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Vehicle Routing and Scheduling Problems with time window constraints – Optimization Based Models

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ABSTRACT

Vehicle Routing Problems have been extensively analyzed to reduce transportation costs. More particularly, the Vehicle Routing Problem with Time Windows (VRPTW) imposes the period of time of customer availability as a constraint, a common characteristic in real world situations. Using minimization of the total distance as the main objective to be fulfilled, this work implements mathematical model and an optimization-based algorithm for the allocation of shipments to vehicles and the design of the minimum-cost vehicle itineraries and schedules that permit all customers to be serviced within their time window, if possible. The results of the implementation of the algorithm using real-world data from the manufacturer are presented.

INTRODUCTION

Time-constrained vehicle routing problems arise in instances where service is provided to customers who impose service deadlines and earliest-service constraints. Consider, for example, the situation where finished products are distributed from a manufacturing plant or a warehouse to a number of wholesalers and vendors. Each customer (wholesaler or vendor) places in advance a fixed-size order and specifies the time interval (time window) for the delivery. The distribution managers of the manufacturing company are responsible for the allocation of shipments to vehicles and the design of the minimum-cost vehicle itineraries and schedules that permit all customers to be serviced within their time window, if possible. We developed a mathematical model and an optimization-based algorithm for the problem described above. The model was implemented to evaluate and optimize the distribution operations of a major manufacturer. In the following sections the model and the algorithm are presented, along with the results of the implementation of the algorithm using real-world data from the manufacturer.

LITERATURE REVIEW

The problem described above is a variant of the general Vehicle Routing Problem (VRP) and the Vehicle Routing and Scheduling Problem with Time Window constraints (VRSPTW). Both problems have been extensively studied in the literature, but their intrinsic complexity has prevented the development of satisfactory algorithms for medium- and large-size problems. Both problems belong to the family of NP-complete problems, which means that an optimal solution cannot be found within polynomial time [N.Christofides, A.Mignozzi &P.Toth (1979)]. As such, the largest problem solved to optimality involves 4 vehicles and 14 customers [A.Kolen, A.Rinnoy Kan & H.Trienekens (1987)]. Most of the early efforts in the field have been concentrated

on case studies developing ad-hoc procedures for the individual problems [H.Pullen & M.Webb (1967)] providing an extensive survey of routing problems and algorithms. More recent efforts include the development of heuristic algorithms able to tackle realistic size problems and provide good qualify solutions [M.M Solomon (1987); E.Baker & J.Schaffer (1986)]. However, although heuristic algorithms have been quite successful in dealing with large-scale problems at minimal computational requirements, they suffer from serious limitations concerning the quality of the obtained solutions; some 4 or 5% deviation of the heuristic solution from optimality could result in millions of dollars of lost profit or unnecessary expenses for a medium-number or large-size company. Furthermore, heuristics are quite specialized and their behaviour and effectiveness depend heavily on the structure of the problem at hand; as such, they exhibit unstable behaviour when dealing with a wide variety of problems. On the other hand, while optimization algorithms can guarantee the optimal solution to a given problem (if one exists), the VRP with time windows does not lead itself to optimal solution approaches due to excessive computational requirements, especially in the case of large-size problems.

The trade-off between the quality of the obtained solution and the computational time requirements motivated the development of an optimization-based heuristic for the VRP with Time Windows. This class of algorithms shows the most potential for growth, and as the time window constrained VRP field matures, it is expected to see an increasing degree of sophistication in the design of successful approximation methods; optimization-based heuristics are a prime candidate in this respect [M.Solomon & J.Desrosiers (1988)]. The positive experience other researchers have had is an additional factor justifying the use of this class of algorithms [M.Fisher and R. Jaikumar (1981)]. These methods are competitive in running time with most standard heuristics that do not incorporate embedded optimization procedures, e.g., the Clark and Wright [G.Clarke &W.Wright (1964)] savings method, and yet provide better cost solutions in almost all instances [T.Magnanti(1981)]. In these lines, the next section presents a mathematical programming-based model and an optimization-based algorithm for shipment routing and scheduling.

THE MODEL AND THE ALGORITHM

The Vehicle Routing and Scheduling Problem with Time Window constraint is formulated as a mixed integer program; the following notation is necessary before the formal statement of the problem.

Let

C_{ij} – Cost of travelling directly from *i* to customer *j*.

q_i - Shipment size of customer *i*.

v_k - Capacity of the vehicle *k*.

t_{ij} - Travel time between customer *i* and *j*.

s_i - Service time at customer *i*.

a_i - Beginning of the time window at *i*

b_i - The end of the time window at *i*

T – Constant larger than the total travel time of any feasible route.

Let us also define the following decision variables:

$$x_{ijk} = \begin{cases} 1 \; if \; vehicle \; k \; travels \; directly \; from \; customer \; i \\ 0 \; otherwise \end{cases}$$

$$y_{ik} = \begin{cases} 1 \; if \; vehicle \; k \; services \; customer \; i \\ 0 \; otherwise \end{cases}$$

t_i = The Arrival time at $customer\ i$

Let

I – Set of delivery points (customer locations)

I° – Set of all customers I plus the depot.

K – Set of vehicles available to serve the customers

N_k–Subset of customers serviced by the vehicle *k*.

The Mixed Integer Programming (MIP) formulation for the Vehicle Routing and Scheduling Problem with Time Windows is stated as follows: (VRSPTW)

$$Min\ F(x,y,t) = \sum_{i\in I}\sum_{j\in I}\sum_{k\in K} c_{ij}x_{ijk} \qquad (1)$$

Subject to;

$$\sum_{i \in I} q_i y_{ik} \leq v_k \quad \forall\, k \in K \qquad (2)$$

$$\sum_k y_{ik} = \begin{cases} k & i = 0 \\ 1 & i = 1,2 \ldots \end{cases} \qquad (3)$$

$$y_{ik} = (0,1) \quad \forall\, i \in I\,, k \in K \qquad (4)$$

$$\sum_{i \in I} x_{ijk} = y_{jk}\ \forall\, j \in I^0 \qquad (5)$$

$$\sum_{i \in I} x_{ijk} = y_{ik}\ \forall\, j \in I^0 \qquad (6)$$

$$x_{ijk} = (0,1) \quad \forall\, i,j \in I^0\,, k \in K \qquad (7)$$

$$t_j \geq t_i + s_i + t_{ij} - (1 - x_{ijk})T \qquad (8)$$

$$a_i \leq t_i \leq b_i \quad \forall\, i \in I \qquad (9)$$

$$t_i \geq 0 \qquad (10)$$

The objective of the mathematical program is to minimize the routing cost of the vehicle delivery operation subject to vehicle capacity and arrival (delivery) time feasibility constraints.

The assignment/clustering binary variables y define feasible assignments of customer orders to vehicles; in other words, they partition the set of customers I into K clusters N_k each cluster to be serviced by a vehicle k. The routing variables x define the sequence that customers are visited by the corresponding vehicle. The time variables t defines the arrival time of the vehicle at each customer.
There are three characteristic groups of constraints in the MIP formulation given above.
Constraints (2) - (4) are the assignment/clustering constraints, dealing with the feasible assignment of customer orders to vehicles and dominated by the assignment variables y. Capacity constraints (2) ensure that the total freight to be carried by vehicle k is within the capacity of the vehicle. Constraints (3) and (4) state that all routes (tours) begin and end at the depot and that each customer i is serviced by one and only one vehicle k.
The second group contains the routing constraints (5) - (7) which include the routing variables x and defines a proper tour for each vehicle through the customers assigned to it. Finally, constraints (8) - (10), involving the time variables t, are the scheduling constraints. Constraint set (8) ensures compatible arrival times between any two consecutive customers i and j. In other words, if vehicle k visits customer j right after customer i (x_{ijk} = 1 and (l - x_{ijk}) = 0), then the arrival time at customer j should be at least as much as the arrival time t_i at customer i, plus the service time s_i (unloading time) at customer i, plus the travel time t_{ij} between customers i and j . Constraints (9) enforce the service time windows, ensuring that all arrival times fall between the corresponding time windows.

For a typical medium to large-size application the number of customers varies from 100 to 1,000 and the number of vehicles from 10 to 50. A problem with 100 customers and 10 vehicles would generate more than 100,000 variables and more than 100,000 constraints; for a 500-customer and 25-vehicle problem the numbers would be approximately 6,275,000 variables and 6,290,000 constraints. Clearly, the size of the problem becomes very quickly intractable for a conventional integer-programming algorithm. This motivated the development of specialized algorithms in which most of the constraints are handled explicitly.

The solution of the VRSPTW involves the relaxation of the time window constraints (9). Any violation of the constraints means that the vehicle arrives and delivers earlier or later than the predefined time interval. Treated as such, the time window constraints constitute "hard" constraints, which should not be violated in any feasible solution. Alternatively, we could allow the violation of the constraints at a cost, in a manner similar to dualizing

constraints in a Lagrangian relaxation context. In this case constraints (9) are perceived as "soft" time window constraints and not all customers are required to be serviced within their time windows.

In the VRSPTW formulation, constraints (9) define hard time windows. Introducing the constraints into the objective function, multiplied by then appropriate penalty coefficients, we create a new soft time window model. The result is a variant of the time constrained VRP, namely the Vehicle Routing Problem with Soft Time Window constraints (VRPSTW), which can be stated as follows:

(VRPSTW) $Min\ \bar{F}(x,y,t) = \sum_{i\in I}\sum_{j\in I}\sum_{k\in K}\bar{c}_{ij}x_{ijk}$ (11)

Subject to; (2) – (8), (10), Where

$$\bar{c}_{ij} = w_D d_{ij} + w_T u_{ij} \quad (12)$$

$$u_{ij} = \frac{1}{2}\left[(a_i - t_i)\, p_e + (t_i - b_i)\, p_l\right] + \frac{1}{2}\left[(a_j - t_j)\, p_e + (t_j - b_j)\, p_l\right] \quad (13)$$

$$(a_i - t_i) = \begin{cases} (a_i - t_i) & if\ a_i > t_i \\ 0 & otherwise \end{cases} \quad (14)$$

$$(t_i - b_i) = \begin{cases} (t_i - b_i) & if\ t_i > b_i \\ 0 & otherwise \end{cases} \quad (15)$$

Where

$d_{ij} = Euclidean\ distance\ between\ i\ and\ j$
$w_D = Spatial\ weighing\ factor$
$w_T = temporal\ weighing\ factor$
$p_e = early\ delivery\ penlaty\ coefficient$
$p_l = late\ delivery\ penalty\ coefficient$

The objective function (11) minimizes the total delivery cost, which consists of spatial costs (total distance traveled) and temporal costs (penalties for early or late deliveries). Constraints (14) and (15) represent early and late delivery penalties, respectively. For each customer i there exist three mutually exclusive possibilities:
The vehicle can deliver early ($a_i > t_i$);
The vehicle can deliver late ($t_i > b_i$); or
The vehicle can deliver within the customer's time window ($a_i < t_i < b_i$). In the last case no delivery penalties are imposed. Consider Constraint (15), for example; if the vehicle delivers after the closing of the time window at customer i, then the arrival time t_i is greater than b_i, which means that term ($(t_i - b_i)$) is positive. In this case a late delivery penalty is imposed, which is equal to$(a_i - t_i)\, p_l$. Early penalties are imposed in the same manner through (14), if$a_i > t_i$.

Once the delivery penalties have been calculated at both ends of a link
(i j), they are incorporated in the cost structure of the link through the temporal cost coefficient u_{ij}, as shown in (13). Note that the penalties at each end of the link (if any), are multiplied by 1/2 to avoid double counting. If both $a_i \le t_i \le b_i$ and $a_j \le t_j \le b_j$, then u_{ij} = 0 and no penalties are imposed on link (i,j). Finally, the spatial and temporal costs d_{ij} and u_{ij} are incorporated in the objective function (11) through the composite cost coefficient$\bar{c}_{ij}$. It is clear from (12) that the cost coefficients $\bar{c}$ are weighted sums of the spatial (routing) component and the temporal (scheduling) component of the problem, with weights w_D aad w_T respectively. Throughout this study, both components have been equally weighted with w_D = w_T = 1.

IMPLEMENTATION OF THE (VRPSTW) MODEL

The VRPSTW model was implemented using traffic data from a manufacturer to analyze the efficiency of the manufacturer's distribution operations as they were performed at that time, to investigate and propose alternative strategies and scenarios to improve these operations, and to identify additional opportunities.

The problem faced by the manufacturer consisted of simultaneously allocating a set of shipments to a fleet of vehicles and routing them at least cost, subject to constraints on when certain shipments must be delivered. This is an application of the classical vehicle routing problem in a time constrained environment. The system in use was semi computerized. Shipments orders arrive at the traffic department through a computerized system, either from other departments of the company or directly from customers. Each order specifies the size and the destination of the shipment, and a requested ship date. The ship date is defined in accordance with customer needs, availability of the products in the requested quantities, and the necessary order process time and transit time of the vehicle. When an order arrives at the traffic department, it is classified either as a "must ship" or as a regular shipment. Shipments classified as "must ship" have to be scheduled for shipping on the same day; regular shipments are scheduled for the same day or the next day. Regular shipments are considered those with a requested ship date at least three days in the future, while "must ship" are shipments with ship date the day before, the current day, and the day after. In other words, each shipment should either leave the warehouse on the same day or wait until the next day. The consolidation managers receive a list daily with all candidate shipments and assign them to the available vehicles. They issue the bill of lading for each order and the warehouse is notified to send the shipment out.

The VRPSTW model was used to produce daily shipment schedules and vehicle tours for the manufacturer; the model solutions were compared to the actual schedules developed manually by the company dispatchers. Although order consolidation and shipping was a daily operation in the company, planning horizons that vary from one day to five days were used. The one-day-long planning horizon simulated the actual operation at the company, where the consolidation of shipments is performed on a daily basis. More specifically the study addressed the following questions:

• How well does the original problem of the manufacturer represented by the VRPSTW model?

• How do the tours produced by the model compare to the tours currently in use and manually produced by the manufacturer?

The data sets used to provide answers to the questions above are described next.

DESCRIPTION OF THE DATA AND THE PERFORMANCE CRITERIA

The study was carried out using data provided by the traffic department of the manufacturer. The data covered the activities during the month of August 2012 and listed daily shipments out of a major distribution center of the company in the Midwest. The destination points serviced by the center were located in the Northeast and the Midwest regions of the Indore. The data included shipment code number, destination city and zip code, shipment size (volume) in truckloads, requested ship date, and list of shipments consolidated together. The shipment code numbers were used to identify each order and list the shipments consolidated on the same vehicle. The city and zip code identified the location of the destination. These data were used to find the longitude and the latitude of the destination and create the distance matrix between all customers and the distribution center. The travel time between any pair of customers was calculated assuming an average speed of 40 mph, suggested by the company. Furthermore, following industry regulations, we assumed that a driver would be on duty at most 10 hours a day.

The requested ship date defined the time window for each shipment. If the shipment was classified as a "must ship," then the time window of the customer was one day long and it was centered on the middle of the ship date. If the shipment was a regular one, the time window became two days long, and it was centered on the beginning of the second day.

Note that some shipments could not possibly be delivered on time, due to the way the ship dates were defined by the company. For example, if a "must ship" load appeared for consolidation and shipping on a given day and the destination was located farther than a day's drive, then the shipment could not be delivered on time. Some data cleanup was necessary before running the computational experiments. In several instances, the consolidation data were either unclear or controversial in terms of which shipments were assigned to which vehicle. When recovery of bad data was not possible, these shipments were excluded from the data base.

The performance of the VRPSTW model on the data provided by the manufacturer and the evaluation of the current distribution system was based on the following criteria:

- Total distance traveled.
- Number of vehicles used.
- Percentage of violated windows.
- Total waiting (idle) time, which was the total amount of time that the vehicles had to wait before making a delivery, if they arrived early at a customer and early delivery was not desirable.

The performance criteria are listed in order of importance for the purpose of this study. The total distance traveled was an indirect measurement of the routing cost which could be calculated multiplying the distance by the cost-per-mile coefficient of the operation. That was the most important measurement for the company, since it was interested in examining the efficiency of the routing operations, as they were performed at that time. The second most important performance criterion was the number of vehicles required to satisfy a given volume of loads. Although we did not consider directly fixed costs associated with the use of a vehicle, it was clear that solutions with fewer vehicles would keep the cost of the operation down, if they could provide adequate level of service. Finally, the number of violated windows and the total waiting time were used as indicators of the level of service provided and the quality of the obtained solution.

Although on-time delivery was not a crucial factor for the company at that point (given the way that time windows were defined), solutions with few violated windows would be quite desirable. By the same token, the total waiting time indicated the efficient use of vehicles and drivers, and characterized the performance of the outbound distribution operation.

The evaluation of the VRPSTW model and the current operations at the company, based on the data and the performance criteria presented, is described next.

RESULTS AND PERFORMANCE EVALUATION

The data obtained from the traffic department of the manufacturer covered the period between August 6 and August 24, 2012. We selectively used data from the first and third week of the month. The basic criterion for the selection of the data was the number of activities per day. We selected data samples that would create interesting and realistically sized problems, without imposing excessive computational requirements. Secondary criteria were the completeness and clarity of the data. Table 1 shows the three weeks of August for which complete daily data were available; it lists the dates and number of orders with ship date that day. In order to evaluate the model and the field operations, we needed to compare the actual tours to those produced by the VRPSTW model. However, the actual tours were not directly available, and they had to be extracted from the consolidation data. We assumed that the clustering was given by the manufacturer's consolidation data, but then we applied a branch and bound optimization algorithm to obtain the routing. Thus, the tours we used as basis to compare the results from the VRPSTW model might have actually been better than the ones run in the field. However, given that at most four shipments were consolidated on the same vehicle, the routing problem was trivial to solve even manually and hence we did not expect significant discrepancies between the tours that were actually run and the ones we considered as field tours.

TABLE -1
ACTIVITY DATA FOR THE DISTRIBUTION CENTER

1st Week		2nd Week		3rd Week	
Date	**Order**	**Date**	**Order**	**Date**	**Order**
6th August	**36**	**13th August**	**32**	**20th August**	**23**
7th August	**45**	**14th August**	**25**	**21th August**	**27**
8th August	**31**	**15th August**	**16**	**22th August**	**11**
9th August	**34**	**16th August**	**21**	**23th August**	**9**
10th August	**32**	**17th August**	**28**	**24th August**	**22**
Total	**178**	**Total**	**122**	**Total**	**92**

TABLE-2
DATES SELECTCD FOR TOE STUDY

Daily	
Dates	**Orders**
6th August	**36**
7th August	**45**
20th August	**23**
22th August	**11**
24th August	**22**

After obtaining the field tours we ran the VRPSTW model on the same data. The results are shown in Tables 3 through 7. Tables 3 to 7 list the results for the daily problems. For each time period shown, the first line indicates the date, the number of orders shipped out, and the total volume of the shipments in truckloads. The line marked "Field" presents the solutions implemented by the manufacturer in the field. The next lines show the solutions proposed by the VRPSTW model. The first one lists the solution using the same number of vehicles with the field solution. The next lists the solution using fewer vehicles, where possible, and the last one lists the solution with fewer violated Windows and routing cost comparable, if not better, than the actual one. Columns 3 and 4 show the number of vehicles used and the vehicle utilization for each solution. The total number of miles travelled is listed in column 5, and the percent improvement of the model solution over the field solution is listed next. Columns 7 and 8 list the number and the percentage of shipments delivered outside the time windows, and the last column gives the total waiting (idle) time for all vehicles.

TABLE 3
ROUTING SOLUTIONS FOR 6th AUGUST

Period: 6th August	**No. of orders: 36**	**Demand: 15.822**
	Field	**Model**
Trucks	**18**	**18**
V.U	**0.879**	**0.879**
Total Miles	**18090**	**17798.2**
% improvement	**-**	**1.62**
Violated Windows	**5 (12%)**	**5 (12%)**

Idle	**184**	**106**

TABLE 4
ROUTING SOLUTIONS FOR 7th AUGUST

Period: 7th August	**No. of orders: 45**	**Demand: 18.978**
	Field	**Model**
Trucks	**22**	**22**
V.U	**0.863**	**0.863**
Total Miles	**19283**	**19270**
% improvement	**-**	**0.07**
Violated Windows	**0 (0%)**	**1 (2%)**
Idle	**50**	**52**

TABLE 5
ROUTING SOLUTIONS FOR 20th AUGUST

Period: 20th August	**No. of orders: 23**	**Demand: 9.921**
	Field	**Model**
Trucks	**12**	**12**
V.U	**0.828**	**0.828**
Total Miles	**13266**	**13138**
% improvement	**-**	**0.96**
Violated Windows	**3 (11%)**	**3 (11%)**
Idle	**0**	**0**

TABLE 6
ROUTING SOLUTIONS FOR 22nd AUGUST

Period: 22nd August	**No. of orders: 11**	**Demand: 4.765**
	Field	**Model**
Trucks	**7**	**7**
V.U	**0.837**	**0.837**
Total Miles	**8699**	**8463**
% improvement	**-**	**2.43**
Violated Windows	**2 (9.5%)**	**3 (14%)**
Idle	**0**	**0**

TABLE 7
ROUTING SOLUTIONS FOR 24th AUGUST

Period: 24th August	**No. of orders: 22**	**Demand: 7.873**
	Field	**Model**
Trucks	**11**	**11**

V.U	**0.830**	**0.830**
Total Miles	**10819**	**9969**
% improvement	**-**	**8.55**
Violated Windows	**2 (7.7%)**	**2 (7.7%)**
Idle	**29**	**55**

It is clear from the obtained solutions that the optimization-based heuristic developed for the time-window constrained VRP performed quite well when applied to real-world situations. The model not only managed to match the routing performed by experts in the field, but even outperformed them in most of the cases. One can see a Tables 3 and 4 that the model not only produced lower cost solutions using the same number of vehicles, but it did that using fewer vehicles and without affecting the level of service. The realized improvements varied from 0.07% to 855%, but in most instances ranged between 1% and 2%. As far as the rest of the performance criteria were concerned, the model matched and often exceeded the field solutions.

In terms of computational times, a detailed analysis of the computational efficiency of the algorithm is available. The analysis was performed on solving problems with 100 customers and 10 to 21 vehicles. Other important parameters were the tightness and the position of the time windows. The solution times varied between 3.0 and 850.0 seconds, depending on the structure and the characteristics of each problem. In general, the computational requirements of the algorithm are exponentially proportional to the size of the problem (number of customers and number of vehicles) and proportional to the tightness of the time windows. Furthermore, the computational efficiency is determined by the desired quality of the solution. Recall that the algorithm is an iterative heuristic, which means that, in

general, the longer it runs the better the solution is. Sensitivity analysis has shown that the algorithm gets within 5% of the best solution in 50 iterations, 2.4% within 75 iterations, and 1.6% within 100 iterations.

CONCLUSIONS

The optimization-based heuristic algorithm for the VRPSTW was successfully implemented in shipment consolidation, vehicle routing, and scheduling problems faced by a major manufacturer. The study demonstrated that the model successfully handled realistic problems and facilitated the evaluation of the manual consolidation system used by the manufacturer. The model adjusted well to the real world environment and was able to deal with realistic problems on a strategic and tactical planning level. Minor adjustments concerning exact travel distance and time between destinations, as well as observation of industry regulations on driver assignment on routes would be necessary if the model were to be used on an operational level. Comparing the routing solutions applied in the field to the solutions developed by the model, it was found that the latter produced better solutions in terms of mileage, number of vehicles, vehicle utilization, and level of service. This should be expected, since the quality of the solutions depended extensively on clustering. Recall that usually there were no more than four customers per vehicle, which made the routing part almost trivial. Hence, the improvements were largely due to improved clustering; one should naturally expect a mathematical model to perform better than humans in this area, especially for problems of non-trivial size. The use of planning horizons more than one day long verified the importance of advanced booking information. The superiority of the model solutions was more evident on the longer horizon problems.

Another task that could be undertaken successfully would be to use the model to provide on-line answers on the feasibility and profitability of a shipment. For example, a customer might call to request a delivery on a given date. The model could investigate whether a delivery is possible on the given date and, furthermore,

what would be the cost of such a delivery, taking into account what other shipments are due on or around the requested date in the "neighbourhood" of the customer. Clearly, the VRPSTW model has the capability to improve the daily operations at the traffic department of the company. The advantage of using a mathematical

programming-based algorithm was evident even for relatively small problems, with a many as 20 customers. That was especially true when the time component increased the complexity of the problem introducing a third dimension. We strongly believe that the model can provide substantial support to distribution managers on a tactical, strategic, and even operational level.

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