ON SOMEWHAT PAIRWISE FUZZY FAINTLY \( \omega \)-CONTINUOUS FUNCTIONS

M. Sudha
E.Roja and M.K.Uma
Department of Mathematics
Sri Sarada College for Women, Salem-16
TamilNadu, India.

Abstract
In this paper the concept of somewhat pairwise fuzzy faintly \( \omega \)-continuous functions and somewhat pairwise fuzzy faintly \( \omega \)-open functions are introduced. Some interesting properties of these functions are investigated besides giving some characterizations of these functions.

Keywords
Fuzzy \( \omega \)-open set, fuzzy \( \omega \)-continuous function, somewhat pairwise fuzzy faintly \( \omega \)-continuous function, somewhat pairwise fuzzy faintly \( \omega \)-open function, pairwise fuzzy \( \theta \) -dense set, pairwise fuzzy \( \theta^* \) -dense set.


1. Introduction
The fuzzy concept has invaded almost all branches of Mathematics ever since the introduction of fuzzy sets by Zadeh [10]. Fuzzy sets have applications in many fields such as information [7] and control [8]. The theory of fuzzy topological spaces was introduced and developed by Chang [3] and since then various notions in classical topology have been extended to fuzzy topological spaces. The concept of somewhat continuous functions was introduced by Karl. R.Gentry and Hughes.B.Hoyle III in [5]. In 1989 Kandil [4] introduced the concept of fuzzy bitopological spaces. Uma, Roja and Balasubramanian introduced the concept of somewhat pairwise fuzzy continuous functions [9]. Anjan Mukherjee introduced the concept of fuzzy faintly continuous functions in [1]. The concept of \( \omega \)-continuous mappings was introduced and studied by Sheik John in [6]. In this paper we introduce a new class of fuzzy set called fuzzy \( \omega \)-open set and a new form of fuzzy \( \theta \)-open set labelled as fuzzy \( \theta^* \)-open set. Also we study some of its properties and characterizations with suitable examples.

2. Preliminaries
We recall the following definitions which we used in this paper.

Definition 2.1 [9]
Let \(( X, T_1, T_2 )\) and \(( Y, S_1, S_2 )\) be any two bitopological spaces. A function \( f : ( X, T_1, T_2 ) \rightarrow ( Y, S_1, S_2 )\) is called pairwise* fuzzy continuous if for each \( S_i\)-fuzzy open set or \( S_j\)-fuzzy open set \( \lambda \) in \(( Y, S_1, S_2 )\), the inverse image \( f^{-1}( \lambda )\) is a \( T_i\)-fuzzy open or \( T_j\)-fuzzy open set in \(( X, T_1, T_2 )\).

Definition 2.2 [9]
Let \(( X, T_1, T_2 )\) and \(( Y, S_1, S_2 )\) be any two bitopological spaces. A function \( f : ( X, T_1, T_2 ) \rightarrow ( Y, S_1, S_2 )\) is called somewhat pairwise fuzzy continuous if for each \( S_i\)-fuzzy open set or \( S_j\)-fuzzy open set \( \lambda \) in \(( Y, S_1, S_2 )\), the inverse image \( f^{-1}( \lambda )\) is a \( T_i\)-fuzzy open or \( T_j\)-fuzzy open set in \(( X, T_1, T_2 )\).

Definition 2.3 [6]
Let \(( X, T_1, T_2 )\) and \(( Y, S_1, S_2 )\) be any two bitopological spaces. A function \( f : ( X, T_1, T_2 ) \rightarrow ( Y, S_1, S_2 )\) is called somewhat pairwise fuzzy continuous if \( \lambda \in S_1 \) or \( \lambda \in S_2 \) and \( f^{-1}( \lambda ) \neq 0 \Rightarrow \) there exists \( \mu \in T_1 \) or \( \mu \in T_2 \) such that \( \mu \neq 0 \) and \( \mu \leq f^{-1}( \lambda )\).

Definition 2.4 [1]
Let \( f : ( X, T ) \rightarrow ( Y, S )\) be a function from the fuzzy topological space \(( X, T )\) to the fuzzy topological space \(( Y, S )\). \( f\) is called fuzzy faintly continuous if \( f^{-1}( \lambda )\) is fuzzy open for every fuzzy \( \theta\)-open set \( \lambda \) in \( Y\).
A subset $A$ of a topological space $(X, T)$ is called $\omega$-closed in $(X, T)$ if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi-open in $(X, T)$.

A subset $A$ is called $\omega$-open in $(X, T)$ if its complement, $A^c$, is $\omega$-closed.

**Definition 2.5 [2]**
Let $(X, T)$ and $(Y, S)$ be any two fuzzy topological spaces. For a mapping $f : (X, T) \rightarrow (Y, S)$, the graph $g : X \times Y \rightarrow X \times Y$ of $f$ is defined as $g(x) = (x, f(x))$, for each $x \in X$.

### 3.1 Somewhat pairwise fuzzy faintly $\omega$-continuous functions
In this section we investigate some properties of Somewhat pairwise fuzzy faintly $\omega$-continuous functions and we also obtain characterizations of these functions.

**Definition 3.1.1**
Let $(X, T)$ be a fuzzy topological space. A fuzzy set $\lambda \in \mathcal{I}^X$ is called fuzzy $\omega$-open in $(X, T)$ if $\text{int}(\lambda)$ is fuzzy semi-closed in $(X, T)$.

**Definition 3.1.2**
Let $(X, T)$ be a fuzzy topological space. A fuzzy set $\lambda \in \mathcal{I}^X$ is called fuzzy $\omega$-open in $(X, T)$ if $\text{int}(\lambda) \neq \emptyset$ implies that there exists a $T_1$-fuzzy $\omega$-open or $T_2$-fuzzy $\omega$-open set $\mu$ such that $\mu \neq 0$ and $\mu \leq f^{-1}(\lambda)$.

**Definition 3.1.3**
Let $(X, T)$ and $(Y, S)$ be any two fuzzy topological spaces. A map $f : (X, T) \rightarrow (Y, S)$ is called $\omega$-continuous if $f^{-1}(\lambda)$ is fuzzy $\omega$-open in $(X, T)$ for every fuzzy open set $\lambda$ in $(Y, S)$.

**Definition 3.1.4**
Let $(X, T_1, T_2)$ and $(Y, S_1, S_2)$ be any two fuzzy bitopological spaces. A function $f : (X, T_1, T_2) \rightarrow (Y, S_1, S_2)$ is called pairwise fuzzy $\omega$-continuous if for each $\lambda \in S_1$ or $\lambda \in S_2$ the inverse image $f^{-1}(\lambda)$ is $T_1$-fuzzy $\omega$-open or $T_2$-fuzzy $\omega$-open in $(X, T_1, T_2)$.

**Definition 3.1.5**
Let $(X, T_1, T_2)$ and $(Y, S_1, S_2)$ be any two fuzzy bitopological spaces. A function $f : (X, T_1, T_2) \rightarrow (Y, S_1, S_2)$ is called somewhat pairwise fuzzy $\omega$-continuous if $\lambda \in S_1$ or $\lambda \in S_2$ and $f^{-1}(\lambda) \neq \emptyset$ implies that there exists a $T_1$-fuzzy $\omega$-open or $T_2$-fuzzy $\omega$-open set $\mu$ such that $\mu \neq 0$ and $\mu \leq f^{-1}(\lambda)$.

**Definition 3.1.6**
Let $(X, T_1, T_2)$ and $(Y, S_1, S_2)$ be any two fuzzy bitopological spaces. A function $f : (X, T_1, T_2) \rightarrow (Y, S_1, S_2)$ is called pairwise fuzzy faintly $\omega$-continuous if $f^{-1}(\lambda) \in T_1$ or $f^{-1}(\lambda) \in T_2$ for every $S_1$-fuzzy $\theta$-open or $S_2$-fuzzy $\theta$-open set $\lambda$ in $(Y, S_1, S_2)$.

**Definition 3.1.7**
Let $(X, T_1, T_2)$ and $(Y, S_1, S_2)$ be any two fuzzy bitopological spaces. A function $f : (X, T_1, T_2) \rightarrow (Y, S_1, S_2)$ is called pairwise fuzzy faintly $\omega$-continuous if for every $S_1$-fuzzy $\theta$-open or $S_2$-fuzzy $\theta$-open set $\lambda$, the inverse image $f^{-1}(\lambda)$ is $T_1$-fuzzy $\omega$-open or $T_2$-fuzzy $\omega$-open.

**Definition 3.1.8**
Let $(X, T_1, T_2)$ and $(Y, S_1, S_2)$ be any two fuzzy bitopological spaces. A function $f : (X, T_1, T_2) \rightarrow (Y, S_1, S_2)$ is called somewhat pairwise fuzzy faintly $\omega$-continuous if for every $S_1$-fuzzy $\theta$-open or $S_2$-fuzzy $\theta$-open set $\lambda$ with $f^{-1}(\lambda) \neq \emptyset$ there exists a $T_1$-fuzzy $\omega$-open or $T_2$-fuzzy $\omega$-open set $\mu$ such that $\mu \neq 0$ and $\mu \leq f^{-1}(\lambda)$.

*Every pairwise fuzzy $\omega$-continuous function is somewhat pairwise fuzzy $\omega$-continuous, but the converse need not be true as the following example shows.*

**Example 3.1.1**
Let $X = \{a, b\}$. Define $T_1 = \{0, 1, \delta_1, \delta_2, \delta_3\}$, $T_2 = \{0, 1, \delta_2\}$, $S_1 = \{\delta_1, \delta_2, \gamma_1, \gamma_2\}$, $S_2 = \{0, 1\}$ where $\delta_1, \delta_2, \delta_3, \gamma_1, \gamma_2 : X \rightarrow \{0, 1\}$ are such that $\delta_1(a) = 0.3, \delta_1(b) = 0.4, \delta_2(a) = 0.61, \delta_2(b) = 0.41, \delta_3(a) = 0.7, \delta_3(b) = 0.6$ and $\gamma_1(a) = 0.59, \gamma_1(b) = 0.56, \gamma_2(a) = 0.41, \gamma_2(b) = 0.44$.

Clearly, $T_1, T_2, S_1$ and $S_2$ are fuzzy topologies on $X$. Define $f : (X, T_1, T_2)$
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\( (X, S_1, S_2) \) as \( f(a) = b \), \( f(b) = a \). Then, \( f \) is not pairwise fuzzy faintly \( \omega \)-continuous since the inverse image \( f^{-1}(\gamma_1) \) is not \( T_1 \)-fuzzy \( \omega \)-open or \( T_2 \)-fuzzy \( \omega \)-open. But \( f \) is somewhat pairwise fuzzy faintly \( \omega \)-continuous since for the \( S_1 \)-fuzzy \( \theta^* \)-open set \( \gamma_1 \) with \( f^{-1}(\gamma_1) \neq \emptyset \), \( \delta_1 \neq 0 \) is \( T_1 \)-fuzzy \( \omega \)-open and \( \delta_1 \leq f^{-1}(\gamma_1) \).

Every pairwise fuzzy faintly continuous function is somewhat pairwise fuzzy faintly \( \omega \)-continuous but the converse need not be true as the following example shows.

Example 3.1.2

Let \( X = \{a, b, c\} \). Define \( T_1 = \{0, 1, \lambda_1, \lambda_2\} \), \( T_2 = \{0, 1\} \), \( S_1 = \{0, 1, \mu_1, \mu_2\} \), \( S_2 = \{0, 1, \mu_3\} \) where \( \lambda_1, \lambda_2, \mu_1, \mu_2, \mu_3 : X \to [0, 1] \) are such that \( \lambda_1(a) = 0.02, \lambda_1(b) = 0, \lambda_2(b) = 1 \), \( \mu_1(a) = 0.1, \mu_1(b) = 0.1, \mu_2(a) = 0.9, \mu_2(b) = 0.9 \) and \( \mu_3(a) = 0.2, \mu_3(b) = 0.3 \). Clearly, \( T_1, T_2, S_1 \) and \( S_2 \) are fuzzy topologies on \( X \). Define \( g : (X, T_1, T_2) \to (X, S_1, S_2) \) as \( g(a) = b \), \( g(b) = a \). Since, \( g(\mu_2) \) is not \( T_1 \)-fuzzy open or \( T_2 \)-fuzzy open, \( g \) is not pairwise fuzzy faintly continuous. But \( g \) is somewhat pairwise fuzzy faintly \( \omega \)-continuous since the fuzzy set \( \gamma : X \to [0, 1] \) defined as \( \gamma(a) = 0.1, \gamma(b) = 0 \) is \( T_1 \)-fuzzy \( \omega \)-open and \( \gamma \leq g(\mu_2) \), for the \( S_1 \)-fuzzy \( \theta^* \)-open set \( S_2 \).

Every pairwise fuzzy faintly continuous mapping is pairwise fuzzy faintly \( \omega \)-continuous, but the converse need not be true as the following example shows.

Example 3.1.3

In Example 7.3.2, \( g \) is not pairwise fuzzy faintly continuous. But \( g \) is pairwise fuzzy faintly \( \omega \)-continuous, since for the \( S_1 \)-fuzzy \( \theta^* \)-open set \( \mu_2, g^{-1}(\mu_2) \) is \( T_1 \)-fuzzy \( \omega \)-open.

The following diagram gives the interrelations:

\[
\begin{array}{ccc}
\text{f is pairwise fuzzy faintly continuous} & \rightarrow & \text{f is somewhat pairwise fuzzy faintly \( \omega \)-continuous} \\
\leftrightarrow & & \leftrightarrow \\
\text{f is pairwise fuzzy faintly \( \omega \)-continuous} & \leftarrow & \text{f is somewhat pairwise fuzzy faintly \( \omega \)-continuous}
\end{array}
\]

Definition 3.1.9

A fuzzy set \( \lambda \) in a fuzzy bitopological space \( (X, T_1, T_2) \) is called pairwise fuzzy \( \omega \)-dense (resp. \( \theta^* \)-dense) set if there exists no \( T_1 \)-fuzzy \( \omega \)-closed (resp. \( \theta^* \)-closed) or \( T_2 \)-fuzzy \( \omega \)-closed (resp. \( \theta^* \)-closed) set \( \mu \) in \( (X, T_1, T_2) \) such that \( \lambda < \mu < 1 \).

Example 3.1.4

Let \( X = \{a, b, c\} \). Define \( T_1 = \{0, 1, \delta, \gamma\} \), \( T_2 = \{0, 1, \gamma\} \) where \( \delta, \gamma : X \to [0, 1] \) are such that \( \delta(a) = 0, \delta(b) = 1/4, \delta(c) = 1/3 \) and \( \gamma(a) = 1, \gamma(b) = 1/3, \gamma(c) = 1/3 \). Clearly \( T_1 \) and \( T_2 \) are fuzzy topologies on \( X \). Define a fuzzy set \( \lambda : X \to [0, 1] \) such that \( \lambda(a) = 1, \lambda(b) = 3/4, \lambda(c) = 2/3 \). Clearly \( \lambda \) is a pairwise fuzzy \( \omega \)-dense set.

Example 3.1.5

Let \( X = \{a, b, c\} \). Define \( T_1 = \{0, 1, \lambda, \mu\} \), \( T_2 = \{0, 1, \gamma\} \) where \( \lambda, \mu, \gamma : X \to [0, 1] \) are defined as \( \lambda(a) = 1/2, \lambda(b) = 3/4, \lambda(c) = 2/3, \mu(a) = 1/2, \mu(b) = 1/4, \mu(c) = 1/3 \) and \( \gamma(a) = 1/3, \gamma(b) = 1/4, \gamma(c) = 1/3 \). Clearly \( T_1 \) and \( T_2 \) are fuzzy topologies on \( X \). Let \( \eta : X \to [0, 1] \) be such that \( \eta(a) = 3/4, \eta(b) = 3/4, \eta(c) = 2/3 \). Clearly \( \eta \) is a pairwise fuzzy \( \theta^* \)-dense set.

Notation 3.1.1

(a) \( \theta^* - \text{int}_{S_1}^{\mu} (\lambda) \) and \( \theta^* - \text{int}_{S_2}^{\mu} (\lambda) \) denotes the \( S_1 \)-fuzzy \( \theta^* \)-interior and \( S_2 \)-fuzzy \( \theta^* \)-interior of a fuzzy set \( \lambda \) in a fuzzy bitopological space \( (X, S_1, S_2) \).
(b) \( \theta^* - \text{cl}_{S_1} (\lambda) \) and \( \theta^* - \text{cl}_{S_2} (\lambda) \) denotes the \( S_1 \)-fuzzy \( \theta^* \)-closure and \\
\( S_2 \)-fuzzy \( \theta^* \)-closure of a fuzzy set \( \lambda \) in a fuzzy bitopological space \( (X, S_1, S_2) \).

**Notation 3.1.2**

(a) \( \omega - \text{int}_{T_1} (\lambda) \) and \( \omega - \text{int}_{T_2} (\lambda) \) denotes the \( T_1 \)-fuzzy \( \omega \)-interior and \\
\( T_2 \)-fuzzy \( \omega \)-interior of a fuzzy set \( \lambda \) in a fuzzy bitopological space \( (X, T_1, T_2) \).

(b) \( \omega - \text{cl}_{T_1} (\lambda) \) and \( \omega - \text{cl}_{T_2} (\lambda) \) denotes the \( T_1 \)-fuzzy \( \omega \)-closure and \\
\( T_2 \)-fuzzy \( \omega \)-closure of a fuzzy set \( \lambda \) in a fuzzy bitopological space \( (X, T_1, T_2) \).

**Proposition 3.1.1**

Let \( (X, T_1, T_2) \) and \( (Y, S_1, S_2) \) be any two fuzzy bitopological spaces. Let \( f : (X, T_1, T_2) \to (Y, S_1, S_2) \) be any function. Then the following conditions are equivalent:

(a) \( f \) is somewhat pairwise fuzzy faintly \( \omega \)-continuous.

(b) If \( \lambda \) is \( S_1 \)-fuzzy \( \theta^* \)-closed or \( S_2 \)-fuzzy \( \theta^* \)-closed set such that \\
\( f^{-1} (\lambda) \neq 1 \) then there exists a proper \( T_1 \)-fuzzy \( \omega \)-closed or \\
\( T_2 \)-fuzzy \( \omega \)-closed set \( \mu \) such that \( \mu \leq f^{-1} (\lambda) \).

(c) If \( \lambda \) is a pairwise fuzzy \( \omega \)-dense set in \( (X, T_1, T_2) \) then \( f (\lambda) \) is a pairwise fuzzy \( \theta^* \)-dense set in \( (Y, S_1, S_2) \).

**Proposition 3.1.2**

Let \( (X, T_1, T_2) \) and \( (Y, S_1, S_2) \) be any two fuzzy bitopological spaces and \( f : (X, T_1, T_2) \to (Y, S_1, S_2) \) be a somewhat pairwise fuzzy faintly \( \omega \)-continuous function. Let \( A \subset X \) be such that \( \chi_A \land \mu \neq 0 \) for all \\
\( 0 \neq \mu \in T_1 \cup T_2 \). Let \( T_1/A \) and \( T_2/A \) be the induced fuzzy topologies on \( A \). Then \( f/A : (A, T_1/A, T_2/A) \to (Y, S_1, S_2) \) is somewhat pairwise fuzzy faintly \( \omega \)-continuous.

**Proposition 3.1.3**

Let \( (X, T_1, T_2) \) and \( (Y, S_1, S_2) \) be any two fuzzy bitopological spaces and \( X = A \cup B \) where \( A \) and \( B \) are subsets of \( X \) such that \( \chi_A \land \chi_B \in T_1 \cap T_2 \). Let \( f : (X, T_1, T_2) \to (Y, S_1, S_2) \) be such that \( f/A \) and \( f/B \) are somewhat pairwise fuzzy faintly \( \omega \)-continuous. Then \( f \) is somewhat pairwise fuzzy faintly \( \omega \)-continuous.

3.2 Somewhat pairwise fuzzy faintly \( \omega \)-open functions

In this section we investigate some properties of Somewhat pairwise fuzzy faintly \( \omega \)-open functions and we also obtain characterizations of these functions.

**Definition 3.2.1**

Let \( (X, T_1, T_2) \) and \( (Y, S_1, S_2) \) be any two fuzzy bitopological spaces. A mapping \( f : (X, T_1, T_2) \to (Y, S_1, S_2) \) is called pairwise fuzzy \( \omega \)-open if for every \( T_1 \)-fuzzy \( \omega \)-open or \( T_2 \)-fuzzy \( \omega \)-open set \( \lambda \), the image \( f(\lambda) \) is \( S_1 \)-fuzzy \( \omega \)-open or \( S_2 \)-fuzzy \( \omega \)-open.

**Definition 3.2.2**

Let \( (X, T_1, T_2) \) and \( (Y, S_1, S_2) \) be any two fuzzy bitopological spaces. A mapping \( f : (X, T_1, T_2) \to (Y, S_1, S_2) \) is called somewhat pairwise fuzzy \( \omega \)-open if for every \( T_1 \)-fuzzy \( \omega \)-open or \( T_2 \)-fuzzy \( \omega \)-open set \( \lambda \) with \( \lambda \neq 0 \), there exists a \( S_1 \)-fuzzy \( \omega \)-open or \( S_2 \)-fuzzy \( \omega \)-open set \( \mu \) such that \( \mu \neq 0 \) and \( \mu \leq f(\lambda) \).

**Definition 3.2.3**

Let \( (X, T_1, T_2) \) and \( (Y, S_1, S_2) \) be any two fuzzy bitopological spaces. A mapping \( f : (X, T_1, T_2) \to (Y, S_1, S_2) \) is called somewhat pairwise fuzzy faintly \( \omega \)-open if for every \( T_1 \)-fuzzy \( \omega \)-open or \( T_2 \)-fuzzy \( \omega \)-open set \( \lambda \) with \( \lambda \neq 0 \), there exists a \( S_1 \)-fuzzy \( \theta^* \)-open or \( S_2 \)-fuzzy \( \theta^* \)-open set \( \mu \) such that \( \mu \neq 0 \) and \( \mu \leq f(\lambda) \). That is, \( \theta^* - \text{int}_{S_1} (f(\lambda)) \neq 0 \) or \( \theta^* - \text{int}_{S_2} (f(\lambda)) \neq 0 \). Every pairwise fuzzy \( \omega \)-open function is somewhat pairwise fuzzy \( \omega \)-open, but the converse need not be true as shown in the following example.
Example 3.2.1
Let \( X = \{ a, b, c \} \). Define \( T_1 = \{ \emptyset, \{1\}, \{1,2\} \}, \ T_2 = \{ \emptyset, 1 \} \),
\( S_1 = \{ 0, 1, 2 \}, S_2 = \{ 0, 1, 2 \} \) where \( \mu_1, \mu_2, \delta_1, \delta_2 : X \rightarrow [0,1] \) are defined as \( \mu_1(a) = 0.1, \mu_1(b) = 0.2, \mu_2(a) = 0.9, \mu_2(b) = 0.8, \delta_1(a) = 0.1, \delta_1(b) = 0.3, \delta_2(a) = 0.3. \) Clearly \( T_1, T_2, S_1, \) and \( S_2 \) are fuzzy topologies on \( X \). Let \( f : ( X, T_1, T_2 ) \rightarrow ( X, S_1, S_2 ) \) be the identity function. Let \( \lambda : X \rightarrow [0,1] \) be such that \( \lambda(a) = 0.2, \lambda(b) = 0.2. \) Now, \( \lambda \) is \( T_1 \)-fuzzy \( \omega \)-open but \( \lambda = f(\lambda) \) is not \( S_1 \)-fuzzy \( \omega \)-open or \( S_2 \)-fuzzy \( \omega \)-open. Therefore, \( f \) is not pairwise fuzzy \( \omega \)-open. Since \( \lambda = f(\lambda) \) is \( S_1 \)-fuzzy \( \omega \)-open and \( \delta_1 \) is \( S_1 \)-fuzzy \( \omega \)-open such that \( \delta_1 \neq 0 \) and \( \delta_1 \leq \lambda = f(\lambda) \). \( f \) is somewhat pairwise fuzzy \( \omega \)-open.

Every pairwise fuzzy faintly \( \omega \)-open function is somewhat pairwise fuzzy faintly \( \omega \)-open but the converse need not be true as the following example shows.

Example 3.2.2
Let \( X = \{ a, b \} \). Define \( T_1 = \{ \emptyset, 1, \gamma \}, T_2 = \{ \emptyset, 1, \} \),
\( S_1 = \{ 0, 1, \mu_1, \mu_2 \}, S_2 = \{ 0, 1, \delta \} \) where \( \lambda, \gamma, \mu_1, \mu_2, \delta : X \rightarrow [0,1] \) are defined as \( \lambda(a) = 0.02, \lambda(b) = 0, \gamma(a) = 0.98, \gamma(b) = 1, \mu_1(a) = 0.05, \mu_1(b) = 0, \mu_2(a) = 0.95, \mu_2(b) = 1, \) and \( \delta(a) = 0.5, \delta(b) = 0.5. \) Clearly \( T_1, T_2, S_1, \) and \( S_2 \) are fuzzy topologies on \( X \). Let \( f : ( X, T_1, T_2 ) \rightarrow ( X, S_1, S_2 ) \) be the identity function. Let \( \eta : X \rightarrow [0,1] \) be such that \( \eta(a) = 0.96, \eta(b) = 1. \) Then, \( \eta \) is \( T_1 \)-fuzzy \( \omega \)-open but \( f(\eta) = \eta \) is not \( S_1 \)-fuzzy \( \theta^* \)-open or \( S_2 \)-fuzzy \( \theta^* \)-open. Therefore, \( f \) is not pairwise fuzzy faintly \( \omega \)-open. Since \( \mu_2 \neq 0 \) is \( S_2 \)-fuzzy \( \theta^* \)-open and \( \mu_2 \leq f(\eta) \), \( f \) is somewhat pairwise fuzzy faintly \( \omega \)-open.

Proposition 3.2.1
Let \( ( X, T_1, T_2 ) \) and \( ( Y, S_1, S_2 ) \) be any fuzzy bitopological spaces if \( f : ( X, T_1, T_2 ) \rightarrow ( Y, S_1, S_2 ) \) and \( g : ( Y, S_1, S_2 ) \rightarrow ( Z, R_1, R_2 ) \) are somewhat pairwise fuzzy faintly \( \omega \)-open functions then \( g \circ f : ( X, T_1, T_2 ) \rightarrow ( Z, R_1, R_2 ) \) is a somewhat pairwise fuzzy faintly \( \omega \)-open function.

Proposition 3.2.2
Let \( ( X, T_1, T_2 ) \) and \( ( Y, S_1, S_2 ) \) be any two fuzzy bitopological spaces and let \( f : ( X, T_1, T_2 ) \rightarrow ( Y, S_1, S_2 ) \) be a one-to-one and onto function. Then the following conditions are equivalent:
(a) \( f \) is somewhat pairwise fuzzy faintly \( \omega \)-open.
(b) If \( \lambda \) is a pairwise fuzzy \( \theta \)-dense set in \( ( Y, S_1, S_2 ) \), then \( f^{-1}(\lambda) \) is a pairwise fuzzy \( \omega \)-dense set in \( ( X, T_1, T_2 ) \).

Proposition 3.2.3
Let \( ( X, T_1, T_2 ) \) and \( ( Y, S_1, S_2 ) \) be any two fuzzy bitopological spaces and let \( f : ( X, T_1, T_2 ) \rightarrow ( Y, S_1, S_2 ) \) be a one-to-one and onto function. Then the following conditions are equivalent:
(a) \( f \) is somewhat pairwise fuzzy faintly \( \omega \)-open.
(b) If \( \lambda \) is a \( T_1 \)-fuzzy \( \omega \)-closed or \( T_2 \)-fuzzy \( \omega \)-closed set in \( ( X, T_1, T_2 ) \) such that \( f(\lambda) \neq 1 \), then there exists a \( S_1 \)-fuzzy \( \theta^* \)-closed or \( S_2 \)-fuzzy \( \theta^* \)-closed set \( \mu \) in \( ( Y, S_1, S_2 ) \) such that \( \mu \neq 1 \) and \( \mu > f(\lambda) \).

Definition 3.2.4
A fuzzy bitopological space \( ( X, T_1, T_2 ) \) is called a pairwise fuzzy \( D \)-space if every non-zero fuzzy set \( \lambda \) in \( T_1 \) or \( \lambda \) in \( T_2 \) of \( ( X, T_1, T_2 ) \) is dense in \( ( X, T_1, T_2 ) \).

Definition 3.2.5
A fuzzy bitopological space \( ( X, T_1, T_2 ) \) is called a pairwise fuzzy \( D_{\theta^*} \)-space (resp. \( D_0 \)-space) if every non-zero fuzzy \( T_1 \)-fuzzy \( \omega \)-open (resp. \( \theta^* \)-open) or \( T_2 \)-fuzzy \( \omega \)-open (resp. \( \theta^* \)-open) set in \( ( X, T_1, T_2 ) \) is pairwise fuzzy \( \omega \)-dense (resp. \( \theta^* \)-dense) in \( ( X, T_1, T_2 ) \).
Proposition 3.2.4
Let \( f : (X, T_1, T_2) \to (Y, S_1, S_2) \) be somewhat pairwise fuzzy faintly \( \omega \)-continuous. Suppose \( (X, T_1, T_2) \) is a pairwise fuzzy \( D_\omega \)-space. Then \( (Y, S_1, S_2) \) is a pairwise fuzzy \( D_\omega^* \)-space.

Proposition 3.2.5
Let \( (X, T_1, T_2) \) and \( (Y, S_1, S_2) \) be any two fuzzy bitopological spaces. Let \( X = A \cup B \) where \( A \) and \( B \) are subsets of \( X \) and \( f : (X, T_1, T_2) \to (Y, S_1, S_2) \) is a function such that \( f/A \) and \( f/B \) are somewhat pairwise fuzzy faintly \( \omega \)-open. Then \( f \) is somewhat pairwise fuzzy faintly \( \omega \)-open.

Proposition 3.2.6
Let \( f : (X, T_1, T_2) \to (Y, S_1, S_2) \) be any function from a fuzzy bitopological space \( (X, T_1, T_2) \) to another fuzzy bitopological space \( (Y, S_1, S_2) \). If the graph \( g : X \to X \times Y \) of \( f \) is somewhat pairwise fuzzy faintly \( \omega \)-continuous then \( f \) is also somewhat pairwise fuzzy faintly \( \omega \)-continuous.

Proposition 3.2.7
Let \( f : (X, T_1, T_2), (X_1, S_1, S_2) \) and \( (X_2, R_1, R_2) \) be any three fuzzy bitopological spaces. Let \( p_i : X_1 \times X_2 \to X_i \) \((i = 1, 2)\) be the projection mappings. If \( f : X \to X_1 \times X_2 \) is a somewhat pairwise fuzzy \( \omega \)-continuous function then \( p_i \circ f \) is also somewhat pairwise fuzzy \( \omega \)-continuous function for \( i = 1,2 \).

Acknowledgement
The authors thank the referees for their valuable suggestions regarding the betterment of the paper.

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