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On general Eulerian integral of certain products of special functions and a class of multivariable polynomial V

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ABSTRACT
The object of this paper is to establish an general Eulerian integral involving the product of two multivariable Aleph-functions, multivariable I-function defined by Nambisan et al [1] and a class of multivariable polynomials which provide unification and extension of numerous results. We will study the case concerning the multivariable I-function defined by Sharma et al [3].

Keywords: Eulerian integral, multivariable I-function, Lauricella function of several variables, multivariable H-function, multivariable Aleph-function, class of multivariable polynomials

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1.Introduction

The object of this paper is to establish an general Eulerian integral involving the multivariable Aleph-functions , a expansion of multivariable I-function defined by Nambisan et al [1] and a class of multivariable polynomials defined by Srivastava et al [4] which provide unification and extension of numerous results.

First time, we define the multivariable I-function by :

$$\bar{I}(z_1,\cdots,z_r) = \bar{I}^{0,n:m_1,n_1;\cdots;m_r,n_r}_{p,q:p_1,q_1;\cdots;p_r,q_r}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_{n+1,p}: \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_{m+1,q}: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}_j^{(1)},\gamma_j^{(1)};1)_{1,n_1},(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{n_1+1,p_1};\cdots;(c_j^{(r)},\gamma_j^{(r)};1)_{1,n_r},(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{n_r+1,p_r} \\ \\ (\mathrm{d}_j^{(1)},\delta_j^{(1)};1)_{1,m_1},(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{m_1+1,q_1};\cdots;(d_j^{(r)},\delta_j^{(r)};1)_{1,m_r},(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{m_r+1,q_r} \end{array}\right) \quad (1.1)$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi_1(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \quad (1.2)$$

where $\phi_1(s_1,\cdots,s_r),\theta_i(s_i),i=1,\cdots,r$ are given by :

$$\phi_1(s_1,\cdots,s_r) = \frac{1}{\prod_{j=n+1}^{p}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)\prod_{j=m+1}^{q}\Gamma^{B_j}\left(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}s_j\right)} \quad (1.3)$$

$$\theta_i(s_i) = \frac{\prod_{j=1}^{n_i} \Gamma\left(1 - c_j^{(i)} + \gamma_j^{(i)} s_i\right) \prod_{j=1}^{m_i} \Gamma\left(d_j^{(i)} - \delta_j^{(i)} s_i\right)}{\prod_{j=n_i+1}^{p_i} \Gamma^{C_j^{(i)}}\left(c_j^{(i)} - \gamma_j^{(i)} s_i\right) \prod_{j=m_i+1}^{q_i} \Gamma^{D_j^{(i)}}\left(1 - d_j^{(i)} - \delta_j^{(i)} s_i\right)} \tag{1.4}$$

$i = 1, \cdots, r$

Serie representation

If $z_i \neq 0; i = 1, \cdots, r$

$\delta_{h_i}^{(i)}(d_j^{(i)} + k_i) \neq \delta_j^{(i)}(\delta_{h_i}^{(i)} + \eta_i) for j \neq h_i, j, h_i = 1, \cdots, m_i (i = 1, \cdots, r), k_i, \eta_i = 0, 1, 2, \cdots (i = 1, \cdots, r)$, then

$$\bar{I}(z_1, \cdots, z_r) = \sum_{h_1=1}^{m_1} \cdots \sum_{h_r=1}^{m_r} \sum_{k_1=0}^{\infty} \cdots \sum_{k_r=0}^{\infty} \left[\phi_1\left(\frac{dh_1^{(1)} + k_1}{\delta h_1^{(1)}}, \cdots, \frac{dh_r^{(r)} + k_r}{\delta h_r^{(r)}}\right)\right]_{j \neq h_i} \prod_{i=1}^{r} \frac{(-)^{k_i}}{\delta h_i^{(i)} k_i!} z_i^{\frac{dh_i + k_i}{\delta h_i}} \tag{1.5}$$

This result can be proved on computing the residues at the poles :

$$s_i = \frac{dh_i^{(i)} + k_i}{\delta h_i^{(i)}}, (h_i = 1, \cdots, m_i, k_i = 0, 1, 2, \cdots) for i = 1, \cdots, r \tag{1.6}$$

We may establish the the asymptotic expansion in the following convenient form :

$I(z_1, \cdots, z_r) = 0(\, |z_1|^{\alpha_1}, \cdots, |z_r|^{\alpha_r}\,), max(\, |z_1|, \cdots, |z_r|\,) \rightarrow 0$

$\bar{I}(z_1, \cdots, z_r) = 0(\, |z_1|^{\beta_1}, \cdots, |z_r|^{\beta_r}\,), min(\, |z_1|, \cdots, |z_r|\,) \rightarrow \infty$

where $k = 1, \cdots, r : \alpha_k = min[Re(d_j^{(k)}/\delta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k = max[Re((c_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \cdots, n_k$$

We will note $\eta_{h_i,k_i} = \dfrac{dh_i^{(i)} + k_i}{\delta h_i}$, $(h_i = 1, \cdots, m_i, k_i = 0, 1, 2, \cdots) for i = 1, \cdots, r$ (1.7)

The Aleph-function of several variables generalize the multivariable I-function defined by Sharma and Ahmad [3] , itself is an a generalisation of G and H-functions of several variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $\aleph(z'_1, \cdots, z'_s) = \aleph^{0,\mathfrak{n}':m'_1,n'_1,\cdots,m'_s,n'_s}_{p'_i,q'_i,\tau_i;R:p'_{i^{(1)}},q'_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p'_{i^{(s)}},q'_{i^{(s)}};\tau_{i^{(s)}};R^{(s)}} \left(\begin{array}{c} \text{z'}_1 \\ \cdot \\ \cdot \\ \cdot \\ \text{z'}_s \end{array}\right|$

$[(\text{a'}_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)})_{1,\mathfrak{n}'}]$ $,[\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \cdots, \alpha_{ji}^{(r)})_{\mathfrak{n}'+1,p_i}]$:

.... $,[\tau_i(b_{ji}; \beta_{ji}^{(1)}, \cdots, \beta_{ji}^{(r)})_{m'+1,q_i}]$:

$[(\text{c'}_j^{(1)}, \gamma_j'^{(1)})_{1,n'_1}], [\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)}, \gamma_{ji^{(1)}}^{(1)})_{n'_1+1,p_i'^{(1)}}]; \cdots ; [(\text{c'}_j^{(s)}, \gamma_j'^{(s)})_{1,n'_s}], [\tau_{i^{(s)}}(c_{ji^{(s)}}^{(s)}, \gamma_{ji^{(s)}}^{(s)})_{n'_s+1,p_i'^{(s)}}]$

$\left.[(\text{d'}_j^{(1)}, \delta_j'^{(1)})_{1,m'_1}], [\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}, \delta_{ji^{(1)}}^{(1)})_{m'_1+1,q_i'^{(1)}}]; \cdots ; [(\text{d'}_j^{(s)}, \delta_j'^{(s)})_{1,m'_s}], [\tau_{i^{(s)}}(d_{ji^{(s)}}^{(s)}, \delta_{ji^{(s)}}^{(s)})_{m'_s+1,q_i'^{(s)}}]\right)$

$$=\frac{1}{(2\pi\omega)^s}\int_{L'_1}\cdots\int_{L'_s}\psi(t_1,\cdots,t_s)\prod_{k=1}^{s}\zeta_k(t_k){z'_k}^{t_k}\,\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{1.8}$$

with $\omega=\sqrt{-1}$

$$\psi(t_1,\cdots,t_s)=\frac{\prod_{j=1}^{\mathfrak{n}'}\Gamma(1-a'_j+\sum_{k=1}^{r}\alpha_j^{(k)}t_k)}{\sum_{i=1}^{R}[\tau_i\prod_{j=\mathfrak{n}'+1}^{p'_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}t_k)\prod_{j=1}^{q'_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}t_k)]} \tag{1.9}$$

and $$\zeta_k(t_k)=\frac{\prod_{j=1}^{M_k}\Gamma(b''^{(k)}_j-\beta''^{(k)}_j x_k)\prod_{j=1}^{N_k}\Gamma(1-a''^{(k)}_j+\alpha''^{(k)}_j x_k)}{\sum_{i^{(k)}=1}^{r^{(k)}}[\iota_{i^{(k)}}\prod_{j=M_k+1}^{Q_{i^{(k)}}}\Gamma(1-b''^{(k)}_{ji^{(k)}}+\beta''^{(k)}_{ji^{(k)}}x_k)\prod_{j=N_k+1}^{P_{i^{(k)}}}\Gamma(a''^{(k)}_{ji^{(k)}}-\alpha''^{(k)}_{ji^{(k)}}x_k)]} \tag{1.10}$$

Suppose , as usual , that the parameters

$a'_j, j=1,\cdots,p'; b'_j, j=1,\cdots,q';$

$c_{ji^{(k)}}^{(k)}, j=n_k+1,\cdots,p_{i^{(k)}}; \mathfrak{c}_\mathfrak{j}^{(k)}, j=1,\cdots,n_k;$

$d_j^{(k)}, j=1,\cdots,m_k; d_{ji^{(k)}}^{(k)}, j=m_k+1,\cdots,q_{i^{(k)}};$

with $k=1\cdots,s, i=1,\cdots,R\ , i^{(k)}=1,\cdots,R^{(k)}$

are complex numbers , and the $\alpha's,\beta's,\gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)}=\sum_{j=1}^{\mathfrak{n}'}{\alpha'}_j^{(k)}+\tau_i\sum_{j=\mathfrak{n}'+1}^{p'_i}\alpha_{ji}^{(k)}+\sum_{j=1}^{n'_k}{\gamma'}_j^{(k)}+\tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}-\tau_i\sum_{j=1}^{q'_i}\beta_{ji}^{(k)}-\sum_{j=1}^{m'_k}{\delta'}_j^{(k)}$$

$$-\tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)}\leqslant 0 \tag{1.11}$$

The reals numbers τ_i are positives for $i=1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)}=1$ to $R^{(k)}$

The contour L_k is in the s_k-p lane and run from $\sigma-i\infty$ to $\sigma+i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma({d'}_j^{(k)}-{\delta'}_j^{(k)}s_k)$ with $j=1$ to m_k are separated from those o $\Gamma(1-a'_j+\sum_{i=1}^{r}\alpha_j^{(k)}s_k)$f with $j=1$ to n and $\Gamma(1-{c'}_j^{(k)}+{\gamma'}_j^{(k)}s_k)$ with $j=1$ to n'_k to the left of the contour L'_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|arg z'_k|<\frac{1}{2}A_i^{(k)}\pi$, where

$$A_i^{(k)}=\sum_{j=1}^{\mathfrak{n}'}{\alpha'}_j^{(k)}-\tau_i\sum_{j=\mathfrak{n}'+1}^{p'_i}\alpha_{ji}^{(k)}-\tau_i\sum_{j=1}^{q'_i}\beta_{ji}^{(k)}+\sum_{j=1}^{n'_k}{\gamma'}_j^{(k)}-\tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{m'_k}\delta'^{(k)}_j - \tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta^{(k)}_{ji^{(k)}} > 0, \quad \text{with } k=1\cdots,s, i=1,\cdots,R\ , i^{(k)}=1,\cdots,R^{(k)} \tag{1.12}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z'_1,\cdots,z'_s) \ = 0(\ |z'_1|^{\alpha'_1},\cdots,|z'_s|^{\alpha'_s}\ ), max(\ |z'_1|,\cdots,|z'_s|\ ) \rightarrow 0$$

$$\aleph(z'_1,\cdots,z'_s) = 0(\ |z'_1|^{\beta'_1},\cdots,|z_s|^{\beta'_s}\ ), min(\ |z'_1|,\cdots,|z'_s|\ ) \rightarrow \infty$$

where $k=1,\cdots,s: \alpha'_k = min[Re(d'^{(k)}_j/\delta'^{(k)}_j)], j=1,\cdots,m'_k$ and

$$\beta'_k = max[Re((c'^{(k)}_j - 1)/\gamma'^{(k)}_j)], j=1,\cdots,n'_k$$

We will use these following notations in this paper

$$U = p'_i, q'_i, \tau_i; R\ ;\ V = m'_1, n'_1; \cdots; m'_r, n'_r \tag{1.13}$$

$$\mathrm{W} = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}, \cdots, p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)} \tag{1.14}$$

$$A = \{(a'_j; \alpha'^{(1)}_j, \cdots, \alpha'^{(r)}_j)_{1,n'}\}, \{\tau_i(a_{ji}; \alpha^{(1)}_{ji}, \cdots, \alpha^{(r)}_{ji})_{n+1,p_i}\} \tag{1.15}$$

$$B = \{\tau_i(b_{ji}; \beta^{(1)}_{ji}, \cdots, \beta^{(r)}_{ji})_{m+1,q_i}\} \tag{1.16}$$

$$C = \{(c'^{(1)}_j; \gamma'^{(1)}_j)_{1,n_1}\}, \tau_{i^{(1)}}(c^{(1)}_{ji^{(1)}}; \gamma^{(1)}_{ji^{(1)}})_{n'_1+1,p_{i^{(1)}}}\}, \cdots,$$
$$\{(c'^{(s)}_j; \gamma'^{(s)}_j)_{1,n'_s}\}, \tau_{i^{(s)}}(c^{(s)}_{ji^{(s)}}; \gamma^{(s)}_{ji^{(s)}})_{n'_s+1,p_{i^{(s)}}}\} \tag{1.17}$$

$$D = \ \{(d'^{(1)}_j; \delta'^{(1)}_j)_{1,m'_1}\}, \tau_{i^{(1)}}(d^{(1)}_{ji^{(1)}}; \delta^{(1)}_{ji^{(1)}})_{m'_1+1,q_{i^{(1)}}}\}, \cdots$$
$$, \{(d'^{(s)}_j; \delta'^{(s)}_j)_{1,m'_s}\}, \tau_{i^{(s)}}(d^{(s)}_{ji^{(s)}}; \delta^{(s)}_{ji^{(s)}})_{m'_s+1,q_{i^{(s)}}}\} \tag{1.18}$$

The multivariable Aleph-function write :

$$\aleph(z_1,\cdots,z_r) = \aleph^{0,\mathfrak{n}':V}_{U:W}\left(\begin{array}{c|c} \text{z'}_1 & \text{A : C} \\ \cdot & \cdot\ \cdot\ \cdot \\ \cdot & \\ \text{z'}_s & \text{B : D} \end{array}\right) \tag{1.19}$$

Consider the Aleph-function of s variables

$$\aleph(z''_1,\cdots,z''_u) = \ \aleph^{0,N:M_1,N_1,\cdots,M_u,N_u}_{P_i,Q_i,\iota_i;r':P_{i^{(1)}},Q_{i^{(1)}},\iota_{i^{(1)}};r^{(1)};\cdots;P_{i^{(u)}},Q_{i^{(u)}};\iota_{i^{(u)}};r^{(u)}}\left(\begin{array}{c|} \text{z"}_1 \\ \cdot \\ \cdot \\ \cdot \\ \text{z"}_u \end{array}\right.$$

$$\begin{array}{ll} [(\mathrm{u}_j; \mu^{(1)}_j, \cdots, \mu^{(u)}_j)_{1,N}] & ,[\iota_i(u_{ji}; \mu^{(1)}_{ji}, \cdots, \mu^{(u)}_{ji})_{N+1,P_i}] : \\ \text{..........................} & ,[\iota_i(v_{ji}; \upsilon^{(1)}_{ji}, \cdots, \upsilon^{(u)}_{ji})_{M+1,Q_i}] : \end{array}$$

$$\left.\begin{matrix}[(\mathrm{a}"_j^{(1)};\alpha_j''^{(1)})_{1,N_1}],[\iota_{i^{(1)}}(a_{ji^{(1)}}^{(1)};\alpha_{ji^{(1)}}^{(1)})_{N_1+1,P_i^{(1)}}];\cdots;\ [(\mathrm{a}_j^{(u)});\alpha_j^{(u)})_{1,N_u}],[\iota_{i^{(u)}}(a_{ji^{(u)}}^{(u)};\alpha_{ji^{(u)}}^{(u)})_{N_u+1,P_i^{(u)}}]\\ [(\mathrm{b}"_j^{(1)};\beta_j''^{(1)})_{1,M_1}],[\iota_{i^{(1)}}(b_{ji^{(1)}}^{(1)};\beta_{ji^{(1)}}^{(1)})_{M_1+1,Q_i^{(1)}}];\cdots;\ [(\mathrm{b}_j^{(u)});\beta_j^{(u)})_{1,M_u}],[\iota_{i^{(u)}}(b_{ji^{(u)}}^{(u)};\beta_{ji^{(u)}}^{(u)})_{M_u+1,Q_i^{(u)}}]\end{matrix}\right)$$

$$=\frac{1}{(2\pi\omega)^u}\int_{L_1''}\cdots\int_{L_u''}\zeta(x_1,\cdots,x_u)\prod_{k=1}^{u}\phi_k(x_k)z_k''^{x_k}\,\mathrm{d}x_1\cdots\mathrm{d}x_u \tag{1.20}$$

with $\omega=\sqrt{-1}$

$$\zeta(x_1,\cdots,x_u)=\frac{\prod_{j=1}^{N}\Gamma(1-u_j''+\sum_{k=1}^{u}\mu_j^{(k)}x_k)}{\sum_{i=1}^{r'}[\iota_i\prod_{j=N+1}^{P_i}\Gamma(u_{ji}-\sum_{k=1}^{u}\mu_{ji}^{(k)}x_k)\prod_{j=1}^{Q_i}\Gamma(1-v_{ji}''+\sum_{k=1}^{u}v_{ji}^{(k)}x_k)]} \tag{1.21}$$

and

$$\phi_k'(x_k)=\frac{\prod_{j=1}^{M_k}\Gamma(b_j''^{(k)}-\beta_j''^{(k)}x_k)\prod_{j=1}^{N_k}\Gamma(1-a_j''^{(k)}+\alpha_j''^{(k)}x_k)}{\sum_{i^{(k)}=1}^{r^{(k)}}[\iota_{i^{(k)}}\prod_{j=M_k+1}^{Q_{i^{(k)}}}\Gamma(1-b''_{ji^{(k)}}{}^{(k)}+\beta''_{ji^{(k)}}{}^{(k)}x_k)\prod_{j=N_k+1}^{P_{i^{(k)}}}\Gamma(a_{ji^{(k)}}^{(k)}-\alpha_{ji^{(k)}}^{(k)}x_k)]} \tag{1.22}$$

Suppose , as usual , that the parameters

$u_j, j=1,\cdots,P; v_j, j=1,\cdots,Q;$

$a_j''^{(k)}, j=1,\cdots,N_k; a_{ji^{(k)}}^{(k)}, j=n_k+1,\cdots,P_{i^{(k)}};$

$b_{ji^{(k)}}^{(k)}, j=m_k+1,\cdots,Q_{i^{(k)}}; b_j^{(k)}, j=1,\cdots,M_k;$

with $k=1\cdots,u, i=1,\cdots,r'\ , i^{(k)}=1,\cdots,r^{(k)}$

are complex numbers , and the $\alpha's,\beta's,\gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i'^{(k)}=\sum_{j=1}^{N}\mu_j^{(k)}+\iota_i\sum_{j=N+1}^{P_i}\mu_{ji}^{(k)}+\sum_{j=1}^{N_k}\alpha_j''^{(k)}+\iota_{i^{(k)}}\sum_{j=N_k+1}^{P_{i^{(k)}}}\alpha_{ji^{(k)}}^{(k)}-\iota_i\sum_{j=1}^{Q_i}v_{ji}^{(k)}-\sum_{j=1}^{M_k}\beta_j''^{(k)}$$

$$-\iota_{i^{(k)}}\sum_{j=M_k+1}^{Q_{i^{(k)}}}\beta_{ji^{(k)}}^{(k)}\leqslant 0 \tag{1.23}$$

The reals numbers τ_i are positives for $i=1,\cdots,u$, $\iota_{i^{(k)}}$ are positives for $i^{(k)}=1\cdots r^{(k)}$

The contour L_k is in the t_k-p lane and run from $\sigma-i\infty$ to $\sigma+i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(b_j''^{(k)}-\beta_j''^{(k)}t_k)$ with $j=1$ to M_k are separated from those o $\Gamma(1-u_j''+\sum_{i=1}^{s}\mu_j^{(k)}t_k)$f with $j=1$ to N and $\Gamma(1-a_j''^{(k)}+\alpha_j''^{(k)}t_k)$ with $j=1$ to N_k to the left of the contour L_k'' . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k''|<\frac{1}{2}B_i^{(k)}\pi$, where

$$B_i^{(k)} = \sum_{j=1}^{N} \mu_j^{(k)} - \iota_i \sum_{j=N+1}^{P_i} \mu_{ji}^{(k)} - \iota_i \sum_{j=1}^{Q_i} \upsilon_{ji}^{(k)} + \sum_{j=1}^{N_k} \alpha_j''^{(k)} - \iota_{i^{(k)}} \sum_{j=N_k+1}^{P_{i^{(k)}}} \alpha_{ji^{(k)}}^{(k)}$$

$$+ \sum_{j=1}^{M_k} \beta_j''^{(k)} - \iota_{i^{(k)}} \sum_{j=M_k+1}^{q_{i^{(k)}}} \beta_{ji^{(k)}}^{(k)} > 0, \quad \text{with } k = 1, \cdots, s, i = 1, \cdots, u \;, i^{(k)} = 1, \cdots, r^{(k)} \tag{1.24}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1'', \cdots, z_u'') = 0(|z_1''|^{\alpha_1''}, \cdots, |z_u''|^{\alpha_s''}), max(|z_1''|, \cdots, |z_u''|) \to 0$$

$$\aleph(z_1'', \cdots, z_u'') = 0(|z_1''|^{\beta_1''}, \cdots, |z_u''|^{\beta_s''}), min(|z_1''|, \cdots, |z_u''|) \to \infty$$

where $k = 1, \cdots, z : \alpha_k'' = min[Re(b_j''^{(k)} / \beta_j''^{(k)})], j = 1, \cdots, M_k$ and

$$\beta_k'' = max[Re((a_j''^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, N_k$$

We will use these following notations in this paper

$$U' = P_i, Q_i, \iota_i; r' ; \; V' = M_1, N_1; \cdots ; M_u, N_u \tag{1.25}$$

$$W' = P_{i^{(1)}}, Q_{i^{(1)}}, \iota_{i^{(1)}}; r^{(1)}, \cdots, P_{i^{(u)}}, Q_{i^{(u)}}, \iota_{i^{(u)}}; r^{(u)} \tag{1.26}$$

$$A' = \{(u_j; \mu_j^{(1)}, \cdots, \mu_j^{(u)})_{1,N}\}, \{\iota_i(u_{ji}; \mu_{ji}^{(1)}, \cdots, \mu_{ji}^{(u)})_{N+1,P_i}\} \tag{1.27}$$

$$B' = \{\iota_i(v_{ji}; \upsilon_{ji}^{(1)}, \cdots, \upsilon_{ji}^{(u)})_{M+1,Q_i}\} \tag{1.28}$$

$$C' = (a_j''^{(1)}; \alpha_j''^{(1)})_{1,N_1} \;, \iota_{i^{(1)}} (a_{ji^{(1)}}^{(1)}; \alpha_{ji^{(1)}}^{(1)})_{N_1+1,P_{i^{(1)}}}, \cdots,$$

$$(a_j^{(u)}; \alpha_j^{(u)})_{1,N_u} \;, \iota_{i^{(u)}} (a_{ji^{(u)}}^{(u)}; \alpha_{ji^{(u)}}^{(u)})_{N_u+1,P_{i^{(u)}}} \tag{1.29}$$

$$D' = (b_j''^{(1)}; \beta_j''^{(1)})_{1,M_1}, \iota_{i^{(1)}} (b_{ji^{(1)}}^{(1)}; \beta_{ji^{(1)}}^{(1)})_{M_1+1,Q_{i^{(1)}}}, \cdots,$$

$$(b_j^{(u)}; \beta_j^{(u)})_{1,M_u}, \iota_{i^{(u)}} (\beta_{ji^{(u)}}^{(u)}; \beta_{ji^{(u)}}^{(u)})_{M_u+1,Q_{i^{(u)}}} \tag{1.30}$$

The multivariable Aleph-function write :

$$\aleph(z_1'', \cdots, z_u'') = \aleph_{U':W'}^{0,N:V'} \left(\begin{array}{c|c} z'_1 & A' : C' \\ \cdot & \cdot \; \cdot \; \cdot \\ \cdot & \\ z'u & B' : D' \end{array} \right) \tag{1.31}$$

Srivastava [4] introduced and defined a general class of multivariable polynomials as follows

$$S_{N_1,\cdots,N_v}^{M_1,\cdots,M_v}[y_1, \cdots, y_v] = \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_v=0}^{[N_v/M_v]} \frac{(-N_1)_{M_1 K_1}}{K_1!} \cdots \frac{(-N_v)_{M_v K_v}}{K_v!}$$

$$A[N_1,K_1;\cdots;N_v,K_v]y_1^{K_1}\cdots y_v^{K_v} \tag{1.32}$$

where $M_1,\cdots,M_v$ are arbitrary positive integers and the coefficients are $A[N_1,K_1;\cdots;N_v,K_v]$ arbitrary constants, real or complex.

2. Integral representation of Lauricella function of several variables

The Lauricella function $F_D^{(k)}$ is defined as

$$F_D^{(k)}\left[a,b_1,\cdots,b_k;c;x_1,\cdots,x_k\right] = \frac{\Gamma(c)}{\Gamma(a)\prod_{j=1}^k\Gamma(b_j)}\frac{1}{(2\pi\omega)^k}\int_{L_1}\cdots\int_{L_k}\frac{\Gamma\left(a+\sum_{j=1}^k\zeta_j\right)\Gamma(b_1+\zeta_1),\cdots,\Gamma(b_k+\zeta_k)}{\Gamma\left(c+\sum_{j=1}^k\zeta_j\right)}$$

$$\prod_{j=1}^k\Gamma(-\zeta_j)(-x_j)^{\zeta_i}\,\mathrm{d}\zeta_1\cdots\mathrm{d}\zeta_k \tag{2.1}$$

where $max\left[|arg(-x_1)|,\cdots,|arg(-x_k)|\right]<\pi, c\neq 0,-1,-2,\cdots$.

In order to evaluate a number of integrals of multivariable Aleph-functions, we first establish the formula

$$\int_a^b(t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^k(f_jt+g_j)^{\sigma_j}\mathrm{d}t = (b-a)^{\alpha+\beta-1}B(\alpha,\beta)\prod_{j=1}^k(af_j+g_j)^{\sigma_j}$$

$$\times F_D^{(k)}\left[\alpha,-\sigma_1,\cdots,-\sigma_k;\alpha+\beta;-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right] \tag{2.2}$$

where $a,b\in\mathbb{R}(a<b),\alpha,\beta,f_i,g_i,\sigma_i\in\mathbb{C},(i=1,\cdots,k)$; $min(Re(\alpha),Re(\beta))>0$ and

$$\max_{1\leqslant j\leqslant k}\left\{\left|\frac{(b-a)f_i}{af_i+g_i}\right|\right\}<1$$

$F_D^{(k)}$ is a Lauricella's function of k-variables, see Srivastava et al ([5], page 60)

The formula (2.2) can be establish by expanding $\prod_{j=1}^k(f_jt+g_j)^{\sigma_j}$ by means of the formula :

$$(1-z)^{-\alpha}=\sum_{r=0}^\infty\frac{(\alpha)_r}{r!}z^r(|z|<1) \tag{2.3}$$

integrating term by term with the help of the integral given by Saigo and Saxena [2, page 93, eq.(3.2)] and applying the definition of the Lauricella function $F_D^{(k)}$ [5, page 60].

3. Eulerian integral

In this section , we evaluate a general Eulerian integral with the product of two multivariable Aleph-functions and the I-function defined by Nambisan et al [1]. We given the expansion serie concerning the last function.

We note : $X_1=V;V';1,0;\cdots;1,0;Y_1=W;W';0,1;\cdots;0,1$ (3.1)

V,V',W and W' are defined respectively by (1.13), (1.14), (1.25) and (1.26).

$$A_1 = \left[1 - a - \sum_{l=1}^{r} \lambda_l \eta_{h_l,k_l} - \sum_{k=0}^{v} K_k \lambda'_k ; \gamma_1, \cdots, \gamma_s, \gamma'_1, \cdots, \gamma'_u, 1, \cdots, 1\right] \tag{3.2}$$

$$A_2 = \left[1 - b - \sum_{l=1}^{r} \mu_l \eta_{h_l,k_l} - \sum_{k=1}^{v} K_k \mu'_k ; \tau_1, \cdots, \tau_s, \tau'_1, \cdots, \tau'_u, 1, \cdots, 1\right] \tag{3.3}$$

$$A_3 = \left[1 + \rho_j + \sum_{l=1}^{r} a_j^{(l)} \eta_{h_l,k_l} + \sum_{k=1}^{v} K_k b_j^{(k)} ; c_j^{(1)}, \cdots, c_j^{(s)}, {c'_j}^{(1)}, \cdots, {c'_j}^{(u)}, 0, \cdots, 1, 0, \cdots, 0\right]_{1,h} \tag{3.4}$$

j

$$A_4 = \left[1 + \rho_j + \sum_{l=1}^{r} a_j^{(l)} \eta_{h_l,k_l} + \sum_{k=1}^{v} K_k b_j^{(k)} ; c_j^{(1)}, \cdots, c_j^{(s)}, {c'_j}^{(1)}, \cdots, {c'_j}^{(u)}, 0, \cdots, 0\right]_{1,h} \tag{3.5}$$

$$A_5 = \left[1 - a - b - \sum_{l=1}^{r} (\lambda_l + \mu_l) \eta_{g_l,k_l} - \sum_{k=1}^{v} K_k (\lambda'_k + \mu'_k) ; \gamma_1 + \tau_1, \cdots, \gamma_s + \tau_s,\right.$$

$$\left.\gamma'_u + \tau'_u, 1, \cdots, 1\right] \tag{3.6}$$

$$C_1 = C; C'; (1,0), \cdots, (1,0) ; D_1 = D; D'; (0,1), \cdots, (0,1) \tag{3.7}$$

$$P_1 = (n-m)^{a+b-1} \left\{ \prod_{j=1}^{h} (p_j m + q_j)^{\rho_j} \right\} \tag{3.8}$$

$$P_2 = (n-m)^{\sum_{l=1}^{r} (\lambda_l + \mu_l) \eta_{G_l,g_l} + \sum_{l=1}^{v} (\lambda'_l + \mu') K_l} \left\{ \prod_{j=1}^{h} (p_j m + q_j)^{\sum_{l=1}^{r} a_j^{(l)} \eta_{g_l,h_l} + \sum_{l=1}^{v} a_j^{\prime(l)} K_l} \right\} \tag{3.9}$$

$$A_v = \frac{(-N_1)_{M_1 K_1}}{K_1!} \cdots \frac{(-N_v)_{M_v K_v}}{K_v!} A[N_1, K_1; \cdots; N_v, K_v] \tag{3.10}$$

We have the following result

$$\int_m^n (t-m)^{a-1} (n-t)^{b-1} \prod_{j=1}^{h} (p_j t + q_j)^{\rho_j} S_{N_1,\cdots,N_v}^{M_1,\cdots,M_v} \left(\begin{array}{c} \mathrm{Z}_1 (t-m)^{\lambda'_1} (n-t)^{\mu'_1} \prod_{j=1}^{h} (p_j t + q_j)^{b'_j} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{Z}_v (t-m)^{\lambda'_v} (n-t)^{\mu'_v} \prod_{j=1}^{h} (p_j t + q_j)^{b_j^{(r)}} \end{array} \right)$$

$$\bar{I}\begin{pmatrix} \mathrm{z}_1(t-m)^{\lambda_1}(n-t)^{\mu_1}\prod_{j=1}^{h}(p_jt+q_j)^{a'_j} \\ . \\ . \\ . \\ \mathrm{z}_r(t-m)^{\lambda_r}(n-t)^{\mu_r}\prod_{j=1}^{h}(p_jt+q_j)^{a_j^{(r)}} \end{pmatrix} \aleph \begin{pmatrix} \mathrm{z'}_1(t-m)^{\gamma_1}(n-t)^{\tau_1}\prod_{j=1}^{h}(p_jt+q_j)^{-c_j^{(1)}} \\ . \\ . \\ . \\ \mathrm{z'}_s(t-m)^{\gamma_s}(n-t)^{\tau_s}\prod_{j=1}^{h}(p_jt+q_j)^{-c_j^{(s)}} \end{pmatrix}$$

$$\aleph\begin{pmatrix} \mathrm{z''}_1(t-m)^{\gamma'_1}(n-t)^{\tau'_1}\prod_{j=1}^{h}(p_jt+q_j)^{-c_j'^{(1)}} \\ . \\ . \\ . \\ \mathrm{z''}_u(t-m)^{\gamma'_u}(n-t)^{\tau'_u}\prod_{j=1}^{h}(p_jt+q_j)^{-c_j'^{(u)}} \end{pmatrix}\mathrm{d}t$$

$$= P_1\sum_{h_1=1}^{m_1}\cdots\sum_{h_r=1}^{m_r}\sum_{k_1=0}^{\infty}\cdots\sum_{k_r=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_v=0}^{[N_v/M_v]}\prod_{i=1}^{r}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i^{\eta_{h_i,k_i}}\prod_{k=1}^{v}Z^kP_2[\phi_1\left(\eta_{h_1,k_1},\cdots,\eta_{h_r,k_r}\right)]_{j\neq h_i}A_v$$

$$\aleph^{0,n;0,N;h+2:X_1}_{U;U;h+2,h+1:Y_1}\left(\begin{array}{c|c} \frac{z'_1(n-m)^{\gamma_1+\tau_1}}{\prod_{j=1}^{h}(mf_j+q_j)^{c_j^{(1)}}} & \\ \cdots & \mathrm{A}\ ;\mathrm{A'};\ \mathrm{A}_1,A_2,A_3:C_1 \\ \cdots & \\ \frac{z'_s(n-m)^{\gamma_s+\tau_s}}{\prod_{j=1}^{h}(mf_j+q_j)^{c_j^{(s)}}} & . \\ \cdots & . \\ \frac{z''_1(n-m)^{\gamma'_1+\tau'_1}}{\prod_{j=1}^{h}(mf_j+q_j)^{c_j'^{(1)}}} & . \\ \cdots & . \\ \cdots & . \\ \frac{z''_u(n-m)^{\gamma'_u+\tau'_u}}{\prod_{j=1}^{h}(mf_j+q_j)^{c_j'^{(u)}}} & . \\ \cdots & . \\ \frac{(n-m)p_1}{p_1m+q_1} & \mathrm{B};\mathrm{B'}\ ;\ \mathrm{A}_4,A_5:D_1 \\ \cdots & \\ \cdots & \\ \frac{(n-m)p_h}{p_hm+q_h} & \end{array}\right) \tag{3.11}$$

$A, A', B, B', X_1, Y_1, C_1$, D_1 and $\phi_1$are defined respectively by (1.15),(1.27),(1.16),(1.28),(3.1), (3.7) and (1.3)

Provided

(A) See the section 1

(B) $m, n \in \mathbb{N}, \gamma_i, \tau_i, \gamma'_i, \tau'_i, c_j^{(i)}, c_j'^{(i)}, \lambda_l, \lambda'_k, \mu'_k, \mu_l, a_j^{(l)}, b_j^{(k)} \in \mathbb{R}^+, \rho_j \in \mathbb{R}, p_i, q_i \in \mathbb{C}$

$(C)\ \max_{1\leqslant j\leqslant k}\left\{\left|\frac{(n-m)p_i}{mp_i+q_i}\right|\right\}<1$

(D) $$Re\big[a+\sum_{l=1}^{r}\lambda_l\min_{1\leqslant j\leqslant m_l}\frac{d_j^{(l)}}{\delta_j^{(l)}}+\sum_{i=1}^{s}\gamma_i\min_{1\leqslant j\leqslant m'_i}\frac{d_j'^{(i)}}{\delta_j'^{(i)}}+\sum_{i=1}^{u}\gamma'_i\min_{1\leqslant j\leqslant m''_i}\frac{b_j''^{(i)}}{\beta_j''^{(i)}}\big]>0$$

$$Re\big[b+\sum_{l=1}^{r}\mu_l\min_{1\leqslant j\leqslant m'_l}\frac{d_j^{(l)}}{\delta_j^{(l)}}+\sum_{i=1}^{s}\tau_i\min_{1\leqslant j\leqslant m'_i}\frac{d_j'^{(i)}}{\delta_j'^{(i)}}+\sum_{i=1}^{u}\tau'_i\min_{1\leqslant j\leqslant m''_i}\frac{b_j''^{(i)}}{\beta_j''^{(i)}}\big]>0$$

(E) $$U_i^{(k)}=\sum_{j=1}^{\mathfrak{n}'}\alpha'^{(k)}_j+\tau_i\sum_{j=\mathfrak{n}'+1}^{p'_i}\alpha_{ji}^{(k)}+\sum_{j=1}^{n'_k}\gamma'^{(k)}_j+\tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}-\tau_i\sum_{j=1}^{q'_i}\beta_{ji}^{(k)}-\sum_{j=1}^{m'_k}\delta'^{(k)}_j$$

$$-\tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)}\leqslant 0$$

$$U_i'^{(k)}=\sum_{j=1}^{N}\mu_j^{(k)}+\iota_i\sum_{j=N+1}^{P_i}\mu_{ji}^{(k)}+\sum_{j=1}^{N_k}\alpha''^{(k)}_j+\iota_{i^{(k)}}\sum_{j=N_k+1}^{P_{i^{(k)}}}\alpha_{ji^{(k)}}^{(k)}-\iota_i\sum_{j=1}^{Q_i}\upsilon_{ji}^{(k)}-\sum_{j=1}^{M_k}\beta''^{(k)}_j$$

$$-\iota_{i^{(k)}}\sum_{j=M_k+1}^{Q_{i^{(k)}}}\beta_{ji^{(k)}}^{(k)}\leqslant 0$$

(F) $\left|arg\left(z'_i\prod_{j=1}^{h}(p_jt+q_j)^{-c_j^{(i)}}\right)\right|<\frac{1}{2}A_i^{(k)}\pi$ **,** $A_i^{(k)}$ is defined by (1.12) and

$\left|arg\left(z''_i\prod_{j=1}^{h}(p_jt+q_j)^{-c_j'^{(i)}}\right)\right|<\frac{1}{2}B_i^{(k)}\pi$, $B_i^{(k)}$ is defined by (1.24)

Proof

To prove (3.11), first, we express in serie the multivariable I-function defined by Nambisan et al [1] with the help of (1.5), a class of multivariable polynomials defined by Srivastava $S_{N_1,\cdots,N_v}^{M_1,\cdots,M_v}[.]$, in serie with the help of (1.32), the Aleph-functions of s-variables and u-variables in terms of Mellin-Barnes type contour integral with the help of (1.8) and (1.20) respectively. Now interchange the order of summations and integrations (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process). We collect the power of (p_jt+q_j) with $j=1,\cdots,h$ and use the equations (2.1) and (2.2) and we obtain h-Mellin-Barnes contour integral and interpreting $(s+u+h)-$Mellin-barnes contour integral in multivariable Aleph-function, we obtain the desired result.

Remark :

If a) $\gamma_1=\cdots=\gamma_s=\gamma'_1=\cdots,\gamma'_u=0$ and $\lambda_1=\cdots=\lambda_r=\lambda'_1=\cdots=\lambda'_v=0$

b) $\tau_1=\cdots=\tau_s=\tau'_1=\cdots=\tau'_u=0$ and $\mu_1=\cdots=\mu_r=\mu'_1=\cdots=\mu'_v=0$

we obtain the similar formulas that (3.11) with the corresponding simplifications.

4. Particular case

1) $\tau,\tau_1,\cdots,\tau_s,\iota,\iota_1,\cdots,\iota_u\rightarrow 1$, the Aleph-function of s-variables and the Aleph-function of u-variables reduces respectively to I-function of s-variables and I-function of u-variables defined by Sharma et al [3],and we have :

$$\int_m^n (t-m)^{a-1}(n-t)^{b-1}\prod_{j=1}^h (p_jt+q_j)^{\rho_j} S_{N_1,\cdots,N_v}^{M_1,\cdots,M_v}\begin{pmatrix} \mathrm{Z}_1(t-m)^{\lambda'_1}(n-t)^{\mu'_1}\prod_{j=1}^h(p_jt+q_j)^{b'_j} \\ . \\ . \\ . \\ \mathrm{Z}_v(t-m)^{\lambda'_v}(n-t)^{\mu'_v}\prod_{j=1}^h(p_jt+q_j)^{b_j^{(r)}} \end{pmatrix}$$

$$I\begin{pmatrix} \mathrm{z}_1(t-m)^{\lambda_1}(n-t)^{\mu_1}\prod_{j=1}^h(p_jt+q_j)^{a'_j} \\ . \\ . \\ . \\ \mathrm{z}_r(t-m)^{\lambda_r}(n-t)^{\mu_r}\prod_{j=1}^h(p_jt+q_j)^{a_j^{(r)}} \end{pmatrix} I\begin{pmatrix} \mathrm{z'}_1(t-m)^{\gamma_1}(n-t)^{\tau_1}\prod_{j=1}^h(p_jt+q_j)^{-c_j^{(1)}} \\ . \\ . \\ . \\ \mathrm{z'}_s(t-m)^{\gamma_s}(n-t)^{\tau_s}\prod_{j=1}^h(p_jt+q_j)^{-c_j^{(s)}} \end{pmatrix}$$

$$I\begin{pmatrix} \mathrm{z''}_1(t-m)^{\gamma'_1}(n-t)^{\tau'_1}\prod_{j=1}^h(p_jt+q_j)^{-c_j'^{(1)}} \\ . \\ . \\ . \\ \mathrm{z''}_u(t-m)^{\gamma'_u}(n-t)^{\tau'_u}\prod_{j=1}^h(p_jt+q_j)^{-c_j'^{(u)}} \end{pmatrix}\mathrm{d}t$$

$$= P_1\sum_{h_1=1}^{m_1}\cdots\sum_{h_r=1}^{m_r}\sum_{k_1=0}^{\infty}\cdots\sum_{k_r=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_v=0}^{[N_v/M_v]}\prod_{i=1}^r\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i^{\eta_{h_i,k_i}}\prod_{k=1}^v Z^k P_2[\phi_1(\eta_{h_1,k_1},\cdots,\eta_{h_r,k_r})]_{j\neq h_i}A_v$$

$$I_{U;U;h+2,h+1:Y_1}^{0,n;0,N;h+2:X_1}\left(\begin{array}{c|c} \frac{z'_1(n-m)^{\gamma_1+\tau_1}}{\prod_{j=1}^h(mf_j+q_j)^{c_j^{(1)}}} & \mathrm{A};\mathrm{A'};\ \mathrm{A}_1, A_2, A_3 : C_1 \\ \cdots & . \\ \cdots & . \\ \frac{z'_s(n-m)^{\gamma_s+\tau_s}}{\prod_{j=1}^h(mf_j+q_j)^{c_j^{(s)}}} & . \\ \cdots & . \\ \frac{z''_1(n-m)^{\gamma'_1+\tau'_1}}{\prod_{j=1}^h(mf_j+q_j)^{c_j'^{(1)}}} & . \\ \cdots & . \\ \cdots & . \\ \frac{z''_u(n-m)^{\gamma'_u+\tau'_u}}{\prod_{j=1}^h(mf_j+q_j)^{c_j'^{(u)}}} & \\ \cdots & \\ \frac{(n-m)p_1}{p_1m+q_1} & \mathrm{B};\mathrm{B'};\ \mathrm{A}_4, A_5 : D_1 \\ \cdots & \\ \cdots & \\ \frac{(n-m)p_h}{p_hm+q_h} & \end{array}\right) \qquad (4.1)$$

under the same conditions that (3.11) with $\tau, \tau_1, \cdots, \tau_s, \iota, \iota_1, \cdots, \iota_u \to 1$

5. Conclusion

In view of the generality of the multivariable Aleph-function, the multivariable I-function defined by Nambisan et al [1] and a general class of polynomials on specializing the various parameters, involved therein, we can obtain from our results, several results involving remarkably wide variety of useful special functions of several variables and one variable, for example the multivariable H-function defined by Srivastava et al [6]. The results presented in this document would at once yield a very large number of results involving a large variety of special functions occuring in the problems of mechanics and mathematical physics.

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