

## Fractional derivatives involving multivariable I-function II

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### ABSTRACT

In this document, we derive two key formulas for the fractional derivatives of the multivariable I-function defined by Prasad [2] which is defined by a multiple contour integral of Mellin-Barnes type and general a class of polynomial of several variables. Several particular cases are given

Keywords: Fractional derivative, multivariable I-function, multivariable H-function, general class of polynomials, Srivastava-Daoust function, generalized Leibnitz rule.

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### 1. Introduction and preliminaries.

Srivastava et al [3] have obtained a number of key formulas for the fractional derivatives of the multivariable H-function defined by Srivastava et al [6]. The main of this paper is obtained two formulas for the fractional derivatives of product of the multivariable I-function defined by Prasad [2] and a class of polynomial of several variables. The multivariable I-function of r-variables is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1, z_2, \dots, z_r) = I_{p_2, q_2, p_3, q_3; \dots; p_r, q_r; p^{(1)}, q^{(1)}; \dots; p^{(r)}, q^{(r)}}^{0, n_2; 0, n_3; \dots; 0, n_r; m^{(1)}, n^{(1)}; \dots; m^{(r)}, n^{(r)}} \left( \begin{array}{c|c} z_1 & (a_{2j}; \alpha'_{2j}, \alpha''_{2j})_{1, p_2}; \dots; \\ \cdot & \\ \cdot & \\ \cdot & (b_{2j}; \beta'_{2j}, \beta''_{2j})_{1, q_2}; \dots; \\ z_r & \end{array} \right)$$

$$(a_{rj}; \alpha_{rj}^{(1)}, \dots, \alpha_{rj}^{(r)})_{1, p_r} : (a_j^{(1)}, \alpha_j^{(1)})_{1, p^{(1)}}; \dots; (a_j^{(r)}, \alpha_j^{(r)})_{1, p^{(r)}} \Bigg) \tag{1.1}$$

$$(b_{rj}; \beta_{rj}^{(1)}, \dots, \beta_{rj}^{(r)})_{1, q_r} : (b_j^{(1)}, \beta_j^{(1)})_{1, q^{(1)}}; \dots; (b_j^{(r)}, \beta_j^{(r)})_{1, q^{(r)}} \Bigg)$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \phi(s_1, \dots, s_r) \prod_{i=1}^r \theta_i(t_i) z_i^{t_i} dt_1 \dots dt_r \tag{1.2}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [2]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$$|argz_i| < \frac{1}{2} \Omega_i \pi, \text{ where}$$

$$\Omega_i = \sum_{k=1}^{n^{(i)}} \alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}} \alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}} \beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}} \beta_k^{(i)} + \left( \sum_{k=1}^{n_2} \alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2} \alpha_{2k}^{(i)} \right) + \dots +$$

$$\left( \sum_{k=1}^{n_s} \alpha_{sk}^{(i)} - \sum_{k=n_s+1}^{p_s} \alpha_{sk}^{(i)} \right) - \left( \sum_{k=1}^{q_2} \beta_{2k}^{(i)} + \sum_{k=1}^{q_3} \beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_s} \beta_{sk}^{(i)} \right) \quad (1.3)$$

where  $i = 1, \dots, r$

The complex numbers  $z_i$  are not zero. Throughout this document, we assume the existence and absolute convergence conditions of the multivariable I-function. We may establish the asymptotic expansion in the following convenient form :

$$I(z_1, \dots, z_r) = O(|z_1|^{\alpha_1}, \dots, |z_r|^{\alpha_r}), \max(|z_1|, \dots, |z_r|) \rightarrow 0$$

$$I(z_1, \dots, z_r) = O(|z_1|^{\beta_1}, \dots, |z_r|^{\beta_r}), \min(|z_1|, \dots, |z_r|) \rightarrow \infty$$

where  $k = 1, \dots, r : \alpha'_k = \min[Re(b_j^{(k)} / \beta_j^{(k)})], j = 1, \dots, m_k$  and

$$\beta'_k = \max[Re((a_j^{(k)} - 1) / \alpha_j^{(k)})], j = 1, \dots, n_k$$

We will use these following notations in this paper :

$$U_r = p_2, q_2; p_3, q_3; \cdots; p_{r-1}, q_{r-1}; V_r = 0, n_2; 0, n_3; \cdots; 0, n_{r-1} \quad (1.4)$$

$$W_r = (p^{(1)}, q^{(1)}); \cdots; (p^{(r)}, q^{(r)}); X_r = (m^{(1)}, n^{(1)}); \cdots; (m^{(r)}, n^{(r)}) \quad (1.5)$$

$$A = (a_{2k}; \alpha_{2k}^{(1)}, \alpha_{2k}^{(2)}); \cdots; (a_{(r-1)k}; \alpha_{(r-1)k}^{(1)}, \alpha_{(r-1)k}^{(2)}, \cdots, \alpha_{(r-1)k}^{(r-1)}) \quad (1.6)$$

$$B = (b_{2k}; \beta_{2k}^{(1)}, \beta_{2k}^{(2)}); \cdots; (b_{(r-1)k}; \beta_{(r-1)k}^{(1)}, \beta_{(r-1)k}^{(2)}, \cdots, \beta_{(r-1)k}^{(r-1)}) \quad (1.7)$$

$$\mathfrak{A} = (a_{rk}; \alpha_{rk}^{(1)}, \alpha_{rk}^{(2)}, \cdots, \alpha_{rk}^{(r)}) : \mathfrak{B} = (b_{rk}; \beta_{rk}^{(1)}, \beta_{rk}^{(2)}, \cdots, \beta_{rk}^{(r)}) \quad (1.8)$$

$$A_1 = (a_k^{(1)}, \alpha_k^{(1)})_{1,p^{(1)}}; \cdots; (a_k^{(r)}, \alpha_k^{(r)})_{1,p^{(r)}}; B_1 = (b_k^{(1)}, \beta_k^{(1)})_{1,p^{(1)}}; \cdots; (b_k^{(r)}, \beta_k^{(r)})_{1,p^{(r)}} \quad (1.9)$$

The multivariable I-function of r-variables write :

$$I(z_1, \dots, z_r) = I_{U_r; 0, n_r; X_r}^{V_r; 0, n_r; X_r} \left( \begin{array}{c|c} z_1 & A; \mathfrak{A}; A_1 \\ \cdot & \\ \cdot & \\ \cdot & \\ z_r & B; \mathfrak{B}; B_1 \end{array} \right) \quad (1.10)$$

$$I(z'_1, z'_2, \dots, z'_s) = I_{p'_2, q'_2, p'_3, q'_3; \dots; p'_s, q'_s; p'^{(1)}, q'^{(1)}; \dots; p'^{(s)}, q'^{(s)}}^{0, n'_2; 0, n'_3; \dots; 0, n'_s; m'^{(1)}, n'^{(1)}; \dots; m'^{(s)}, n'^{(s)}} \left( \begin{array}{c|c} z'_1 & (a'_{2j}; \alpha'_{2j}^{(1)}, \alpha'_{2j}^{(2)})_{1, p'_2}; \cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ z'_s & (b'_{2j}; \beta'_{2j}^{(1)}, \beta'_{2j}^{(2)})_{1, q'_2}; \cdots; \end{array} \right)$$

$$\left( \begin{array}{l} (a'_{sj}; \alpha'^{(1)'}_{sj}, \dots, \alpha'^{(s)'}_{sj})_{1,p'_s} : (a'^{(1)'}_j, \alpha'^{(1)'}_j)_{1,p'(1)}; \dots; (a'^{(s)'}_j, \alpha'^{(s)'}_j)_{1,p'(s)} \\ (b'_{sj}; \beta'^{(1)'}_{sj}, \dots, \beta'^{(s)'}_{sj})_{1,q'_s} : (b'^{(1)'}_j, \beta'^{(1)'}_j)_{1,q'(1)}; \dots; (b'^{(s)'}_j, \beta'^{(s)'}_j)_{1,q'(s)} \end{array} \right) \quad (1.11)$$

$$= \frac{1}{(2\pi\omega)^s} \int_{L_1} \cdots \int_{L_s} \psi(t_1, \dots, t_s) \prod_{i=1}^s \xi_i(t_i) z_i^{t_i} dt_1 \cdots dt_s \quad (1.12)$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [2]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

where  $|\arg z'_i| < \frac{1}{2} \Omega'_i \pi$ ,

$$\begin{aligned} \Omega'_i = & \sum_{k=1}^{n'(i)} \alpha'_k{}^{(i)} - \sum_{k=n'(i)+1}^{p'(i)} \alpha'_k{}^{(i)} + \sum_{k=1}^{m'(i)} \beta'_k{}^{(i)} - \sum_{k=m'(i)+1}^{q'(i)} \beta'_k{}^{(i)} + \left( \sum_{k=1}^{n'_2} \alpha'_{2k}{}^{(i)} - \sum_{k=n_2+1}^{p'_2} \alpha'_{2k}{}^{(i)} \right) \\ & + \cdots + \left( \sum_{k=1}^{n'_s} \alpha'_{sk}{}^{(i)} - \sum_{k=n'_s+1}^{p'_s} \alpha'_{sk}{}^{(i)} \right) - \left( \sum_{k=1}^{q'_2} \beta'_{2k}{}^{(i)} + \sum_{k=1}^{q'_3} \beta'_{3k}{}^{(i)} + \cdots + \sum_{k=1}^{q'_s} \beta'_{sk}{}^{(i)} \right) \end{aligned} \quad (1.13)$$

where  $i = 1, \dots, s$

The complex numbers  $z_i$  are not zero. Throughout this document, we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z'_1, \dots, z'_s) = O(|z'_1|^{\alpha'_1}, \dots, |z'_s|^{\alpha'_s}), \max(|z'_1|, \dots, |z'_s|) \rightarrow 0$$

$$I(z'_1, \dots, z'_s) = O(|z'_1|^{\beta'_1}, \dots, |z'_s|^{\beta'_s}), \min(|z'_1|, \dots, |z'_s|) \rightarrow \infty$$

where  $k = 1, \dots, s : \alpha''_k = \min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \dots, m'_k$  and

$$\beta''_k = \max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \dots, n'_k$$

We will use these following notations in this paper :

$$U_s = p'_2, q'_2; p'_3, q'_3; \dots; p'_{s-1}, q'_{s-1}; V_s = 0, n_2; 0, n_3; \dots; 0, n_{s-1} \quad (1.14)$$

$$W_s = (p'^{(1)}, q'^{(1)}); \dots; (p'^{(s)}, q'^{(s)}); X_s = (m'^{(1)}, n'^{(1)}); \dots; (m'^{(s)}, n'^{(s)}) \quad (1.15)$$

$$A' = (a'_{2k}; \alpha'_{2k}{}^{(1)}, \alpha'_{2k}{}^{(2)}); \dots; (a'_{(s-1)k}; \alpha'_{(s-1)k}{}^{(1)}, \alpha'_{(s-1)k}{}^{(2)}, \dots, \alpha'_{(s-1)k}{}^{(s-1)}) \quad (1.16)$$

$$B' = (b'_{2k}; \beta'_{2k}^{(1)}, \beta'_{2k}^{(2)}); \dots; (b'_{(s-1)k}; \beta'_{(s-1)k}^{(1)}, \beta'_{(s-1)k}^{(2)}, \dots, \beta'_{(s-1)k}^{(s-1)}) \quad (1.17)$$

$$\mathfrak{A}' = (a'_{sk}; \alpha'_{sk}^{(1)}, \alpha'_{sk}^{(2)}, \dots, \alpha'_{sk}^{(s)}) : \mathfrak{B}' = (b'_{sk}; \beta'_{sk}^{(1)}, \beta'_{sk}^{(2)}, \dots, \beta'_{sk}^{(s)}) \quad (1.18)$$

$$A'_1 = (a'_k{}^{(1)}, \alpha'_k{}^{(1)})_{1,p^{(1)}}; \dots; (a'_k{}^{(s)}, \alpha'_k{}^{(s)})_{1,p^{(s)}}; B'_1 = (b'_k{}^{(1)}, \beta'_k{}^{(1)})_{1,p^{(1)}}; \dots; (b'_k{}^{(s)}, \beta'_k{}^{(s)})_{1,p^{(s)}} \quad (1.19)$$

The multivariable I-function write :

$$I(z'_1, \dots, z'_s) = I_{U_s; p'_s, q'_s; W_s}^{V_s; 0, n'_s; X_s} \left( \begin{matrix} z'_1 \\ \vdots \\ \vdots \\ z'_s \end{matrix} \middle| \begin{matrix} A' ; \mathfrak{A}' ; A'_1 \\ \vdots \\ B' ; \mathfrak{B}' ; B'_1 \end{matrix} \right) \quad (1.20)$$

The Riemann-Liouville fractional derivative (or integral ) of order  $\mu$  is defined as follows [1 ,page 49]

$$D_x^\mu f(x) = \begin{cases} \frac{1}{\Gamma(-\mu)} \int_0^x (x-t)^{-\mu-1} f(t) dt, Re(\mu) < 0 \\ \frac{d^m}{dx^m} [D_x^{\mu-m} \{f(x)\}], 0 \leq m, m \in \mathbb{N} \end{cases} \quad (1.21)$$

We have the following fractional derivative formula [1,page 67, eq. (4.4.4)] is also required :

$$D_x^\mu (x^\lambda) = \frac{\Gamma(\lambda+1)}{\Gamma(\lambda-\mu+1)} x^{\lambda-\mu}, Re(\lambda) > -1 \quad (1.22)$$

The generalized Leibnitz formula for fractional calculus is required in the following form [1,page 76, eq.(5.5.2)]

$$D_x^\mu [f(x)g(x)] = \sum_{l=0}^{\infty} \binom{\mu}{l} D_x^{\mu-l} [f(x)] D_x^l [g(x)] \quad (1.23)$$

$\mu$  is a real or complex arbitrary number.

$$\text{We have the binomial formula : } (x+a)^\lambda = a^\lambda \sum_{m=0}^{\infty} \binom{\lambda}{m} \left(\frac{x}{a}\right)^m ; \left|\frac{x}{a}\right| < 1 \quad (1.24)$$

Srivastava and Garg [4] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1, \dots, h_u} [z_1, \dots, z_u] = \sum_{R_1, \dots, R_u=0}^{h_1 R_1 + \dots + h_u R_u \leq L} (-L)_{h_1 R_1 + \dots + h_u R_u} B(E; R_1, \dots, R_u) \frac{z_1^{R_1} \dots z_u^{R_u}}{R_1! \dots R_u!} \quad (1.25)$$

## 2. Main results

### Theorem 1

$$D_x^\mu \{x^k(x^v + \zeta^v)^\lambda S_L^{h_1, \dots, h_u} [w_1 x^{\rho_1} (x^v + \zeta^v)^{b_1}, \dots, w_u x^{\rho_u} (x^v + \zeta^v)^{b_u}]\}$$

$$I [z_1 x^{a_1} (x^v + \zeta^v)^{\sigma_1}, \dots, z_r x^{a_r} (x^v + \zeta^v)^{\sigma_r}] = \zeta^{\lambda v} x^{k-\mu} \sum_{m=0}^{\infty} \sum_{R_1, \dots, R_u=0}^{h_1 R_1 + \dots + h_u R_u \leq L} \zeta^{v \sum_{i=1}^u b_i h_i} x^{\sum_{i=1}^u \rho_i h_i}$$

$$(-L)_{h_1 R_1 + \dots + h_u R_u} B(E; R_1, \dots, R_u) \frac{w_1^{R_1} \dots w_u^{R_u}}{R_1! \dots R_u!} I_{U_r: p_r+2, q_r+2; W_r}^{V_r; 0, n_r+2; X_r} \left( \begin{array}{c} z_1 \zeta^{v \sigma_1} x^{a_1} \\ \vdots \\ z_r \zeta^{v \sigma_r} x^{a_r} \end{array} \right)$$

$$\left. \begin{array}{l} A; (-\lambda - \sum_{i=1}^u R_i b_i; \sigma_1, \dots, \sigma_r), \quad (-k-mv - \sum_{i=1}^u R_i b_i; a_1, \dots, a_r), \mathfrak{A}; A_1 \\ B; (1-\lambda - \sum_{i=1}^u R_i b_i; \sigma_1, \dots, \sigma_r), (\mu - mv - k - \sum_{i=1}^u R_i b_i; a_1, \dots, a_r), \mathfrak{B}; B_1 \end{array} \right) \quad (2.1)$$

Provided that :

$$a) \min\{v, b_i, \rho_i, \sigma_j, a_j, \} > 0, i = 1, \dots, u; j = 1, \dots, r; \left| \arg \left( \frac{x^v}{\zeta^v} \right) \right| < \pi$$

$$b) \operatorname{Re} \left[ k + \sum_{i=1}^r a_i \min_{1 \leq j \leq m^{(i)}} \frac{b_j^{(i)}}{\beta_j^{(i)}} \right] > -1$$

$$c) |\arg(z_i)| < \frac{1}{2} \Omega_i \pi \quad (i = 1, \dots, r) \text{ where } \Omega_i \text{ is defined by (1.3)}$$

## Theorem 2

$$D_x^\mu \{x^k(x^v + \zeta^v)^\lambda S_L^{h_1, \dots, h_u} [w_1 x^{\rho_1} (x^v + \zeta^v)^{b_1}, \dots, w_u x^{\rho_u} (x^v + \zeta^v)^{b_u}]\}$$

$$I [z_1 x^{a_1} (x^v + \zeta^v)^{\sigma_1}, \dots, z_r x^{a_r} (x^v + \zeta^v)^{\sigma_r}] I^* [y_1 x^{\mu_1}, \dots, y_s x^{\mu_s}] \}$$

$$= \zeta^{\lambda v} x^{k-\mu} \sum_{l, m=0}^{\infty} \binom{\mu}{l} \frac{(-x^v/\zeta^v)^m}{m!} \sum_{R_1, \dots, R_u=0}^{h_1 R_1 + \dots + h_u R_u \leq L} (-L)_{h_1 R_1 + \dots + h_u R_u} B(E; R_1, \dots, R_u) \frac{w_1^{R_1} \dots w_u^{R_u}}{R_1! \dots R_u!}$$

$$\begin{aligned}
& \zeta^v \sum_{i=1}^u b_i h_i x^{\sum_{i=1}^u \rho_i h_i} I_{U_r: p_r+2, q_r+2; W_r}^{V_r; 0, n_r+2; X_r} \left( \begin{array}{c} z_1 \zeta^{v \sigma_1} x^{a_1} \\ \vdots \\ z_r \zeta^{v \sigma_r} x^{a_r} \end{array} \middle| \begin{array}{l} \text{A}; (-\lambda - m - \sum_{i=1}^u R_i b_i; \sigma_1, \dots, \sigma_r), \\ \text{B}; (1 - \lambda - m - \sum_{i=1}^u R_i b_i; \sigma_1, \dots, \sigma_r), \end{array} \right. \\
& \left. \begin{array}{l} (-mv - \sum_{i=1}^u R_i b_i; a_1, \dots, a_r), \mathfrak{A}; A_1 \\ (\mu - mv - \sum_{i=1}^u R_i b_i; a_1, \dots, a_r), \mathfrak{B}; B_1 \end{array} \right) I_{U_s: p'_s+1, q'_s+1; W_s}^{V_s; 0, n'_s+1; X_s} \left( \begin{array}{c} y_1 x^{\mu_1} \\ \vdots \\ y_s x^{\mu_s} \end{array} \middle| \right. \\
& \left. \begin{array}{l} \text{A}'; (-k; \mu_1, \dots, \mu_s), \mathfrak{A}'; A'_1 \\ \text{B}'; (\mu - k - l; \mu_1, \dots, \mu_s), \mathfrak{B}'; B'_1 \end{array} \right) \tag{2.2}
\end{aligned}$$

where  $I^*(., \dots, .)$  is the  $s$ -variables I-function (occurring on the right-hand side) without the additional parameters

$(-k; \mu_1, \dots, \mu_s)$  and  $(\mu - k - l; \mu_1, \dots, \mu_s)$

Provided that :

$$\text{a) } \min\{v, b_i, \rho_i, \sigma_j, a_j, \mu_k\} > 0, i = 1, \dots, u; j = 1, \dots, r; k = 1, \dots, s; \left| \arg \left( \frac{x^v}{\zeta^v} \right) \right| < \pi$$

$$\text{b) } \operatorname{Re} \left[ k + \sum_{i=1}^s \mu_i \min_{1 \leq j \leq m^{(i)}} \frac{b_j^{(i)}}{\beta_j^{(i)}} \right] > -1 ; \sum_{i=1}^r a_i \min_{1 \leq j \leq m^{(i)}} \frac{b_j^{(i)}}{\beta_j^{(i)}} > -1;$$

$$\text{c) } |\arg(z_i)| < \frac{1}{2} \Omega_i \pi \quad (i = 1, \dots, r) \text{ and } |\arg(w_i)| < \frac{1}{2} \Omega'_i \pi \quad (i = 1, \dots, s)$$

where  $\Omega_i^{(k)}$  and  $\Omega'_i^{(k)}$  are defined respectively by (1.3) and (1.13)

### Proof

To prove (2.1) we first replace a class of polynomial  $S_L^{h_1, \dots, h_u}[\cdot]$  in serie with the help of (1.25), the multivariable I-function occurring on the left-hand side by its Mellin-Barnes contour integral, collect the power of  $x$  and  $(x^v + \zeta^v)$  and apply the binomial expansion with the help of (1.24). Changing the order of integration and summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process). We can apply the formula (1.22) and interpret the resulting Mellin-Barnes contour integral as an I-function of  $r$ -variables.

To prove (2.2), we make use of the generalized Leibniz rule for fractional derivatives with the help of (1.23).

with  $f(x) = I^* [w_1 x^{\mu_1}, \dots, w_s x^{\mu_s}]$  and

$$g(x) = (x^v + \zeta^v)^\lambda S_L^{h_1, \dots, h_u} [w_1 x^{\rho_1} (x^v + \zeta^v)^{b_1}, \dots, w_u x^{\rho_u} (x^v + \zeta^v)^{b_u}]$$

$$I [z_1 x^{a_1} (x^v + \zeta^v)^{\sigma_1}, \dots, z_r x^{a_r} (x^v + \zeta^v)^{\sigma_r}]$$

and apply two special cases of the formula (2.1) when  $\mu \rightarrow \mu - l; \lambda = 0, \mu_i = a_i$  and  $\sigma_i \rightarrow 0; i = 1, \dots, r$  or when  $\mu = m (m \in \mathbb{N}_0)$ , we arrive at (2.2), the  $s$ -variables  $I^*$  involved in (2.2), being identical.

### 3. Multivariable H-function

In this section, we use the multivariable H-function defined by Srivastava et al [6].

Let  $U_r = V_r = A = B = 0$ , the multivariable I-function degenerates in multivariable H-function

#### Lemme 1

$$D_x^\mu \{x^k (x^v + \zeta^v)^\lambda S_L^{h_1, \dots, h_u} [w_1 x^{\rho_1} (x^v + \zeta^v)^{b_1}, \dots, w_u x^{\rho_u} (x^v + \zeta^v)^{b_u}]\}$$

$$H [z_1 x^{a_1} (x^v + \zeta^v)^{\sigma_1}, \dots, z_r x^{a_r} (x^v + \zeta^v)^{\sigma_r}] \left. \right\} = \zeta^{\lambda v} x^{k-\mu} \sum_{m=0}^{\infty} \sum_{R_1, \dots, R_u=0}^{h_1 R_1 + \dots + h_u R_u \leq L} \zeta^{v \sum_{i=1}^u b_i h_i} x^{\sum_{i=1}^u \rho_i h_i}$$

$$(-L)_{h_1 R_1 + \dots + h_u R_u} B(E; R_1, \dots, R_u) \frac{w_1^{R_1} \dots w_u^{R_u}}{R_1! \dots R_u!} H_{p_r+2, q_r+2; W_r}^{0, n_r+2; X_r} \left( \begin{matrix} z_1 \zeta^{v \sigma_1} x^{a_1} \\ \cdot \\ \cdot \\ \cdot \\ z_r \zeta^{v \sigma_r} x^{a_r} \end{matrix} \right)$$

$$\left. \begin{matrix} (-\lambda - \sum_{i=1}^u R_i b_i; \sigma_1, \dots, \sigma_r), & (-k-mv - \sum_{i=1}^u R_i b_i; a_1, \dots, a_r), \mathfrak{A}; A_1 \\ (1-\lambda - \sum_{i=1}^u R_i b_i; \sigma_1, \dots, \sigma_r), & (\mu - mv - k - \sum_{i=1}^u R_i b_i; a_1, \dots, a_r), \mathfrak{B}; B_1 \end{matrix} \right) \quad (3.1)$$

under the same conditions and notations that (2.1) with  $U_r = V_r = A = B = 0$

Let  $U_s = V_s = U_r = V_r = A = B = A' = B' = 0$ , the two multivariable I-functions degenerate in two multivariable H-functions.

#### Lemme 2

$$D_x^\mu \{x^k (x^v + \zeta^v)^\lambda S_L^{h_1, \dots, h_u} [w_1 x^{\rho_1} (x^v + \zeta^v)^{b_1}, \dots, w_u x^{\rho_u} (x^v + \zeta^v)^{b_u}]\}$$

$$H [z_1 x^{a_1} (x^v + \zeta^v)^{\sigma_1}, \dots, z_r x^{a_r} (x^v + \zeta^v)^{\sigma_r}] H^* [y_1 x^{\mu_1}, \dots, y_s x^{\mu_s}] \left. \right\}$$

$$\begin{aligned}
&= \zeta^{\lambda v} x^{k-\mu} \sum_{l,m=0}^{\infty} \binom{\mu}{l} \frac{(-x^v/\zeta^v)^m}{m!} \sum_{R_1, \dots, R_u=0}^{h_1 R_1 + \dots + h_u R_u \leq L} (-L)_{h_1 R_1 + \dots + h_u R_u} B(E; R_1, \dots, R_u) \frac{w_1^{R_1} \dots w_u^{R_u}}{R_1! \dots R_u!} \\
&\zeta^{v \sum_{i=1}^u b_i h_i} x^{\sum_{i=1}^u \rho_i h_i} H_{p_r+2, q_r+2; W_r}^{0, n_r+2; X_r} \left( \begin{array}{c} z_1 \zeta^{v \sigma_1} x^{a_1} \\ \vdots \\ z_r \zeta^{v \sigma_r} x^{a_r} \end{array} \middle| \begin{array}{l} (-\lambda - m - \sum_{i=1}^u R_i b_i; \sigma_1, \dots, \sigma_r), \\ (1-\lambda - m - \sum_{i=1}^u R_i b_i; \sigma_1, \dots, \sigma_r), \end{array} \right. \\
&\left. \begin{array}{l} (-mv - \sum_{i=1}^u R_i b_i; a_1, \dots, a_r), \mathfrak{A}; A_1 \\ (\mu - mv - \sum_{i=1}^u R_i b_i; a_1, \dots, a_r), \mathfrak{B}; B_1 \end{array} \right) H_{p'_s+1, q'_s+1; W_s}^{0, n'_s+1; X_s} \left( \begin{array}{c} y_1 x^{\mu_1} \\ \vdots \\ y_s x^{\mu_s} \end{array} \middle| \right. \\
&\left. \begin{array}{l} A'; (-k; \mu_1, \dots, \mu_s), \mathfrak{A}'; A'_1 \\ B'; (\mu - k - l; \mu_1, \dots, \mu_s), \mathfrak{B}'; B'_1 \end{array} \right) \tag{3.2}
\end{aligned}$$

under the same conditions and notations that (2.2) with  $U_s = V_s = U_r = V_r = A = B = A' = B' = 0$

#### 4. Particular cases

$$\text{If } B(L; R_1, \dots, R_u) = \frac{\prod_{j=1}^{\bar{A}} (a_j)_{R_1 \theta'_j + \dots + R_u \theta_j^{(u)}} \prod_{j=1}^{B'} (b'_j)_{R_1 \phi'_j} \dots \prod_{j=1}^{B^{(u)}} (b_j^{(u)})_{R_u \phi_j^{(u)}}}{\prod_{j=1}^{\bar{C}} (c_j)_{m_1 \psi'_j + \dots + m_u \psi_j^{(u)}} \prod_{j=1}^{D'} (d'_j)_{R_1 \delta'_j} \dots \prod_{j=1}^{D^{(u)}} (d_j^{(u)})_{R_u \delta_j^{(u)}}} \tag{4.1}$$

then the general class of multivariable polynomial  $S_L^{R_1, \dots, R_u} [x_1, \dots, x_u]$  reduces to generalized Lauricella function defined by Srivastava et al [5].

$$F_{\bar{C}:D'; \dots; D^{(u)}}^{1+\bar{A}:B'; \dots; B^{(u)}} \left( \begin{array}{c} w_1 \\ \dots \\ w_u \end{array} \middle| \begin{array}{l} [(-L); R_1, \dots, R_u] [(a); \theta', \dots, \theta^{(u)}] : [(b'); \phi']; \dots; [(b^{(u)}); \phi^{(u)}] \\ [(c); \psi', \dots, \psi^{(u)}] : [(d'); \delta']; \dots; [(d^{(u)}); \delta^{(u)}] \end{array} \right) \tag{4.2}$$

and we have the formulas

#### Lemme 3

$$\begin{aligned}
& D_x^\mu \left\{ x^k (x^v + \zeta^v)^\lambda F_{\bar{C}; D'; \dots; D^{(u)}}^{1+\bar{A}; B'; \dots; B^{(u)}} \left( \begin{array}{c} w_1 x_1^{\rho_1} (x^v + \zeta^v)^{b_1} \\ \vdots \\ w_u x_u^{\rho_u} (x^v + \zeta^v)^{b_u} \end{array} \right) \right. \\
& \left. [(-L); R_1, \dots, R_u] [(a); \theta', \dots, \theta^{(u)}] : [(b'); \phi']; \dots; [(b^{(u)}); \phi^{(u)}] \right. \\
& \quad \left. [(c); \psi', \dots, \psi^{(u)}] : [(d'); \delta']; \dots; [(d^{(u)}); \delta^{(u)}] \right) \\
& I [z_1 x^{a_1} (x^v + \zeta^v)^{\sigma_1}, \dots, z_r x^{a_r} (x^v + \zeta^v)^{\sigma_r}] \Big\} = \zeta^{\lambda v} x^{k-\mu} \sum_{m=0}^{\infty} \sum_{R_1, \dots, R_u=0}^{h_1 R_1 + \dots + h_u R_u \leq L} \zeta^{v \sum_{i=1}^u b_i h_i} x^{\sum_{i=1}^u \rho_i h_i} \\
& (-L)_{h_1 R_1 + \dots + h_u R_u} B(E; R_1, \dots, R_u) \frac{w_1^{R_1} \dots w_u^{R_u}}{R_1! \dots R_u!} I_{U_r; p_r+2, q_r+2; W_r}^{V_r; 0, n_r+2; X_r} \left( \begin{array}{c} z_1 \zeta^{v \sigma_1} x^{a_1} \\ \vdots \\ z_r \zeta^{v \sigma_r} x^{a_r} \end{array} \right) \\
& A; (-\lambda - \sum_{i=1}^u R_i b_i; \sigma_1, \dots, \sigma_r), \quad (-k-mv - \sum_{i=1}^u R_i b_i; a_1, \dots, a_r), \mathfrak{A}; A_1 \\
& B; (1-\lambda - \sum_{i=1}^u R_i b_i; \sigma_1, \dots, \sigma_r), (\mu - mv - k - \sum_{i=1}^u R_i b_i; a_1, \dots, a_r), \mathfrak{B}; B_1
\end{aligned} \tag{4.3}$$

$$\text{where } B(L; R_1, \dots, R_u) = \frac{\prod_{j=1}^{\bar{A}} (a_j)_{R_1 \theta'_j + \dots + R_u \theta_j^{(u)}} \prod_{j=1}^{B'} (b'_j)_{R_1 \phi'_j} \dots \prod_{j=1}^{B^{(u)}} (b_j^{(u)})_{R_u \phi_j^{(u)}}}{\prod_{j=1}^{\bar{C}} (c_j)_{m_1 \psi'_j + \dots + m_u \psi_j^{(u)}} \prod_{j=1}^{D'} (d'_j)_{R_1 \delta'_j} \dots \prod_{j=1}^{D^{(u)}} (d_j^{(u)})_{R_u \delta_j^{(u)}}} \tag{4.4}$$

under the same conditions that (2.1)

**Lemme 4**

$$D_x^\mu \left\{ x^k (x^v + \zeta^v)^\lambda F_{\bar{C}; D'; \dots; D^{(u)}}^{1+\bar{A}; B'; \dots; B^{(u)}} \left( \begin{array}{c} w_1 x_1^{\rho_1} (x^v + \zeta^v)^{b_1} \\ \vdots \\ w_u x_u^{\rho_u} (x^v + \zeta^v)^{b_u} \end{array} \right) \right.$$

$$\left[ \begin{array}{l} (-L); R_1, \dots, R_u [(a); \theta', \dots, \theta^{(u)}] : [(b'); \phi']; \dots : [(b^{(u)}); \phi^{(u)}] \\ [(c); \psi', \dots, \psi^{(u)}] : [(d'); \delta']; \dots : [(d^{(u)}); \delta^{(u)}] \end{array} \right)$$

$$I [z_1 x^{a_1} (x^v + \zeta^v)^{\sigma_1}, \dots, z_r x^{a_r} (x^v + \zeta^v)^{\sigma_r}] I^* [y_1 x^{\mu_1}, \dots, y_s x^{\mu_s}] \}$$

$$= \zeta^{\lambda v} x^{k-\mu} \sum_{l,m=0}^{\infty} \binom{\mu}{l} \frac{(-x^v/\zeta^v)^m}{m!} \sum_{R_1, \dots, R_u=0}^{h_1 R_1 + \dots + h_u R_u \leq L} (-L)_{h_1 R_1 + \dots + h_u R_u} B(E; R_1, \dots, R_u) \frac{w_1^{R_1} \dots w_u^{R_u}}{R_1! \dots R_u!}$$

$$\zeta^{v \sum_{i=1}^u b_i h_i} x^{\sum_{i=1}^u \rho_i h_i} I_{U_r; p_r+2, q_r+2; W_r}^{V_r; 0, n_r+2; X_r} \left( \begin{array}{l} z_1 \zeta^{v \sigma_1} x^{a_1} \\ \vdots \\ z_r \zeta^{v \sigma_r} x^{a_r} \end{array} \middle| \begin{array}{l} A; (-\lambda - m - \sum_{i=1}^u R_i b_i; \sigma_1, \dots, \sigma_r), \\ B; (1 - \lambda - m - \sum_{i=1}^u R_i b_i; \sigma_1, \dots, \sigma_r), \end{array} \right)$$

$$\left( \begin{array}{l} (-mv - \sum_{i=1}^u R_i b_i; a_1, \dots, a_r), \mathfrak{A}; A_1 \\ (\mu - mv - \sum_{i=1}^u R_i b_i; a_1, \dots, a_r), \mathfrak{B}; B_1 \end{array} \right) I_{U_s; p'_s+1, q'_s+1; W_s}^{V_s; 0, n'_s+1; X_s} \left( \begin{array}{l} y_1 x^{\mu_1} \\ \vdots \\ y_s x^{\mu_s} \end{array} \right)$$

$$\left( \begin{array}{l} A'; (-k; \mu_1, \dots, \mu_s), \mathfrak{A}'; A'_1 \\ B'; (\mu - k - l; \mu_1, \dots, \mu_s), \mathfrak{B}'; B'_1 \end{array} \right) \tag{4.2}$$

$$\text{where } B(L; R_1, \dots, R_u) = \frac{\prod_{j=1}^{\bar{A}} (a_j)_{R_1 \theta'_j + \dots + R_u \theta_j^{(u)}} \prod_{j=1}^{B'} (b'_j)_{R_1 \phi'_j} \dots \prod_{j=1}^{B^{(u)}} (b_j^{(u)})_{R_u \phi_j^{(u)}}}{\prod_{j=1}^{\bar{C}} (c_j)_{m_1 \psi'_j + \dots + m_u \psi_j^{(u)}} \prod_{j=1}^{D'} (d'_j)_{R_1 \delta'_j} \dots \prod_{j=1}^{D^{(u)}} (d_j^{(u)})_{R_u \delta_j^{(u)}}} \tag{4.6}$$

Under the same conditions that (2.2)

### 3. Conclusion

In this paper we have evaluated two formulas concerning the fractional derivatives of product of the multivariable I-function defined by Prasad [2] and a class of polynomial of several variables. The two formulae established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several

variables can be obtained.

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