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METHODS FOR IDENTIFICATION OF CONFOUNDED EFFECTS IN FACTORIAL EXPERIMENTS

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Abstract

In this paper,we have studied some new methods of identification of confounding in (2^2),(2^3) and (2^4) symmetrical factorial experiments. The methods are verified and substantiated using practical examples. These methods can be extended for the case of 2^n and 3^n factorial experiments.

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1 Introduction

When the number of factors or number of levels of the factors increase, the number of treatment combinations increase very rapidly and it is not possible to accommodate all these treatment combinations in a single homogeneous block. For example, a 2^6 factorial experiments would have 64 homogeneity blocks and blocks of 64 plots are quite large and it is impossible to ensure homogeneity within them.A new technique is therefore necessary for designing experiments with a large number of treatments. One such device is to take block of size less than the number of treatments and have more than one block per replication. The treatment combinations are then divided into as many groups as the number of blocks per replication. The different groups of treatments are allocated to the blocks. There are many ways of grouping the treatments into as many groups as the number of blocks per replication.

The grouping of treatments combinations must be done in such a way that only the unimportant effects are confused with the block effects and other import anded effects could be evolved compare significantly. This technique is called confounding.

When there are two or more replicates in factorial experiments, the question arises normally, whether the same set of interactions are confounded in all replicates or different sets of interactions are confounded in different replicates. This leads to two types confounding in factorial experiments namely (1). Partial and (2). Total confounding. If the different sets of interaction are confounded in different replicates,it is known as partial confounding. Here in this case all the effects could be estimated including the confounded effects could be estimated. Here partial confounded effect can be estimated from the replicates in which is not confounded.In complete confounding the same effect confounded in all the replicates are sames here. It can not be estimated.In confounding the grouping of the treatment combinations is based on the effects which we want to can confounded. As stated earlier. There are many methods are available for in the estimate grouping the treatment combinations.Here we have evolved some new methodology for finding a confounded effect in a particular replicates in case of 2^2, 2^3 and 2^n factorial experiments.

2 Main Results

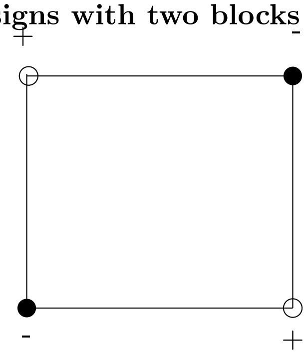
2.1 **Table of sign method**

In this method was introduced reference [5].The method is apparent from the table of plus and minus sign. All treatment combinations that have on plus on assigned one block and minus sign assigned to another block. The treatment combinations with blocks are run in random error. Therefore, the higher order interaction is confounded.

2.2 **Geometric Method**

We are introduced four methods for identification of confounding in factorial experiments is discuss one by one.First for Geometric Method. Let us suppose that, we to run a single replicate of the 2^2 design. Each of the $2^2 = (2 \times 2) = 4$ replicate of the 2^2 treatments combinations requires a quantity of raw material. In each batch raw material is only large for two treatment combinations to be tested. Thus, two batches of raw material are considered as blocks then, we must sign two of the four treatment combinations to each block. The geometric view indicates that treatment combinations on opposing diagonals are assigned to different blocks. Suppose we estimate the main effect of A and B just as if no blocking had occurred $A = \frac{1}{2}[\text{ ab + a - b - (1)}]$ and $B = \frac{1}{2}[\text{ ab + b - a - (1)}]$ both A and B are unaffected by blocking since each estimate there of one plus(+) and one (-) treatment combination from each block. That is any difference between blocks I and block II will carried out out. Now consider AB interaction $A\ B = \frac{1}{2}[\text{ ab - a - b + (1)}]$. Since the two treatment combinations with the plus sign [ab and (1)] are in block I and the two with the sign minus sign [a and b] are in block II are given below,

2^2 **Designs with two blocks in Geometric Method**



Block I	Block II
(1)	a
ab	b

the block (Null) effect and the AB interaction are identical that is AB is confounded with blocks.

2.3 **ODD (or) EVEN METHOD**

In odd or even method of confounding are the key block will contains the even number of treatments while the other block will contain odd number of treatments. In each every single odd number of treatments is multiply by we get the treatment combinations is confounded.

2.4 **Multiplication Method**

It is a another method for identification of confounding of factorial experiments. This method is based on the algebraic multiplication of single treatments in other block (block II). The treatment combination obtained by such algebraic multiplication gives rise to the identification the factorial effect confounded in the factorial design.

2.5 **Defining contrast Method**

The method used the linear combinations as L= $a_1 \times_1 + a_2 \times_2 +,...,+a_k \times_k$ (i = 1 to k) Where, $\times_i$ is the i^{th} factor appearing in a particular treatment combination ; a_i is the exponent appearing on the i^{th} factor in the effect to be confounded for the 2^k system, we have $\alpha_i = 0$ or 1 and $\times_i = 0$ or 1. The above equation is called defining contrast. Treatment combinations that produce the same block value of L (Mod 2) will be placed in the same block. Since the only possible values of L(Mod2)are 0 and 1. This will assign the treatment combinations to exactly two blocks.

In other words confounding problem the blocks can be sub grouped into two or more incomplete blocks. The most general method of their sub grouping is called defining contrast rule. This method uses the linear combination defined by

L = $\sum P_i \times_i$, where $\times_i$ (i=1tok) i^{th} factor appearing in the particular treatment combinations ; P_i

is the exponent (index) of the i^{th} factor in the confounded factorial effect and 'n' denotes the number of factors in the design.

3 Applications

3.1 Table of sign method

Let us consider 2^3 is a factorial with small number of treatment combinations but for illustration purpose, this example has been considered. Let the three factors be A,B,C each two levels.

3.1.1 Table of plus and minus signs for the 2^3

Treatment Effects→ Treatment combination ↓	(1)	A	B	C	AB	AC	ABC
(1)	-	-	-	+	+	+	-
(a)	+	-	-	-	-	+	+
(b)	-	+	-	-	+	-	+
(ab)	+	+	-	+	-	-	-
(c)	-	-	+	+	-	-	+
(ac)	+	-	+	-	+	-	-
(bc)	-	+	+	-	-	+	-
(abc)	+	+	+	+	+	+	+

A= (abc)+(ac)+(ab)+(a)-(bc)-(c)-(b)-(1)
B= (abc)+(bc)+(ab)+(b)-(ac)-(c)-(a)-(1)
C= (abc)+(bc)+(ac)+(c)-(ab)-(b)-(a)-(1)
AB= (abc)+(c)+(ab)+(1)-(bc)-(ac)-(b)-(a)
AC= (abc)+(ac)+(b)+(1)-(bc)-(c)-(ab)-(a)
BC= (abc)+(bc)+(a)+(1)-(ac)-(c)-(ab)-(b)
ABC= (abc)+(c)+(b)+(a)-(bc)-(ac)-(ab)-(1)
Let the higher order interaction is confounded. Thus in order to confound the interaction ABC with block all the treatment combinations with positive sign are allocated at random in one block and those with negative signs in the other block. Thus the following arrangement gives ABC confounded with blocks and hence we loose information on ABC.

Block I	Block II
(1)	a
ab	b
ac	c
bc	abc

It can be observed that the contrast estimating ABC is identical to the contrast estimating block effects.

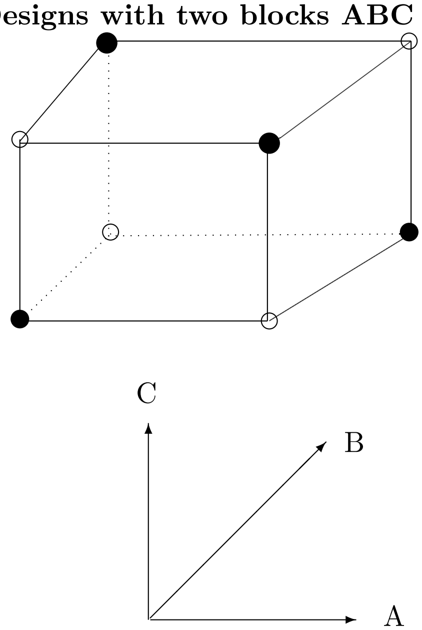
3.2 **Geometric Method**

Let three factors A,B and C each with two levels 0 and 1. $2^3 = 2\times2\times2 = 8$ treatment combinations are $000, 100, 010, 110, 001, 101, 011, 111$; $a_0b_0c_0$, $a_1b_0c_0$, $a_0b_1c_0$, $a_1b_1c_0$, $a_0b_0c_1$, $a_1b_0c_1$, $a_0b_1c_1$, $a_1b_1c_1$; (1), a, b, ab, c, ac, bc, abc. Where (1) is general mean effect; A,B,C are called main effects; AB, BC, AC are called two factor interaction effect and ABC are third order interaction effect. Already discussed in section 3 the Geometric Method of 2^2.The same way 2^3. The main effects are

$A = \frac{1}{4}[$ (a-1)(b+1)(c+1)] ; $B = \frac{1}{4}[$ (a+1)(b-1)(c+1)] and $C = \frac{1}{4}[$ (a+1)(b+1)(c-1)]

Since the four treatment combinations with the plus sign [a,b,c and abc] are in block I and the four with the minus sign [-ab, -ac, -bc and (1)] are in block II are given below,

2^3 **Designs with two blocks ABC Confounded**



Block I	Block II
(1)	a
ab	b
ac	c
bc	abc

The block (Null) effect and the ABC interaction are identical that is ABC is confounded with blocks.The same way we are using the method to any factorial experiments.

3.3 **Odd (or) Even Method**

In 2^n factorial experiment the identification of confounding (complete) of higher order interactions can easily be done by a typical method known as odd or even method. In this method of confounding, the key block will contains the even number of treatments while the other block will contain odd number of treatments, in each treatment combinations. for example, Consider a 2^4 factorial experiments treatment combinations are given below,

Block I	Block II
(1)	a
ab	b
ac	c
ad	d
bc	abc
bd	abd
cd	bcd
abcd	acd

In this problem of confounding, the key block contains the treatment combinations having even number of treatments namely ab, bc, ad, bd, cd and ac. The other block contains the treatment combinations having odd numbers of treatments, namely a, b,c, d, abc, abd, bcd, acd and abcd . The odd single numbers multiply (a$\times$ b$\times$c $\times$ d)we get the factorial effect abcd. Therefore, main effect ABCD is confounded.

3.4 **Defining contrast Method**

It is another method for Defining contrast Method for identification of confounding. For example 2^3 factorial experiment. The the defining contrast will be

L $= \sum P_i \times_i$, where $\times_i$, for i = 1,2,3.

L $= P_1 \times_1 + P_2 \times_2 + P_3 \times_3$, Put $P_1 = P_2 = P_3 = 1$. L $= \times_1 + \times_2 + \times_3$. Factor in $2^3 = 2 \times 2 \times 2 =$ 8 treatment combinations is (1), a, b, ab, c, ac, bc, abc ;000, 100, 010, 110, 001, 101, 011, 111. These 8 treatment combinations can be divided into two sub groups (Incomplete blocks) by means of two sets of

values of L. Namely, 0(Mod 2) and 1(Mod 2).
L(1) $= 0+0+0=0$; L(a) $= 1+0+0=1$; L(b) $= 0+1+0=1$; L(ab) $= 1+1+0=0$;
L(c) $= 0+0+1=1$; L(ac) $= 1+0+1=0$; L(bc) $= 0+1+1=0$; L(abc) $= 1+1+1=1$
Thus, the 8 treatment combinations are sub groups into two incomplete blocks are given below

L = 0 (Mod 2)	L = 1 (Mod 2)
(1)	a
ab	b
bc	c
ac	abc

In block I is only even number of treatment and block II is odd number of treatments. Therefore odd numbers is multiply get a abc. The higer order interaction effect ABC is confounded in this method.

3.5 **Multiplication Method**

In another multiplication method of identification of confounding of any factorial effect in a confounded design. This method is based on the algebraic multiplication of single treatments in other block (block II). The treatment combination obtained by such algebraic multiplication gives rise to the identification the factorial effect confounded in the factorial design. The plan and yields of a 2^4 factorial experiment on beans plan and yields of a 2^4 factorial experiment

Replicate I	Replicate I	Replicate II	Replicate II	Replicate III	Replicate III
Block 1	Block 2	Block 1	Block 2	Block 1	Block 2
(1)	a	(1)	a	(1)	a
ab	b	ab	b	c	ac
cd	acd	acd	cd	abd	bd
abcd	bcd	bcd	abcd	abcd	bcd
c	ac	c	d	c	d
abc	bc	abc	abd	bc	cd
d	ad	ad	ac	ad	ab
abd	bd	bd	bc	acd	abc

Replicate IV	Replicate IV
Block 1	Block 2
(1)	a
bc	abc
abd	bd
acd	cd
b	d
c	bcd
ad	ab
abcd	ac

In this problem of partial confounding. Replicate I can be identified by locating single treatment in the other than key blocks namely a×b ; c×d and a×b×c×d. Multiplying these two and four treatment combinations, gives the treatment ab, cd and abcd. This leads to the identification the factorial effect AB, CD and ABCD is confounded.The same way, In replication II The factorial effect ABD, CD and ABC is confounded ;In replication III The factorial effect AD, CD and AB is confounded ;In replication IV The factorial effect AD, BCD and ABC is confounded. Finally in this problem is partial confounding.

4 Conclusion

In this paper we have discussed four methods of confounded effects. namely (1). Geometric Method,(2). Odd (or) Even Method, (3). Defining contrast Method and (4). Multiplication Methods.There are very easy to understand and easy of apply problems in symmetrical factorial experiments. These methodology an be extended to deal with 2^n and 3^n factorial experiments.

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