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A general and unified integral II

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ABSTRACT
The object of the paper is to obtain a general and unified integral involving the product of multivariable I-functions and Fox's H-function with general arguments. Several cases are also included.

Keywords :Multivariable I-functions,multivariable H-function , H-function,

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1. Introduction

The multivariable I-function is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,z_2,...z_r) = I^{0,N_2;0,N_3;\cdots;0,N_r:m',n';\cdots;M^{(s)},N^{(r)}}_{P_2,Q_2,P_3,Q_3;\cdots;P_r,Q_r:P',Q';\cdots;P^{(r)},Q^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{A}_{2j};\gamma'_{2j},\gamma''_{2j})_{1,P_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{B}_{2j};\delta'_{2j},\delta''_{2j})_{1,Q_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{A}_{rj};\gamma'_{rj},\cdots,\gamma^{(r)}_{rj})_{1,P_r}:(A'_j,\gamma'_j)_{1,P'};\cdots;(A^{(r)}_j,\gamma^{(r)}_j)_{1,P^{(r)}} \\ \\ (\mathrm{B}_{rj};\delta'_{rj},\cdots,\delta^{(r)}_{rj})_{1,Q_r}:(B'_j,\delta'_j)_{1,Q'};\cdots;(B^{(r)}_j,\delta^{(r)}_j)_{1,Q^{(r)}} \end{array}\right) \tag{1.1}$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(t_i)z_i^{t_i}\mathrm{d}t_1\cdots\mathrm{d}t_r \tag{1.2}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [3]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_i| < \frac{1}{2}\Omega_i\pi$, where

$$\Omega_i=\sum_{k=1}^{N^{(i)}}\gamma_k^{(i)}-\sum_{k=N^{(i)}+1}^{P^{(i)}}\gamma_k^{(i)}+\sum_{k=1}^{M^{(i)}}\delta_k^{(i)}-\sum_{k=M^{(i)}+1}^{Q^{(i)}}\delta_k^{(i)}+\left(\sum_{k=1}^{N_2}\alpha_{2k}^{(i)}-\sum_{k=N_2+1}^{P_2}\gamma_{2k}^{(i)}\right)+\cdots+$$

$$\left(\sum_{k=1}^{N_r}\gamma_{rk}^{(i)}-\sum_{k=N_r+1}^{P_r}\gamma_{rk}^{(i)}\right)-\left(\sum_{k=1}^{Q_2}\delta_{2k}^{(i)}+\sum_{k=1}^{Q_3}\delta_{3k}^{(i)}+\cdots+\sum_{k=1}^{Q_r}\delta_{rk}^{(i)}\right) \tag{1.3}$$

where $i = 1, \cdots, r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1, \cdots, z_r) = 0(|z_1|^{\gamma_1'}, \cdots ,|z_r|^{\gamma_r'}) , max(|z_1|, \cdots , |z_r|) \rightarrow 0$$

$$I(z_1, \cdots, z_r) \ = 0(|z_1|^{\delta_1'}, \cdots , |z_r|^{\beta_r'}) , min(|z_1, \cdots ,|z_r|) \rightarrow \infty$$

where $k = 1, \cdots, r : \gamma_k' = min[Re(B_j^{(k)}/\delta_j^{(k)})], j = 1, \cdots, M_k$ and

$$\delta_k' = max[Re((A_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \cdots, N_k$$

We will use these following notations in this paper :

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

Serie representation of multivariable I-function of several variables is given by

$$I(y_1, \cdots, y_r) = \sum_{G_1, \cdots, G_r = 0}^{\infty} \sum_{g_1=0}^{M_1} \cdots \sum_{g_r=0}^{M_r} \frac{(-)^{G_1 + \cdots + G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!} \phi(\eta_{G_1, g_1}, \cdots, \eta_{G_r, g_r})$$

$$\times \ \theta_1(\eta_{G_1,g_1}) \cdots \theta_r(\eta_{G_r,g_r}) y_1^{-\eta_{G_1,g_1}} \cdots y_r^{-\eta_{G_r,g_r}} \tag{1.4}$$

Where $\psi(., \cdots, .), \theta_i(.), i = 1, \cdots, r$ are defined by Prasad (see integral (1.2))

$$\eta_{G_1, g_1} = \frac{B_{g_1}^{(1)} + G_1}{\delta_{g_1}^{(1)}}, \cdots, \ \eta_{G_r, g_r} = \frac{B_{g_r}^{(r)} + G_r}{\delta_{g_r}^{(r)}}$$

which is valid under the conditions $\delta_{g_i}^{(i)}[B_j^i + p_i] \neq \delta_j^{(i)}[B_{g_i}^i + G_i]$ (1.5)

for $j \neq M_i, M_i = 1, \cdots \eta_{G_i, g_i}; P_i, N_i = 0, 1, 2, \cdots, ; y_i \neq 0, i = 1, \cdots, r$ (1.6)

In the document , we will note :

$$G(\eta_{G_1,g_1}, \cdots, \eta_{G_r,g_r}) = \phi(\eta_{G_1,g_1}, \cdots, \eta_{G_r,g_r}) \theta_1(\eta_{G_1,g_1}) \cdots \theta_r(\eta_{G_r,g_r}) \tag{1.7}$$

where $\phi(\eta_{G_1,g_1}, \cdots, \eta_{G_r,g_r}), \theta_1(\eta_{G_1,g_1}), \cdots, \theta_r(\eta_{G_r,g_r})$ are given in (1.2)

We will use these following notations in this paper :

$$U_1 = P_2, Q_2; P_3, Q_3; \cdots ; P_{r-1}, q_{r-1}; V_1 = 0, N_2; 0, N_3; \cdots ; 0, N_{r-1} \tag{1.8}$$

$$W_1 = (P', Q'); \cdots ; (P^{(r)}, Q^{(r)}); X_1 = (M', N'); \cdots ; (M^{(r)}, N^{(r)}) \tag{1.9}$$

$$A_1 = (A_{2k}, \gamma_{2k}^{(1)}, \gamma_{2k}^{(2)}); \cdots ; (A_{(r-1)k}, \gamma_{(r-1)k}^{(1)}, \gamma_{(r-1)k}^{(2)}, \cdots , \gamma_{(r-1)k}^{(r-1)}) \tag{1.10}$$

$$B_1 = (B_{2k}, \delta_{2k}^{(1)}, \delta_{2k}^{(2)}); \cdots ; (B_{(r-1)k}, \delta_{(r-1)k}^{(1)}, \delta_{(r-1)k}^{(2)}, \cdots , \delta_{(r-1)k}^{(r-1)}) \tag{1.11}$$

$$\mathfrak{A}_1 = (A_{rk}; \gamma_{rk}^{(1)}, \gamma_{rk}^{(2)}, \cdots , \gamma_{rk}^{(r)}) : \mathfrak{B}_1 = (B_{rk}; \delta_{rk}^{(1)}, \delta_{rk}^{(2)}, \cdots , \delta_{rk}^{(r)}) \tag{1.12}$$

$$A_1' = (A_k', \gamma_k')_{1,P'}; \cdots ; (a_k^{(r)}, \gamma_k^{(r)})_{1,P^{(r)}}; B_1' = (B_k', \delta_k')_{1,P'}; \cdots ; (B_k^{(r)}, \delta_k^{(r)})_{1,p^{(r)}} \tag{1.13}$$

The multivariable I-function of r-variables write :

$$I(z_1, \cdots , z_r) = I_{U_1:P_r,Q_r;W_1}^{V_1;0,N_r;X_1} \left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A}_1; \mathfrak{A}_1; A_1' \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_s & \mathrm{B}_1; \mathfrak{B}_1; B_1' \end{array} \right) \tag{1.14}$$

The multivariable I-function of s-variables is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1, z_2, ...z_s) = I_{p_2,q_2,p_3,q_3;\cdots;p_s,q_s:p',q';\cdots;p^{(s)},q^{(s)}}^{0,n_2;0,n_3;\cdots;0,n_r:m',n';\cdots;m^{(s)},n^{(s)}} \left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_{2j}; \alpha_{2j}', \alpha_{2j}'')_{1,p_2}; \cdots ; \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_s & (\mathrm{b}_{2j}; \beta_{2j}', \beta_{2j}'')_{1,q_2}; \cdots ; \end{array} \right.$$

$$\left. \begin{array}{l} (\mathrm{a}_{sj}; \alpha_{sj}', \cdots , \alpha_{sj}^{(s)})_{1,p_s} : (a_j', \alpha_j')_{1,p'}; \cdots ; (a_j^{(s)}, \alpha_j^{(s)})_{1,p^{(s)}} \\ \\ (\mathrm{b}_{sj}; \beta_{sj}', \cdots , \beta_{sj}^{(s)})_{1,q_s} : (b_j', \beta_j')_{1,q'}; \cdots ; (b_j^{(s)}, \beta_j^{(s)})_{1,q^{(s)}} \end{array} \right) \tag{1.15}$$

$$= \frac{1}{(2\pi\omega)^s} \int_{L_1} \cdots \int_{L_s} \xi(t_1, \cdots , t_s) \prod_{i=1}^{s} \phi_i(t_i) z_i^{t_i} \mathrm{d}t_1 \cdots \mathrm{d}t_s \tag{1.16}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [3]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_i| < \frac{1}{2}\Omega'_i\pi$, where

$$\Omega'_i = \sum_{k=1}^{n^{(i)}} \alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}} \alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}} \beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}} \beta_k^{(i)} + \left(\sum_{k=1}^{n_2} \alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2} \alpha_{2k}^{(i)}\right) + \cdots +$$

$$\left(\sum_{k=1}^{n_s} \alpha_{sk}^{(i)} - \sum_{k=n_s+1}^{p_s} \alpha_{sk}^{(i)}\right) - \left(\sum_{k=1}^{q_2} \beta_{2k}^{(i)} + \sum_{k=1}^{q_3} \beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_s} \beta_{sk}^{(i)}\right) \tag{1.17}$$

where $i = 1, \cdots, s$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1, \cdots, z_s) = 0(|z_1|^{\gamma'_1}, \cdots, |z_s|^{\gamma'_s}) , max(|z_1|, \cdots, |z_s|) \rightarrow 0$$

$$I(z_1, \cdots, z_s) = 0(|z_1|^{\delta'_1}, \cdots, |z_s|^{\beta'_s}) , min(|z_1, \cdots, |z_s|) \rightarrow \infty$$

where $k = 1, \cdots, z : \alpha'_k = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta'_k = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, n_k$$

We will use these following notations in this paper :

$$U = p_2, q_2; p_3, q_3; \cdots; p_{s-1}, q_{s-1}; V = 0, n_2; 0, n_3; \cdots; 0, n_{s-1} \tag{1.18}$$

$$W = (p', q'); \cdots; (p^{(s)}, q^{(s)}); X = (m', n'); \cdots; (m^{(s)}, n^{(s)}) \tag{1.19}$$

$$A = (a_{2k}, \alpha_{2k}^{(1)}, \alpha_{2k}^{(2)}); \cdots; (a_{(s-1)k}, \alpha_{(s-1)k}^{(1)}, \alpha_{(s-1)k}^{(2)}, \cdots, \alpha_{(s-1)k}^{(s-1)}) \tag{1.20}$$

$$B = (b_{2k}, \beta_{2k}^{(1)}, \beta_{2k}^{(2)}); \cdots; (b_{(s-1)k}, \beta_{(s-1)k}^{(1)}, \beta_{(s-1)k}^{(2)}, \cdots, \beta_{(s-1)k}^{(s-1)}) \tag{1.21}$$

$$\mathfrak{A} = (a_{sk}; \alpha_{sk}^{(1)}, \alpha_{sk}^{(2)}, \cdots, \alpha_{sk}^{(s)}) : \mathfrak{B} = (b_{sk}; \beta_{sk}^{(1)}, \beta_{sk}^{(2)}, \cdots, \beta_{sk}^{(s)}) \tag{1.22}$$

$$A' = (a'_k, \alpha'_k)_{1,p'}; \cdots; (a_k^{(s)}, \alpha_k^{(s)})_{1,p^{(s)}}; B' = (b'_k, \beta'_k)_{1,p'}; \cdots; (b_k^{(s)}, \beta_k^{(s)})_{1,p^{(s)}} \tag{1.23}$$

The multivariable I-function write :

$$I(z_1,\cdots,z_s) = I^{V;0,n_s;X}_{U:p_s,q_s;W}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A}\ ;\mathfrak{A};\mathrm{A'} \\ \cdot & \\ \cdot & \\ \cdot & \mathrm{B};\ \mathfrak{B};\ \mathrm{B'} \\ \mathrm{z}_s & \end{array}\right) \tag{1.24}$$

Braaksma [1] has shown that if

$$\beta_h(b_j+\lambda) \neq \beta_l(b_h+r) \text{ for } j\neq h; j,h=1,\cdots,m; \lambda, r = 0,1,2,\cdots \tag{1.25}$$

in which case the poles of $\prod_{j=1}^{m}\Gamma(b_j+\beta_j s)$ are simple, and the conditions

i) $\delta > 0, x\neq 0$ where $\delta = \sum_{j=1}^{q}\beta_j - \sum_{j=1}^{p}\alpha_j$

ii) $\delta = 0$ and $0<|x|<D^{-1}$, where $D=\prod_{j=1}^{p}(\alpha_j)^{\alpha_i}\prod_{j=1}^{q}(\beta_j)^{-\beta_i}$

are satisfied, then

$$H^{m,n}_{p,q}(x) = \sum_{h=1}^{m}\sum_{r=0}^{\infty}\frac{\prod_{j=1,j\neq k}^{m}\Gamma(b_j-\beta_j\xi_{h,r})\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j\xi_{h,r})}{\prod_{j=m+1}^{q}\Gamma(1-b_j+\beta_j\xi_{h,r})\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j\xi_{h,r})\beta_h r!}(-)^r x^{\xi_{h,r}} \tag{1.26}$$

where $\xi_{h,r} = \{(b_h+r)/\beta_h\}$ (1.27)

we note $g(\xi_{h,r}) = \dfrac{\prod_{j=1,j\neq k}^{m}\Gamma(b_j-\beta_j\xi_{h,r})\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j\xi_{h,r})}{\prod_{j=m+1}^{q}\Gamma(1-b_j+\beta_j\xi_{h,r})\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j\xi_{h,r})\beta_h r!}$ (1.28)

For more details, see Braaksma [1] .

2. Main integral

In this paper, we have , $r=s$. We have the general unified integral

Theorem

$$\int_0^\infty x^{k_1}(1+tx)^{-k_2}H^{k,0}_{p,q}\left[\mathrm{e}x^{w_1}(1+tx)^{-w_2}\middle| \begin{array}{c}(\mathrm{g}_{j'},G_{j'})_{1,P}\\(\mathrm{h}_{j'},H_{j'})_{1,Q}\end{array}\right] I^{V_1;0,N_r;X_1}_{U_1:P_r,Q_r;W_1}\left(\begin{array}{c}\mathrm{a}_1x^{\lambda'_1}(1+tx)^{-\lambda'_2}\\ \cdot \\ \cdot \\ \mathrm{a}_r x^{\lambda_1^{(r)}}(1+tx)^{-\lambda_2^{(r)}}\end{array}\right)$$

$$I^{V;0,n_r;X}_{U:p_r,q_r r;W}\left(\begin{array}{c}\mathrm{y}_1x^{\mu'_1}(1+tx)^{-\mu'_2}\\ \cdot \\ \cdot \\ \mathrm{y}_r x^{\mu_1^{(r)}}(1+tx)^{-\mu_2^{(r)}}\end{array}\right)\mathrm{d}x = \sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{M_1}\cdots\sum_{g_r=0}^{M_r}\sum_{j=1}^{k}\sum_{w=0}^{\infty}\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}$$

$$G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})a_1^{\eta_{G_1,g_1}}\cdots a_r^{\eta_{G_r,g_r}}t^{-(k_1+\sum_{i=1}^r\lambda_1^{(i)}\eta_{G_i,g_i})}\frac{(-)^w g(\eta_{j,w})e^{\eta_{j,w}}t^{-w_1\eta_{j,w}}}{w!H_j}$$

$$I_{U:p_r+2,q_r+1;W}^{V;0,n_r+2;X}\left(\begin{array}{c}\mathrm{y}_1 t^{-\mu_1'}\\ \cdot\\ \cdot\\ \mathrm{y}_r t^{-\mu_1^{(r)}}\end{array}\middle|\begin{array}{l}\mathrm{A}\ ;\ (\text{-k}_1-\sum_{i=1}^r\eta_{G_i,g_i}\lambda_1^{(i)}-w_1\eta_{j,w};\mu_1',\cdots,\mu_1^{(r)}),\\ \cdots\\ \mathrm{B}\ ;(1\text{-k}_2-\sum_{i=1}^r\eta_{G_i,g_i}\lambda_2^{(i)}-w_2\eta_{j,w};\mu_2',\cdots,\mu_2^{(r)}),\end{array}\right.$$

$$\left.\begin{array}{c}(2+\mathrm{k}_1-k_2+\sum_{i=1}^r\eta_{G_i,g_i}(\lambda_1^{(i)}-\lambda_2^{(i)})+(w_1-w_2)\eta_{j,w};\mu_1'-\mu_2',\cdots,\mu_1^{(r)}-\mu_2^{(r)}),\mathfrak{A}:A'\\ \cdots\\ \mathfrak{B}\ ;\mathrm{B}'\end{array}\right) \quad (2.1)$$

$\phi(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r}),\theta_1(\eta_{G_1,g_1}),\cdots,\theta_r(\eta_{G_r,g_r})$ are given respectively in (1.2) and (1.3) and

$$g(\eta_{h_j,w})=\frac{\prod_{j'=1,j'\neq j}^k\Gamma(h_{j'}-H_{j'}\eta_{j,w})}{\prod_{j'=k+1}^q\Gamma(1-h_{j'}+H_{j'}\eta_{j,w})\prod_{j'=1}^p\Gamma(g_{j'}-G_{j'}\eta_{j,w})} \quad (2.2)$$

where $\eta_{j,w}=\dfrac{h_j+w}{H_j},j=1,\cdots,r$ (2.3)

The condition of validity of (2.1) are :

a) $t>0;0\leqslant w_1\leqslant w_2;0\leqslant\lambda_1^{(i)}\leqslant\lambda_2^{(i)};0\leqslant\mu_1^{(i)}\leqslant\mu_2^{(i)},i=1,\cdots,r$

b) $0<Re(k_1)<Re(k_2)$

c) $Re[k_2+w_2\min_{1\leqslant j\leqslant k}\dfrac{h_j}{H_j}+\sum_{i=1}^r\lambda_2^{(i)}\min_{1\leqslant j\leqslant M_i}\dfrac{B_j^{(i)}}{\delta_j^{(i)}}+\sum_{i=1}^r\mu_2^{(i)}\min_{1\leqslant j\leqslant m_i}\dfrac{b_j^{(i)}}{\beta_j^{(i)}}]>$

$$>Re[k_1+w_1\min_{1\leqslant j\leqslant k}\frac{h_j}{H_j}+\sum_{i=1}^r\lambda_1^{(i)}\min_{1\leqslant j\leqslant M_i}\frac{B_j^{(i)}}{\delta_j^{(i)}}+\sum_{i=1}^r\mu_1^{(i)}\min_{1\leqslant j\leqslant m_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}]+1>0$$

d) $|arge|<\dfrac{1}{2}\pi A'$, where $A'=\sum_{j'=1}^k H_j-\sum_{j'=k+1}^q H_j-\sum_{j'=1}^p G_j>0$

e) $\delta=\sum_{j'=1}^q H_j-\sum_{j'=1}^p G_j>0$

f) $|arga_i|<\dfrac{1}{2}\Omega_i\pi$, where Ω_i is defined by (1.3) ; $i=1,\cdots,r$

g) $|argy_i| < \frac{1}{2}\Omega'_i\pi$, where Ω'_i is defined by (1.17) ; $i = 1, \cdots, r$

h) The series on the right-hand side of (2.1) is assumed to be absolutely convergent

Proof

To evaluate the integral (2.1) ,we first use the serie expansions as follows for $I_{U_1:P_r,Q_r;W_1}^{V_1;0,N_r;X_1}\left(\begin{array}{c}\mathrm{a}_1x^{\lambda'_1}(1+tx)^{-\lambda'_2}\\ \cdot\\ \cdot\\ \mathrm{a}_rx^{\lambda_1^{(r)}}(1+tx)^{-\lambda_2^{(r)}}\end{array}\right)$

with the help of the equation (1.6),and change the order of integration ans summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process), we thus find that

$$\text{L.H.S} = \sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{M_1}\cdots\sum_{g_r=0}^{M_r}\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}G(\eta_{G_1,g_1},\cdots\eta_{G_r,g_r})\, a_1^{\eta_{G_1,g_1}}\cdots a_r^{\eta_{G_r,g_r}}\, I \tag{2.4}$$

where $I = \int_0^{\infty} x^{k_1+\sum_{i=1}^r \lambda_1^{(i)}\eta_{G_i,g_i}}(1+tx)^{-k_2-\sum_{i=1}^r \lambda_2^{(i)}\eta_{G_i,g_i}} I_{U:p_r,q_rr;W}^{V;0,n_r;X}\left(\begin{array}{c}\mathrm{y}_1x^{\mu'_1}(1+tx)^{-\mu'_2}\\ \cdot\\ \cdot\\ \mathrm{y}_rx^{\mu_1^{(r)}}(1+tx)^{-\mu_2^{(r)}}\end{array}\right)$

$$H_{p,q}^{k,0}\left[\mathrm{e}x^{w_1}(1+tx)^{-w_2}\left|\begin{array}{c}(\mathrm{g}_{j'},G_{j'})_{1,P}\\ (\mathrm{h}_{j'},H_{j'})_{1,Q}\end{array}\right.\right]\mathrm{d}x \tag{2.5}$$

For evaluating the integral I , we express the multivariable I-function in Mellin Barnes contour integral with the help of the equation (1.16) and change the order of the $x-$integral and the $(t_1,\cdots,t_r)-$integral (which is permissible under the conditions stated with (2.1)), evaluating the $x-$integral with the help of a result due to Koul ([2],1973,page 368), we get

$$I = \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\zeta(t_1,\cdots,t_r)\prod_{k=1}^{s}\phi_k(t_k)\quad t^{-(k_1+\sum_{i=1}^r\lambda_1^{(i)}+\sum_{i=1}^r\mu_1^{(i)}t_i)}$$

$$H_{p+2,q+1}^{k,2}\left[ct^{-w_1}\left|\begin{array}{c}(\mathrm{g}_{j'},G_{j'})_{1,P},\,(\text{-k}_1-\sum_{i=1}^r\eta_{G_i,g_i}\lambda_1^{(i)}-\sum_{j=1}^r\mu_1^{(i)}t_j;w_1)\\ \cdots\qquad\cdots\\ (\mathrm{h}_{j'},H_{j'})_{1,Q},(\text{1-k}_2-\sum_{i=1}^r\eta_{G_i,g_i}\lambda_2^{(i)}-\sum_{j=1}^r\mu_2^{(i)}t_t;w_2)\end{array}\right.\right.$$

$$\left.\begin{array}{c}(2+\mathrm{k}_1-k_2+\sum_{i=1}^r\eta_{G_i,g_i}(\lambda_1^{(i)}-\lambda_2^{(i)})+(w_1-w_2)\eta_{j,w}+\sum_{i=1}^r(u_1^{(i)}-\mu_2^{(i)})t_i);w_2-w_1)\\ \cdots\\ \cdots\end{array}\right]$$

$$\prod_{k=1}^{r} z_k^{t_k}\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{2.6}$$

where $t\neq 0$

Now using the serie representation (1.26) in (2.6), and substituting the value of I in (2.4). Finally interpreting the result thus obtained with the Mellin-barnes contour integral with the help of (1.9), we arrive at the desired result.

3. Particular cases

The integral (2.1) is a general nature. On specialization the various parameters in it, we can obtain a large number of known or new (finite and infinite) integrals. We give below some special cases of (2.1) which are of interest cases.

First integral

Taking $w_2=0$ in (2.1) and the right-hand side with the help of the results (Srivastava et al [6], 1982),page14, Eq (2.3.4) and (2.2.16) ; page12, Eq (2.3.4) and (6.4.8), and making some simplifications, we get

Corollary 1

$$\int_0^\infty x^{k_1}(1+tx)^{-k_2}\,H_{p,q}^{k,0}\left[\mathrm{cx}^{w_1}\middle|\begin{matrix}(\mathrm{g}_{j'},G_{j'})_{1,P}\\(\mathrm{h}_{j'},H_{j'})_{1,Q}\end{matrix}\right]I_{U_1:P_r,Q_r;W_1}^{V_1;0,N_r;X_1}\left(\begin{matrix}\mathrm{a}_1x^{\lambda'_1}(1+tx)^{-\lambda'_2}\\ \cdot\\ \cdot\\ \mathrm{a}_rx^{\lambda_1^{(r)}}(1+tx)^{-\lambda_2^{(r)}}\end{matrix}\right)$$

$$I_{U:p_r,q_rr;W}^{V;0,n_r;X}\left(\begin{matrix}\mathrm{y}_1x^{\mu'_1}(1+tx)^{-\mu'_2}\\ \cdot\\ \cdot\\ \mathrm{y}_rx^{\mu_1^{(r)}}(1+tx)^{-\mu_2^{(r)}}\end{matrix}\right)\mathrm{d}x=\frac{(e)^{\frac{-(k_1+1)}{w_1}}}{w_1}\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{M_1}\cdots\sum_{g_r=0}^{M_r}\sum_{w=0}^{\infty}$$

$$\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})\,(a_1c)^{-\frac{\lambda'_1}{w_1}\eta_{G_1,g_1}}\cdots(a_rc)^{-\frac{\lambda_1^{(r)}}{w_1}\eta_{G_r,g_r}}\frac{(-)^w\left(te^{-\frac{1}{w_1}}\right)^w}{w!}$$

$$I_{U:p_r+q+1,q_r+q+1;W}^{V;0,n_r+q+1;X}\left(\begin{matrix}\mathrm{y}_1e^{-\mu'_1/w_1}\\ \cdot\\ \cdot\\ \mathrm{y}_rt^{-\mu_1^{(r)}/w_1}\end{matrix}\middle|\begin{matrix}\mathrm{A}\ ;\ (1\text{-k}_2-w-\sum_{i=1}^r\eta_{G_i,g_i}\lambda_1^{(i)};\mu'_1,\cdots,\mu_1^{(r)}),\\ \cdots\\ \mathrm{B}\ ;\ (1\text{-k}_2-\sum_{i=1}^r\eta_{G_i,g_i}\lambda_2^{(i)};\mu'_2,\cdots,\mu_2^{(r)}),\end{matrix}\right.$$

$$\left.\begin{matrix}\left[1-h_{j'}-(k_1+\sum_{i=1}^r\lambda_1^{(i)}\eta_{G_i,g_i}+w+1)\frac{H_{j'}}{w_1};\frac{\mu'_1}{w_1},\cdots,\frac{\mu_1^{(r)}}{w_1}\right]_{1,q},\mathfrak{A};A'\\ \cdots\\ \left[1-g_{j'}-(k_1+\sum_{i=1}^r\lambda_1^{(i)}\eta_{G_i,g_i}+w+1)\frac{G_{j'}}{w_1};\frac{\mu'_1}{w_1}g_1,\cdots,\frac{\mu_1^{(r)}}{w_1}g_r\right]_{1,p},\mathfrak{B};B'\end{matrix}\right) \tag{3.1}$$

The condition of validity of (3.1) are :

a) $t > 0; 0 \leqslant w_1; 0 \leqslant \lambda_1^{(i)} \leqslant \lambda_2^{(i)}; 0 \leqslant \mu_1^{(i)} \leqslant \mu_2^{(i)}, i = 1, \cdots, r$

b) $0 < Re(k_1) < Re(k_2)$

c) $Re[k_1 + w_1 \min_{1 \leqslant j \leqslant k} \frac{h_j}{H_j} + \sum_{i=1}^{r} \lambda_1^{(i)} \min_{1 \leqslant j \leqslant M_i} \frac{B_j^{(i)}}{\delta_j^{(i)}} + \sum_{i=1}^{r} \mu_1^{(i)} \min_{1 \leqslant j \leqslant m_i} \frac{b_j^{(i)}}{\beta_j^{(i)}}] + 1 > 0$

d) $|arge| < \frac{1}{2}\pi A'$, where $A' = \sum_{j'=1}^{k} H_j - \sum_{j'=k+1}^{q} H_j - \sum_{j'=1}^{p} G_j > 0$

e) $\delta = \sum_{j'=1}^{q} H_j - \sum_{j'=1}^{p} G_j > 0$

f) $|arga_i| < \frac{1}{2}\Omega_i \pi$, where Ω_i is defined by (1.3) ; $i = 1, \cdots, r$

g) $|argy_i| < \frac{1}{2}\Omega_i' \pi$, where Ω_i' is defined by (1.17) ; $i = 1, \cdots, r$

h) The series on the right-hand side of (2.1) is assumed to be absolutely convergent

Second integral

If we let $t \rightarrow 0$ in (2.1) , put $n_r = N_r = 0$ and make slight changes in the parapeters , we obtain

Corollary 2

$$\int_0^{\infty} x^{\rho-1} H_{p,q}^{k,0} \left[\mathrm{c}x^{w} \middle| \begin{matrix} (\mathrm{g}_{j'}, G_{j'})_{1,P} \\ (\mathrm{h}_{j'}, H_{j'})_{1,Q} \end{matrix} \right] I_{U_1:P_r,Q_r;W_1}^{V_1;0,N_r;X_1} \begin{pmatrix} \mathrm{a}_1 x^{\lambda_1'} \\ \cdot \\ \cdot \\ \mathrm{a}_r x^{\lambda_1^{(r)}} \end{pmatrix} I_{U:p_r,q_r r;W}^{V;0,n_r;X} \begin{pmatrix} \mathrm{y}_1 x^{\sigma_1'} \\ \cdot \\ \cdot \\ \mathrm{y}_r x^{\sigma_1^{(r)}} \end{pmatrix} \mathrm{d}x$$

$$= \frac{(e)^{\frac{-\rho}{w}}}{w} \sum_{G_1,\cdots,G_r=0}^{\infty} \sum_{g_1=0}^{M_1} \cdots \sum_{g_r=0}^{M_r} \sum_{w=0}^{\infty} \frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!} G'(\eta_{G_1,g_1}, \cdots, \eta_{G_r,g_r})$$

$$(a_1 c)^{-\frac{\lambda_1'}{w}\eta_{G_1,g_1}} \cdots (a_r c)^{-\frac{\lambda_1^{(r)}}{w}\eta_{G_r,g_r}} I_{U:p_r+q,q_r+q;W}^{V;0,q;X} \left(\begin{matrix} \mathrm{y}_1 e^{-\sigma_1'/w_1} \\ \cdot \\ \cdot \\ \mathrm{y}_r t^{-\sigma_1^{(r)}/w_1} \end{matrix} \right|$$

$$\left.\begin{array}{c} \mathrm{A}\ ;\Big[1-h_{j'}-(\rho+\sum_{i=1}^{r}\lambda_1^{(i)}\eta_{G_i,g_i})\frac{H_{j'}}{w};\frac{\sigma'}{w},\cdots,\frac{\sigma^{(r)}}{w}\Big]_{1,q};A' \\ \cdots \\ \mathrm{B}\ ;\ \Big[1-g_{j'}-(\rho+\sum_{i=1}^{r}\lambda_1^{(i)}\eta_{G_i,g_i})\frac{G_{j'}}{w};\frac{\sigma'}{w}g_1,\cdots,\frac{\sigma^{(r)}}{w}g_r\Big]_{1,p},\mathfrak{B};B' \end{array}\right) \tag{3.2}$$

where $G'(\eta_{G_1,g_1},\cdots\eta_{G_r,g_r})=G(\eta_{G_1,g_1},\cdots\eta_{G_r,g_r})_{\mathfrak{n}=0}$; $U_{P+q,Q+p}=P_i+q;Q_i+p;\iota_i;r'$

The condition of validity of (3.2) are :

a) $min\{\lambda',\cdots,\lambda^{(r)},\sigma',\cdots,\sigma^{(r)},w,Re(\rho)\}>0$

b) $0<Re(k_1)<Re(k_2)$

c) $Re[\rho+w\min\limits_{1\leqslant j\leqslant k}\dfrac{h_j}{H_j}+\sum\limits_{i=1}^{r}\lambda_1^{(j)}\min\limits_{1\leqslant j\leqslant M_i}\dfrac{B_j^{(i)}}{\delta_j^{(i)}}+\sum\limits_{i=1}^{r}\sigma^{(j)}\min\limits_{1\leqslant j\leqslant m_i}\dfrac{b_j^{(i)}}{\beta_j^{(i)}}]+1>0$

d) $|arge|<\dfrac{1}{2}\pi A'$, where $A'=\sum\limits_{j'=1}^{k}H_j-\sum\limits_{j'=k+1}^{q}H_j-\sum\limits_{j'=1}^{p}G_j>0$

e) $\delta=\sum\limits_{j'=1}^{q}H_j-\sum\limits_{j'=1}^{p}G_j>0$

f) $|arga_i|<\dfrac{1}{2}\Omega_i\pi$, where Ω_i is defined by (1.3) ; $i=1,\cdots,r$

g) $|argy_i|<\dfrac{1}{2}\Omega_i'\pi$, where Ω_i' is defined by (1.17) ; $i=1,\cdots,r$

h) The series on the right-hand side of (3.2) is assumed to be absolutely convergent

Third integral

Corollary 3

Putting $t=1$, replacing x by $x/(u-x)$, k_2 by k_1+k_2+2 , $\lambda_2^{(i)}$ by $\lambda_1^{(i)}+\lambda_2^{(i)}$, w_2 by w_1+w_2, $\mu_2^{(i)}$ by $\mu_1^{(i)}+\mu_2^{(i)}$, $a_1y_iu^{-\mu_2^{(i)}}$ by y_i and cu^{-w_2} by c, we get from (2.1) the following finite integral which itself is quite general in nature.

$$\int_0^u x^{k_1}(u-x)^{k_2}\,H_{p,q}^{k,0}\left[\mathrm{c}x^{w_1}(u-x)^{w_2}\left|\begin{array}{c}(\mathrm{g}_{j'},G_{j'})_{1,P}\\(\mathrm{h}_{j'},H_{j'})_{1,Q}\end{array}\right.\right]I_{U_1:P_r,Q_r;W_1}^{V_1;0,N_r;X_1}\left(\begin{array}{c}\mathrm{a}_1x^{\lambda_1'}(u-x)^{-\lambda_2'}\\ \cdot\\ \cdot\\ \mathrm{a}_rx^{\lambda_1^{(r)}}(u-x)^{-\lambda_2^{(r)}}\end{array}\right)$$

$$I_{U:p_r,q_rr;W}^{V;0,n_r;X}\left(\begin{array}{c} \mathrm{y}_1x^{\mu'_1}(u-x)^{-\mu'_2} \\ \cdot \\ \cdot \\ \mathrm{y}_r x^{\mu_1^{(r)}}(u-x)^{-\mu_2^{(r)}} \end{array}\right)\mathrm{d}x = u^{k_1+k_2+1}\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{M_1}\cdots\sum_{g_r=0}^{M_r}\sum_{j=1}^{k}\sum_{w=0}^{\infty}$$

$$\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})(a_1u)^{(\lambda'1+\lambda'2)\eta_{G_1,g_1}}\cdots(a_ru)^{(\lambda_1^{(r)}+\lambda_2^{(r)})\eta_{G_r,g_r}}$$

$$\frac{(-)^w g(\eta_{j,w})e^{\eta_{j,w}}t^{-w_1\eta_{j,w}}}{w!H_j} I_{U:p_r+2,q_r+1;W}^{V;0,n_r+2;X}\left(\begin{array}{c} \mathrm{y}_1u^{\mu'_1+\mu'_2} \\ \cdot \\ \cdot \\ \mathrm{y}_r t^{\mu_1^{(r)}+\mu_2^{(r)}} \end{array}\right|$$

$$\begin{array}{l} \mathrm{A}\ ;\ (\text{-k}_1-\sum_{i=1}^r\eta_{G_i,g_i}\lambda_1^{(i)}-w_1\eta_{j,w};\mu'_1,\cdots,\mu_1^{(r)}), \\ \quad\quad\quad\quad \cdots \\ \quad\quad\quad\quad \mathrm{B}\ ; \end{array}$$

$$\left.\begin{array}{c} (\text{-k}_2-\sum_{i=1}^r\eta_{G_i,g_i}\lambda_2^{(i)}-w_2\eta_{j,w};\mu'_2,\cdots,\mu_2^{(r)}),\mathfrak{A};A' \\ \cdots \\ (1\text{-k}_1-k_2-\sum_{i=1}^r(\lambda_1^{(i)}+\lambda_2^{(i)})-(w_1+w_2)\eta_{j,w};\mu'_1+\mu'_2,\cdots,\mu_1^{(r)}+\mu_2^{(r)}),\mathfrak{B};B' \end{array}\right) \tag{3.4}$$

a) $min\{\lambda'_1,\cdots,\lambda_1^{(r)},\lambda'_2,\cdots,\lambda_2^{(r)},w_1,w_2,\mu'_1,\cdots,\mu_1^{(r)},\mu'_2,\cdots,\mu_2^{(r)}Re(k_1+1),Re(k_2+1)\}>0$

b)$Re[Re(k_1+k_2)+(w_1+w_2)\min_{1\leqslant j\leqslant k}\frac{h_j}{H_j}+\sum_{i=1}^r(\lambda_1^{(i)}+\lambda_2^{(i)})\min_{1\leqslant j\leqslant M_i}\frac{B_j^{(i)}}{\delta_j^{(i)}}+\sum_{i=1}^r(\mu_1^{(i)}+\mu_2^{(i)})\min_{1\leqslant j\leqslant m_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}]+1>0$

c) $|arge|<\frac{1}{2}\pi A'$, where $A'=\sum_{j'=1}^{k}H_j-\sum_{j'=k+1}^{q}H_j-\sum_{j'=1}^{p}G_j>0$

d) $|arga_i|<\frac{1}{2}\Omega_i\pi$, where Ω_i is defined by (1.3) ; $i=1,\cdots,r$

e) $|argy_i|<\frac{1}{2}\Omega'_i\pi$, where Ω'_i is defined by (1.17) ; $i=1,\cdots,r$

f) The series on the right-hand side of (3.2) is assumed to be absolutely convergent

g) $\delta=\sum_{j'=1}^{q}H_j-\sum_{j'=1}^{p}G_j>0$

The quantities $.U_1,V_1,W_1,X_1,A_1,B_1,\mathfrak{A}_1,\mathfrak{B}_1,A'_1$ and B'_1 are defined to (1.8) of (1.13) and the quantities $U,V,W,X,A,B,\mathfrak{A},\mathfrak{B},A'$ B'

$U, V, W, X, A, B, \mathfrak{A}, \mathfrak{B}, A'$ and B' are defined by the equations (1.18) to (1;23)

4. Multivariable H-function

If $U = V = A = B = U_1 = V_1 = A_1 = B_1 = 0$, the multivariable I-function defined by Prasad degenere in multivariable H-function defined by Srivastava et al [5]. Our integral contain two multivariable H-functions. We have :

Integral 1

$$\int_0^\infty x^{k_1}(1+tx)^{-k_2} H_{p,q}^{k,0}\left[\mathrm{e}x^{w_1}(1+tx)^{-w_2}\left| \begin{matrix} (\mathrm{g}_{j'}, G_{j'})_{1,P} \\ (\mathrm{h}_{j'}, H_{j'})_{1,Q} \end{matrix}\right.\right] H_{P_r,Q_r;W_1}^{;0,N_r;X_1}\left(\begin{matrix} \mathrm{a}_1 x^{\lambda'_1}(1+tx)^{-\lambda'_2} \\ \cdot \\ \cdot \\ \mathrm{a}_r x^{\lambda_1^{(r)}}(1+tx)^{-\lambda_2^{(r)}} \end{matrix}\right)$$

$$H_{p_r,q_r;W}^{0,n_r;X}\left(\begin{matrix} \mathrm{y}_1 x^{\mu'_1}(1+tx)^{-\mu'_2} \\ \cdot \\ \cdot \\ \mathrm{y}_r x^{\mu_1^{(r)}}(1+tx)^{-\mu_2^{(r)}} \end{matrix}\right)\mathrm{d}x = \sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{M_1}\cdots\sum_{g_r=0}^{M_r}\sum_{j=1}^{k}\sum_{w=0}^{\infty}\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}$$

$$G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})a_1^{\eta_{G_1,g_1}}\cdots a_r^{\eta_{G_r,g_r}}t^{-(k_1+\sum_{i=1}^r\lambda_1^{(i)}\eta_{G_i,g_i})}\frac{(-)^w g(\eta_{j,w})e^{\eta_{j,w}}t^{-w_1\eta_{j,w}}}{w!H_j}$$

$$H_{p_r+2,q_r+1;W}^{0,n_r+2;X}\left(\begin{matrix} \mathrm{y}_1 t^{-\mu'_1} \\ \cdot \\ \cdot \\ \mathrm{y}_r t^{-\mu_1^{(r)}} \end{matrix}\left| \begin{matrix} (\text{-k}_1-\sum_{i=1}^r\eta_{G_i,g_i}\lambda_1^{(i)}-w_1\eta_{j,w};\mu'_1,\cdots,\mu_1^{(r)}), \\ \cdots \\ (1\text{-k}_2-\sum_{i=1}^r\eta_{G_i,g_i}\lambda_2^{(i)}-w_2\eta_{j,w};\mu'_2,\cdots,\mu_2^{(r)}), \end{matrix}\right.\right.$$

$$\left.\begin{matrix}(2+\mathrm{k}_1-k_2+\sum_{i=1}^r\eta_{G_i,g_i}(\lambda_1^{(i)}-\lambda_2^{(i)})+(w_1-w_2)\eta_{j,w};\mu'_1-\mu'_2,\cdots,\mu_1^{(r)}-\mu_2^{(r)}),\mathfrak{A}:A' \\ \cdots \\ \mathfrak{B}\ ;\mathrm{B}' \end{matrix}\right) \quad (4.1)$$

under the same notations and conditions that (2.1) with $U = V = A = B = U_1 = V_1 = A_1 = B_1 = 0$

Integral 2

$$\int_0^u x^{k_1}(u-x)^{k_2} H_{p,q}^{k,0}\left[\mathrm{c}x^{w_1}(u-x)^{w_2}\left| \begin{matrix} (\mathrm{g}_{j'}, G_{j'})_{1,P} \\ (\mathrm{h}_{j'}, H_{j'})_{1,Q} \end{matrix}\right.\right] H_{P_r,Q_r;W_1}^{0,N_r;X_1}\left(\begin{matrix} \mathrm{a}_1 x^{\lambda'_1}(u-x)^{-\lambda'_2} \\ \cdot \\ \cdot \\ \mathrm{a}_r x^{\lambda_1^{(r)}}(u-x)^{-\lambda_2^{(r)}} \end{matrix}\right)$$

$$H^{0,n_r;X}_{p_r,q_r r;W}\left(\begin{array}{c} y_1 x^{\mu'_1}(u-x)^{-\mu'_2} \\ \cdot \\ \cdot \\ y_r x^{\mu_1^{(r)}}(u-x)^{-\mu_2^{(r)}} \end{array}\right)\mathrm{d}x = u^{k_1+k_2+1}\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{M_1}\cdots\sum_{g_r=0}^{M_r}\sum_{j=1}^{k}\sum_{w=0}^{\infty}$$

$$\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})(a_1u)^{(\lambda'1+\lambda'2)\eta_{G_1,g_1}}\cdots(a_ru)^{(\lambda_1^{(r)}+\lambda_2^{(r)})\eta_{G_r,g_r}}$$

$$\frac{(-)^w g(\eta_{j,w})e^{\eta_{j,w}}t^{-w_1\eta_{j,w}}}{w!H_j}H^{0,n_r+2;X}_{p_r+2,q_r+1;W}\left(\begin{array}{c} y_1 u^{\mu'_1+\mu'_2} \\ \cdot \\ \cdot \\ y_r t^{\mu_1^{(r)}+\mu_2^{(r)}} \end{array}\right|$$

$$(\text{-k}_1-\textstyle\sum_{i=1}^r\eta_{G_i,g_i}\lambda_1^{(i)}-w_1\eta_{j,w};\mu'_1,\cdots,\mu_1^{(r)}),$$
$$\cdots$$

$$\left.\begin{array}{c}(\text{-k}_2-\sum_{i=1}^r\eta_{G_i,g_i}\lambda_2^{(i)}-w_2\eta_{j,w};\mu'_2,\cdots,\mu_2^{(r)}),\mathfrak{A};A' \\ \cdots \\ (\text{1-k}_1-k_2-\sum_{i=1}^r(\lambda_1^{(i)}+\lambda_2^{(i)})-(w_1+w_2)\eta_{j,w};\mu'_1+\mu'_2,\cdots,\mu_1^{(r)}+\mu_2^{(r)}),\mathfrak{B};B'\end{array}\right) \tag{4.2}$$

under the same notations and conditions that (3.4) with $U=V=A=B=U_1=V_1=A_1=B_1=0$

Remark :we have the similar integral with the H-function of two variables, for more details, see Srivastava et al [4]

6. Conclusion

In this paper we have evaluate a general unified integral involving the product of two multivariable Aleph-functions, and the Fox's H-functionThe integral established in this paper is of very general nature as it contains multivariable Aleph-function, which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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