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SECOND ORDER FUZZY W-HAUSDORFF SPACES

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ABSTRACT

In this paper the definition of W-Hausdorff space introduced by Warren,R.H.[2] is extended to second order fuzzy topological spaces in two different ways. It is shown that these two concepts are hereditary and productive. The relations between first order and second order concepts are studied. The behaviour of these concepts with regard to $i(\hat{\delta})$, $i_\varepsilon(\hat{\delta})$, $i^*(\hat{\delta})$, $\omega_2(\tau)$, $(\omega_2(\tau))_\varepsilon$ and $\omega_2^*(\tau)$ are also analysed.

INTRODUCTION

A **fuzzy set** on a set X is a map defined on X with values in I, where I is the closed unit interval [0, 1]. Equivalently fuzzy sets which are named as first order fuzzy sets in this study deal with crisply defined membership functions or degrees of membership. It is doubtful whether, for instance, human beings have or can have a crisp image of membership functions in their minds. Zadeh [8] therefore suggested the notion of a fuzzy set whose membership function itself is a fuzzy set. This leads to the following definition of a second order fuzzy set or a fuzzy set of type 2.
A **second order fuzzy set** on a nonempty set X is a map from X to I^I.

First order fuzzy sets are denoted by f, g, h, . . . and second order fuzzy sets are denoted by $\hat{f}, \hat{g}, \hat{h}, \ldots$.

In this paper the terms 'fuzzy set' and 'first order fuzzy set' are used synonymously.
Whenever a fuzzy set is considered without mentioning the order, it always refers to a first order fuzzy set.

Similar terminology applies to all concepts related to first order fuzzy sets.
Fundamental definitions and properties of second order fuzzy sets and second order fuzzy topological spaces are introduced in [5]. Six important and interesting connections $\not\subset_1$, $\not\subset_2$, $\not\subset_3$, $\not\subset_4$, $\not\subset_5$ and $\not\subset_6$ between first order and second order fuzzy topological spaces are also studied in [5].
Connections between crisp topological spaces and second order fuzzy topological spaces are given in [6].
With every crisp topology τ on a nonempty set X, three second order fuzzy topologies $\omega_2(\tau)$, $(\omega_2(\tau)_\varepsilon$ and $\omega_2^*(\tau)$ on X are associated. Also with every second order fuzzy topology $\hat{\delta}$ on a nonempty set X, three crisp topologies $i(\hat{\delta})$, $i_\varepsilon(\hat{\delta})$ and $i^*(\hat{\delta})$ on X are associated [6].
In this paper the definition of W-Hausdorff space introduced by Warren,R.H.[2] is extended to second order fuzzy topological spaces in two different ways. It is shown that these two concepts are hereditary and productive. The relations between first order and second order concepts are studied. The behaviour of these concepts with regard to $i(\hat{\delta})$, $i_\varepsilon(\hat{\delta})$, $i^*(\hat{\delta})$, $\omega_2(\tau)$, $(\omega_2(\tau))_\varepsilon$ and $\omega_2^*(\tau)$ are also analysed.

SECTION: 1 FUNDAMENTAL DEFINITIONS AND NOTATIONS

Definition : 1.1[5]

A second order Chang fuzzy topology $\hat{\delta}$ on a nonempty set X is a collection of second order fuzzy sets on X satisfying the following conditions :

(i) $\hat{0}, \hat{1} \in \hat{\delta}$ where, for any $x \in X$

$\hat{0}(x)$ = the zero function **0** on I $\hat{1}(x)$ = the constant function **1** on I

(ii) $\hat{f}_\lambda \in \hat{\delta}$ for each $\lambda \in \Lambda$ implies $(\bigvee_{\lambda \in \Lambda} \hat{f}_\lambda) \in \hat{\delta}$

(iii) $\hat{f}_i \in \hat{\delta}$ for $i = 1, 2, \ldots, m$ implies $(\bigwedge_{i=1}^{m} \hat{f}_i) \in \hat{\delta}$

The pair $(X, \hat{\delta})$ is called a **second order Chang fuzzy topological space**.

A **second order Lowen fuzzy topology** $\hat{\delta}$ on X is defined by replacing axiom (i) in the above definition by axiom (i)′.

(i)′ All constant function $\hat{\alpha} \in \hat{\delta}$ where, for any $x \in X$, $\hat{\alpha}(x)$ = the constant function α on I. The pair $(X, \hat{\delta})$ is called a **second order Lowen fuzzy topological space**. The elements of $\hat{\delta}$ are called **second order fuzzy open sets.**

Definition : 1.2[5]

Every first order fuzzy topology $\delta = \{f_\lambda / \lambda \in \Lambda\}$ on a non-empty set X defines a second order fuzzy topology $\hat{\delta} = \{\hat{f}_\lambda / f_\lambda \in \delta\}$ on X where $\hat{f}_\lambda(x)(\alpha) = f_\lambda(x)$, for every $x \in X$ and for every $\alpha \in I$. The correspondence $\delta \to \hat{\delta}$ is denoted as ϕ_1.

Let $\hat{\delta} = \{\hat{f}_\lambda / \lambda \in \Lambda\}$ be a second order fuzzy topology (Lowen) on X.

Fix $\alpha \in I$. The collection δ_α =the distinct elements of the collection $\{(f_\lambda)_\alpha / \hat{f}_\lambda \in \hat{\delta}\}$ defines a first order fuzzy topology on X where $(f_\lambda)_\alpha(x) = \hat{f}_\lambda(x)(\alpha)$, for every $x \in X$. ϕ_3 denotes the correspondence $\hat{\delta} \to \delta_\alpha$.

The collection $S^\diamond = \{(f_\lambda)^\diamond / \hat{f}_\lambda \in \hat{\delta}\}$ is a subbase for a first order fuzzy topology $\delta^\diamond$ on X where $(f_\lambda)^\diamond(x) = \bigvee_{\alpha \in I} \hat{f}_\lambda(x)(\alpha)$, for every $x \in X$. ϕ_4 denotes the correspondence $\hat{\delta} \to \delta^\diamond$

Given a second order fuzzy topology on a nonempty set X, the association ϕ_6, gives a way of getting another second order fuzzy topology on the same set X. That is, given a second order fuzzy topology $\hat{\delta} = \{\hat{f}_\lambda / \lambda \in \Lambda\}$ on a nonempty set X, the collection $\hat{\delta}_c = \{(\hat{f}_\lambda)_c / \hat{f}_\lambda \in \hat{\delta}\}$ is also a second order fuzzy topology on X.

Definition : 1.3 [6]

Let $(X, \hat{\delta})$ be a second order fuzzy topological space. Define

(1) $i_\varepsilon(\hat{\delta})$ to be the topology generated by the collection $\{(A_{\hat{f}})_\varepsilon / \hat{f} \in \hat{\delta}\}$ where for $\varepsilon \in (0, 1)$, $(A_{\hat{f}})_\varepsilon = \{x \in X / \hat{f}(x)^{-1}(\varepsilon, 1] = I\}$.

(2) $i^*(\hat{\delta})$ to be the topology generated by the collection $\{A_{\hat{f}} / \hat{f} \in \hat{\delta}\}$ where $(A_{\hat{f}}) = \{x \in X / \hat{f}(x)^{-1}(\varepsilon, 1] = I$ for some $\varepsilon \in (0, 1)\}$

(3) $i_\varepsilon(\hat{\delta})$ to be the topology having the collection $\{(A_{\hat{f}})_\varepsilon / \hat{f} \in \hat{\delta}, \varepsilon \in (0,1)\}$ as a sub basis.

Definition : 1.4 [6]

Let (X, τ) be a topological space. Define

(1) $(\omega_2(\tau))_\varepsilon$ to be the second order fuzzy topology generated by $\hat{K}_\varepsilon$ where for $\varepsilon \in (0, 1)$, $\hat{K}_\varepsilon = \{\hat{f} \in (I^I)^X / (A_{\hat{f}})_\varepsilon \in \tau\}$

(2) $\omega_2(\tau)$ to be the second order fuzzy topology generated by $\hat{K}$ where

$\hat{K} = \{\hat{f} \in (I^I)^X / (A_{\hat{f}})_\varepsilon \in \tau$, for every $\varepsilon \in (0, 1)\}$.

(3) $\omega_2{}^*(\tau)$ to be the second order fuzzy topology generated by $\hat{K}_*$ where

$\hat{K}_* = \{\hat{f} \in (I^I)^X / A_{\hat{f}} \in \tau\}$.

Definition : 1.5[2]

A fuzzy topological spaces (X, δ) is said to be **fuzzy W-Hausdorff** if $\forall$ x, y $\in$ X, x $\neq$ y, there exist f, g $\in \delta$ such that f (x) = 1, g (y) =1 and f Λ g = 0.

SECTION: 2 Second order fuzzy W-Hausdorff Spaces

Definition : 2.1

A second order fuzzy topological spaces $(X, \hat{\delta})$ is said to be **second order fuzzy W-Hausdorff of type 1, denoted by $(W - H)_1$,** if for every x, y $\in$ X, x $\neq$ y, there exist $\hat{f}, \hat{g} \in \hat{\delta}$ such that $\hat{f}(x) =$ the constant function **1,** $\hat{g}(y) =$ the constant function **1** and $\hat{f} \Lambda_1 \hat{g} = \hat{0}$ where $\hat{f} \Lambda_1 \hat{g} = \hat{0}$ means given x$\in$X, either $\hat{f}(x)=0$ or $\hat{g}(x)=0$

Definition : 2.2

A **second order fuzzy W-Hausdorff of type 2, denoted by $(W - H)_2$,** is defined by replacing the condition $\hat{f} \Lambda_1 \hat{g} = \hat{0}$ in the above definition by $\hat{f} \Lambda_2 \hat{g} = \hat{0}$ where $\hat{f} \Lambda_2 \hat{g} = \hat{0}$ means given x$\in$X and $\alpha \in$ I, either $\hat{f}(x)(\alpha)=0$ or $\hat{g}(x)(\alpha)=0$

Note

$(X, \hat{\delta})$ is $(W - H)_1 \Rightarrow (X, \hat{\delta})$ is $(W - H)_2$ since $\hat{f} \Lambda_1 \hat{g} \Rightarrow \hat{f} \Lambda_2 \hat{g}$.

But the converse need not be true.

Theorem : 2.3

Subspace of a $(W - H)_1$ space is $(W - H)_1$.

Proof

Let $(X, \hat{\delta})$ be a $(W - H)_1$ space. Let Y be a subspace of X.

Consider $y_1, y_2 \in Y$ such that $y_1 \neq y_2$. Then $y_1, y_2 \in X$. There exist $\hat{f}, \hat{g} \in \hat{\delta}$ such that $\hat{f}(y_1) = 1$, $\hat{g}(y_2) = 1$ and $\hat{f} \Lambda_1 \hat{g} = \hat{0}$.

$\therefore (\hat{f} / Y), (\hat{g} / Y) \in (\hat{\delta} / Y)$. Also $(\hat{f} / Y)(y_1) = 1$, $(\hat{g} / Y)(y_2) = 1$ And $(\hat{f} / Y) \Lambda (\hat{g} / Y) = \hat{0}$.

$\therefore (Y, \hat{\delta} /Y)$ is $(W - H)_1$.

Theorem : 2.4

Product of two $(W - H)_1$ space is $(W - H)_1$.

Proof

Let $(X, \hat{\delta}_1)$, $(Y, \hat{\delta}_2)$ be two $(W - H)_1$ spaces.

Consider two distinct points(x_1,y_1), $(x_2 ,y_2) \in X \times Y$. Either $x_1 \neq x_2$ or $y_1 \neq y_2$.

Assume $x_1 \neq x_2$. $\therefore$ There exist $\hat{f}, \hat{g} \in \hat{\delta}_1$ such that $\hat{f}(x_1) = \mathbf{1}$, $\hat{g}(x_2) = \mathbf{1}$ and $\hat{f} \Lambda_1 \hat{g} = \hat{0}$.

Therefore $\hat{f}\hat{1}$, $\hat{g}\hat{1} \in \hat{\delta}_1 \times \hat{\delta}_2$. Also $(\hat{f}\hat{1})(x_1,y_1) = \mathbf{1}$ and $(\hat{g}\hat{1})(x_2,y_2) = \mathbf{1}$

For any $(x,y) \in X \times Y$, $(\hat{f}\hat{1})(x,y) \neq \mathbf{0} \Rightarrow \hat{f}(x)(\alpha) \neq 0$ for some $\alpha \in$ I

$\Rightarrow \hat{f}(x) \neq \mathbf{0} \Rightarrow \hat{g}(x) \neq \mathbf{0} \Rightarrow (\hat{g}\hat{1})(x,y) = \mathbf{0} \quad \therefore (\hat{f}\hat{1}) \Lambda_1 (\hat{g}\hat{1}) = \hat{0}$

The proof in the other case is similar. Hence $(X \times Y, \hat{\delta}_1 \times \hat{\delta}_2)$ is $(W - H)_1$.

Definition : 2.5

Let $\{(X_\lambda, \hat{\delta}_\lambda) / \lambda \in \Lambda\}$ be a family of second order fuzzy topological spaces and $X = \prod_{\lambda\in\Lambda} X_\lambda$. The product topology on X is the one with basic second order fuzzy open sets of the form $\prod_{\lambda\in\Lambda} \hat{f}_\lambda$ where $\hat{f}_\lambda \in \hat{\delta}_\lambda$ and $\hat{f}_\lambda = \hat{1}$ except for finitely many λ's.

Here $(\prod_{\lambda\in\Lambda} \hat{f}_\lambda)((x_\lambda)_{\lambda\in\Lambda})(\alpha) = \Lambda_{\lambda\in\Lambda}\, \hat{f}_\lambda(x_\lambda)(\alpha)$, for every $(x_\lambda)_{\lambda\in\Lambda} \in \prod_{\lambda\in\Lambda} X_\lambda$ and for every $\alpha \in I$.

Theorem: 2.6

Arbitrary product of $(W - H)_1$ spaces is $(W - H)_1$ in the product topology.

Proof

Let $\{(X_\lambda, \hat{\delta}_\lambda) / \lambda \in \Lambda\}$ be a collection of $(W - H)_1$ spaces.

Let $X = \prod_{\lambda\in\Lambda} X_\lambda$ and $\hat{\delta} = \prod_{\lambda\in\Lambda} \hat{\delta}_\lambda$

Consider two distinct points $(x_\lambda)_{\lambda\in\Lambda}, (y_\lambda)_{\lambda\in\Lambda} \in \prod_{\lambda\in\Lambda} X_\lambda$. $\therefore x_\mu \neq y_\mu$ for some $\mu \in \Lambda$.

$\therefore$ There exist $\hat{f}_\mu, \hat{g}_\mu \in \hat{\delta}_\mu$ such that $\hat{f}_\mu(x_\mu) = \mathbf{1}$, $\hat{g}_\mu(y_\mu) = \mathbf{1}$ and $\hat{f}_\mu \Lambda_1 \hat{g}_\mu = \hat{0}$

Let $\hat{f} = \prod_{\lambda\in\Lambda} \hat{f}_\lambda$ where $\hat{f}_\lambda = \hat{1}$ for $\lambda \neq \mu$ and $\hat{g} = \prod_{\lambda\in\Lambda} \hat{g}_\lambda$ where $\hat{g}_\lambda = \hat{1}$ for $\lambda \neq \mu$.

Then $\hat{f}, \hat{g} \in \prod_{\lambda\in\Lambda} \hat{\delta}_\lambda$. Also $\hat{f}((x_\lambda)_{\lambda\in\Lambda}) = \hat{f}_\mu(x_\mu) = \mathbf{1}$, $\hat{g}((y_\lambda)_{\lambda\in\Lambda}) = \hat{g}_\mu(y_\mu) = \mathbf{1}$ and $\hat{f} \Lambda_1 \hat{g} = \hat{0}$

Hence $(X, \hat{\delta})$ is $(W - H)_1$.

Theorem: 2.7

(X, δ) is fuzzy W-Hausdorff iff $(X, \hat{\delta})$ is $(W - H)_1$ where $(X, \hat{\delta})$ is got from (X, δ) through the association ϕ_1.

Proof

Given δ, $\hat{\delta} = \{\hat{f} / f \in \delta\}$ where $\hat{f}(x)(\alpha) = f(x)$.

Assume (X, δ) is fuzzy W-Hausdorff. Consider $x, y \in X$ such that $x \neq y$. There exist $f, g \in \delta$ such that $f(x) = 1$, $g(y) = 1$ and $f \Lambda g = \mathbf{0}$. $\therefore \hat{f}, \hat{g} \in \hat{\delta}$. Also $\hat{f}(x) = \mathbf{1}$, $\hat{g}(y) = \mathbf{1}$

Claim: $f \Lambda g = \mathbf{0} \Rightarrow \hat{f} \Lambda_1 \hat{g} = \hat{0}$

For $z \in X$, $\hat{f}(z) \neq \mathbf{0} \Rightarrow \hat{f}(z)(\alpha) \neq 0$ for some $\alpha \in I$

$\Rightarrow f(z) \neq 0 \quad \Rightarrow g(z) = 0$ ($\because f \Lambda g = \mathbf{0}$) $\quad \Rightarrow \hat{g}(z) = \mathbf{0} \quad \therefore \hat{f} \Lambda_1 \hat{g} = \hat{0}$. Hence $(X, \hat{\delta})$ is $(W - H)_1$.

To prove the converse, it is enough to observe that $\hat{f} \Lambda_1 \hat{g} = \hat{0} \Rightarrow f \Lambda g = \mathbf{0}$.

Theorem: 2.8

If $(X, \hat{\delta})$ is $(W - H)_1$ then for every $\alpha \in I$, (X, δ_α) is fuzzy W-Hausdorff where (X, δ_α) is got from $(X, \hat{\delta})$ through the association ϕ_3.

Proof

Given $\hat{\delta}$, $\delta_\alpha = \{f_\alpha / \hat{f} \in \hat{\delta}\}$ where $f_\alpha(x) = \hat{f}(x)(\alpha)$, $\forall\, x \in X$.

Consider $x, y \in X$ such that $x \neq y$.

There exist $\hat{f}, \hat{g} \in \hat{\delta}$ such that $\hat{f}(x) = \mathbf{1}$, $\hat{g}(y) = \mathbf{1}$ and $\hat{f} \Lambda_1 \hat{g} = \hat{0}$

$\therefore f_\alpha, g_\alpha \in \delta_\alpha$. Also $f_\alpha(x) = 1$ and $g_\alpha(y) = 1$.

Claim: $\hat{f} \Lambda_1 \hat{g} = \hat{0}$ implies $f_\alpha \Lambda g_\alpha = \mathbf{0}$

For $z \in X$, $f_\alpha(z) \neq 0 \Rightarrow \hat{f}(z)(\alpha) \neq 0 \Rightarrow \hat{f}(z) \neq \mathbf{0} \Rightarrow \hat{g}(z) = \mathbf{0} \Rightarrow g_\alpha(z) = 0 \quad \therefore \quad f_\alpha \Lambda g_\alpha = \mathbf{0}$

Hence $(X, \delta\alpha)$ is fuzzy W-Hausdorff.

Theorem: 2.9

If $(X, \hat{\delta})$ is $(W - H)_1$, then $(X, \delta^\diamond)$ is fuzzy W-Hausdorff where $(X, \delta^\diamond)$ is got from $(X, \hat{\delta})$ through the association ϕ_4

Proof

Given $\hat{\delta}$, a subbase for $\delta^\diamond$ is $S^\diamond = \{f^\diamond / \hat{f} \in \hat{\delta}\}$ where $f^\diamond(x) = \bigvee_{\alpha\in I} \hat{f}_\lambda(x)(\alpha)$, $\forall\, x \in X$.

Hence, to prove the theorem it is enough to observe that for $x \in X$, $\hat{f}(x) = \mathbf{1} \Rightarrow f^\diamond(x) = 1$ and $\hat{f} \Lambda_1 \hat{g} = \hat{0} \Rightarrow f^\diamond \Lambda g^\diamond = \mathbf{0}$.

Theorem: 2.10

$(X, \hat{\delta})$ is $(W - H)_1$ iff $(X, (\hat{\delta})_c)$ is $(W - H)_1$ where $(X, (\hat{\delta})_c)$ is got from $(X, \hat{\delta})$ through the association ϕ_6.

Proof

The result follows by noting that for any $\hat{f}, \hat{g} \in \hat{\delta}$, $\hat{f} \Lambda_1 \hat{g} \in \hat{0}$ iff $(\hat{f})_c \Lambda_1 (\hat{g})_c = \hat{0}$.

Theorem: 2.11

If $(X, \hat{\delta})$ is $(W - H)_1$, then

(1) For $\varepsilon \in (0,1)$, $(X, i_\varepsilon(\hat{\delta}))$ is Hausdorff.
(2) $(X, i^*(\hat{\delta}))$ is Hausdorff.
(3) $(X, i(\hat{\delta}))$ is Hausdorff.

Proof

(1) Consider $x,y \in X$ such that $x \neq y$. there exist $\hat{f}, \hat{g} \in \hat{\delta}$ such that $\hat{f}(x)=1$, $\hat{g}(y)=1$ and $\hat{f} \wedge_1 \hat{g}=\hat{0}$.
$\hat{f}(x)=\mathbf{1} \Rightarrow (\hat{f}(x))^{-1}(\varepsilon,1] = I \quad \Rightarrow x \in (A_{\hat{f}})_\varepsilon$
Similarly $\hat{g}(y)=\mathbf{1} \Rightarrow y \in (A_{\hat{g}})_\varepsilon$
$\hat{f}, \hat{g} \in \hat{\delta} \Rightarrow (A_{\hat{f}})_\varepsilon , (A_{\hat{g}})_\varepsilon \in i_\varepsilon(\hat{\delta})$
Next to prove $(A_{\hat{f}})_\varepsilon \cap (A_{\hat{g}})_\varepsilon = \varphi$
$z \in (A_{\hat{f}})_\varepsilon \cap (A_{\hat{g}})_\varepsilon$
$\Rightarrow \hat{f}(z) \neq \mathbf{0}$ and $\hat{g}(z) \neq \mathbf{0}$ which is a contradiction since $\hat{f} \wedge_1 \hat{g}=\hat{0}$
Hence $(X, i_\varepsilon(\hat{\delta}))$ is Hausdorff.
Proofs of (2) and (3) are similar.

Theorem: 2.12

If (X, τ) is Hausdorff, then

(1) For $\varepsilon \in (0,1)$, $(X,(\omega_2(\tau))_\varepsilon)$ is $(W - H)_1$.
(2) $(X,\omega_2(\tau))$ is $(W - H)_1$.
(3) $(X,\omega_2^*(\tau))$ is $(W - H)_1$.

Proof

(1) For $x, y \in X$, $x \neq y$, there exist $U, V \in \tau$ such that $x \in U$, $y \in V$ and $U \cap V=\varphi$.
Then $\hat{\chi}_u(x) = \mathbf{1}$, $\hat{\chi}_v(y) = \mathbf{1}$
Next to prove $\hat{\chi}_u \wedge_1 \hat{\chi}_v = \hat{0}$
For $z \in X$, $\hat{\chi}_u(z) \neq \mathbf{0} \Rightarrow \hat{\chi}_u(z)=\mathbf{1} \Rightarrow z \in U \Rightarrow z \notin V \Rightarrow \hat{\chi}_v(z)=\mathbf{0} \quad \therefore \hat{\chi}_u \wedge_1 \hat{\chi}_v=\hat{0}$
Since $(A_{\hat{\chi}_u})_\varepsilon = U \in \tau$, $\hat{\chi}_u \in \hat{K}_\varepsilon$ which is a base of $(\omega_2(\tau))_\varepsilon$. Similarly $\hat{\chi}_v \in \hat{K}_\varepsilon$. Hence $(X,(\omega_2(\tau))_\varepsilon)$ is $(W - H)_1$. Proofs of (2) and (3) are similar.

Note: 2.13

The theorems 2.3 to 2.12 proved for the separation axiom $(W - H)_1$ have exact parallels for the separation axiom $(W - H)_2$ also.

However, the following two results are to be used to prove the theorems parallel to 2.7 and 2.10 for the separation axiom $(W - H)_2$.

(1) $f \wedge g = \mathbf{0}$ iff $\hat{f} \wedge_2 \hat{g} = \hat{0}$
(2) $\hat{f} \wedge_2 \hat{g} = \hat{0}$ iff $(\hat{f})_c \wedge_2 (\hat{g})_c = \hat{0}$

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