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A NEW VIEW ON COMPACTNESS IN SMOOTH FUZZY TOPOLOGICAL SPACES

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Abstract

In this paper, new definitions of fuzzy closure and fuzzy interior of an r-fuzzy set which satisfy almost all properties of the corresponding definitions in smooth fuzzy topological spaces in the sense of Sostak [8] and Ramadan [5] are established. As a consequence of these definitions the concept of $\tilde{g}$ -P-fuzzy compact space is introduced and established.

Keywords: $\tilde{g}$ -P-fuzzy compact spaces, $\tilde{g}$ -P-fuzzy almost compact spaces and $\tilde{g}$ -P-fuzzy nearly compact spaces.

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1. Introduction and Preliminaries

The concept of fuzzy set was introduced by Zadeh [12] in his classical paper. Fuzzy sets have applications in many fields such as information [7] and control [11]. Chang [1] introduced the notion of a fuzzy topology. Later Lowen [3] redefined what is now known as startified fuzzy topology. Sostak [8] introduced the notion of fuzzy topology as an extension of Chang an Lowen's fuzzy topology. Later on he has developed the theory of fuzzy topological spaces in [9] and [10]. Defining the closure and the interior of a fuzzy set in fuzzy topological space, a different approach for the compactness structure of fuzzy topological space was introduced in [2]. The concept of $\tilde{g}$ -open set was discussed by Rajesh and Erdal Ekici [4]. The purpose of this paper is to study a new family $\tilde{g}$ -P (T). In this connection, $\tilde{g}$ -P (T) fuzzy closure and $\tilde{g}$ -P (T) fuzzy interior of a fuzzy set for which $\tilde{g}$ -P (T) satisfy almost all properties of the corresponding definitions in smooth fuzzy topological spaces in the sense of Sostak [8] and Ramadan [5]. As a consequence of these definitions the topological concepts, especially various types of compactness are established.

Throughout this paper, let X be a nonempty set, I = [0, 1] and I_0 = (0, 1]. For $\alpha \in I$, $\overline{\alpha}$ (x) = α for all $x \in X$.

Definition 1.1 [8] A function $T : I^X \to I$ is called a smooth fuzzy topology on X if it satisfies the following conditions :

(1) $T(\overline{0}) = T(\overline{1}) = 1$,

(2) $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$ for any $\mu_1, \mu_2 \in I^X$,

(3) $T\left(\bigvee_{j \in \Gamma} \mu_j\right) \geq \bigwedge_{j \in \Gamma} T(\mu_j)$ for any $\{\mu_j\}_{j \in \Gamma} \in I^X$.

The pair (X, T) is called a smooth fuzzy topological space.

Remark 1.1 Let (X, T) be a smooth fuzzy topological space. Then for each $r \in I_0$, $T_r = \{ \mu \in I^X : T(\mu) \geq r \}$ is Chang's fuzzy topology on X.

Definition 1.2 [6] Let (X, T) be a smooth fuzzy topological space. For each $\lambda \in I^X$, $r \in I_0$, an operator $C_T : I^X \times I_0 \to I^X$ is defined as follows:

$C_T (\lambda, r) = \wedge \{ \mu : \mu \geq \lambda, T (\overline{1} - \mu) \geq r \}$. For $\lambda, \mu \in I^X$ and $r, s \in I_0$, it satisfies the following conditions:

(1) $C_T (\overline{0}, r) = \overline{0}$.
(2) $\lambda \leq C_T (\lambda, r)$.
(3) $C_T (\lambda, r) \vee C_T (\mu, r) = C_T (\lambda \vee \mu, r)$.
(4) $C_T (\lambda, r) \leq C_T (\lambda, s)$, if $r \leq s$.
(5) $C_T (C_T (\lambda, r), r) = C_T (\lambda, r)$.

Proposition 1.1 [6] Let (X, T) be a smooth fuzzy topological space. For each $\lambda \in I^X$, $r \in I_0$, an operator $I_T : I^X \times I_0 \to I^X$ is defined as follows:

$I_T (\lambda, r) = \vee \{ \mu : \mu \leq \lambda, T (\mu) \geq r \}$. For $\lambda, \mu \in I^X$ and $r, s \in I_0$, it satisfies the following conditions:

(1) $I_T (\overline{1} - \lambda, r) = \overline{1} - C_T (\lambda, r)$.
(2) $I_T (\overline{1}, r) = \overline{1}$.
(3) $\lambda \geq I_T (\lambda, r)$.
(4) $I_T (\lambda, r) \wedge I_T (\mu, r) = I_T (\lambda \wedge \mu, r)$.
(5) $I_T (\lambda, r) \geq I_T (\lambda, s)$, if $r \leq s$.
(6) $I_T (I_T (\lambda, r), r) = I_T (\lambda, r)$.

Definition 1.3 Let (X, T) be a smooth fuzzy topological space. For $\lambda \in I^X$ and $r \in I_0$, λ is called

(1) r-fuzzy $\hat{g}$ -closed if $C_T (\lambda, r) \leq \mu$ whenever $\lambda \leq \mu$ and μ is r-fuzzy semi-open. The complement of an r-fuzzy $\hat{g}$ -closed set is said to be an r-fuzzy $\hat{g}$ -open set.

(2) r-fuzzy *g-closed if $C_T (\lambda, r) \leq \mu$ whenever $\lambda \leq \mu$ and μ is r-fuzzy $\hat{g}$ -open. The complement of an r-fuzzy *g-closed set is said to be an r-fuzzy *g-open set.

(3) r-fuzzy $^{\#}$g-semi-closed (briefly, $^{\#}$fgs-closed) if $SC_T (\lambda, r) \leq \mu$ whenever $\lambda \leq \mu$ and μ is r-fuzzy *g-open. The complement of an r-fuzzy $^{\#}$g-semi-closed set is said to be an r-fuzzy $^{\#}$g-semi-open set (briefly, r-$^{\#}$fgs-open).

(4) r-fuzzy $\tilde{g}$ -closed if $C_T (\lambda, r) \leq \mu$ whenever $\lambda \leq \mu$ and μ is r-$^{\#}$fgs-open. The complement of an r-fuzzy $\tilde{g}$ -closed set is said to be an r-fuzzy $\tilde{g}$ -open set.

2. VARIOUS TYPES OF $\tilde{g}$ -P-FUZZY COMPACTNESS AND CONCERNED RELATIONS

In this section, various types of $\tilde{g}$ -P-fuzzy compactness are discussed. Also, some interesting properties and characterizations are studied.

Definition 2.1 Let (X, T) be a smooth fuzzy topological space. For $r \in I_0$, the family $\tilde{g}$ -P (T) is defined by $\tilde{g}$ -P (T) = { $\lambda \in I^X$: λ is an r-fuzzy $\tilde{g}$ -open set with $\lambda \geq \overline{1} - \lambda$ } $\bigcup$ { $\lambda \in I^X$: $\lambda = \overline{0}$ }.

Definition 2.2 Let (X, T) be a smooth fuzzy topological space. For $r \in I_0$, the family $\tilde{g}$ -P (T) satisfies the following two properties:

(1) $\overline{0}, \overline{1} \in \tilde{g}$ -P (T)
(2) $\underset{j \in J}{\vee} \lambda_j \in \tilde{g}$ -P (T), for $\lambda_j \in \tilde{g}$ -P (T), $j \in J$.

The pair (X, $\tilde{g}$ -P (T)) is called a smooth $\tilde{g}$ -P-fuzzy topological space.

Remark 2.1 Let (X, $\tilde{g}$ -P (T)) be a smooth $\tilde{g}$ -P-fuzzy topological space, $r \in I_0$. The family $\tilde{g}$ -R (T) is defined by $\tilde{g}$ -R (T) = { $\lambda \in I^X$: λ is r-fuzzy $\tilde{g}$ -closed with $\overline{1} - \lambda \geq \lambda$ } $\bigcup$ { $\lambda \in I^X$: $\lambda = \overline{1}$ }. It is straightforward that $\tilde{g}$ -R (T) = { $\mu \in I^X$: $\overline{1} - \mu \in \tilde{g}$ -P (T) } and it satisfies the properties:

(1) $\overline{0}, \overline{1} \in \tilde{g}$ -R (T).
(2) $\underset{j \in J}{\wedge} \lambda_j \in \tilde{g}$ -R (T), for $\lambda_j \in \tilde{g}$ -R (T), $j \in J$.

Definition 2.3 Let (X, $\tilde{g}$ -P (T)) be a smooth $\tilde{g}$ -P-fuzzy topological space. For any $\lambda \in I^X$ and $r \in I_0$,

(1) $\tilde{g}$-$I_{P(T)}(\lambda, r) = \vee \{ \mu \in I^X : \mu \le \lambda;\ \mu \in \tilde{g}\text{-}P(T) \}$ is called $\tilde{g}$-$P(T)$ fuzzy interior of λ.

(2) $\tilde{g}$-$C_{R(T)}(\lambda, r) = \wedge \{ \mu \in I^X : \mu \ge \lambda; \mu \in \tilde{g}\text{-}R(T) \}$ is called $\tilde{g}$-$R(T)$ fuzzy closure of λ.

Remark 2.2 Let $(X, \tilde{g}\text{-}P(T))$ be a smooth $\tilde{g}$-P-fuzzy topological space. For any $\lambda \in I^X$ and $r \in I_0$,

(1) If $\lambda \in \tilde{g}\text{-}P(T)$ then $\lambda = \tilde{g}\text{-}I_{P(T)}(\lambda, r)$.

(2) If $\lambda \in \tilde{g}\text{-}R(T)$ then $\lambda = \tilde{g}\text{-}C_{R(T)}(\lambda, r)$.

Definition 2.4 Let $(X, \tilde{g}\text{-}P(T))$ and $(Y, \tilde{g}\text{-}P(S))$ be any two smooth $\tilde{g}$-P-fuzzy topological spaces and $f: (X, \tilde{g}\text{-}P(T)) \to (Y, \tilde{g}\text{-}P(S))$ be a function. Then

(1) f is said to be a $\tilde{g}$-P-fuzzy continuous function if, for each $\lambda \in \tilde{g}\text{-}P(S)$, $f^{-1}(\lambda) \in \tilde{g}\text{-}P(T)$.

(2) f is called a $\tilde{g}$-P-fuzzy open function if, for each $\lambda \in \tilde{g}\text{-}P(T)$, $f(\lambda) \in \tilde{g}\text{-}P(S)$.

Proposition 2.1 Let $(X, \tilde{g}\text{-}P(T))$ and $(Y, \tilde{g}\text{-}P(S))$ be any two smooth $\tilde{g}$-P-fuzzy topological spaces, $r \in I_0$. For a function, $f: (X, \tilde{g}\text{-}P(T)) \to (Y, \tilde{g}\text{-}P(S))$, the following statements are equivalent:

(a) f is a $\tilde{g}$-P-fuzzy continuous function.

(b) For every $\lambda \in I^X$, $f(\tilde{g}\text{-}C_{R(T)}(\lambda, r)) \le \tilde{g}\text{-}C_{R(S)}(f(\lambda), r)$.

(c) For every $\lambda \in I^Y$, $f^{-1}(\tilde{g}\text{-}C_{R(S)}(\lambda, r)) \ge \tilde{g}\text{-}C_{R(T)}(f^{-1}(\lambda), r)$.

(d) For every $\lambda \in I^Y$, $f^{-1}(\tilde{g}\text{-}I_{P(S)}(\lambda, r)) \le \tilde{g}\text{-}I_{P(T)}(f^{-1}(\lambda), r)$.

Proof: (a) $\Rightarrow$ (b). Let $\lambda \in I^X$ then, $\overline{1} - (\tilde{g}\text{-}C_{R(S)}(f(\lambda), r)) \in \tilde{g}\text{-}P(S)$, Since f is $\tilde{g}$-P-fuzzy continuous, $f^{-1}(\overline{1} - (\tilde{g}\text{-}C_{R(S)}(f(\lambda), r))) \in \tilde{g}\text{-}P(T)$. Since $\lambda \le f^{-1}(f(\lambda)) \le f^{-1}(\tilde{g}\text{-}C_{R(S)}(f(\lambda), r))$, it follows that $\tilde{g}\text{-}C_{R(T)}(\lambda, r) \le f^{-1}(\tilde{g}\text{-}C_{R(S)}(f(\lambda), r))$. Hence, $f(\tilde{g}\text{-}C_{R(T)}(\lambda, r)) \le \tilde{g}\text{-}C_{R(S)}(f(\lambda), r)$.

(b) $\Rightarrow$ (c). Let $\lambda \in I^Y$. Then by (b), $f(\tilde{g}\text{-}C_{R(T)}(f^{-1}(\lambda), r)) \le \tilde{g}\text{-}C_{R(S)}(f(f^{-1}(\lambda)), r) \le \tilde{g}\text{-}C_{R(S)}(\lambda, r)$. Hence, $\tilde{g}\text{-}C_{R(T)}(f^{-1}(\lambda), r) \le f^{-1}(\tilde{g}\text{-}C_{R(S)}(\lambda, r))$.

(c) $\Rightarrow$ (d). Let $\lambda \in I^Y$. By (c), $f^{-1}(\tilde{g}\text{-}C_{R(S)}(\overline{1} - \lambda, r)) \ge \tilde{g}\text{-}C_{R(T)}(f^{-1}(\overline{1} - \lambda), r) = \tilde{g}\text{-}C_{R(T)}(\overline{1} - f^{-1}(\lambda), r))$. Then, $f^{-1}(\tilde{g}\text{-}C_{R(S)}(\overline{1} - \lambda, r)) \ge \overline{1} - \tilde{g}\text{-}I_{P(T)}(f^{-1}(\lambda), r))$.

Hence, $\tilde{g}\text{-}I_{P(T)}(f^{-1}(\lambda), r)) \ge \overline{1} - f^{-1}(\tilde{g}\text{-}C_{R(S)}(\overline{1} - \lambda, r)) \ge \overline{1} - f^{-1}(\overline{1} - \tilde{g}\text{-}I_{P(S)}(\lambda, r)) \ge f^{-1}(\tilde{g}\text{-}I_{P(S)}(\lambda, r))$.

(d) $\Rightarrow$ (a). Let $\lambda \in \tilde{g}\text{-}P(S)$. Then, $\tilde{g}\text{-}I_{P(S)}(\lambda, r) = \lambda$, $f^{-1}(\lambda) = f^{-1}(\tilde{g}\text{-}I_{P(S)}(\lambda, r)) \le \tilde{g}\text{-}I_{P(T)}(f^{-1}(\lambda), r)$. Hence by definition 2.3, $f^{-1}(\lambda) = \tilde{g}\text{-}I_{P(T)}(f^{-1}(\lambda), r)$. Thus, $f^{-1}(\lambda) \in \tilde{g}\text{-}P(T)$. Therefore f is $\tilde{g}$-P-fuzzy continuous.

Definition 2.5 Let $(X, \tilde{g}\text{-}P(T))$ be a smooth fuzzy topological space. A $\tilde{g}$-$P(T)$ covering of $(X, \tilde{g}\text{-}P(T))$ is the collection $\{ \lambda_j \in \tilde{g}\text{-}P(T), \ j \in J \}$ such that $\overline{1} = \bigvee_{j \in J} \lambda_j$.

Definition 2.6 A smooth $\tilde{g}$-P-fuzzy topological space $(X, \tilde{g}\text{-}P(T))$ is called $\tilde{g}$-P-fuzzy compact if every $\tilde{g}$-$P(T)$ covering of $(X, \tilde{g}\text{-}P(T))$ has a finite sub-cover.

Definition 2.7 A smooth $\tilde{g}$-P-fuzzy topological space $(X, \tilde{g}\text{-}P(T))$ is called $\tilde{g}$-P-fuzzy almost compact if for any $\tilde{g}$-$P(T)$ covering of $(X, \tilde{g}\text{-}P(T))$, there exists a finite subset J_o of J such that $\bigvee_{j \in J_0} \tilde{g}\text{-}C_{R(T)}(\lambda_j, r) = \overline{1}$, $r \in I_0$.

Definition 2.8 A smooth $\tilde{g}$ -P-fuzzy topological space (X, $\tilde{g}$ -P (T)) is called $\tilde{g}$ -P-fuzzy nearly compact if for any $\tilde{g}$ -P (T) covering of (X, $\tilde{g}$ -P (T)), there exists a finite subset J_o of J such that $\underset{j\in J_0}{\vee} \tilde{g}\text{-}I_{P(T)}(\tilde{g}\text{-}C_{R(T)}(\lambda_j, r), r) = \overline{1}$, $r \in I_0$.

Definition 2.9 A smooth $\tilde{g}$ -P-fuzzy topological space (X, $\tilde{g}$ -P (T)) is called $\tilde{g}$ -P-fuzzy S-closed if for any $\tilde{g}$ -P (T) covering of (X, $\tilde{g}$ -P (T)), there exists a finite subset J_o of J such that $\underset{j\in J_0}{\vee} C_{R(T)}(\lambda_j, r) = \overline{1}$, $r \in I_0$.

Proposition 2.2 Let (X, $\tilde{g}$ -P (T)) and (Y, $\tilde{g}$ -P (S)) be any two smooth $\tilde{g}$ -P-fuzzy topological spaces, $r \in I_0$. Let any function $f : (X, \tilde{g}\text{-}P(T)) \to (Y, \tilde{g}\text{-}P(S))$ be a surjective and $\tilde{g}$ -P-fuzzy continuous function. Then for $r \in I_0$, the following statements are true:

(a) If (X, $\tilde{g}$ -P (T)) is $\tilde{g}$ -P-fuzzy almost compact, then so is (Y, $\tilde{g}$ -P (S)).

(b) If (X, $\tilde{g}$ -P (T))is $\tilde{g}$ -P-fuzzy nearly compact, then (Y, $\tilde{g}$ -P (S)) is $\tilde{g}$ -P-fuzzy almost compact.

Definition 2.10 Let (X, $\tilde{g}$ -P (T)) be a smooth $\tilde{g}$ -P-fuzzy topological space and $r \in I_0$. Then (X, $\tilde{g}$ -P (T)) is called $\tilde{g}$ -P-fuzzy regular if each r-fuzzy $\tilde{g}$ -open set λ in $\tilde{g}$ -P(T) can be written in the form of $\lambda = \vee \{ \mu \in I^X : \mu \in \tilde{g}\text{-}P(T), \tilde{g}\text{-}C_{R(T)}(\mu, r) \le \lambda \}$.

Proposition 2.3 If a smooth $\tilde{g}$ -P-fuzzy topological space (X, $\tilde{g}$ -P (T)) is $\tilde{g}$ -P-fuzzy almost compact and $\tilde{g}$ -P-fuzzy regular then it is $\tilde{g}$ -P-fuzzy compact.

Proposition 2.4 If a smooth $\tilde{g}$ -P-fuzzy topological space (X, $\tilde{g}$ -P (T)) is $\tilde{g}$ -P-fuzzy nearly compact and $\tilde{g}$ -P-fuzzy regular then it is $\tilde{g}$ -P-fuzzy compact.

Proposition 2.5 Let (X, $\tilde{g}$ -P (T)) and (Y, $\tilde{g}$ -P (S)) be any two smooth $\tilde{g}$ -P-fuzzy topological spaces and any function $f : (X, \tilde{g}\text{-}P(T)) \to (Y, \tilde{g}\text{-}P(S))$ be a surjective and $\tilde{g}$ -P-fuzzy continuous function. If (X, $\tilde{g}$ -P (T)) is $\tilde{g}$ -P-fuzzy almost compact then (Y, $\tilde{g}$ -P (S)) is $\tilde{g}$ -P-fuzzy S-closed.

Proposition 2.6 Let (X, $\tilde{g}$ -P (T)) and (Y, $\tilde{g}$ -P (S)) be any two smooth $\tilde{g}$ -P-fuzzy topological spaces and any function $f : (X, \tilde{g}\text{-}P(T)) \to (Y, \tilde{g}\text{-}P(S))$ be surjective and $\tilde{g}$ -P-fuzzy continuous. If (X, $\tilde{g}$ -P (T)) is $\tilde{g}$ -P-fuzzy nearly compact then (Y, $\tilde{g}$ -P (S)) is $\tilde{g}$ -P-fuzzy S-closed.

Proposition 2.7 Let (X, $\tilde{g}$ -P (T)) be a smooth $\tilde{g}$ -P-fuzzy topological space. Then the following statements are true:

(a) If (X, $\tilde{g}$ -P (T)) is $\tilde{g}$ -P-fuzzy compact, then it is $\tilde{g}$ -P-fuzzy almost compact.

(b) If (X, $\tilde{g}$ -P (T)) is $\tilde{g}$ -P-fuzzy compact, then it is P-fuzzy $\tilde{g}$ -closed.

Proposition 2.8 Let (X, $\tilde{g}$ -P (T)) be a smooth $\tilde{g}$ -P-fuzzy topological space. If (X, $\tilde{g}$ -P (T)) is $\tilde{g}$ -P-fuzzy almost compact, then it is P-fuzzy $\tilde{g}$ -closed as well.

Proposition 2.9 For every $\tilde{g}$ -P-fuzzy almost compact space is $\tilde{g}$ -P-fuzzy nearly compact.

Definition 2.11 Let (X, $\tilde{g}$ -P (T)) and (Y, $\tilde{g}$ -P (S)) be any two smooth $\tilde{g}$ -P-fuzzy topological spaces, $r \in I_0$. Any function $f : (X, \tilde{g}\text{-}P(T)) \to (Y, \tilde{g}\text{-}P(S))$ is called $\tilde{g}$ -P-fuzzy weakly continuous if and only if $f^{-1}(\lambda) \le \tilde{g}\text{-}I_{P(T)}(f^{-1}(\tilde{g}\text{-}C_{R(T)}(\lambda_j, r)), r)$ for each $\lambda \in \tilde{g}$ -P(S).

Proposition 2.10 Let (X, $\tilde{g}$ -P (T)) and (Y, $\tilde{g}$ -P (S)) be any two smooth $\tilde{g}$ -P-fuzzy topological spaces. If $f : (X, \tilde{g}\text{-}P(T)) \to (Y, \tilde{g}\text{-}P(S))$ is a $\tilde{g}$ -P-fuzzy weakly continuous function and (X, $\tilde{g}$ -P (T)) is a $\tilde{g}$ -P-fuzzy compact space, then (Y, S) is $\tilde{g}$ -P-fuzzy almost compact, $r \in I_0$.

Definition 2.11 Let (X, $\tilde{g}$ -P (T)) be a smooth $\tilde{g}$ -P-fuzzy topological space. For $r \in I_0$ and $\lambda \in \tilde{g}$ -P(T) is called r-$\tilde{g}$ -P-fuzzy regular open set if $\tilde{g}$ -$I_{P(T)}$ ($\tilde{g}$ -$C_{R(T)}$ (λ, r)), r) = λ.

Proposition 2.11 For any smooth $\tilde{g}$ -P-fuzzy topological space (X, $\tilde{g}$ -P (T)), $r \in I_0$, the following statements are equivalent :

(a) (X, $\tilde{g}$ -P (T)) is $\tilde{g}$ -P-fuzzy nearly compact.

(b) For any family J of r-$\tilde{g}$ -P-fuzzy regular closed sets with the property that $\underset{i \in F}{\wedge} \lambda_i \neq \bar{0}$, $\lambda_i \in \tilde{g}$ -P(T) and for any finite subfamily $F \leq J$, it follows that $\underset{i \in F}{\wedge} \lambda_i \neq \bar{0}$.

Proof: (a) $\Rightarrow$ (b). Let J be a family of r-$\tilde{g}$ -P-fuzzy regular closed sets such that $\underset{i \in F}{\wedge} \lambda_i \neq \bar{0}$, for any finite subfamily F of J. Suppose that $\underset{i \in F}{\wedge} \lambda_i = \bar{0}$. Then, $\bar{1} - \underset{i \in J}{\wedge} \lambda_i = \bar{1}$, which implies that $\underset{i \in J}{\vee} (\bar{1} - \lambda_i) = \bar{1}$. By (a), there is a finite subfamily F of J such that $\underset{i \in F}{\vee} (\bar{1} - \lambda_i) = \bar{1}$. Hence $\underset{i \in F}{\wedge} \lambda_i = \bar{0}$, which is a contradiction. Therefore, $\underset{i \in J}{\wedge} \lambda_i \neq \bar{0}$.

(b) $\Rightarrow$ (a). Let J be a family of r-$\tilde{g}$ -P-fuzzy regular open sets such that $\underset{i \in J}{\vee} \lambda_i = \bar{1}$, $\lambda_i \in \tilde{g}$ -P(T). Then { ($\bar{1} - \lambda_i$) / $i \in J$ } is a family of r-$\tilde{g}$ -P-fuzzy regular closed sets. Suppose that for any subfamily F of J, $\underset{i \in F}{\vee} \lambda_i \neq \bar{1}$. Then $\underset{i \in F}{\wedge} (\bar{1} - \lambda_i) \neq \bar{0}$. By (b), $\underset{i \in J}{\wedge} (\bar{1} - \lambda_i) \neq \bar{0}$, which implies $\underset{i \in J}{\vee} \lambda_i \neq \bar{1}$, which is a contradiction. Therefore there exists a subfamily F of J such that $\underset{i \in F}{\vee} \lambda_i = \bar{1}$. Hence (X, $\tilde{g}$ -P (T)) is $\tilde{g}$ -P-fuzzy nearly compact.

3. FUZZY $\tilde{g}$ -P (T) FILTER AND ITS CHARACTERIZATIONS

In this section fuzzy $\tilde{g}$ -P (T) ($\tilde{g}$ -R (T)) filter of a smooth $\tilde{g}$ -P-fuzzy topological space is introduced and its characterizations are studied.

Definition 3.1 A fuzzy $\tilde{g}$ -P (T) ($\tilde{g}$ -R (T)) filter $\tilde{g}$ -F of a smooth $\tilde{g}$ -P-fuzzy topological space (X, $\tilde{g}$ -P (T)) is a non-empty collection of $\lambda \in I^X$ with the following properties, $r \in I_0$.

(1) $\lambda \in \tilde{g}$ -F is an r-fuzzy $\tilde{g}$ -open (r-fuzzy $\tilde{g}$ -open) set in $\tilde{g}$ -P (T) ($\tilde{g}$ -R (T)).

(2) $\bar{0} \notin$ F, $\bar{1} \in$ F.

(3) If $\mu_1, \mu_2 \in$ F then $\mu_1 \wedge \mu_2 \in$ F.

(4) If $\mu \in$ F and γ is an r-fuzzy $\tilde{g}$ -open (r-fuzzy $\tilde{g}$ -open) set in $\tilde{g}$ -P (T) ($\tilde{g}$ -R (T)) with $\mu \leq \gamma$ then $\gamma \in$ F.

Definition 3.2 A fuzzy $\tilde{g}$ -P (T) ($\tilde{g}$ -R (T)) filter U on a smooth $\tilde{g}$ -P-fuzzy topological space (X, $\tilde{g}$ -P (T)) is a fuzzy $\tilde{g}$ -P (T) ($\tilde{g}$ -R (T)) ultra filter if there is no other $\tilde{g}$ -P (T) ($\tilde{g}$ -R (T)) filter finer than $\tilde{g}$ -F, $r \in I_0$.

Definition 3.3 A fuzzy $\tilde{g}$ -R (T) filter $\tilde{g}$ -F in a smooth $\tilde{g}$ -P-fuzzy topological space (X, $\tilde{g}$ -P (T)) is called a fuzzy $\tilde{g}$ -R (T) prime filter, if γ and μ are any two r-fuzzy $\tilde{g}$ -closed sets such that $\gamma \vee \mu \in$ F, then $\gamma \in$ F or $\mu \in$ F, $r \in I_0$.

Proposition 3.1 For any smooth $\tilde{g}$ -P-fuzzy topological space (X, $\tilde{g}$ -P (T)) and $r \in I_0$, the following statements are equivalent :

(a) (X, $\tilde{g}$ -P (T)) is $\tilde{g}$ -P-fuzzy compact.

(b) Every fuzzy $\tilde{g}$ -R (T) filter F satisfies $\underset{\mu \in F}{\wedge} \mu \neq \bar{0}$.

(c) Every fuzzy $\tilde{g}$ -R (T) prime filter F satisfies $\wedge_{\mu \in F} \mu \neq \overline{0}$.

(d) Every fuzzy $\tilde{g}$ -R (T) ultra filter U satisfies $\wedge_{\mu \in U} \mu \neq \overline{0}$.

Proof: (a) $\Rightarrow$ (b). Suppose $\wedge_{\mu \in F} \mu = \overline{0}$. Then, $\wedge_{\mu \in F} (\overline{1} - \mu) = \overline{1}$. Since $\overline{1} - \mu$ is r-fuzzy $\tilde{g}$ -open and (X, $\tilde{g}$ -P (T)) is $\tilde{g}$ -P-fuzzy compact, there must exists a finite sub collection { $\overline{1} - \mu_1$, $\overline{1} - \mu_2$, ..., $\overline{1} - \mu_n$ } such that $\overline{1}$ = ($\overline{1} - \mu_1$) $\vee$ ($\overline{1} - \mu_2$) $\vee$ .....$\vee$ ($\overline{1} - \mu_n$), that is, $\mu_1 \wedge \mu_2 \wedge$ .....$\wedge \mu_n = \overline{0}$, a contradiction.

(b) $\Rightarrow$ (c). Follows from the fact that every fuzzy $\tilde{g}$ -R (T) prime filter is fuzzy $\tilde{g}$ -R (T) filter.

(c) $\Rightarrow$ (d). The proof is obvious.

(d) $\Rightarrow$ (a). Suppose H is a family of r-fuzzy $\tilde{g}$ -closed sets with finite intersection property. For each $\gamma \in H$, consider a family G_γ = { μ : μ is r-fuzzy $\tilde{g}$ -closed in $\tilde{g}$ -R (T), $\mu \geq \gamma$ }. Clearly, $\gamma \in G_\gamma$. Let G = $\bigcup_{\gamma \in H} \{ G_\gamma \}$. Since H has the finite intersection property, G also has the property. Thus there exists a fuzzy $\tilde{g}$ -R (T) ultra filter U such that H $\subset$ G $\subset$ U.

Hence, $\wedge_{\mu \in H} \mu \geq \wedge_{\mu \in G} \mu \geq \wedge_{\mu \in U} \mu$. By hypothesis, $\wedge_{\mu \in U} \mu \neq \overline{0}$ and therefore, $\wedge_{\mu \in H} \mu \neq \overline{0}$. This proves that (X, $\tilde{g}$ -P (T)) is $\tilde{g}$ -P-fuzzy compact.

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