Int Jr. of Mathematical Sciences & Applications
Vol. 6, No. 2, (July-December 2016)
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ISSN No: 2230-9888
www.ijmsa.yolasite.com

On general Eulerian integral of certain products of special functions and a class of multivariable polynomial VI

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ABSTRACT
The object of this paper is to establish an general Eulerian integral involving the product of two multivariable I-functions defined by Prasad [1], the multivariable I-function defined by Nambisan et al [2] and a class of multivariable polynomials which provide unification and extension of numerous results. We will study the particular case concerning the multivariable H-function.

Keywords: Eulerian integral, multivariable I-function, Lauricella function of several variables, multivariable H-function, class of multivariable polynomials.

2010 Mathematics Subject Classification. 33C60, 82C31

1.Introduction

The object of this paper is to establish an general Eulerian integral involving the multivariable I-functions defined by Prasad [1], a expansion of the multivariable I-function defined by Nambisan et al [2] and a class of multivariable polynomials defined by Srivastava et al [4] which provide unification and extension of numerous results.

First time, we define the multivariable $\bar{I}$ -function by :

$$\bar{I}(z_1,\cdots,z_r) = \bar{I}^{0,n:m_1,n_1;\cdots;m_r,n_r}_{p,q:p_1,q_1;\cdots;p_r,q_r}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_{n+11,p} : \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_{m+1,q} : \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}_j^{(1)},\gamma_j^{(1)};1)_{1,n_1},(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{n_1+1,p_1};\cdots;(c_j^{(r)},\gamma_j^{(r)};1)_{1,n_r},(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{n_r+1,p_r} \\ (\mathrm{d}_j^{(1)},\delta_j^{(1)};1)_{1,m_1},(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{m_1+1,q_1};\cdots;(d_j^{(r)},\delta_j^{(r)};1)_{1,m_r},(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{m_r+1,q_r} \end{array}\right) \tag{1.1}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi_1(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.2}$$

where $\phi_1(s_1,\cdots,s_r),\theta_i(s_i),i=1,\cdots,r$ are given by :

$$\phi_1(s_1,\cdots,s_r) = \frac{1}{\prod_{j=n+1}^{p}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)\prod_{j=m+1}^{q}\Gamma^{B_j}\left(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}s_j\right)} \tag{1.3}$$

$$\theta_i(s_i) = \frac{\prod_{j=1}^{n_i}\Gamma\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)\prod_{j=1}^{m_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)}{\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}-\delta_j^{(i)}s_i\right)} \tag{1.4}$$

$i = 1, \cdots, r$

Serie representation

If $z_i \neq 0; i = 1, \cdots, r$

$\delta_{h_i}^{(i)}(d_j^{(i)}+k_i) \neq \delta_j^{(i)}(\delta_{h_i}^{(i)}+\eta_i) for j \neq h_i, j, h_i = 1, \cdots, m_i (i = 1, \cdots, r), k_i, \eta_i = 0, 1, 2, \cdots (i = 1, \cdots, r)$, then

$$\bar{I}(z_1, \cdots, z_r) = \sum_{h_1=1}^{m_1}\cdots\sum_{h_r=1}^{m_r}\sum_{k_1=0}^{\infty}\cdots\sum_{k_r=0}^{\infty}\left[\phi_1\left(\frac{dh_1^{(1)}+k_1}{\delta h_1^{(1)}}, \cdots, \frac{dh_r^{(r)}+k_r}{\delta h_r^{(r)}}\right)\right]_{j\neq h_i}\prod_{i=1}^{r}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i^{\frac{dh_i+k_i}{\delta h_i}} \tag{1.5}$$

This result can be proved on computing the residues at the poles :

$$s_i = \frac{dh_i^{(i)}+k_i}{\delta h_i^{(i)}}, (h_i = 1, \cdots, m_i, k_i = 0, 1, 2, \cdots) for i = 1, \cdots, r \tag{1.6}$$

We may establish the the asymptotic expansion in the following convenient form :

$\bar{I}(z_1, \cdots, z_r) = 0(|z_1|^{\alpha_1}, \cdots, |z_r|^{\alpha_r}), max(|z_1|, \cdots, |z_r|) \rightarrow 0$

$\bar{I}(z_1, \cdots, z_r) = 0(|z_1|^{\beta_1}, \cdots, |z_r|^{\beta_r}), min(|z_1|, \cdots, |z_r|) \rightarrow \infty$

where $k = 1, \cdots, r : \alpha_k = min[Re(d_j^{(k)}/\delta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k = max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})], j = 1, \cdots, n_k$$

We will note $\eta_{h_i,k_i} = \dfrac{dh_i^{(i)}+k_i}{\delta h_i}$, $(h_i = 1, \cdots, m_i, k_i = 0, 1, 2, \cdots) for i = 1, \cdots, r$ (1.7)

The multivariable I-function of r-variables is defined by Prasad [1] in term of multiple Mellin-Barnes type integral :

$$I(z'_1, \cdots, z'_s) = I^{0,n_2;0,n_3;\cdots;0,n_s:m^{(1)},n^{(1)};\cdots;m^{(s)},n^{(s)}}_{p_2,q_2,p_3,q_3;\cdots;p_s,q_s:p^{(1)},q^{(1)};\cdots;p^{(s)},q^{(s)}}\left(\begin{array}{c|c} z'_1 & (a_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ z'_s & (b_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{l}(a_{sj};\alpha_{sj}^{(1)},\cdots,\alpha_{sj}^{(s)})_{1,p_s} : (a_j^{(1)},\alpha_j^{(1)})_{1,p^{(1)}};\cdots;(a_j^{(s)},\alpha_j^{(s)})_{1,p^{(s)}} \\ \\ (b_{rj};\beta_{sj}^{(1)},\cdots,\beta_{sj}^{(s)})_{1,q_s} : (b_j^{(1)},\beta_j^{(1)})_{1,q^{(1)}};\cdots;(b_j^{(s)},\beta_j^{(s)})_{1,q^{(s)}}\end{array}\right) \tag{1.8}$$

$$= \frac{1}{(2\pi\omega)^s} \int_{L'_1} \cdots \int_{L'_s} \phi(t_1, \cdots, t_s) \prod_{i=1}^{s} \zeta_i(t_i) z_i'^{t_i} \mathrm{d}t_1 \cdots \mathrm{d}t_s \tag{1.9}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [1]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|arg z'_i| < \frac{1}{2}\Omega_i \pi$, where

$$\Omega_i = \sum_{k=1}^{n^{(i)}} \alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}} \alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}} \beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}} \beta_k^{(i)} + \left(\sum_{k=1}^{n_2} \alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2} \alpha_{2k}^{(i)} \right) + \cdots +$$

$$\left(\sum_{k=1}^{n_s} \alpha_{sk}^{(i)} - \sum_{k=n_s+1}^{p_s} \alpha_{sk}^{(i)} \right) - \left(\sum_{k=1}^{q_2} \beta_{2k}^{(i)} + \sum_{k=1}^{q_3} \beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_s} \beta_{sk}^{(i)} \right) \tag{1.10}$$

where $i = 1, \cdots, s$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z'_1, \cdots, z'_s) = 0(|z'_1|^{\alpha_1}, \cdots, |z'_s|^{\alpha_r}), max(|z'_1|, \cdots, |z'_s|) \rightarrow 0$$

$$I(z'_1, \cdots, z'_s) \ = 0(|z'_1|^{\beta_1}, \cdots, |z'_s|^{\beta_r}), min(|z'_1|, \cdots, |z'_s|) \rightarrow \infty$$

where $k = 1, \cdots, r : \alpha'_k = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta'_k = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, n_k$$

We will use these following notations in this section :

$$I(z''_1, \cdots, z''_u) = I_{p'_2,q'_2,p'_3,q'_3;\cdots;p'_u,q'_u:p'^{(1)},q'^{(1)};\cdots;p'^{(u)},q'^{(u)}}^{0,n'_2;0,n'_3;\cdots;0,n'_u:m'^{(1)},n'^{(1)};\cdots;m'^{(u)},n'^{(u)}} \left(\begin{array}{c|c} \mathrm{z''}_1 & (\mathrm{a'}_{2j}; \alpha'^{(1)}_{2j}, \alpha'^{(2)}_{2j})_{1,p'_2}; \cdots ; \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z''}_u & (\mathrm{b'}_{2j}; \beta'^{(1)}_{2j}, \beta'^{(2)}_{2j})_{1,q'_2}; \cdots ; \end{array} \right.$$

$$\left.\begin{matrix} (\text{a'}_{uj};\alpha'^{(1)}_{uj},\cdots,\alpha'_{uj}{}^{(u)})_{1,p'_u}:(a'^{(1)}_j,\alpha'^{(1)}_j)_{1,p'^{(1)}};\cdots;(a'_j{}^{(u)},\alpha'_j{}^{(u)})_{1,p'^{(u)}} \\ \\ (\text{b'}_{uj};\beta'^{(1)}_{uj},\cdots,\beta'_{uj}{}^{(u)})_{1,q'_u}:(b'^{(1)}_j,\beta'^{(1)}_j)_{1,q'^{(1)}};\cdots;(b'_j{}^{(u)},\beta'_j{}^{(u)})_{1,q'^{(u)}} \end{matrix}\right) \tag{1.11}$$

$$=\frac{1}{(2\pi\omega)^u}\int_{L''_1}\cdots\int_{L''_u}\psi(x_1,\cdots,x_u)\prod_{i=1}^{u}\xi_i(x_i)z''^{x_i}_i\mathrm{d}x_1\cdots\mathrm{d}x_u \tag{1.12}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [1]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

where $|argz''_i|<\frac{1}{2}\Omega''_i\pi$,

$$\Omega'_i=\sum_{k=1}^{n'^{(i)}}\alpha'_k{}^{(i)}-\sum_{k=n'^{(i)}+1}^{p'^{(i)}}\alpha'_k{}^{(i)}+\sum_{k=1}^{m'^{(i)}}\beta'_k{}^{(i)}-\sum_{k=m^{(i)}+1}^{q'^{(i)}}\beta'_k{}^{(i)}+\left(\sum_{k=1}^{n'_2}\alpha'_{2k}{}^{(i)}-\sum_{k=n_2+1}^{p'_2}\alpha'_{2k}{}^{(i)}\right)$$

$$+\cdots+\left(\sum_{k=1}^{n'_u}\alpha'_{uk}{}^{(i)}-\sum_{k=n'_u+1}^{p'_u}\alpha'_{uk}{}^{(i)}\right)-\left(\sum_{k=1}^{q'_2}\beta'_{2k}{}^{(i)}+\sum_{k=1}^{q'_3}\beta'_{3k}{}^{(i)}+\cdots+\sum_{k=1}^{q'_u}\beta'_{uk}{}^{(i)}\right) \tag{1.13}$$

where $i=1,\cdots,u$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z''_1,\cdots,z''_u)=0(\,|z''_1|^{\alpha'_1},\cdots,|z''_u|^{\alpha'_s}\,)\,,max(\,|z''_1|\,,\cdots,|z''_u|\,)\rightarrow 0$$

$$I(z''_1,\cdots,z''_u)\;=0(\,|z''_1|^{\beta'_1}\;,\cdots,|z''_u|^{\beta'_s}\,)\,,min(\,|z''_1|\,,\cdots,|z''_u|\,)\rightarrow\infty$$

where $k=1,\cdots,z:\alpha''_k=min[Re(b'^{(k)}_j/\beta'^{(k)}_j)],j=1,\cdots,m'_k$ and

$$\beta''_k=max[Re((a'^{(k)}_j-1)/\alpha'^{(k)}_j)],j=1,\cdots,n'_k$$

Srivastava [4] introduced and defined a general class of multivariable polynomials as follows

$$S^{M_1,\cdots,M_v}_{N_1,\cdots,N_v}[y_1,\cdots,y_v]=\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_v=0}^{[N_v/M_v]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_v)_{M_vK_v}}{K_v!}$$

$$A[N_1,K_1;\cdots;N_v,K_v]y_1^{K_1}\cdots y_v^{K_v} \tag{1.14}$$

where $M_1,\cdots,M_v$ are arbitrary positive integers and the coefficients are $A[N_1,K_1;\cdots;N_v,K_v]$ arbitrary constants, real or complex.

2. Integral representation of Lauricella function of several variables

The Lauricella function $F_D^{(k)}$ is defined as

$$F_D^{(k)}\left[a,b_1,\cdots,b_k;c;x_1,\cdots,x_k\right]=\frac{\Gamma(c)}{\Gamma(a)\prod_{j=1}^k\Gamma(b_j)}\frac{1}{(2\pi\omega)^k}\int_{L_1}\cdots\int_{L_k}\frac{\Gamma\left(a+\sum_{j=1}^k\zeta_j\right)\Gamma(b_1+\zeta_1),\cdots,\Gamma(b_k+\zeta_k)}{\Gamma\left(c+\sum_{j=1}^k\zeta_j\right)}$$

$$\prod_{j=1}^k\Gamma(-\zeta_j)(-x_j)^{\zeta_i}\,\mathrm{d}\zeta_1\cdots\mathrm{d}\zeta_k \tag{2.1}$$

where $max\left[|arg(-x_1)|,\cdots,|arg(-x_k)|\right]<\pi, c\neq 0,-1,-2,\cdots$.

In order to evaluate a number of integrals of multivariable I-functions, we first establish the formula

$$\int_a^b(t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^k(f_jt+g_j)^{\sigma_j}\mathrm{d}t=(b-a)^{\alpha+\beta-1}B(\alpha,\beta)\prod_{j=1}^k(af_j+g_j)^{\sigma_j}$$

$$\times F_D^{(k)}\left[\alpha,-\sigma_1,\cdots,-\sigma_k;\alpha+\beta;-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right] \tag{2.2}$$

where $a,b\in\mathbb{R}(a<b),\alpha,\beta,f_i,g_i,\sigma_i\in\mathbb{C},(i=1,\cdots,k)$; $min(Re(\alpha),Re(\beta))>0$ and

$$\max_{1\leqslant j\leqslant k}\left\{\left|\frac{(b-a)f_i}{af_i+g_i}\right|\right\}<1$$

$F_D^{(k)}$ is a Lauricella's function of k-variables, see Srivastava et al ([5], page 60)

The formula (2.2) can be establish by expanding $\prod_{j=1}^k(f_jt+g_j)^{\sigma_j}$ by means of the formula :

$$(1-z)^{-\alpha}=\sum_{r=0}^{\infty}\frac{(\alpha)_r}{r!}z^r(|z|<1) \tag{2.3}$$

integrating term by term with the help of the integral given by Saigo and Saxena [3, page 93, eq.(3.2)] and applying the definition of the Lauricella function $F_D^{(k)}$ [5, page 60].

3. Eulerian integral

In this section , we evaluate a general Eulerian integral with the product of two multivariable I-functions defined by Prasad [1] and the I-function defined by Nambisan et al [2]. We given the expansion serie concerning the last function.

Let $U=p_2,q_2;p_3,q_3;\cdots;p_{s-1},q_{s-1};p'_2,q'_2;p'_3,q'_3;\cdots;p'_{u-1},q'_{u-1};0,0;\cdots;0,0$ (3.1)

$$V=0,n_2;0,n_3;\cdots;0,n_{s-1};0,n'_2;0,n'_3;\cdots;0,n'_{u-1};0,0;\cdots;0,0 \tag{3.2}$$

$$X=m^{(1)},n^{(1)};\cdots;m^{(s)},n^{(s)};m'^{(1)},n'^{(1)};\cdots;m'^{(u)},n'^{(u)};1,0;\cdots;1,0 \tag{3.3}$$

$$Y = p^{(1)}, q^{(1)}; \cdots ; p^{(s)}, q^{(s)}; p'^{(1)}, q'^{(1)}; \cdots ; p'^{(u)}, q'^{(u)}; 0, 1; \cdots ; 0, 1 \tag{3.4}$$

$$A = (a_{2k}; \alpha_{2k}^{(1)}, \alpha_{2k}^{(2)}); \cdots ; (a_{(s-1)k}; \alpha_{(s-1)k}^{(1)}, \alpha_{(s-1)k}^{(2)}, \cdots , \alpha_{(s-1)k}^{(s-1)}) ; (a'_{2k}; \alpha'^{(1)}_{2k}, \alpha'^{(2)}_{2k}); \cdots ;$$
$$(a'_{(u-1)k}; \alpha'^{(1)}_{(u-1)k}, \alpha'^{(2)}_{(u-1)k}, \cdots , \alpha'^{(u-1)}_{(u-1)k}) \tag{3.5}$$

$$B = (b_{2k}; \beta_{2k}^{(1)}, \beta_{2k}^{(2)}); \cdots ; (b_{(s-1)k}; \beta_{(s-1)k}^{(1)}, \beta_{(s-1)k}^{(2)}, \cdots , \beta_{(s-1)k}^{(s-1)}) ; (b'_{2k}; \beta'^{(1)}_{2k}, \beta'^{(2)}_{2k}); \cdots ;$$

$$(b'_{(u-1)k}; \beta'^{(1)}_{(u-1)k}, \beta'^{(2)}_{(u-1)k}, \cdots , \beta'^{(u-1)}_{(u-1)k}) \tag{3.6}$$

$$\mathfrak{A} = (a_{sk}; \alpha_{sk}^{(1)}, \alpha_{sk}^{(2)}, \cdots , \alpha_{sk}^{(s)}, 0, \cdots , 0, 0 \cdots , 0) \tag{3.7}$$

$$\mathfrak{A}' = (a'_{uk}; 0, \cdots , 0, \alpha'^{(1)}_{uk}, \alpha'^{(2)}_{uk}, \cdots , \alpha'^{(u)}_{uk}, 0, \cdots , 0) \tag{3.8}$$

$$\mathfrak{B} = (b_{sk}; \beta_{sk}^{(1)}, \beta_{sk}^{(2)}, \cdots , \beta_{sk}^{(s)}, 0, \cdots , 0, 0, \cdots , 0) \tag{3.9}$$

$$\mathfrak{B}' = (b'_{uk}; 0, \cdots , 0, \beta'^{(1)}_{uk}, \beta'^{(2)}_{uk}, \cdots , \beta'^{(u)}_{uk}, 0, \cdots , 0) \tag{3.10}$$

$$\mathfrak{A}_2 = (a_k^{(1)}, \alpha_k^{(1)})_{1,p^{(1)}}; \cdots ; (a_k^{(s)}, \alpha_k^{(s)})_{1,p^{(s)}}; (a'^{(1)}_k, \alpha'^{(1)}_k)_{1,p^{(1)}}; \cdots ; (a'^{(u)}_k, \alpha'^{(u)}_k)_{1,p'^{(u)}};$$
$$(1, 0); \cdots ; (1, 0) \tag{3.11}$$

$$\mathfrak{B}_2 = (b_k^{(1)}, \beta_k^{(1)})_{1,p^{(1)}}; \cdots ; (b_k^{(s)}, \beta_k^{(s)})_{1,p^{(s)}}; (b'^{(1)}_k, \beta'^{(1)}_k)_{1,p'^{(1)}}; \cdots ; (b'^{(u)}_k, \beta'^{(u)}_k)_{1,p'^{(u)}};$$

$$(0, 1); \cdots ; (0, 1) \tag{3.12}$$

$$A_1 = \left[1 - a - \sum_{l=1}^{r} \lambda_l \eta_{h_l,k_l} - \sum_{k=0}^{v} K_k \lambda'_k; \gamma_1, \cdots , \gamma_s, \gamma'_1, \cdots , \gamma'_u, 1, \cdots , 1\right] \tag{3.13}$$

$$A_2 = \left[1 - b - \sum_{l=1}^{r} \mu_l \eta_{h_l,k_l} - \sum_{k=1}^{v} K_k \mu'_k; \tau_1, \cdots , \tau_s, \tau'_1, \cdots , \tau'_u, 1, \cdots , 1\right] \tag{3.14}$$

$$A_3 = \left[1 + \rho_j + \sum_{l=1}^{r} a_j^{(l)} \eta_{h_l,k_l} + \sum_{k=1}^{v} K_k b_j^{(k)}; c_j^{(1)}, \cdots , c_j^{(s)}, {c'_j}^{(1)}, \cdots , {c'_j}^{(u)}, 0, \cdots , \underset{\text{j}}{1}, 0, \cdots , 0\right]_{1,h} \tag{3.15}$$

$$A_4 = \left[1 + \rho_j + \sum_{l=1}^{r} a_j^{(l)} \eta_{h_l,k_l} + \sum_{k=1}^{v} K_k b_j^{(k)}; c_j^{(1)}, \cdots , c_j^{(s)}, {c'_j}^{(1)}, \cdots , {c'_j}^{(u)}, 0, \cdots , 0\right]_{1,h} \tag{3.16}$$

$$A_5 = \left[1-a-b-\sum_{l=1}^{r}(\lambda_l+\mu_l)\eta_{g_l,k_l}-\sum_{k=1}^{v}K_k(\lambda'_k+\mu'_k);\gamma_1+\tau_1,\cdots,\gamma_s+\tau_s,\right.$$

$$\left.\gamma'_u+\tau'_u,1,\cdots,1\right] \tag{3.17}$$

$$P_1=(n-m)^{a+b-1}\left\{\prod_{j=1}^{h}(p_jm+q_j)^{\rho_j}\right\} \tag{3.18}$$

$$P_2=(n-m)^{\sum_{l=1}^{r}(\lambda_l+\mu_l)\eta_{G_l,g_l}+\sum_{l=1}^{v}(\lambda'_l+\mu')K_l}\left\{\prod_{j=1}^{h}(p_jm+q_j)^{-\sum_{l=1}^{r}a_j^{(l)}\eta_{g_l,h_l}-\sum_{l=1}^{v}a_j'^{(l)}K_l}\right\} \tag{3.19}$$

$$A_v=\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_v)_{M_vK_v}}{K_v!}A[N_1,K_1;\cdots;N_v,K_v] \tag{3.20}$$

We have the following result

$$\int_m^n(t-m)^{a-1}(n-t)^{b-1}\prod_{j=1}^{h}(p_jt+q_j)^{\rho_j}S_{N_1,\cdots,N_v}^{M_1,\cdots,M_v}\begin{pmatrix}\mathrm{Z}_1(t-m)^{\lambda'_1}(n-t)^{\mu'_1}\prod_{j=1}^{h}(p_jt+q_j)^{b'_j}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{Z}_v(t-m)^{\lambda'_v}(n-t)^{\mu'_v}\prod_{j=1}^{h}(p_jt+q_j)^{b_j^{(r)}}\end{pmatrix}$$

$$\bar{I}\begin{pmatrix}\mathrm{z}_1(t-m)^{\lambda_1}(n-t)^{\mu_1}\prod_{j=1}^{h}(p_jt+q_j)^{a'_j}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r(t-m)^{\lambda_r}(n-t)^{\mu_r}\prod_{j=1}^{h}(p_jt+q_j)^{a_j^{(r)}}\end{pmatrix}I\begin{pmatrix}\mathrm{z'}_1(t-m)^{\gamma_1}(n-t)^{\tau_1}\prod_{j=1}^{h}(p_jt+q_j)^{-c_j^{(1)}}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z'}_s(t-m)^{\gamma_s}(n-t)^{\tau_s}\prod_{j=1}^{h}(p_jt+q_j)^{-c_j^{(s)}}\end{pmatrix}$$

$$I\begin{pmatrix}\mathrm{z''}_1(t-m)^{\gamma'_1}(n-t)^{\tau'_1}\prod_{j=1}^{h}(p_jt+q_j)^{-c_j'^{(1)}}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z''}_u(t-m)^{\gamma'_u}(n-t)^{\tau'_u}\prod_{j=1}^{h}(p_jt+q_j)^{-c_j'^{(u)}}\end{pmatrix}\mathrm{d}t$$

$$= P_1 \sum_{h_1=1}^{m_1} \cdots \sum_{h_r=1}^{m_r} \sum_{k_1=0}^{\infty} \cdots \sum_{k_r=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_v=0}^{[N_v/M_v]} \prod_{i=1}^{r} \frac{(-)^{k_i}}{\delta h_i^{(i)} k_i!} z_i^{\eta_{h_i,k_i}} \prod_{k=1}^{v} Z^k P_2[\phi_1(\eta_{h_1,k_1}, \cdots, \eta_{h_r,k_r})]_{j \neq h_i} A_v$$

$$I^{V;0,n'+n''+h+2;X}_{U:p'+p''+h+2,q'+q''+h+1;Y} \left(\begin{array}{c|c} \frac{z'_1(n-m)^{\gamma_1+\tau_1}}{\prod_{j=1}^{h}(mf_j+q_j)^{c_j^{(1)}}} & \\ \cdots & \\ \cdots & \mathrm{A} \ ;\mathfrak{A}_1, \mathrm{A}_1, A_2, A_3 : \mathfrak{A}_2 \\ \frac{z'_s(n-m)^{\gamma_s+\tau_s}}{\prod_{j=1}^{h}(mf_j+q_j)^{c_j^{(s)}}} & \cdot \\ \cdots & \cdot \\ \frac{z''_1(n-m)^{\gamma'_1+\tau'_1}}{\prod_{j=1}^{h}(mf_j+q_j)^{c'^{(1)}_j}} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{z''_u(n-m)^{\gamma'_u+\tau'_u}}{\prod_{j=1}^{h}(mf_j+q_j)^{c'^{(u)}_j}} & \cdot \\ \cdots & \cdot \\ \frac{(n-m)p_1}{p_1 m+q_1} & \mathrm{B};\mathfrak{B}_1, \ \mathrm{A}_4, A_5 : \mathfrak{B}_2 \\ \cdots & \\ \cdots & \\ \frac{(n-m)p_h}{p_h m+q_h} & \end{array} \right) \quad (3.21)$$

$\phi_1$is defined respectively by (1.3)

Provided that

(A) $a'_{ij}, b'_{ik}, \in \mathbb{C} \ (i = 1, \cdots, s; j = 1, \cdots, p'_i; k = 1, \cdots, q'_i); a'^{(i)}_j, b'^{(k)}_j \in \mathbb{C}$

$(i = 1, \cdots, s; j = 1, \cdots, p'^{(i)}; k = 1, \cdots, q'^{(i)})$

$a''_{ij}, b''_{ik}, \in \mathbb{C} \ (i = 1, \cdots, u'; j = 1, \cdots, p''_i; k = 1, \cdots, q''_i); a''^{(i)}_j, b''^{(k)}_j, \in \mathbb{C}$

$(i = 1, \cdots, u; j = 1, \cdots, p''^{i)}; k = 1, \cdots, q''^{(i)})$

$\alpha'^{(k)}_{ij}, \beta'^{(k)}_{ij} \in \mathbb{R}^+ \ (i = 1, \cdots, s, j = 1, \cdots, p'_i, k = 1, \cdots, s) \, ; \alpha'^{(i)}_j, \beta'^{(i)}_i \ \in \mathbb{R}^+ \ (i = 1, \cdots, s; j = 1, \cdots, p'_i)$

$\alpha''^{(k)}_{ij}, \beta''^{(k)}_{ij} \in \mathbb{R}^+ \ (i = 1, \cdots, u, j = 1, \cdots, p''_i, k = 1, \cdots, u) \, ; \alpha''^{(i)}_j, \beta''^{(i)}_i \ \in \mathbb{R}^+ \ (i = 1, \cdots, u; j = 1, \cdots, p''_i)$

(B) $m, n \in \mathbb{N}, \gamma_i, \tau_i, \gamma'_i, \tau'_i, c^{(i)}_j, c'^{(i)}_j, \lambda_l, \lambda'_k, \mu'_k, \mu_l, a^{(l)}_j, b^{(k)}_j \in \mathbb{R}^+, \rho_j \in \mathbb{R}, p_i, q_i \in \mathbb{C}$

$(C) \max_{1 \leqslant j \leqslant k} \left\{ \left| \frac{(n-m)p_i}{mp_i + q_i} \right| \right\} < 1$

(D) $Re\big[a+\sum_{j=1}^{r}\lambda_j \min_{1\leqslant k\leqslant m_i}\frac{d_k^{(j)}}{\delta_k^{(j)}}+\sum_{j=1}^{s}\gamma_j \min_{1\leqslant k\leqslant m'_i}\frac{b_k^{(j)}}{\beta_k^{(j)}}+\sum_{j=1}^{u}\gamma'_j \min_{1\leqslant k\leqslant m''_i}\frac{b_k'^{(j)}}{\beta_k'^{(j)}}\big]>0$

$$Re\big[b+\sum_{j=1}^{r}\mu_j \min_{1\leqslant k\leqslant m'_i}\frac{d_k^{(j)}}{\delta_k^{(j)}}+\sum_{j=1}^{s}\tau_j \min_{1\leqslant k\leqslant m'_i}\frac{b_k^{(j)}}{\beta_k^{(j)}}+\sum_{j=1}^{u}\tau'_j \min_{1\leqslant k\leqslant m''_i}\frac{b_k'^{(j)}}{\beta_k'^{(j)}}\big]>0$$

(E) $\left|arg\left(z'_i\prod_{j=1}^{h}(p_jt+q_j)^{-c_j^{(i)}}\right)\right|<\frac{1}{2}\Omega_i\pi$ **,** Ω_i is defined by (1.10) and

$\left|arg\left(z''_i\prod_{j=1}^{h}(p_jt+q_j)^{-c_j'^{(i)}}\right)\right|<\frac{1}{2}\Omega'_i\pi$, Ω'_i is defined by (1.13)

Proof

To prove (3.21), first, we express in serie the multivariable I-function defined by Nambisan et al [2] with the help of (1.5), a class of multivariable polynomials defined by Srivastava $S_{N_1,\cdots,N_v}^{M_1,\cdots,M_v}[.]$ in serie with the help of (1.14), the I-functions of s-variables and u-variables defined by Prasad [1] in terms of Mellin-Barnes type contour integral with the help of (1.2) and (1.8) respectively. Now interchange the order of summations and integrations (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process). We collect the power of (p_jt+q_j) with $j=1,\cdots,h$ and use the equations (2.1) and (2.2) and we obtain h-Mellin-Barnes contour integral and interpreting $(s+u+h)-$Mellin-barnes contour integral in multivariable I-function, we obtain the desired result.

Remark :

If a) $\gamma_1=\cdots=\gamma_s=\gamma'_1=\cdots,\gamma'_u=0$ and $\lambda_1=\cdots=\lambda_r=\lambda'_1=\cdots=\lambda'_v=0$

b) $\tau_1=\cdots=\tau_s=\tau'_1=\cdots=\tau'_u=0$ and $\mu_1=\cdots=\mu_r=\mu'_1=\cdots=\mu'_v=0$

we obtain the similar formulas that (3.21) with the corresponding simplifications.

4.Particular case

1) Let $U=V=A=B=0$, the multivariable I-functions defined by Prasad reduces to multivariable H-functions defined by Srivastava et al [6] and we have.

$$\int_m^n(t-m)^{a-1}(n-t)^{b-1}\prod_{j=1}^{h}(p_jt+q_j)^{\rho_j}S_{N_1,\cdots,N_v}^{M_1,\cdots,M_v}\begin{pmatrix}\mathrm{Z}_1(t-m)^{\lambda'_1}(n-t)^{\mu'_1}\prod_{j=1}^{h}(p_jt+q_j)^{b'_j}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{Z}_v(t-m)^{\lambda'_v}(n-t)^{\mu'_v}\prod_{j=1}^{h}(p_jt+q_j)^{b_j^{(r)}}\end{pmatrix}$$

$$\bar{I}\begin{pmatrix} z_1(t-m)^{\lambda_1}(n-t)^{\mu_1}\prod_{j=1}^{h}(p_jt+q_j)^{a'_j} \\ . \\ . \\ . \\ z_r(t-m)^{\lambda_r}(n-t)^{\mu_r}\prod_{j=1}^{h}(p_jt+q_j)^{a_j^{(r)}} \end{pmatrix} H\begin{pmatrix} z'_1(t-m)^{\gamma_1}(n-t)^{\tau_1}\prod_{j=1}^{h}(p_jt+q_j)^{-c_j^{(1)}} \\ . \\ . \\ . \\ z'_s(t-m)^{\gamma_s}(n-t)^{\tau_s}\prod_{j=1}^{h}(p_jt+q_j)^{-c_j^{(s)}} \end{pmatrix}$$

$$H\begin{pmatrix} z''_1(t-m)^{\gamma'_1}(n-t)^{\tau'_1}\prod_{j=1}^{h}(p_jt+q_j)^{-c_j'^{(1)}} \\ . \\ . \\ . \\ z''_u(t-m)^{\gamma'_u}(n-t)^{\tau'_u}\prod_{j=1}^{h}(p_jt+q_j)^{-c_j'^{(u)}} \end{pmatrix}\mathrm{d}t$$

$$= P_1\sum_{h_1=1}^{m_1}\cdots\sum_{h_r=1}^{m_r}\sum_{k_1=0}^{\infty}\cdots\sum_{k_r=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_v=0}^{[N_v/M_v]}\prod_{i=1}^{r}\frac{(-)^{k_i}}{\delta h_i^{(i)}k_i!}z_i^{\eta_{h_i,k_i}}\prod_{k=1}^{v}Z^kP_2[\phi_1(\eta_{h_1,k_1},\cdots,\eta_{h_r,k_r})]_{j\neq h_i}A_v$$

$$H^{0,n'+n''+h+2;X}_{p'+p''+h+2,q'+q''+h+1;Y}\left(\begin{array}{c|c} \frac{z'_1(n-m)^{\gamma_1+\tau_1}}{\prod_{j=1}^{h}(mf_j+q_j)^{c_j^{(1)}}} & \\ \cdot\;\cdot\;\cdot & \\ \cdot\;\cdot\;\cdot & \mathfrak{A}_1, \mathrm{A}_1, A_2, A_3 : \mathfrak{A}_2 \\ \frac{z'_s(n-m)^{\gamma_s+\tau_s}}{\prod_{j=1}^{h}(mf_j+q_j)^{c_j^{(s)}}} & \cdot \\ \cdot\;\cdot\;\cdot & \cdot \\ \frac{z''_1(n-m)^{\gamma'_1+\tau'_1}}{\prod_{j=1}^{h}(mf_j+q_j)^{c_j'^{(1)}}} & \cdot \\ \cdot\;\cdot\;\cdot & \cdot \\ \cdot\;\cdot\;\cdot & \cdot \\ \frac{z''_u(n-m)^{\gamma'_u+\tau'_u}}{\prod_{j=1}^{h}(mf_j+q_j)^{c_j'^{(u)}}} & \cdot \\ \cdot\;\cdot\;\cdot & \cdot \\ \frac{(n-m)p_1}{p_1m+q_1} & \mathfrak{B}_1, \mathrm{A}_4, A_5 : \mathfrak{B}_2 \\ \cdot\;\cdot\;\cdot & \\ \cdot\;\cdot\;\cdot & \\ \frac{(n-m)p_h}{p_hm+q_h} & \end{array}\right) \tag{4.1}$$

under the same conditions and notations that (3.21) with $U = V = A = B = 0$

Remark:

By the following similar procedure, the results of this document can be extented to product of any finite number of multivariable I-functions defined by Prasad [1] and a class of multivariable polynomials defined by Srivastava [4].

5. Conclusion

In view of the generality of the multivariable I-function defined by Prasad [1], the multivariable I-function defined by Nambisan et al [1] and a general class of polynomials on specializing the various parameters, involved therein, we can obtain from our results, several results involving remarkably wide variety of useful special functions of several variables and one variable, for example the multivariable H-function defined by Srivastava et al [6]. The results presented in this document would at once yield a very large number of results involving a large variety of special functions occuring in the problems of mechanics and mathematical physics.

REFERENCES

[1] Prasad Y.N. Multivariable I-function , Vijnana Parishad Anusandhan Patrika 29 (1986) , page 231-237.

[2] Prathima J. Nambisan V. and Kurumujji S.K. A Study of I-function of Several Complex Variables, International Journalof Engineering Mathematics Vol(2014) , 2014 page 1-12

[3] Saigo M. and Saxena R.K. Unified fractional integral formulas for the multivariable H-function I. J.Fractional Calculus 15 (1999), page 91-107.

[4] Srivastava H.M. A multilinear generating function for the Konhauser set of biorthogonal polynomials suggested by Laguerre polynomial, Pacific. J. Math. 177(1985), page183-191

[5] Srivastava H.M. and Manocha H.L : A treatise of generating functions. Ellis. Horwood.Series. Mathematics and Applications 1984

[6] H.M. Srivastava And R.Panda. Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24(1975), p.119-137.