_0^{\,2}) + c - 3B_0 d_0\sin\alpha - 3B_0\cos\alpha - 2fd_0\}\cos k\cos v \end{aligned} \tag{9}$$

Using (9) and (8), We get –

$$\begin{aligned} &(n-1)\frac{\partial A_1}{\partial \theta} - 2anB_1 = \mu\sin\theta + v\cos\theta \\ &and\ a(n-1)\frac{\partial B_1}{\partial \theta} - 2nA_1 = -\mu\cos\theta - v\sin\theta \end{aligned} \tag{10}$$

Where, μ and *v* are given by

$$\begin{aligned} \mu &= 2 - c - fd_o \\ v &= 10Ad_0 + 3B_0\sin\alpha - 3B_0 d_0\cos\alpha + f + cd_0 \end{aligned} \tag{11}$$

The periodic solution with respect to θ of the system of equation (10) can be easily obtained as –

$$\begin{aligned} A_1 &= -\frac{1}{n+1}(\mu\cos\theta + v\sin\theta) \\ B_1 &= \frac{-1}{a(n+1)}(\mu\sin\theta - v\cos\theta) \end{aligned} \tag{12}$$

From (7) and (12), We obtain –

$$\begin{aligned} &\frac{da}{dv} = \frac{-e}{n+1}(\mu\cos v + v\sin\theta) \\ &and\ \frac{d\theta}{dv} = (n-1) - \frac{1}{a(n+1)}(v\cos\theta - \mu\sin\theta) \end{aligned} \tag{13}$$

The system of equation given by (13) can be written in the form –

$$\begin{aligned} \frac{da}{dv} &= \frac{1}{a}\frac{\partial\phi}{\partial\theta} \\ \frac{d\theta}{dv} &= -\frac{1}{a}\frac{\partial\phi}{\partial a} \end{aligned} \tag{14}$$

Where,
$$\phi = \frac{ae}{(n+1)}(v\cos\theta - \mu\sin\theta) - \frac{(n-1)a^2}{2} \tag{15}$$

Clearly, the system of equations given in (14) is in canonical form and hence it admits a first integral of the form:

$$\phi = \phi_0 \tag{16}$$

Where, ϕ_0 = constant of integration.

In order to examine the stability, the integral curve (15) has been plotted in phase plane in the form :

$$(n^2-1)a^2 - 2ae(v\cos\theta - \mu\sin\theta) + co = 0 \tag{17}$$

Where, $co = 2(n+1)\phi_0 =$ constant

The integral curves have been plotted in fig.1 and fig.2 for n=0.957, n=1.25 respectively for different values of parameters involved. Since both curves are closed, so we get the stability.

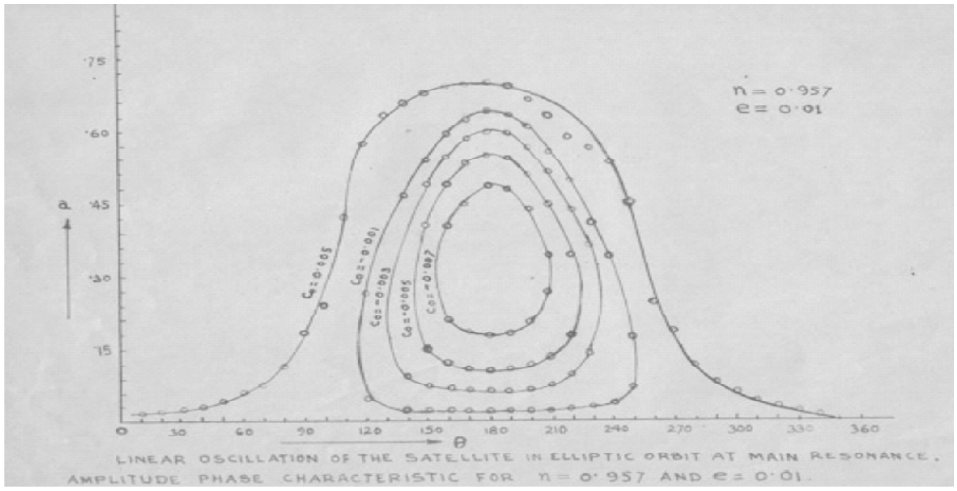

Figure 1

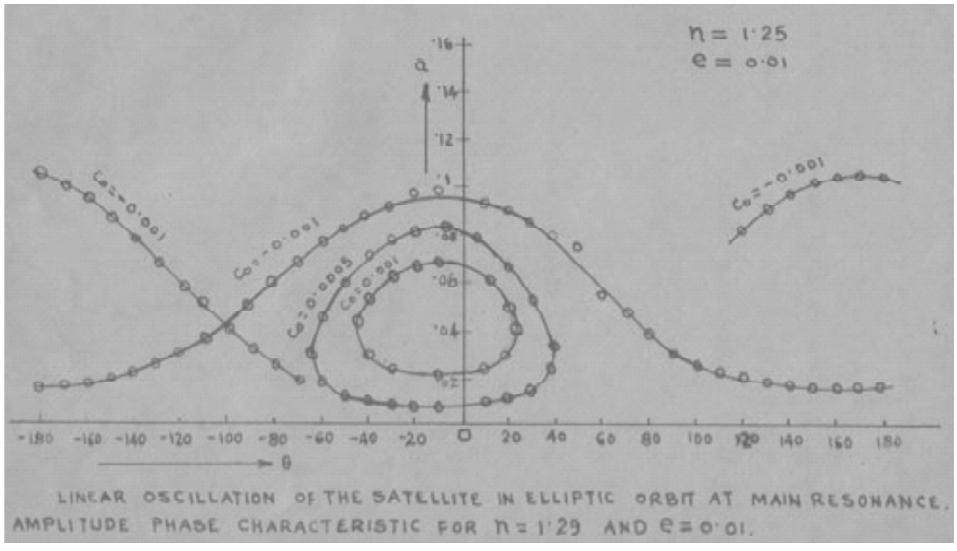

Figure 2

References

[i] Beledsky; V. V., Motion of artificial satellites relative to the centre of mass; Nauka; 1965 (Russian).

[ii] Singh; R.B., Proc. Nat. Aca. Sc. India 46(A) iv; 1976.

[iii) Das; S.K. and Bhattacharya P. K., Effect of magnetic force on the motion of a system of two cable-connected satellites in orbit (1976) Proc. Nat. Ac. Sc; India.