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On some Geometric Principles involved in Space Research and Robotics

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ABSTRACT

Topological spaces and their geometry is central to the management, analysis and design of navigational tools in space programmes. In particular the control of trajectories of space ships, the study of orbits and the dynamics of space ships all of these activities involve the study of the underlying spaces and their curvatures. Similarly when we look at the kinematics of artificial systems like robots moving in space, their movement though designed mechanically, actually involves the study of curves and surfaces. We basically refer to some research work done at NASA and then switch back to some pure mathematical ideas that aid us to solve problems in space research management

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1.0-Overview

The Mathematical study of the astronomical bodies has its roots to antiquity. People were interested in the patterns of the motion of the sun, moon and stars that were visible to us. However rigorous mathematics got applied only in the last few centuries.

In this paper we make an exposition of the basic *'Mathematics'* involved in the space research programmes. We report on some connections between geometry and topology with deep space exploration. Basically Mathematicians and Physicists are interested in predicting the existence and future influences of heavenly bodies [1]. For instance Kepler made many calculations based on Tycho's observations. He gave three basic laws that became fundamental to the study of astronomy. Celestial Mechanics and dynamics were essentially introduced by Newton which were further polished only in the last century by Henri Poincare. Newton's classic works have profound influence on the way people started to think about the Milky Way. In the 19th century some asteroids like Ceres were discovered. Following this Euler and Lagrange used calculus methods to understand the macrolevel dynamics of the celestial bodies. A truly classical work in this direction is that of Henri Poincare we alluded to before. He introduced the concept of topology to describe dynamics.

In the understanding of celestial bodies and their interaction one considers a shape space Q/G . This corresponds to a n-body interaction between the spacecraft and the celestial bodies. Specializing for the case of two bodies, let Q be

the configuration space of the system and G be the group of rigid motions. Then the space we are looking for is the configuration space of S1. Using the principle of conservation of momentum one concludes that Q/G in general is a fiber bundle. An appropriate connection here is the so called mechanical connection that makes the space locally flat.

1.1 Computations and simulations

The role of the modern era of computing is enormous given the kind of computations that go into the mission design. Cosmology and space research are studies that heavily depend on Theoretical physics, Computer Science and Mathematics. In Physics, Astrodynamics and Dynamical astronomy are crucial areas for space research. The theoretical studies are backed by the certain mathematical theories propounded by Scientists working in the areas of Topology and Dynamical systems associated with manifolds G.D.Birkhoff, A.N.Kolmogorov, V.I.Arnold and Moser, Henri Poincare to name a few. We enlist here the main functions of the computing and simulations group in space research.

1)Design and control of spacecraft trajectories.

This involves numerical computing using supercomputers to develop curves and surfaces that fit with the data received from moving spaceships

2)Finding a transfer trajectory

This involves analysis and geometry to find the right kind of projectile so that the objects are placed in the target orbits.

3)Determination of effect of other celestial bodies

The study of heavenly bodies interacting under gravitation and other forces is done by determining 'ephimeris', a listing of the position and velocities of all known celestial bodies as a function of time in a chosen coordinate system.One needs to determine invariant manifolds in this process to make way for an ideal path.

4)Determination of reaction dynamics.

Here the molecular system of each phase is understood and a detailed understanding of the effects of various gases is determined. Doppler corrections have to be applied to account for atmospheric refraction.

Further osculating elements have to be determined, range measurements have to be computed using raw data, elevation angles and simulation techniques. Simulation is mainly used to calibrate the calculations made with respect to the orbit determined.

2.0- Deep space exploration

The major Space Research organizations in the world emphasize a very strong Mathematical base to understand the underlying theories involved in space research. We classify the very important among them into three classes of theoretical studies. 1-Analysis, 2-Geometry/Topology and 3-Statistics.

Though we have listed them in an alphabetical order, we would like to mention geometry first. Scientists and astronomers first looked at the amazing geometry of our universe .Hence the study of geometry/topology should come first as was seen in the historical introduction. A systematic study was first taken up by Kepler.J who understood the structure of certain orbits of the planetary system. However Geometry did not get a complete hold over astronomical studies then. It was Newton who initiated a microscopic study of the physical space. His idea of using calculus for a dynamical system was a revolutionary and today in the era of computational advancement his

contributions can be termed as path breaking. Of late Riemannian geometry is aiding several computations at NASA. At GTDS (Goddard Trajectory Determination system) problems involving orbit determination are solved by analyzing a feedback control system. The underlying problem is essentially a coordinate transformation problem, so that a linearizing coordinate system can be constructed out of data received. The above said geometric tools are very much involved in the set of algorithms used by NASA for the so called Automatic flight controller.

By Analysis we essentially mean Numerical analysis and variational techniques to fit various data sets coming from radar and Doppler systems. Statistics deals with problems of error analysis and the analysis of deviations.

2.1 Algorithm involved

We now give a more detailed description in the form of an algorithm.

Step 1: Setting up a coordinate system.

Step 2 : Determine the geometry of the orbit

Step 3 : Path and surface detection

Step 4 : Error analysis.

The positions captured in step-1 are modified after checking the signals emanated from the space ship. The problem here is that the computer algorithms used assume point masses whereas the construction of the space ship itself has a certain perturbation. Hence the position errors have to be taken into consideration. Step-2 requires the understanding of dynamics of many body interactions. Variational techniques are used then to arrive at a stable solution. The results of step-1 leads to an input to work out the problems in step-2 since the underlying geometry and topology is very much essential to understand the effect of gravitational forces. The determination of the exact geometry of contours in the space involves a lot of statistical estimation since the X-ray data analysis is done by using Gaussian kernels. Finally the last step involves the correction factors to be discounted for the results arrived at.

3.0-Robotics in deep space exploration

Robotic automation is intimately related to deep space exploration when one thinks of expeditions which are automated. The mathematical features of this component are essentially involving control theory. The dynamics of the space ship, kinematics of individual robots and real time manipulations involves linear programming and algebraic geometry. In this section we look at the control problems solved by algebraic geometry. Given the geometry and topology of the space surrounding a target object in deep space one needs to make a precise navigation using the system at hand. There are two basic problems faced by engineers namely the forward kinematic problem and the inverse kinematic problem. The very subtle problems the engineers face in such exploration are precisely inverse kinematic problems. The configuration space we considered earlier makes a presence here in the forward kinematics problem. Here we are given a configuration of the system with all possible joints and we need to arrive at the end configuration system. The configuration space QF is a Cartesian product of the spaces of individual joints of the manipulators. Hence the forward kinematics problem is represented as where is the total configuration space and is the set of all rigid motions of the 3-dimensional Euclidean space. The mapping is essentially a composition of rigid motions due to individual joints. We next consider the notion of a workspace W.

$W=\{g_\alpha : g \in QF\}$

Here $g:QF \rightarrow SE(3)$ is a typical rigid transformation of the robotic system.

This is a space needed to get hold of all desired motions of the robot.

In a robotic system specifying the joint angles specifies the location of all the links. For revolute joints the angle is given by $\alpha_i \in [0,2\pi)$

The configuration space for the entire system is the Cartesian product of all the joint spaces. For instance the above link manipulator has he configuration space S^1 the unit circle since the set of all angles specified leads the unit circle.

In the inverse kinematic problem we are given a desired end configuration and we need to build a robotic system such that the set of all controls leads to the end configuration. It is in this situation algebraic geometry is heavily used. For example in the plane a two-link manipulator has to satisfy the following equations

$x = l_1 \cos \alpha_1 + l_2 \cos(\alpha_1 + \alpha_2)$…….(1)

$y = l_1 \sin \alpha_1 + l_2 \sin(\alpha_1 + \alpha_2)$……(2)

By using the 2 equations one determines answer for the forward problem essentially by solving for α_1 and α_2 given the values x and y. Thus one has a set of equations in more realistic examples and we need to determine the parameters involved. The software used for solving more complicated problems involves the Grobner bases in algebraic geometry. Traditionally inverse kinematics problems are solved either for closed form solutions if such solutions exist or numerically the equations are solved. The set of all solutions leads to a path like a curve or a surface which gives the engineers an idea as to how the systems have to be placed in order to get end results.

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