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An Inventory Model with Stock-Dependent Demand with Two Storages Capacity for Non-Instantaneous Deteriorating Items

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ABSTRACT

This paper deals with two warehouse inventory model of determining the optimal replenishment policy for non-instantaneous deteriorating items partial backlogging and stock-dependent demand. In the model, shortages are allowed and the backlogging rate is variable and dependent on the waiting time for the next replenishment. The necessary and sufficient conditions of the existence and uniqueness of the optimal solution are shown. Numerical example is presented to demonstrate the developed model.

Keywords: Two warehouses, Non-instantaneous deterioration, Stock-dependent demand, Partial backlogging.

AMS Mathematics Classification:

1. INTRODUCTION

Generally, deterioration is defined as the damage, dryness, spoilage, vaporization, etc., that result in decrease of usefulness of the original one. In real life, mostly goods have a span of maintaining quality or the original condition (e.g. food, vegetables, fish, meat, fruit and so on), define, during that time, there was no deterioration occurring. We term the phenomenon as ‘‘non-instantaneous deterioration’’. Last so many years, some researchers have given attention to the situation where the demand rate is dependent on the level of the on-hand inventory. First **Ghare** and **Schrader (1963)** discussed an EOQ inventory model for deteriorating items. **Philip (1974)** developed the model with a three parameter Weibull distribution rate and no shortages. **Shah (1977)** extended the **Philip’s (1974)** model and assumed that shortage allowed. First **Gupta** and **Vrat (1986)** discuss the inventory models for stock-dependent consumption rate. Many researchers such as **Hollier** and **Mak (1983)** and **Wee (1995)** studied the constant partial backlogging rates during the shortage period in their inventory models and many researchers have modified some inventory policies by considering the ‘‘time-proportional partial backlogging rate’’ such as **Abad (2000), Chang (2004). Singh** and **Malik (2010)** developed a demand dependent production inventory model with partial backlogging and two storage capacity. **Singh** and **Malik (2010)** developed an Optimal ordering policy with linear deterioration, exponential demand and two storage capacities.

In this paper we consider an inventory model with two warehouses to determining the optimal replenishment policy for non-instantaneous deteriorating items stock-dependent demand and partial backlogging. In this model, we take shortages are allowed; the backlogging rate is variable and dependent on the waiting time for the next replenishment. The necessary and sufficient conditions of the existence and

uniqueness of the optimal solution are given. Finally, numerical example is presented to demonstrate the developed model.

2. ASSUMPTIONS AND NOTATION

The following assumptions and notation are used in this paper:

(1) The demand rate D(t) at time t is $D(t) = \begin{cases} a + bI(t), & I(t) > 0, \\ a, & I(t) \le 0, \end{cases}$

Where a, b are positive constants and I(t) is the inventory level at time t.

(2) Shortages are allowed to occur. Let B denote this fraction where t is the waiting time up to the next replenishment and assumed that only a fraction of demand is backlogged.

(3) Replenishment rate is infinite and the lead time is zero.

(4) t_1 is the length of time in which the product has no deterioration (i.e., fresh product time). α and β are deterioration in RW and OW respectively.

(5) t_3 is the length of time in which the inventory is no shortage. T is the length of order cycle. Q is the order quantity per cycle. t_1, t_2, T and Q are decision variables.

(6) $A, C_{H_R}, C_{H_O}, C_{D_H}, C_{D_O}, C_S$ *and* C_O denotes the ordering cost per order, inventory holding cost in RW per unit time, inventory holding cost in OW per unit time, deteriorating cost in RW per unit, deteriorating cost in OW per unit, the shortage cost for backlogged items and the unit cost of lost sales, respectively. All of the cost parameters are positive constant.

3. MODEL FORMULATION

The inventory system goes like as R units of item arrive at the inventory system at the beginning of each cycle. During the interval [0, t_1], the inventory levels are positive at RW and OW and decreasing only owing to stock-dependent demand rate in RW. At RW, the inventory is depleted due to the combined effects of demand and deterioration. The inventory level is dropping to zero due to demand and deterioration during the time interval [t_2, t_3]. Then shortage interval keeps to the end of the current order cycle. The whole process is repeated. As described above, the inventory level decreases only owing to stock-dependent demand rate during the time interval [0, t_2]. During the shortage interval [t_3, T], the demand at time t is partially backlogged at fraction B (T-t). The inventory level at RW and OW are governed by the following differential equations:

$$\frac{dI_{R1}(t)}{dt} = -\left[a + b\,I_{R1}(t)\right] \qquad 0 \le t \le t_1 \qquad \text{…. (1)}$$

$$\frac{dI_{R2}(t)}{dt} + \alpha\, I_{R2}(t) = -\left[a + b\,I_{R2}(t)\right] \qquad t_1 \le t \le t_2 \qquad \text{…. (2)}$$

$$\frac{dI_{O1}(t)}{dt} = 0 \qquad 0 \le t \le t_1 \qquad \text{…. (3)}$$

$$\frac{dI_{O2}(t)}{dt} + \beta\, I_{O2}(t) = 0 \qquad t_1 \le t \le t_2 \qquad \text{….(4)}$$

$$\frac{dI_{O3}(t)}{dt} + \beta\, I_{O3}(t) = -\left[a + b\,I_{O3}(t)\right], \qquad t_2 \le t \le t_3 \qquad \text{…..(5)}$$

$$\frac{dI_N(t)}{dt} = -\frac{a}{1+\delta(T-t)}, \qquad t_3 \le t \le T \qquad \text{…..(6)}$$

with the boundary conditions $I_{R1}(0) = R$, $I_{R2}(t_2) = 0$, $I_{O1}(t_1) = W$, $I_{O3}(t_3) = 0$, $I_N(t_3) = 0$ respectively. Solving these differential equations, we get the inventory level as follows:

$$I_{R1}(t) = \mathrm{R}e^{-bt} - \frac{a}{b}\left\{1 - e^{-bt}\right\}, \qquad 0 \le t \le t_1 \qquad \text{…. (7)}$$

$$I_{R2}(t) = \frac{a}{b+\alpha}\left\{e^{(b+\alpha)(t_2-t)} - 1\right\}, \qquad t_1 \le t \le t_2 \qquad \text{…..(8)}$$

$$I_{O1}(t) = W, \qquad 0 \le t \le t_1 \qquad \text{…. (9)}$$

$$I_{O2}(t) = We^{\beta(t_1-t)}, \qquad t_1 \le t \le t_2 \qquad \text{…..(10)}$$

$$I_{O3}(t) = \frac{a}{b+\beta}\left\{e^{(b+\beta)(t_3-t)} - 1\right\}, \qquad t_2 \le t \le t_3 \qquad \text{…..(11)}$$

$$I_N(t) = -\frac{a}{\delta}\left[\log\{1+\delta(T-t_3)\} - \log\{1+\delta(T-t)\}\right] \qquad t_3 \le t \le T \qquad \text{…. (12)}$$

Putting t=T in Equation (12), we obtain the maximum amount of demand backlogged per cycle as follows:

$$S = -I_3(T) = -\frac{a}{\delta}\left[\log\{1+\delta(T-t_3)\}\right] \qquad \text{…. (13)}$$

Considering continuity of I (t) at $t=t_1$, it follows from Equations (7) and (8) that

$$I_{R1}(t_1) = I_{R2}(t_1)$$

$$\Rightarrow \qquad \mathrm{R} = \frac{a}{b+\alpha}\left\{e^{(b+\alpha)(t_2-t_1)} - 1\right\}e^{bt_1} + \frac{a}{b}\left\{e^{bt_1} - 1\right\} \qquad \text{…..(14)}$$

Substituting equation (14) into (7), we get

$$I_{R1}(t) = \frac{a}{b+\alpha}\left\{e^{(b+\alpha)(t_2-t_1)} - 1\right\}e^{-b(t-t_1)} + \frac{a}{b}\left\{e^{-b(t-t_1)} - 1\right\}, \qquad 0 \le t \le t_1 \qquad \text{…. (15)}$$

From Equation (13) and (14), we can obtain the order quantity, Q; as

Q = R+S

$$= \frac{a}{b+\alpha}\left\{e^{(b+\alpha)(t_2-t_1)} - 1\right\}e^{bt_1} + \frac{a}{b}\left\{e^{bt_1} - 1\right\} + \frac{a}{\delta}\left[\log\{1+\delta(T-t_3)\}\right] \qquad \text{…. (16)}$$

According to given conditions at $t=t_2$, $I_{O2}(t_2) = I_{O3}(t_2)$

$$\Rightarrow \quad t_3 = t_2 + \frac{1}{b+\beta}\log\left\{1 + \frac{w(b+\beta)}{a}e^{\beta(t_1-t_2)}\right\} \qquad \text{…..(17)}$$

Next, the total relevant inventory cost per cycle consists of the following elements:

Ordering cost per cycle is A. …(18)

Inventory holding cost per cycle in **RW** (which is denoted by IHC_{RW}) is given by

$$\text{IHC}_{\text{RW}}=C_{H_R}\left\{\int_0^{t_1} I_{R1}(t)dt+\int_{t_1}^{t_2} I_{R2}(t)dt\right\}$$

$$=C_{H_R}\left[\frac{a}{b+\alpha}\left\{e^{(b+\alpha)(t_2-t_1)}-1\right\}\left\{e^{bt_1}-1\right\}+\frac{a}{b^2}\left\{e^{bt_1}-bt_1-1\right\}\right.$$

$$\left.+\frac{a}{(b+\alpha)^2}\left\{e^{(b+\alpha)(t_2-t_1)}-(t_2-t_1)(b+\alpha)-1\right\}\right] \quad \text{....(19)}$$

Inventory holding cost per cycle in **OW** (which is denoted by IHC_{OW}) is given by

$$\text{IHC}_{\text{OW}}=C_{H_O}\left\{\int_0^{t_1} I_{O1}(t)dt+\int_{t_1}^{t_2} I_{O2}(t)dt+\int_{t_2}^{t_3} I_{O3}(t)dt\right\}$$

$$=C_{H_O}\left[W t_1+\frac{W}{\beta}\left[1-e^{-\beta(t_2-t_1)}\right]+\frac{a}{(b+\beta)^2}\left\{e^{(b+\beta)(t_3-t_2)}-(t_3-t_2)(b+\beta)-1\right\}\right] \quad \text{...(20)}$$

Deterioration cost per cycle in **RW** (which is denoted by DC_{RW}) is given by

$$\text{DC}_{\text{RW}}=C_{D_R}\left\{I_{R2}(t_1)-\int_{t_1}^{t_2} D(t)dt\right\}$$

$$=C_{D_R}\left[\frac{a\alpha}{b+\alpha}\left\{e^{(b+\alpha)(t_2-t_1)}-(t_2-t_1)(b+\alpha)-1\right\}\right] \quad \text{...(21)}$$

Deterioration cost per cycle in **OW** (which is denoted by DC_{OW}) is given by

$$\text{DC}_{\text{OW}}=C_{D_O}\left\{I_{O2}(t_1)-\int_{t_1}^{t_2} I_{O2}(t)dt-\int_{t_2}^{t_3} D(t)dt\right\}$$

$$=C_{D_O}\left[W+\frac{W}{\beta}\left[1-e^{-\beta(t_2-t_1)}\right]-a(t_3-t_2)+\frac{ab}{(b+\beta)^2}\left\{1-e^{(b+\beta)(t_3-t_2)}+(t_3-t_2)(b+\beta)\right\}\right] \quad \text{...(22)}$$

Shortage cost per cycle due to **backlog** (which is denoted by SC) is given by

$$\text{SC}=C_S\int_{t_3}^{T}\left[-I_N(t)\right]dt=\frac{a\,C_S}{\delta}\left[(T-t_3)-\frac{1}{\delta}\log\left\{1+\delta(T-t_3)\right\}\right] \quad \text{....(23)}$$

Opportunity cost per cycle due to **lost sales** (which is denoted by OC) is given by

$$\text{OC}=\text{C}_{\text{O}}\int_{t_3}^{T} a\left[1-B(T-t)\right]dt=a\,C_O\left\{(T-t_3)-\frac{1}{\delta}\log\left\{1+\delta(T-t_3)\right\}\right\} \quad \text{.....(24)}$$

Therefore, the total relevant inventory cost per unit time is given by

$\text{TC}(t_1, t_2, T) = (1/T)\,[A+\text{IHC}_{\text{RW}}+\text{IHC}_{\text{OW}}+\text{DC}_{\text{RW}}+\text{DC}_{\text{OW}}+\text{SC}+\text{OC}]$ ...(25)

Substituting Equation (18)–(24) in the above equation (25), we get

$$\text{TC}(t_1, t_2, T)=\frac{a}{T}\left\{\frac{A}{a}+\left[C_{H_R}\left(e^{bt_1}-1+\frac{1}{b+\alpha}\right)+\alpha\,C_{D_R}\right]\left[\frac{1}{b+\alpha}\left\{e^{(b+\alpha)(t_2-t_1)}-1\right\}\right]\right.$$

$$\left.-(t_2-t_1)\left(\alpha\,C_{D_R}+\frac{C_{H_R}}{b+\alpha}\right)+\frac{C_{H_R}}{b^2}\left\{e^{bt_1}-bt_1-1\right\}\right]$$

$$+\left[\frac{\left(C_{H_O}-bC_{D_O}\right)}{(b+\beta)^2}\left\{e^{(b+\beta)(t_3-t_2)}-(t_3-t_2)(b+\beta)-1\right\}\right]$$
$$+\frac{W\left(C_{H_O}+C_{D_O}\right)}{a\beta}\left(1-e^{-\beta(t_2-t_1)}\right)$$
$$+\frac{W}{a}(t_1C_{H_O}+C_{D_O})-C_{D_O}(t_3-t_2)$$
$$+\frac{\left(C_S+\delta C_O\right)}{\delta}\left[(T-t_3)-\frac{1}{\delta}\log\left\{1+\delta(T-t_3)\right\}\right]\Bigg\} \qquad \text{…. (26)}$$

The total relevant inventory cost per unit time is minimum if

$$\frac{\partial TC}{\partial t_1}=0,\frac{\partial TC}{\partial t_2}=0,\frac{\partial TC}{\partial T}=0,$$

$$\frac{\partial^2 TC}{\partial t_1^2},\begin{vmatrix}\frac{\partial^2 TC}{\partial t_1^2} & \frac{\partial^2 TC}{\partial t_1 t_2}\\ \frac{\partial^2 TC}{\partial t_1 t_2} & \frac{\partial^2 TC}{\partial t_2^2}\end{vmatrix} \text{ and } \begin{vmatrix}\frac{\partial^2 TC}{\partial t_1^2} & \frac{\partial^2 TC}{\partial t_1 t_2} & \frac{\partial^2 TC}{\partial T\partial t_1}\\ \frac{\partial^2 TC}{\partial t_1 t_2} & \frac{\partial^2 TC}{\partial t_2^2} & \frac{\partial^2 TC}{\partial t_2\partial T}\\ \frac{\partial^2 TC}{\partial T\partial t_1} & \frac{\partial^2 TC}{\partial t_2\partial T} & \frac{\partial^2 TC}{\partial T^2}\end{vmatrix} \text{ are all positive.}$$

4. NUMERICAL EXAMPLE

In order to illustrate the above solution procedure, consider an inventory system with the following data: A = 250; $C_{H_R}=.5, C_{H_O}=.6; C_{D_R}=1.5, C_{D_O}=1.6$; C_S = 2.5; C_O = 2; δ = 2; a = 600; b = 0.1; α = 0.08, β=.09 and t_1 = 1/12 = 0.0833, t_2=1/8=0.125.

5. CONCLUSIONS

In this paper, we proposed a deterministic two warehouses inventory model for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. Shortages are allowed and the backlogging rate is variable and dependent on the waiting time for the next replenishment. Furthermore, the proposed model can be used in inventory control of certain non-instantaneous deteriorating items such as food items, electronic components such as mobile, machines, circuit, toys and fashionable commodities etc. In the future study, it is hoped to further incorporate the proposed model into more realistic assumptions, such as probabilistic demand and a finite rate of replenishment.

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