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Double integration of certain products of special functions with the Multivariable I-function

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ABSTRACT
The object of this paper is to establish three finite double integrals involving certain product of special functions, the multivariables I-function and a general class of polynomials with general arguments. Our double integrals are quite general in character and a number of doubles integrals can be deduced as particular cases.

KEYWORDS :Multivariable I-function, Aleph-function, finite double integrals, general class of polynomials, M-serie, multivariable H-function.

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1.Introduction and preliminaries.

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The object of this paper is to establish three finite double integrals involving certain product of special functions, the multivariables I-function defined by Prasad [4] and a general class of polynomials with general arguments. We will study the case of multivariable H-function defined by Srivastava et al [8].

The Aleph- function , introduced by Südland [9] et al , however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral :

$$\aleph(z)=\aleph_{P_i,Q_i,c_i;r}^{M,N}\left(\mathrm{z}\ \middle|\ \begin{matrix}(\mathrm{a}_j,A_j)_{1,\mathfrak{n}},[c_i(a_{ji},A_{ji})]_{\mathfrak{n}+1,p_i;r}\\(\mathrm{b}_j,B_j)_{1,m},[c_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right)$$

$$=\frac{1}{2\pi\omega}\int_L\ \Omega_{P_i,Q_i,c_i;r}^{M,N}(s)z^{-s}\mathrm{d}s \tag{1.1}$$

for all z different to 0 and

$$\Omega\,_{P_i,Q_i,c_i;r}^{M,N}(s)=\frac{\prod_{j=1}^{M}\Gamma(b_j+B_js)\prod_{j=1}^{N}\Gamma(1-a_j-A_js)}{\sum_{i=1}^{r}c_i\prod_{j=N+1}^{P_i}\Gamma(a_{ji}+A_{ji}s)\prod_{j=M+1}^{Q_i}\Gamma(1-b_{ji}-B_{ji}s)} \tag{1.2}$$

With :

$$|argz|<\frac{1}{2}\pi\Omega\quad\text{Where }\Omega=\sum_{j=1}^{M}\beta_j+\sum_{j=1}^{N}\alpha_j-c_i(\sum_{j=M+1}^{Q_i}\beta_{ji}+\sum_{j=N+1}^{P_i}\alpha_{ji})>0\quad\text{with } i=1,\cdots,r$$

For convergence conditions and other details of Aleph-function , see Südland et al [9].

Serie representation of Aleph-function is given by Chaurasia et al [1].

$$\aleph_{P_i,Q_i,c_i;r}^{M,N}(z)\ =\sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r}^{M,N}(s)}{B_Gg!}z^{-s} \tag{1.3}$$

With $s = \eta_{G,g} = \dfrac{b_G + g}{B_G}, P_i < Q_i, |z| < 1$ and $\Omega^{M,N}_{P_i,Q_i,c_i;r}(s)$ is given in (1.2) (1.4)

The generalized polynomials defined by Srivastava [7], is given in the following manner :

$$S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}[y_1,\cdots,y_s] \ = \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_s=0}^{[N_s/M_s]} \frac{(-N_1)_{M_1K_1}}{K_1!} \cdots \frac{(-N_s)_{M_sK_s}}{K_s!}$$

$$A[N_1,K_1;\cdots;N_s,K_s]y_1^{K_1}\cdots y_s^{K_s} \tag{1.5}$$

Where $M_1,\cdots,M_s$ are arbitrary positive integers and the coefficients

In the present paper, we use the following notation

$$A_1 = \frac{(-N_1)_{M_1K_1}}{K_1!} \cdots \frac{(-N_s)_{M_sK_s}}{K_s!} A[N_1,K_1;\cdots;N_s,K_s] \tag{1.6}$$

The M-serie is defined, see Sharma [6].

$${}_{p'}M^{\alpha}_{q'}(y) = \sum_{s'=0}^{\infty} \frac{[(a_{p'})]_{s'}}{[(b_{q'})]_{s'}} \frac{y^{s'}}{\Gamma(\alpha s' + 1)} \tag{1.7}$$

Here $\alpha \in \mathbb{C}, Re(\alpha) > 0. [(a_{p'})]_{s'} = (a_1)_{s'}\cdots(a_{p'})_{s'}; [(b_{q'})]_{s'} = (b_1)_{s'}\cdots(b_{q'})_{s'}$.
The serie (1.9) converge if $p' \leqslant q' and |y| < 1.$

The multivariable I-function is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,z_2,...z_r) = I^{0,n_2;0,n_3;\cdots;0,n_r:m',n';\cdots;m^{(r)},n^{(r)}}_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p',q';\cdots;p^{(r)},q^{(r)}} \left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots; \end{array} \right.$$

$$\left. \begin{array}{c} (\mathrm{a}_{rj};\alpha'_{rj},\cdots,\alpha^{(r)}_{rj})_{1,p_r} : (a'_j,\alpha'_j)_{1,p'};\cdots;(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ \\ (\mathrm{b}_{rj};\beta'_{rj},\cdots,\beta^{(r)}_{rj})_{1,q_r} : (b'_j,\beta'_j)_{1,q'};\cdots;(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{array} \right) \tag{1.8}$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \xi(t_1,\cdots,t_r) \prod_{i=1}^{s} \phi_i(s_i) z_i^{s_i} \mathrm{d}s_1 \cdots \mathrm{d}s_r \tag{1.9}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [4]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_i| < \frac{1}{2}\Omega_i\pi$, where

$$\Omega_i = \sum_{k=1}^{n^{(i)}} \alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}} \alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}} \beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}} \beta_k^{(i)} + \left(\sum_{k=1}^{n_2} \alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2} \alpha_{2k}^{(i)}\right) +$$

$$+ \left(\sum_{k=1}^{n_r} \alpha_{rk}^{(i)} - \sum_{k=n_r+1}^{p_r} \alpha_{rk}^{(i)}\right) - \left(\sum_{k=1}^{q_2} \beta_{2k}^{(i)} + \sum_{k=1}^{q_3} \beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_r} \beta_{rk}^{(i)}\right) \tag{1.10}$$

where $i = 1, \cdots, r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$I(z_1, \cdots, z_r) = 0(|z_1|^{\gamma_1'}, \cdots, |z_r|^{\gamma_r'}) , max(|z_1|, \cdots ,|z_r|) \rightarrow 0$

$I(z_1, \cdots, z_r) \ = 0(|z_1|, \cdots, |z_r|^{\beta_s'}) , min(|z_1| , \cdots, |z_r|) \rightarrow \infty$

where $k = 1, \cdots, z : \alpha_k' = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k' = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, n_k$$

We will use these following notations in this paper :

$$U = p_2, q_2; p_3, q_3; \cdots ; p_{r-1}, q_{r-1}; V = 0, n_2; 0, n_3; \cdots ; 0, n_{s-1} \tag{1.11}$$

$$W = (p', q'); \cdots ; (p^{(r)}, q^{(r)}); X = (m', n'); \cdots ; (m^{(r)}, n^{(r)}) \tag{1.12}$$

$$A = (a_{2k}, \alpha_{2k}', \alpha_{2k}''); \cdots ; (a_{(r-1)k}), \alpha'_{(r-1)k}, \alpha_{(r-1)k}'', \cdots, \alpha_{(r-1)k}^{(r-1)}) \tag{1.13}$$

$$B = (b_{2k}, \beta_{2k}', \beta_{2k}''); \cdots ; (b_{(r-1)k}), \beta'_{(r-1)k}, \beta_{(r-1)k}'', \cdots, \beta_{(r-1)k}^{(r-1)}) \tag{1.14}$$

$$\mathfrak{A} = (a_{sk}; \alpha'_{rk}, \alpha_{rk}'', \cdots, \alpha_{rk}^r) : \mathfrak{B} = (b_{rk}; \beta'_{rk}, \beta_{rk}'', \cdots, \beta_{rk}^r) \tag{1.15}$$

$$A' = (a_k', \alpha_k')_{1,p'}; \cdots ; (a_k^{(r)}, \alpha_k^{(r)})_{1,p^{(r)}}; B' = (b_k', \beta_k')_{1,p'}; \cdots ; (b_k^{(r)}, \beta_k^{(r)})_{1,p^{(r)}} \tag{1.16}$$

The multivariable I-function write :

$$I(z_1, \cdots, z_r) = \ I_{U:p_r,q_r;W}^{V;0,n_r;X} \left(\begin{array}{c|c} \mathrm{z}_1 & \text{A ;}\mathfrak{A}\text{;A'} \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & \text{B; }\mathfrak{B}\text{; B'} \end{array} \right) \tag{1.17}$$

2. Finite double integrals

in this section, we evaluate three double finites integrals.

Integral 1

$$\int_0^{\pi/2}\int_0^1 e^{wi(\beta+\gamma)\theta}(sin\theta)^{\beta-1}(cos\theta)^{\gamma-1}x^{\rho-1}(1-x)^{\sigma-1}[ax+b(1-x)]^{-\rho-\sigma}$$

$${}_{p''}M^{\alpha}_{q''}(e^{wi(f+g)\theta}(sin\theta)^{f}(cos\theta)^{g}(ax)^{f'}[b(1-x)]^{g'}[ax+b(1-x)]^{-f'-g'})$$

$$\aleph^{M,N}_{P_i,Q_i,c_i;r'}(e^{wi(u+v)\theta}(sin\theta)^{u}(cos\theta)^{v}(ax)^{u'}[b(1-x)]^{v'}[ax+b(1-x)]^{-u'-v'})$$

$$S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\left(\begin{array}{c} \mathrm{e}^{wi(p'_1+q'_1)\theta}(sin\theta)^{p'_1}(cos\theta)^{q'_1}(ax)^{s'_1}[b(1-x)]^{t'_1}[ax+b(1-x)]^{-s'_1-t'_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{e}^{wi(p'_s+q'_s)\theta}(sin\theta)^{p'_s}(cos\theta)^{q'_s}(ax)^{s'_s}[b(1-x)]^{t'_s}[ax+b(1-x)]^{s'_s-t'_s} \end{array}\right)$$

$$I\left(\begin{array}{c} \mathrm{z}_1 e^{wi(h_1+k_1)\theta}(sin\theta)^{h_1}(cos\theta)^{k_1}(ax)^{h'_1}[b(1-x)]^{k'_1}[ax+b(1-x)]^{-h'_1-k'_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{z}_r e^{wi(h_r+k_r)\theta}(sin\theta)^{h_r}(cos\theta)^{k_r}(ax)^{h'_r}[b(1-x)]^{k'_r}[ax+b(1-x)]^{-h'_r-k'_r} \end{array}\right)\mathrm{d}x\mathrm{d}\theta$$

$$=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{l=0}^{\infty}\ \frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})}{B_G g!}\frac{[(a_{p''})]_l}{[(b_{q''})]_l}\frac{1}{\Gamma(\alpha l+1)}A_1$$

$$exp(i\pi w(\beta+lf+u\eta_{G,g}+p'_1K_1+\cdots+p'_sK_s)/2)a^{-\rho}b^{-\sigma}I^{V;0,n_r+4;X}_{U:p_r+4,q_r+2;W}\left(\begin{array}{c|c} \frac{z_1e^{i\pi wh_1/2}}{a^{h'_1}b^{k'_1}} & \mathrm{A}\ ; \\ \cdot & \cdot \\ \cdot & \cdot \\ \frac{z_re^{i\pi wh_r/2}}{a^{h'_r}b^{k'_r}} & \mathrm{B}\ ; \end{array}\right)$$

$$(1\text{-}\rho-f'l-u'\eta_{G,g}-\textstyle\sum_{i=1}^{s}s'_iK_i:h'_1,\cdots,h'_r),(1-\sigma-g'l-v'\eta_{G,g}-\sum_{i=1}^{s}t'_iK_i:k'_1,\cdots,k'_r)$$
$$\cdot\ \cdot\ \cdot$$
$$(1\text{-}\rho-\sigma-(f'+g')l-(u'+v')\eta_{G,g}-\textstyle\sum_{i=1}^{s}(s'_i+t'_i)K_i::h'_1+k'_1,\cdots,h'_r+k'_r)$$

$$(1\text{-}\beta-fl-u\eta_{G,g}-\textstyle\sum_{i=1}^{s}p'_iK_i:h_1,\cdots,h_r),(1-\gamma-gl-v\eta_{G,g}-\sum_{i=1}^{s}q'_iK_i;k_1,\cdots,k_r)$$
$$\cdot\ \cdot\ \cdot$$
$$(1\text{-}\beta-\gamma-(f+g)l-(u+v)\eta_{G,g}-\textstyle\sum_{i=1}^{s}(p'_i+q'_i)K_i:h_1+k_1,\cdots,h_r+k_r)$$

$$\left.\begin{matrix}\mathfrak{A} : \text{A'} \\ \cdot \cdot \cdot \\ \mathfrak{B} : \text{B'}\end{matrix}\right) a^{-(lf'+u'\eta_{G,g}+s'_1K_1+\cdots+s'_sK_s)} \; b^{-(lg'+v'\eta_{G,g}+t'_1K_1+\cdots+t'_sK_s)} \tag{2.1}$$

Provided that

a) $p'_i, q'_i, s'_i, t'_i > 0, i = 1, \cdots, s; h_i, k_i, h'_i, k'_i > 0, i = 1, \cdots, r; p'' < q''$ and $|ab| < 1$

b) $Re[\rho + u' \min\limits_{1 \leqslant j \leqslant M} \dfrac{b_i}{B_i} + \sum\limits_{i=1}^{r} h'_i \min\limits_{1 \leqslant j \leqslant m_i} \dfrac{b_j^{(i)}}{\beta_j^{(i)}}] > 0$

c) $Re[\sigma + v' \min\limits_{1 \leqslant j \leqslant M} \dfrac{b_i}{B_i} + \sum\limits_{i=1}^{r} k'_i \min\limits_{1 \leqslant j \leqslant m_i} \dfrac{b_j^{(i)}}{\beta_j^{(i)}}] > 0$

d) $Re[\beta + u \min\limits_{1 \leqslant j \leqslant M} \dfrac{b_i}{B_i} + \sum\limits_{i=1}^{r} h_i \min\limits_{1 \leqslant j \leqslant m_i} \dfrac{b_j^{(i)}}{\beta_j^{(i)}}] > 0$

e) $Re[\gamma + v \min\limits_{1 \leqslant j \leqslant M} \dfrac{b_i}{B_i} + \sum\limits_{i=1}^{r} k_i \min\limits_{1 \leqslant j \leqslant m_i} \dfrac{b_j^{(i)}}{\beta_j^{(i)}}] > 0$

f) $|argz_i| < \dfrac{1}{2}\Omega_i\pi$, where Ω_i is defined by (1.10)

g) a, b are such that $ax + b(1-x) \neq 0$

Integral 2

$$\int_0^{\pi/2}\int_0^1 e^{wi(\beta+\gamma)\theta}(sin\theta)^{\beta-1}(cos\theta)^{\gamma-1}x^{f-1}(1-x)^{g-1}[ax+b(1-x)]^{-f-g}F(c,d;f;\frac{ax}{ax+b(1-x)}\Big)$$

$${}_{p''}M_{q''}^{\alpha}(e^{wi(f+g)\theta}(sin\theta)^{f}(cos\theta)^{g}(1-x)^{g'}[ax+b(1-x)]^{-f'-g'})$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}(e^{wi(u+v)\theta}(sin\theta)^{u}(cos\theta)^{v}(1-x)^{v'}[ax+b(1-x)]^{-u'-v'})$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{matrix} \mathrm{e}^{wi(p'_1+q'_1)\theta}(sin\theta)^{p'_1}(cos\theta)^{q'_1}x^{s'_1}[b(1-x)]^{t'_1}[ax+b(1-x)]^{-s'_1-t'_1} \\ \cdot \cdot \cdot \\ \cdot \cdot \cdot \\ \mathrm{e}^{wi(p'_s+q'_s)\theta}(sin\theta)^{p'_s}(cos\theta)^{q'_s}x^{s'_s}[b(1-x)]^{t'_s}[ax+b(1-x)]^{s'_s-t'_s} \end{matrix}\right)$$

$$I\left(\begin{matrix} \mathrm{z}_1 e^{wi(h_1+k_1)\theta}(sin\theta)^{h_1}(cos\theta)^{k_1}[b(1-x)]^{k'_1}[ax+b(1-x)]^{-h'_1-k'_1} \\ \cdot \cdot \cdot \\ \cdot \cdot \cdot \\ \mathrm{z}_r e^{wi(h_r+k_r)\theta}(sin\theta)^{h_r}(cos\theta)^{k_r}[b(1-x)]^{k'_r}[ax+b(1-x)]^{-h'_r-k'_r} \end{matrix}\right)\mathrm{d}x\mathrm{d}\theta$$

$$=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{l=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{B_G g!}A_1\frac{[(a_{p''})]_l}{[(b_{q''})]_l}\frac{1}{\Gamma(\alpha l+1)}\Gamma(f)exp(i\pi w\sum_{k=1}^{s}p'_kK_k/2)$$

$$b^{-(g+lg'+v'\eta_{G,g}+\sum_{k=1}^{s}t'_kK_k)}exp(i\pi w(\beta+lf+u\eta_{G,g})/2)I_{U:p_r+4,q_r+3;W}^{V;0,n_r+4;X}\left(\begin{array}{c}\frac{z_1e^{i\pi wh_1/2}}{b^{k'_1}}\\ \cdot\\ \cdot\\ \frac{z_re^{i\pi wh_r/2}}{b^{k'_r}}\end{array}\right|$$

$$\begin{array}{l}\text{A};(1\text{-g-}\sigma-g'l-v'\eta_{G,g}-\sum_{i=1}^{s}t'_iK_i:k'_1,\cdots,k'_r),\\ \cdots\\ \text{B};(1\text{-f-g+c-}\sigma-g'l-v'\eta_{G,g}-\sum_{i=1}^{s}t'_iK_i:k'_i,\cdots,k'_r),\end{array}$$

$$\begin{array}{c}(1\text{-}\beta-fl-u\eta_{G,g}-\sum_{i=1}^{s}p'_iK_i:h_1,\cdots,h_r),(1-\gamma-gl-v\eta_{G,g}-\sum_{i=1}^{s}q'_iK_i:k_1,\cdots,k_r)\\ \cdots\\ (1\text{-}\beta-\gamma-(f+g)l-(u+v)\eta_{G,g}-\sum_{i=1}^{s}(p'_i+q'_i)K_i:h_1+k_1,\cdots,h_r+k_r)\end{array}$$

$$\left.\begin{array}{l}(1\text{+c+d-f-g-}\sigma-g'l-v'\eta_{G,g}-\sum_{i=1}^{s}t'_iK_i:k'_1,\cdots,k'_r),\mathfrak{A}:\text{A'}\\ \cdots\\ (1\text{-f-g+d-}\sigma-g'l-v'\eta_{G,g}-\sum_{i=1}^{s}t'_iK_i:k'_1,\cdots,k'_r),\ \mathfrak{B}:\text{B'}\end{array}\right)b^{-(g+lg'+v'\eta_{G,g}+\sum_{k=1}^{s}t'_kK_k)}\quad(2.2)$$

Provided that

a) $p'_i,q'_i,s'_i,t'_i>0,i=1,\cdots,s;h_i,k_i,h'_i,k'_i>0,i=1,\cdots,r;p''<q''$ and $|b|<1,Re(f)>0$

b) $Re[g+v'\min_{1\leqslant j\leqslant M}\frac{b_i}{B_i}+\sum_{i=1}^{r}k'_i\min_{1\leqslant j\leqslant m_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}]>0$

c) $Re[\beta+u\min_{1\leqslant j\leqslant M}\frac{b_i}{B_i}+\sum_{i=1}^{r}h_i\min_{1\leqslant j\leqslant m_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}]>0$

d) $Re[\gamma+v\min_{1\leqslant j\leqslant M}\frac{b_i}{B_i}+\sum_{i=1}^{r}k_i\min_{1\leqslant j\leqslant m_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}]>0$

e) $|argz_i|<\frac{1}{2}\Omega_i\pi$, where Ω_i is defined by (1.10)

f) a,b are such that $ax+b(1-x)\neq 0$

Integral 3

$$\int_0^{\pi/2}\int_0^1 e^{wi(\beta+\gamma)\theta}(sin\theta)^{\beta-1}(cos\theta)^{\gamma-1}x^{\rho-1}(1-x)^{\sigma-1}{}_{p''}M_{q''}^{\alpha}(e^{wi(f+g)\theta}(sin\theta)^f(cos\theta)^gx^{f'}(1-x)^{g'})$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{array}{c}\mathrm{e}^{wi(p'_1+q'_1)\theta}(sin\theta)^{p'_1}(cos\theta)^{q'_1}x^{s'_1}(1-x)^{t'_1}\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ \mathrm{e}^{wi(p'_s+q'_s)\theta}(sin\theta)^{p'_s}(cos\theta)^{q'_s}x^{s'_s}(1-x)^{t'_s}\end{array}\right)\aleph_{P_i,Q_i,c_i;r'}^{M,N}\left(\ e^{wi(u+v)\theta}(sin\theta)^u(cos\theta)^v x^{u'}(1-x)^{v'}\right)$$

$$I\left(\begin{array}{c}\mathrm{z}_1 e^{wi(h_1+k_1)\theta}(sin\theta)^{h_1}(cos\theta)^{k_1}x^{h'_1}(1-x)^{k'_1}\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ \mathrm{z}_r e^{wi(h_r+k_r)\theta}(sin\theta)^{h_r}(cos\theta)^{k_r}x^{h'_r}(1-x)^{k'_r}\end{array}\right)\mathrm{d}x\mathrm{d}\theta$$

$$=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{l=0}^{\infty}A_1\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{B_G g!}\frac{[(a_{p''})]_l}{[(b_{q''})]_l}\frac{1}{\Gamma(\alpha l+1)}$$

$$exp(i\pi w(\beta+lf+u\eta_{G,g}+\sum_{k=1}^{s}t'_kK_k)/2)\ I_{U:p_r+4,q_r+2;W}^{V;0,n_r+4;X}\left(\begin{array}{c|c}\mathrm{z}_1 e^{i\pi wh_1/2} & \mathrm{A}\ ;\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_r e^{i\pi wh_r/2} & \mathrm{B};\end{array}\right.$$

$$\begin{array}{c}(1\text{-}\rho-f'l-u'\eta_{G,g}-\sum_{i=1}^s s'_iK_i:h'_1,\cdots,h'_r),(1-\sigma-g'l-v'\eta_{G,g}-\sum_{i=1}^s t'_iK_i:k'_1,\cdots,k'_r)\\ \cdot\ \cdot\ \cdot\\ (1\text{-}\rho-\sigma-(f'+g')l-(u'+v')\eta_{G,g}-\sum_{i=1}^s(s'_i+t'_i)K_i::h'_1+k'_1,\cdots,h'_r+k'_r)\end{array}$$

$$\begin{array}{c}(1\text{-}\beta-fl-u\eta_{G,g}-\sum_{i=1}^s p'_iK_i:h_1,\cdots,h_r),(1-\gamma-gl-v\eta_{G,g}-\sum_{i=1}^s q'_iK_i;k_1,\cdots,k_r)\\ \cdot\ \cdot\ \cdot\\ (1\text{-}\beta-\gamma-(f+g)l-(u+v)\eta_{G,g}-\sum_{i=1}^s(p'_i+q'_i)K_i:h_1+k_1,\cdots,h_r+k_r)\end{array}$$

$$\left.\begin{array}{c}\mathfrak{A}:\ \mathrm{A'}\\ \cdot\ \cdot\ \cdot\\ \mathfrak{B}:\ \mathrm{B'}\end{array}\right) \tag{2.3}$$

Provided that

a) $p'_i,q'_i,s'_i,t'_i>0,i=1,\cdots,s;h_i,k_i,h'_i,k'_i>0,i=1,\cdots,r;p''<q''$

b) $Re[\rho+u'\min_{1\leqslant j\leqslant M}\frac{b_i}{B_i}+\sum_{i=1}^{r}h'_i\min_{1\leqslant j\leqslant m_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}]>0$

c) $Re[\sigma+v'\min_{1\leqslant j\leqslant M}\frac{b_i}{B_i}+\sum_{i=1}^{r}k'_i\min_{1\leqslant j\leqslant m_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}]>0$

d) $Re[\beta+u\min_{1\leqslant j\leqslant M}\frac{b_i}{B_i}+\sum_{i=1}^{r}h_i\min_{1\leqslant j\leqslant m_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}]>0$

e) $Re[\gamma+v\min_{1\leqslant j\leqslant M}\frac{b_i}{B_i}+\sum_{i=1}^{r}k_i\min_{1\leqslant j\leqslant m_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}]>0$

f) $|argz_i|<\frac{1}{2}\Omega_i\pi$, where Ω_i is defined by (1.10)

Proof of (2.1)

To derive (2.1), express the general class of polynomial in series with the help of (1.5), the M-function in serie with the help (1.7), the Aleph-function of one variable in serie with the help (1.3) and the multivariable I-function as given in (1.9).Now interchange the order of summations and integrations (which is permissible under the coonditions stated above), evaluate the inner θ-integral and x-integral with the help of known results given [2] and [3], we arrive at the desired result. We use the similar methods to prove (2.2) and (2.3) by making use of known results [3] and [5].

3. Particular cases

Taking $N_i\to 0$, $i=1,\cdots,r$, the result in (2.3) reduces to the following result.

$$\int_0^{\pi/2}\int_0^1 e^{wi(\beta+\gamma)\theta}(sin\theta)^{\beta-1}(cos\theta)^{\gamma-1}x^{\rho-1}(1-x)^{\sigma-1}{}_{p''}M_{q''}^{\alpha}(e^{wi(f+g)\theta}(sin\theta)^{f}(cos\theta)^{g}x^{f'}(1-x)^{g'})$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}(e^{wi(u+v)\theta}(sin\theta)^{u}(cos\theta)^{v}x^{u'}(1-x)^{v'})$$

$$I\left(\begin{array}{c}\mathrm{z}_1e^{wi(h_1+k_1)\theta}(sin\theta)^{h_1}(cos\theta)^{k_1}x^{h_1'}(1-x)^{k_1'}\\ \cdot\;\cdot\;\cdot\\ \cdot\;\cdot\;\cdot\\ \mathrm{z}_re^{wi(h_r+k_r)\theta}(sin\theta)^{h_r}(cos\theta)^{k_r}x^{h_r'}(1-x)^{k_r'}\end{array}\right)\mathrm{d}x\mathrm{d}\theta I$$

$$=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})[(a_{p''})]_l}{B_G g!\qquad[(b_{q''})]_l}\frac{1}{\Gamma(\alpha l+1)}exp(i\pi w(\beta+lf+u\eta_{G,g})/2)$$

$$I_{U:p_r+4,q_r+2;W}^{V;0,n_r+4;X}\left(\begin{array}{c}\mathrm{z}_1e^{i\pi wh_1/2}\\ \cdot\\ \cdot\\ \mathrm{z}_re^{i\pi wh_r/2}\end{array}\left|\begin{array}{c}(1\text{-}\rho-f'l-u'\eta_{G,g}:h_1',\cdots,h_r'),(1-\sigma-g'l-v'\eta_{G,g}:k_1',\cdots,k_r')\\ \cdot\;\cdot\;\cdot\\ (1\text{-}\rho-\sigma-(f'+g')l-(u'+v')\eta_{G,g}:h_1'+k_1',\cdots,h_r'+k_r')\end{array}\right.\right)$$

$$\begin{array}{c}\mathrm{A}\;;\;(1\text{-}\beta-fl-u\eta_{G,g}-pK_1-p'K_2:h_1,\cdots,h_r),(1-\gamma-gl-v\eta_{G,g}-qK_1-q'K_2:k_1,\cdots,k_r)\\ \cdot\;\cdot\;\cdot\\ \mathrm{B}\;;\;(1\text{-}\beta-\gamma-(f+g)l-(u+v)\eta_{G,g}-(p+q)K_1-(p'+q')K_2:h_1+k_1,\cdots,h_r+k_r)\end{array}$$

$$\left.\begin{matrix}\mathfrak{A} : \text{A'} \\ \cdot\ \cdot\ \cdot \\ \mathfrak{B} : \text{B'}\end{matrix}\right) \tag{3.1}$$

Provided that

a) $h_i, k_i, h'_i, k'_i > 0, i = 1, \cdots, r; p'' < q''$

b) $Re[\rho + u' \min_{1 \leqslant j \leqslant M} \dfrac{b_i}{B_i} + \sum_{i=1}^{r} h'_i \min_{1 \leqslant j \leqslant m_i} \dfrac{b_j^{(i)}}{\beta_j^{(i)}}] > 0$

c) $Re[\sigma + v' \min_{1 \leqslant j \leqslant M} \dfrac{b_i}{B_i} + \sum_{i=1}^{r} k'_i \min_{1 \leqslant j \leqslant m_i} \dfrac{b_j^{(i)}}{\beta_j^{(i)}}] > 0$

d) $Re[\beta + u \min_{1 \leqslant j \leqslant M} \dfrac{b_i}{B_i} + \sum_{i=1}^{r} h_i \min_{1 \leqslant j \leqslant m_i} \dfrac{b_j^{(i)}}{\beta_j^{(i)}}] > 0$

e) $Re[\gamma + v \min_{1 \leqslant j \leqslant M} \dfrac{b_i}{B_i} + \sum_{i=1}^{r} k_i \min_{1 \leqslant j \leqslant m_i} \dfrac{b_j^{(i)}}{\beta_j^{(i)}}] > 0$

f) $|argz_i| < \dfrac{1}{2}\Omega_i \pi$, where Ω_i is defined by (1.10)

The special cases of the double integrals involving the Hermite polynomials, the Laguerre polynomials and the Jacobi polynomials can be obtained by similar methods.

3. Multivariable H-function

If $U = V = A = B = 0$, the multivariable I-function defined by Prasad degenere in multivariable H-function defined by Srivastava et al [8]. We have the following results.

Integral 4

$$\int_0^{\pi/2} \int_0^1 e^{wi(\beta+\gamma)\theta} (sin\theta)^{\beta-1} (cos\theta)^{\gamma-1} x^{\rho-1} (1-x)^{\sigma-1} [ax + b(1-x)]^{-\rho-\sigma}$$

$${}_{p''}M_{q''}^{\alpha}(e^{wi(f+g)\theta} (sin\theta)^f (cos\theta)^g (ax)^{f'} [b(1-x)]^{g'} [ax + b(1-x)]^{-f'-g'})$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}(e^{wi(u+v)\theta} (sin\theta)^u (cos\theta)^v (ax)^{u'} [b(1-x)]^{v'} [ax + b(1-x)]^{-u'-v'})$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix} \mathrm{e}^{wi(p'_1+q'_1)\theta} (sin\theta)^{p'_1} (cos\theta)^{q'_1} (ax)^{s'_1} [b(1-x)]^{t'_1} [ax + b(1-x)]^{-s'_1-t'_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{e}^{wi(p'_s+q'_s)\theta} (sin\theta)^{p'_s} (cos\theta)^{q'_s} (ax)^{s'_s} [b(1-x)]^{t'_s} [ax + b(1-x)]^{s'_s-t'_s} \end{pmatrix}$$

$$H\left(\begin{array}{c} \mathrm{z}_1 e^{wi(h_1+k_1)\theta}(sin\theta)^{h_1}(cos\theta)^{k_1}(ax)^{h'_1}[b(1-x)]^{k'_1}[ax+b(1-x)]^{-h'_1-k'_1} \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{z}_r e^{wi(h_r+k_r)\theta}(sin\theta)^{h_r}(cos\theta)^{k_r}(ax)^{h'_r}[b(1-x)]^{k'_r}[ax+b(1-x)]^{-h'_r-k'_r} \end{array}\right)\mathrm{d}x\mathrm{d}\theta$$

$$=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{l=0}^{\infty}\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})[(a_{p''})]_l}{B_G g!\qquad\qquad [(b_{q''})]_l}\frac{1}{\Gamma(\alpha l+1)}A_1$$

$$exp(i\pi w(\beta+lf+u\eta_{G,g}+p'_1K_1+\cdots+p'_sK_s)/2)a^{-\rho}b^{-\sigma}H^{0,n_r+4;X}_{p_r+4,q_r+2;W}\left(\begin{array}{c} \frac{z_1 e^{i\pi w h_1/2}}{a^{h'_1}b^{k'_1}} \\ \cdot \\ \cdot \\ \frac{z_r e^{i\pi w h_r/2}}{a^{h'_r}b^{k'_r}} \end{array}\right|$$

$$\begin{array}{c}(1\text{-}\rho-f'l-u'\eta_{G,g}-\sum_{i=1}^{s}s'_iK_i:h'_1,\cdots,h'_r),(1-\sigma-g'l-v'\eta_{G,g}-\sum_{i=1}^{s}t'_iK_i:k'_1,\cdots,k'_r) \\ \cdot\cdot\cdot \\ (1\text{-}\rho-\sigma-(f'+g')l-(u'+v')\eta_{G,g}-\sum_{i=1}^{s}(s'_i+t'_i)K_i::h'_1+k'_1,\cdots,h'_r+k'_r)\end{array}$$

$$\begin{array}{c}(1\text{-}\beta-fl-u\eta_{G,g}-\sum_{i=1}^{s}p'_iK_i:h_1,\cdots,h_r),(1-\gamma-gl-v\eta_{G,g}-\sum_{i=1}^{s}q'_iK_i;k_1,\cdots,k_r) \\ \cdot\cdot\cdot \\ (1\text{-}\beta-\gamma-(f+g)l-(u+v)\eta_{G,g}-\sum_{i=1}^{s}(p'_i+q'_i)K_i:h_1+k_1,\cdots,h_r+k_r)\end{array}$$

$$\left.\begin{array}{c}\mathfrak{A}:\mathrm{A'} \\ \cdot\cdot\cdot \\ \mathfrak{B}:\mathrm{B'}\end{array}\right)\ a^{-(lf'+u'\eta_{G,g}+s'_1K_1+\cdots+s'_sK_s)}\ b^{-(lg'+v'\eta_{G,g}+t'_1K_1+\cdots+t'_sK_s)} \tag{4.1}$$

under the same notations and conditions that (2.1) with $U=V=A=B=0$

Integral 5

$$\int_0^{\pi/2}\int_0^1 e^{wi(\beta+\gamma)\theta}(sin\theta)^{\beta-1}(cos\theta)^{\gamma-1}x^{f-1}(1-x)^{g-1}[ax+b(1-x)]^{-f-g}F(c,d;f;\frac{ax}{ax+b(1-x)}\Big)$$

$${}_{p''}M^{\alpha}_{q''}(e^{wi(f+g)\theta}(sin\theta)^f(cos\theta)^g(1-x)^{g'}[ax+b(1-x)]^{-f'-g'})$$

$$\aleph^{M,N}_{P_i,Q_i,c_i;r'}(e^{wi(u+v)\theta}(sin\theta)^u(cos\theta)^v(1-x)^{v'}[ax+b(1-x)]^{-u'-v'})$$

$$S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\left(\begin{array}{c} \mathrm{e}^{wi(p'_1+q'_1)\theta}(sin\theta)^{p'_1}(cos\theta)^{q'_1}x^{s'_1}[b(1-x)]^{t'_1}[ax+b(1-x)]^{-s'_1-t'_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{e}^{wi(p'_s+q'_s)\theta}(sin\theta)^{p'_s}(cos\theta)^{q'_s}x^{s'_s}[b(1-x)]^{t'_s}[ax+b(1-x)]^{s'_s-t'_s} \end{array}\right)$$

$$H\left(\begin{array}{c} \mathrm{z}_1e^{wi(h_1+k_1)\theta}(sin\theta)^{h_1}(cos\theta)^{k_1}[b(1-x)]^{k'_1}[ax+b(1-x)]^{-h'_1-k'_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{z}_re^{wi(h_r+k_r)\theta}(sin\theta)^{h_r}(cos\theta)^{k_r}[b(1-x)]^{k'_r}[ax+b(1-x)]^{-h'_r-k'_r} \end{array}\right)\mathrm{d}x\mathrm{d}\theta$$

$$=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{l=0}^{\infty}\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})}{B_Gg!}A_1\frac{[(a_{p''})]_l}{[(b_{q''})]_l}\frac{1}{\Gamma(\alpha l+1)}\Gamma(f)exp(i\pi w\sum_{k=1}^{s}p'_kK_k/2)$$

$$b^{-(g+lg'+v'\eta_{G,g}+\sum_{k=1}^s t'_kK_k)}exp(i\pi w(\beta+lf+u\eta_{G,g})/2)H^{0,n_r+4;X}_{p_r+4,q_r+3;W}\left(\begin{array}{c} \frac{z_1e^{i\pi wh_1/2}}{b^{k'_1}} \\ \cdot \\ \cdot \\ \frac{z_re^{i\pi wh_r/2}}{b^{k'_r}} \end{array}\right|$$

$$\begin{array}{l} \mathrm{A}\ ;\ (\text{1-g-}\sigma-g'l-v'\eta_{G,g}-\sum_{i=1}^s t'_iK_i:k'_1,\cdots,k'_r), \\ \qquad\qquad\cdot\ \cdot\ \cdot \\ \mathrm{B}\ ;\ (\text{1-f-g+c-}\sigma-g'l-v'\eta_{G,g}-\sum_{i=1}^s t'_iK_{:}k'_i,\cdots,k'_r), \end{array}$$

$$\begin{array}{c} (\text{1-}\beta-fl-u\eta_{G,g}-\sum_{i=1}^s p'_iK_i:h_1,\cdots,h_r),(1-\gamma-gl-v\eta_{G,g}-\sum_{i=1}^s q'_iK_i:k_1,\cdots,k_r) \\ \cdot\ \cdot\ \cdot \\ (\text{1-}\beta-\gamma-(f+g)l-(u+v)\eta_{G,g}-\sum_{i=1}^s(p'_i+q'_i)K_i:h_1+k_1,\cdots,h_r+k_r) \end{array}$$

$$\left.\begin{array}{c} (\text{1+c+d-f-g-}\sigma-g'l-v'\eta_{G,g}-\sum_{i=1}^s t'_iK_i:k'_1,\cdots,k'_r),\mathfrak{A}\ :\mathrm{A}' \\ \cdot\ \cdot\ \cdot \qquad\qquad\qquad \cdot\ \cdot\ \cdot \\ (\text{1-f-g+d-}\sigma-g'l-v'\eta_{G,g}-\sum_{i=1}^s t'_iK_i:k'_1,\cdots,k'_r),\ \ \mathfrak{B}\ :\mathrm{B}' \end{array}\right)b^{-(g+lg'+v'\eta_{G,g}+\sum_{k=1}^s t'_kK_k)} \qquad (4.2)$$

under the same conditions and notations that (2.2) with $U=V=A=B=0$

Integral 3

$$\int_0^{\pi/2}\int_0^1 e^{wi(\beta+\gamma)\theta}(sin\theta)^{\beta-1}(cos\theta)^{\gamma-1}x^{\rho-1}(1-x)^{\sigma-1}{}_{p''}M^{\alpha}_{q''}(e^{wi(f+g)\theta}(sin\theta)^f(cos\theta)^gx^{f'}(1-x)^{g'})$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{array}{c} \mathrm{e}^{wi(p'_1+q'_1)\theta}(sin\theta)^{p'_1}(cos\theta)^{q'_1}x^{s'_1}(1-x)^{t'_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{e}^{wi(p'_s+q'_s)\theta}(sin\theta)^{p'_s}(cos\theta)^{q'_s}x^{s'_s}(1-x)^{t'_s} \end{array}\right)\aleph_{P_i,Q_i,c_i;r'}^{M,N}\left(e^{wi(u+v)\theta}(sin\theta)^{u}(cos\theta)^{v}x^{u'}(1-x)^{v'}\right)$$

$$H\left(\begin{array}{c} \mathrm{z}_1 e^{wi(h_1+k_1)\theta}(sin\theta)^{h_1}(cos\theta)^{k_1}x^{h'_1}(1-x)^{k'_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{z}_r e^{wi(h_r+k_r)\theta}(sin\theta)^{h_r}(cos\theta)^{k_r}x^{h'_r}(1-x)^{k'_r} \end{array}\right)\mathrm{d}x\mathrm{d}\theta$$

$$=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{l=0}^{\infty}A_1\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{B_G g!}\frac{[(a_{p''})]_l}{[(b_{q''})]_l}\frac{1}{\Gamma(\alpha l+1)}$$

$$exp(i\pi w(\beta+lf+u\eta_{G,g}+\sum_{k=1}^{s}t'_k K_k)/2)\ H_{p_r+4,q_r+3;W}^{0,n_r+4;X}\left(\begin{array}{c} \mathrm{z}_1 e^{i\pi w h_1/2} \\ \cdot \\ \cdot \\ \mathrm{z}_r e^{i\pi w h_r/2} \end{array}\right|$$

$$\begin{array}{c}(1\text{-}\rho-f'l-u'\eta_{G,g}-\sum_{i=1}^{s}s'_iK_i:h'_1,\cdots,h'_r),(1-\sigma-g'l-v'\eta_{G,g}-\sum_{i=1}^{s}t'_iK_i:k'_1,\cdots,k'_r) \\ \cdot\ \cdot\ \cdot \\ (1\text{-}\rho-\sigma-(f'+g')l-(u'+v')\eta_{G,g}-\sum_{i=1}^{s}(s'_i+t'_i)K_i::h'_1+k'_1,\cdots,h'_r+k'_r)\end{array}$$

$$\begin{array}{c}(1\text{-}\beta-fl-u\eta_{G,g}-\sum_{i=1}^{s}p'_iK_i:h_1,\cdots,h_r),(1-\gamma-gl-v\eta_{G,g}-\sum_{i=1}^{s}q'_iK_i;k_1,\cdots,k_r) \\ \cdot\ \cdot\ \cdot \\ (1\text{-}\beta-\gamma-(f+g)l-(u+v)\eta_{G,g}-\sum_{i=1}^{s}(p'_i+q'_i)K_i:h_1+k_1,\cdots,h_r+k_r)\end{array}$$

$$\left.\begin{array}{c}\mathfrak{A}:\ \mathrm{A}' \\ \cdot\ \cdot\ \cdot \\ \mathfrak{B}:\ \mathrm{B}'\end{array}\right) \tag{4.3}$$

under the same conditions and notations that (2.3) with $U=V=A=B=0$

6. Conclusion

The I-function of several variables defined by Prasad [4] presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions of several variables such as , multivariable H-function defined by Srivastava et al [8].

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