Int. J. of Mathematical Sciences and Applications,
Vol. 6, No. 2, December 2016

ISSN: 2230-9888
www.journalshub.com

Euler type double integrals involving Kampé de Férier function, general class of polynomials and multivariable I-function defined by Prasad

$F.Y.\ AYANT^1$

1 Teacher in High School , France
E-mail : fredericayant@gmail.com

ABSTRACT
The aim of the present document is to evaluate three double Euler type integrals involving Kampé de Fériet function, general class of polynomials and multivariable Aleph-function. Importance of our findings lies in the fact that they involve the multivariable I-function defined by Prasad [2], which are the sufficiently general in nature and are capable of yielding a large number of simpler and useful results merely by specializing the parameters in them.

KEYWORDS : I-function of several variables, double Euler type integrals, Kampé de Fériet function, general class of polynomials, multivariable H-function, Srivastava-Daoust function.

2010 Mathematics Subject Classification. 33C60, 82C31

1. Introduction and preliminaries.

The object of this document is to evaluate three double Eulerian integrals involving Kampé de Fériet function,general class of polynomial and the I-function of several variables defined by Prasad [2]. The multivariable I-function defined by Prasad [2] is an extension of the multivariable H-function defined by Srivastava et al [5]. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables I-function throughout our present study and will be defined and represented as follows.
The multivariable I-function is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,\cdots,z_r) = I^{0,n_2;0,n_3;\cdots;0,n_r:m',n';\cdots;m^{(r)},n^{(r)}}_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p',q';\cdots;p^{(r)},q^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{a}_{rj};\alpha'_{rj},\cdots,\alpha^{(r)}_{rj})_{1,p_r}:(a'_j,\alpha'_j)_{1,p'};\cdots;(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ \\ (\mathrm{b}_{rj};\beta'_{rj},\cdots,\beta^{(r)}_{rj})_{1,q_r}:(b'_j,\beta'_j)_{1,q'};\cdots;(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{array}\right) \tag{1.1}$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(t_1,\cdots,t_r)\prod_{i=1}^{s}\theta_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.2}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [2]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_i| < \frac{1}{2}\Omega_i\pi$, where

$$\Omega_i = \sum_{k=1}^{n^{(i)}} \alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}} \alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}} \beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}} \beta_k^{(i)} + \left(\sum_{k=1}^{n_2} \alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2} \alpha_{2k}^{(i)} \right) +$$

$$+ \left(\sum_{k=1}^{n_r} \alpha_{rk}^{(i)} - \sum_{k=n_r+1}^{p_r} \alpha_{rk}^{(i)} \right) - \left(\sum_{k=1}^{q_2} \beta_{2k}^{(i)} + \sum_{k=1}^{q_3} \beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_r} \beta_{rk}^{(i)} \right) \tag{1.3}$$

where $i = 1, \cdots, r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1, \cdots, z_r) = 0(|z_1|^{\gamma_1'}, \cdots, |z_r|^{\gamma_r'}), max(|z_1|, \cdots, |z_r|) \rightarrow 0$$

$$I(z_1, \cdots, z_r) = 0(|z_1|, \cdots, |z_r|^{\beta_s'}), min(|z_1| , \cdots, |z_r|) \rightarrow \infty$$

where $k = 1, \cdots, z : \alpha_k' = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k' = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, n_k$$

We will use these following notations in this paper :

$$U = p_2, q_2; p_3, q_3; \cdots ; p_{r-1}, q_{r-1}; V = 0, n_2; 0, n_3; \cdots ; 0, n_{s-1} \tag{1.4}$$

$$W = (p', q'); \cdots ; (p^{(r)}, q^{(r)}); X = (m', n'); \cdots ; (m^{(r)}, n^{(r)}) \tag{1.5}$$

$$A = (a_{2k}, \alpha_{2k}', \alpha_{2k}''); \cdots ; (a_{(r-1)k}, \alpha'_{(r-1)k}, \alpha_{(r-1)k}'', \cdots, \alpha_{(r-1)k}^{(r-1)}) \tag{1.6}$$

$$B = (b_{2k}, \beta_{2k}', \beta_{2k}''); \cdots ; (b_{(r-1)k}, \beta'_{(r-1)k}, \beta_{(r-1)k}'', \cdots, \beta_{(r-1)k}^{(r-1)}) \tag{1.7}$$

$$\mathfrak{A} = (a_{sk}; \alpha'_{rk}, \alpha_{rk}'', \cdots, \alpha_{rk}^r) : \mathfrak{B} = (b_{rk}; \beta'_{rk}, \beta_{rk}'', \cdots, \beta_{rk}^r) \tag{1.8}$$

$$A' = (a_k', \alpha_k')_{1,p'}; \cdots ; (a_k^{(r)}, \alpha_k^{(r)})_{1,p^{(r)}}; B' = (b_k', \beta_k')_{1,q'}; \cdots ; (b_k^{(r)}, \beta_k^{(r)})_{1,q^{(r)}} \tag{1.9}$$

The contracted form is :

$$I(z_1, \cdots, z_r) = I_{U:p_r,q_r;W}^{V;0,n_r;X} \left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A} ;\mathfrak{A}:\mathrm{A}' \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & \mathrm{B}; \mathfrak{B}: \mathrm{B}' \end{array} \right) \tag{1.10}$$

Srivastava and Garg [4] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1,\cdots,h_s}[z_1,\cdots,z_s]=\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)\frac{z_1^{R_1}\cdots z_s^{R_s}}{R_1!\cdots R_s!} \quad (1.11)$$

The coefficients $B(E;R_1,\cdots,R_s)$ are arbitrary constants, real or complex.

We will note

$$B_s=\frac{(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)}{R_1!\cdots R_s!} \quad (1.12)$$

2 . Results required :

Lemme 1

$$\int_0^1 x^{a-1}(1-x)^{b-1}{}_1F_1(c;f;sx)\, F_{G:H}^{D:E}\left[\begin{matrix}\text{(d)}:\ \text{(e')}\ ;\cdots;(e^{(n)});\\ \text{(g)}:\ \text{(h')}\ ;\cdots;(h^{(n)});\end{matrix} r_1x,\cdots,r_nx\right]\mathrm{d}x$$

$$=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}\sum_{m,m_1,\cdots,m_n=0}^{\infty}\frac{(a,m+m_1+\cdots+m_n)((d),m_1+\cdots m_n)(c,m)}{(a+b,m+m_1+\cdots+m_n)((g),m_1+\cdots m_n)(f,m)}$$

$$\frac{((e'),m_1)\cdots((e^{(n)}),m_n)s^m r_1^{m_1}\cdots r_m^{m_n}}{((h'),m_1)\cdots((h^{(n)}),m_n)m!m_1!\cdots m_n!} \quad (2.1)$$

Lemme 2

$$\int_0^1 x^{a-1}(1-x)^{b-1}{}_1F_1(c;f;sx)\, F_{G:H}^{D:E}\left[\begin{matrix}\text{(d)}:\ \text{(e')}\ ;\cdots;(e^{(n)});\\ \text{(g)}:\ \text{(h')}\ ;\cdots;(h^{(n)});\end{matrix} r_1x,r_2,\cdots,r_n\right]\mathrm{d}x$$

$$=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}\sum_{m,m_1,\cdots,m_n=0}^{\infty}\frac{((d),m_1+\cdots+m_n)(a,m)(c,m)}{((g),m_1+\cdots+m_n)(a+b,m+m_1)(f,m)}$$

$$\frac{((e'),m_1)\cdots((e^{(n)}),m_n)s^m r_1^{m_1}\cdots r_m^{m_n}}{((h'),m_1)\cdots((h^{(n)}),m_n)m!m_1!\cdots m_n!} \quad (2.2)$$

Lemme 3

$$\int_0^1 x^{a-1}(1-x)^{b-1}{}_1F_1(c;f;sx)\, F_{G:H}^{D:E}\left[\begin{matrix}\text{(d)}:\ \text{(e')}\ ;\cdots;(e^{(n)});\\ \text{(g)}:\ \text{(h')}\ ;\cdots;(h^{(n)});\end{matrix}\right.$$

$$r_1x,\cdots,r_kx,r_{k+1}(1-x),\cdots,r_n(1-x)\Big]\mathrm{d}x$$

$$=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}\sum_{m,m_1,\cdots,m_n=0}^{\infty}\frac{((d),m_1+\cdots+m_n)(a,m+m_1+\cdots+m_k)(c,m)(b;m_{k+1}+\cdots+m_n)}{(a+b,m+m_1\cdots+m_n)((g),m_1+\cdots+m_n)(f,m)}$$

$$\frac{((e'),m_1)\cdots((e^{(n)}),m_n)s^m r_1^{m_1}\cdots r_m^{m_n}}{((h'),m_1)\cdots((h^{(n)}),m_n)m!m_1!\cdots m_n!} \quad (2.3)$$

For more details , see Exton [1]

Lemme 4

$$\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)\mathrm{d}x=\frac{\pi\Gamma(c)\Gamma(a+b+1/2)\Gamma(c-a-b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)\Gamma(c-a+1/2)\Gamma(c-b+1/2)} \quad (2.4)$$

Where Re(c) > 0, Re(2c-a-b) > -1 , see Vyas and Rathie [6]

3. Main results

Theorem 1

$$\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}(1-y)^{\beta-1}{}_1F_1(\delta;f;s'y)$$

$$F_{G:H}^{D:E}\left[\begin{matrix}\text{(d) : (e') ; }\cdots\text{ ; }(e^{(n)});\\ \text{(g) : (h') ; }\cdots\text{ ; }(h^{(n)});\end{matrix}\ r_1y,\cdots,r_ny\right]S_L^{h_1,\cdots,h_s}\begin{pmatrix}\mathrm{a}_1x^{t_1}y^{n_1}(1-y)^{\phi_1}\\ \cdots\\ \mathrm{a}_sx^{t_s}y^{n_s}(1-y)^{\phi_s}\end{pmatrix}$$

$$I\begin{pmatrix}\mathrm{z}_1x^{\sigma_1}y^{\rho_1}(1-y)^{\lambda_1}\\ \cdots\\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}(1-y)^{\lambda_r}\end{pmatrix}\left|\begin{matrix}\text{A ;}\mathfrak{A}\text{ : A'}\\ \text{B ; }\mathfrak{B}\text{ : B'}\end{matrix}\right.\Bigg)\,\mathrm{d}x\mathrm{d}y=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{m,m_1,\cdots,m_n=0}^{\infty}$$

$$\frac{((d),m_1+\cdots m_n)(\delta,m)((e'),m_1)\cdots((e^{(n)}),m_n)s'^m r_1^{m_1}\cdots r_m^{m_n}}{((g),m_1+\cdots m_n)(f,m)((h'),m_1)\cdots((h^{(n)}),m_n)m!m_1!\cdots m_n!}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}B_s a^{R_1}\cdots a_s^{R_s}$$

$$I_{U:p_r+4,q_r+3;W}^{V;0,n_r+4;X}\left(\begin{matrix}\mathrm{z}_1\\ \cdots\\ \mathrm{z}_r\end{matrix}\ \left|\begin{matrix}\text{A; (1-c -}\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r),\quad\text{(1/2-c +a+b -}\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r),\\ \cdots\qquad\qquad\cdots\\ \text{B; (1/2-c+a-}\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r),\quad\text{(1/2-c+b-}\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r),\end{matrix}\right.\right.$$

$$\begin{matrix}(1-\beta-\sum_{i=1}^s R_i\phi_i;\lambda_1,\cdots,\lambda_r),\\ \cdots\\ (1-\alpha-\beta-\sum_{i=1}^s R_i(n_i+\phi_i)-m-\sum_{i=1}^n m_i;\rho_1+\lambda_1,\cdots,\rho_r+\lambda_r),\end{matrix}$$

$$\left. \begin{array}{l} (1 - \alpha - m - \sum_{i=1}^{n} m_i - \sum_{i=1}^{s} R_i n_i; \rho_1, \cdots, \rho_r), \mathfrak{A} : A' \\ \quad \cdots \\ \mathfrak{B} : \mathrm{B'} \end{array} \right) \tag{3.1}$$

Provided that :

1) $min\{t_i, n_i, \phi_i, \sigma_j; \rho_j, \lambda_j\} > 0; i = 1, \cdots, s; j = 1, \cdots, r$

2) $Re(c + \sum_{i=1}^{r} \sigma_i s_i + \sum_{i=1}^{s} t_i R_i) > 0$; $Re(c - a - b + \sum_{i=1}^{r} \sigma_i s_i + \sum_{i=1}^{s} t_i R_i) > -1/2$

3) $Re(\beta + \sum_{i=1}^{s} \phi_i R_i + \sum_{i=1}^{r} \lambda_i s_i) > 0$; $Re(c - a + \sum_{i=1}^{r} \sigma_i s_i + \sum_{i=1}^{s} t_i R_i) > -1/2$

4) $Re(c - b + \sum_{i=1}^{r} \sigma_i s_i + \sum_{i=1}^{s} t_i R_i) > -1/2$; $Re(\alpha + m + \sum_{i=1}^{n} m_i + \sum_{i=1}^{r} \rho_i s_i + \sum_{i=1}^{s} R_i n_i) > 0$

5) $Re(\alpha + \beta + m + \sum_{i=1}^{n} m_i + \sum_{i=1}^{r} (\rho_i + \lambda_i) s_i + \sum_{i=1}^{s} R_i (n_i + \phi_i)) > 0$

6) $|argz_i| < \frac{1}{2} \Omega_i \pi$ where Ω_i is given in (1.3)

Theorem 2

$$\int_0^1 \int_0^1 x^{c-1} (1-x)^{-1/2} {}_2F_1(a, b; a+b+1/2; x) y^{\alpha-1} (1-y)^{\beta-1} {}_1F_1(\delta; f; s'y)$$

$$F_{G:H}^{D:E} \left[\begin{array}{l} \text{(d) : (e') ; } \cdots ; (e^{(n)}); \\ \text{(g) : (h') ; } \cdots ; (h^{(n)}); \end{array} r_1 y, r_2 \cdots, r_n \right] S_L^{h_1, \cdots, h_s} \begin{pmatrix} \mathrm{a}_1 x^{t_1} y^{n_1} (1-y)^{\phi_1} \\ \cdots \\ \mathrm{a}_s x^{t_s} y^{n_s} (1-y)^{\phi_s} \end{pmatrix}$$

$$I \left(\begin{array}{l} \mathrm{z}_1 x^{\sigma_1} y^{\rho_1} (1-y)^{\lambda_1} \\ \cdots \\ \mathrm{z}_r x^{\sigma_r} y^{\rho_r} (1-y)^{\lambda_r} \end{array} \middle| \begin{array}{l} \text{A ;} \mathfrak{A} : \text{A'} \\ \text{B ; } \mathfrak{B} : \text{B'} \end{array} \right) \mathrm{d}x \mathrm{d}y = \frac{\pi \Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)} \sum_{m, m_1, \cdots, m_n = 0}^{\infty}$$

$$\frac{((d), m_1 + \cdots m_n)(\delta, m)((e'), m_1) \cdots ((e^{(n)}), m_n) s'^m r_1^{m_1} \cdots r_m^{m_n}}{((g), m_1 + \cdots m_n)(f, m)((h'), m_1) \cdots ((h^{(n)}), m_n) m! m_1! \cdots m_n!} \sum_{R_1, \cdots, R_s = 0}^{h_1 R_1 + \cdots h_s R_s \leqslant L} B_s a^{R_1} \cdots a_s^{R_s}$$

$$I_{U:p_r+4, q_r+3; W}^{V;0, n_r+4; X} \left(\begin{array}{c} \mathrm{z}_1 \\ \cdots \\ \mathrm{z}_r \end{array} \middle| \begin{array}{l} \text{A : (1-c -} \sum_{i=1}^{s} R_i t_i; \sigma_1, \cdots, \sigma_r), \quad \text{(1/2-c +a+b -} \sum_{i=1}^{s} R_i t_i; \sigma_1, \cdots, \sigma_r), \\ \quad \cdots \qquad\qquad \cdots \\ \text{B : (1/2-c+a-} \sum_{i=1}^{s} R_i t_i; \sigma_1, \cdots, \sigma_r), \quad \text{(1/2-c+b-} \sum_{i=1}^{s} R_i t_i; \sigma_1, \cdots, \sigma_r), \end{array} \right.$$

$$(1 - \beta - m_1 - \sum_{i=1}^{s} R_i\phi_i; \lambda_1, \cdots, \lambda_r),$$
$$\cdots$$
$$(1- \alpha - \beta - \sum_{i=1}^{s} R_i(n_i + \phi_i) - m - \sum_{i=1}^{n} m_i; \rho_1 + \lambda_1, \cdots, \rho_r + \lambda_r),$$

$$\left. \begin{matrix} (1 - \alpha - m - \sum_{i=1}^{s} R_i n_i; \rho_1, \cdots, \rho_r), \mathfrak{A} : A' \\ \cdots \\ \mathfrak{B} : B' \end{matrix} \right) \tag{3.2}$$

Provided that :

1) $min\{t_i, n_i, \phi_i, \sigma_j; \rho_j, \lambda_j\} > 0; i = 1, \cdots, s; j = 1, \cdots, r$

2) $Re(c + \sum_{i=1}^{r} \sigma_i s_i + \sum_{i=1}^{s} t_i R_i) > 0$; $Re(c - a - b + \sum_{i=1}^{r} \sigma_i s_i + \sum_{i=1}^{s} t_i R_i) > -1/2$

3) $Re(\beta + m_1 + \sum_{i=1}^{s} \phi_i R_i + \sum_{i=1}^{r} \lambda_i s_i) > 0$; $Re(c - a + \sum_{i=1}^{r} \sigma_i s_i + \sum_{i=1}^{s} t_i R_i) > -1/2$

4) $Re(c - b + \sum_{i=1}^{r} \sigma_i s_i + \sum_{i=1}^{s} t_i R_i) > -1/2$; $Re(\alpha + m + \sum_{i=1}^{r} \rho_i s_i + \sum_{i=1}^{s} R_i n_i) > 0$

5) $Re(\alpha + \beta + m + m_1 + \sum_{i=1}^{r} (\rho_i + \lambda_i) s_i + \sum_{i=1}^{s} R_i(n_i + \phi_i)) > 0$

6) $|argz_i| < \frac{1}{2}\Omega_i\pi$ where Ω_i is given in (1.3)

Theorem 3

$$\int_0^1 \int_0^1 x^{c-1}(1-x)^{-1/2} {}_2F_1(a, b; a + b + 1/2; x) y^{\alpha-1}(1-y)^{\beta-1} {}_1F_1(\delta; f; s'y)$$

$$F_{G:H}^{D:E} \left[\begin{matrix} (\mathrm{d}) : (\mathrm{e'}) ; \cdots ; (e^{(n)}); \\ (\mathrm{g}) : (\mathrm{h'}) ; \cdots ; (h^{(n)}); \end{matrix} r_1 x, \cdots, r_k x, r_{k+1}(1-x), \cdots, r_n(1-x) \right]$$

$$S_L^{h_1, \cdots, h_s} \begin{pmatrix} \mathrm{a}_1 x^{t_1} y^{n_1} (1-y)^{\phi_1} \\ \cdots \\ \mathrm{a}_s x^{t_s} y^{n_s} (1-y)^{\phi_s} \end{pmatrix} I \left(\begin{matrix} \mathrm{z}_1 x^{\sigma_1} y^{\rho_1} (1-y)^{\lambda_1} \\ \cdots \\ \mathrm{z}_r x^{\sigma_r} y^{\rho_r} (1-y)^{\lambda_r} \end{matrix} \left| \begin{matrix} \mathrm{A} ; \mathfrak{A} : \mathrm{A'} \\ \mathrm{B} ; \mathfrak{B} : \mathrm{B'} \end{matrix} \right. \right) \mathrm{d}x \mathrm{d}y$$

$$= \frac{\pi\Gamma(a + b + 1/2)}{\Gamma(a + 1/2)\Gamma(b + 1/2)} \sum_{m, m_1, \cdots, m_n = 0}^{\infty} \frac{((d), m_1 + \cdots m_n)(\delta, m)}{((g), m_1 + \cdots m_n)(f, m)}$$

$$\sum_{R_1, \cdots, R_s = 0}^{h_1 R_1 + \cdots h_s R_s \leqslant L} B_s a^{R_1} \cdots a_s^{R_s} \frac{((e'), m_1) \cdots ((e^{(n)}), m_n) s'^m r_1^{m_1} \cdots r_m^{m_n}}{((h'), m_1) \cdots ((h^{(n)}), m_n) m! m_1! \cdots m_n!}$$

$$I^{V;0,n_r+4;X}_{U:p_r+4,q_r+3;W}\left(\begin{array}{c} z_1 \\ \cdots \\ z_r \end{array}\middle| \begin{array}{l} \text{A: } (1\text{-c} -\sum_{i=1}^s R_i t_i;\sigma_1,\cdots,\sigma_r),\quad (1/2\text{-c} +\text{a+b} -\sum_{i=1}^s R_i t_i;\sigma_1,\cdots,\sigma_r), \\ \qquad\cdots \qquad\qquad\qquad\qquad\qquad\cdots \\ \text{B: } (1/2\text{-c+a-}\sum_{i=1}^s R_i t_i;\sigma_1,\cdots,\sigma_r),\quad (1/2\text{-c+b-}\sum_{i=1}^s R_i t_i;\sigma_1,\cdots,\sigma_r), \end{array}\right.$$

$$\begin{array}{c} (1 - \beta - \sum_{i=1}^{k+1} m_i - \sum_{i=1}^s R_i\phi_i;\lambda_1,\cdots,\lambda_r), \\ \cdots \\ (1\text{-}\ \alpha-\beta-\sum_{i=1}^s R_i(n_i+\phi_i)-m-\sum_{i=1}^n m_i;\rho_1+\lambda_1,\cdots,\rho_r+\lambda_r), \end{array}$$

$$\left.\begin{array}{c} (1 - \alpha - m - \sum_{i=1}^k m_i - \sum_{i=1}^s R_i n_i;\rho_1,\cdots,\rho_r),\mathfrak{A}:A' \\ \cdots \\ \mathfrak{B}:\text{B'} \end{array}\right) \tag{3.3}$$

Provided that :

1) $min\{t_i,n_i,\phi_i,\sigma_j;\rho_j,\lambda_j\}>0; i=1,\cdots,s; j=1,\cdots,r$

2) $Re(c+\sum_{i=1}^r\sigma_i s_i+\sum_{i=1}^s t_i R_i)>0$; $Re(c-a-b+\sum_{i=1}^r\sigma_i s_i+\sum_{i=1}^s t_i R_i)>-1/2$

3) $Re(\alpha+m+\sum_{i=1}^k m_i+\sum_{i=1}^s\phi_i R_i+\sum_{i=1}^r\lambda_i s_i)>0$; $Re(c-a+\sum_{i=1}^r\sigma_i s_i+\sum_{i=1}^s t_i R_i)>-1/2$

4) $Re(c-b+\sum_{i=1}^r\sigma_i s_i+\sum_{i=1}^s t_i R_i)>-1/2$; $Re(\beta+\sum_{i=k+1}^r m_i+\sum_{i=1}^r\lambda_i s_i+\sum_{i=1}^s R_i n_i)>0$

5) $Re(\alpha+\beta+m+\sum_{i=1}^n m_i+\sum_{i=1}^r(\rho_i+\lambda_i)s_i+\sum_{i=1}^s R_i(n_i+\phi_i))>0$

6) $|argz_i|<\frac{1}{2}\Omega_i\pi$ where Ω_i is given in (1.3)

Proof of (3.1): The equation (3.1) writes

$$\int_0^1 x^c(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)\Bigg[\int_0^1 y^{\alpha-1}(1-y)^{\beta-1}\ {}_1F_1(\delta;f;s'y)$$

$$F^{D:E}_{G:H}\left[\begin{array}{l}(\text{d}):(\text{e'});\cdots;(e^{(n)});\\(\text{g}):(\text{h'});\cdots;(h^{(n)});\end{array} r_1y,\cdots,r_ny\right]\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L} B_s\left[a_1x^{t_1}y^{n_1}(1-y)^{\phi_1}\right]^{R_1}$$

$$\cdots\quad [a_sx^{t_s}y^{n_s}(1-y)^{\phi_s}]^{R_s}\, I\left(\begin{array}{c} z_1x^{\sigma_1}y^{\rho_1}(1-y)^{\lambda_1} \\ \cdots \\ z_rx^{\sigma_r}y^{\rho_r}(1-y)^{\lambda_r}\end{array}\middle|\begin{array}{c}\text{A };\mathfrak{A}:\text{A'}\\ \text{B };\mathfrak{B}:\text{B'}\end{array}\right)\text{d}y\Bigg]\text{d}x \tag{3.4}$$

We first express the multivariable Aleph-function involving in L.H.S. of (3.4) in terms od Mellin-Barnes contour integral with the help of (1.1) and then interchange the order of integration. We find that L.H.S. of (3.4)

$$\frac{1}{(2\pi\omega)^r}\left(\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}B_s a^{R_1}\cdots a_s^{R_s}\right.$$

$$\left(\int_0^1 x^{c+\sigma_1 s_1+\cdots+\sigma_r s_r}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)\mathrm{d}x\right)$$

$$\left[\int_0^1 y^{a+\rho_1 s_1+\cdots+\rho_r s_r+n_1R_1+\cdots+n_sR_s-1}\,(1-y)^{\lambda_1 s_1+\cdots+\lambda_r s_r+\phi_1R_1+\cdots+\phi_sR_s-1}{}_1F_1(\delta;f;s'y)\right.$$

$$\left.\left.F_{G:H}^{D:E}\left[\begin{matrix}\text{(d) : (e') ; }\cdots\text{ ; }(e^{(n)});\\ \text{(g) : (h') ; }\cdots\text{ ; }(h^{(n)});\end{matrix}r_1y,\cdots,r_ny\right]\mathrm{d}y\right]\mathrm{d}s_1\cdots\mathrm{d}s_r\right) \tag{3.6}$$

Now using the result (2.1) to evaluate the y-integral and (2.4) to evaluate the x-integral and reinterpreting the multiple contour integral so obtained with the help of (3.1) , we arrive at the R.H.S. of (3.1).
The results (3.2) and (3.3) can be proved similarly with the help of results given by (2.2), (2.3) and (2.4).

4. Multivariable I-function

If $U=V=A=B=0$, the multivariable I-function defined by Prasad [2] reduces to the multivariable H-function defined by Srivastava et al [5]. We have the following results.

Corollary 1

$$\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}(1-y)^{\beta-1}{}_1F_1(\delta;f;s'y)$$

$$F_{G:H}^{D:E}\left[\begin{matrix}\text{(d) : (e') ; }\cdots\text{ ; }(e^{(n)});\\ \text{(g) : (h') ; }\cdots\text{ ; }(h^{(n)});\end{matrix}r_1y,\cdots,r_ny\right]S_L^{h_1,\cdots,h_s}\begin{pmatrix}\mathrm{a}_1x^{t_1}y^{n_1}(1-y)^{\phi_1}\\ \cdots\\ \mathrm{a}_sx^{t_s}y^{n_s}(1-y)^{\phi_s}\end{pmatrix}$$

$$H\left(\begin{matrix}\mathrm{z}_1x^{\sigma_1}y^{\rho_1}(1-y)^{\lambda_1}\\ \cdots\\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}(1-y)^{\lambda_r}\end{matrix}\,\middle|\,\begin{matrix}\mathfrak{A}\text{ : A'}\\ \mathfrak{B}\text{ : B'}\end{matrix}\right)\mathrm{d}x\mathrm{d}y=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{m,m_1,\cdots,m_n=0}^{\infty}$$

$$\frac{((d),m_1+\cdots m_n)(\delta,m)((e'),m_1)\cdots((e^{(n)}),m_n)s'^m r_1^{m_1}\cdots r_m^{m_n}}{((g),m_1+\cdots m_n)(f,m)((h'),m_1)\cdots((h^{(n)}),m_n)m!m_1!\cdots m_n!}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}B_s a^{R_1}\cdots a_s^{R_s}$$

$$H_{p_r+4,q_r+3;W}^{0,n_r+4;X}\left(\begin{matrix}\mathrm{z}_1\\ \cdots\\ \mathrm{z}_r\end{matrix}\,\middle|\,\begin{matrix}\text{(1-c -}\sum_{i=1}^{s}R_it_i;\sigma_1,\cdots,\sigma_r),\quad\text{(1/2-c +a+b -}\sum_{i=1}^{s}R_it_i;\sigma_1,\cdots,\sigma_r),\\ \cdots\qquad\cdots\\ \text{(1/2-c+a-}\sum_{i=1}^{s}R_it_i;\sigma_1,\cdots,\sigma_r),\quad\text{(1/2-c+b-}\sum_{i=1}^{s}R_it_i;\sigma_1,\cdots,\sigma_r),\end{matrix}\right.$$

$$(1 \text{ - } \beta-\sum_{i=1}^{s} R_i\phi_i;\lambda_1,\cdots,\lambda_r),$$
$$\cdots$$
$$(1\text{- } \alpha-\beta-\sum_{i=1}^{s} R_i(n_i+\phi_i)-m-\sum_{i=1}^{n} m_i;\rho_1+\lambda_1,\cdots,\rho_r+\lambda_r),$$

$$\left.\begin{matrix}(1 \text{ - } \alpha-m-\sum_{i=1}^{n} m_i-\sum_{i=1}^{s} R_in_i;\rho_1,\cdots,\rho_r),\mathfrak{A}:A' \\ \cdots \\ \mathfrak{B} :\text{B'}\end{matrix}\right) \tag{4.1}$$

under the same condition and notations that (3.1) with $U=V=A=B=0$

Corollary 2

$$\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}(1-y)^{\beta-1}{}_1F_1(\delta;f;sy)$$

$$F_{G:H}^{D:E}\left[\begin{matrix}\text{(d) : (e') ; }\cdots;(e^{(n)}); \\ \text{(g) : (h') ; }\cdots;(h^{(n)});\end{matrix} r_1y,r_2\cdots,r_n\right] S_L^{h_1,\cdots,h_s}\begin{pmatrix}\text{a}_1x^{t_1}y^{n_1}(1-y)^{\phi_1} \\ \cdots \\ \text{a}_sx^{t_s}y^{n_s}(1-y)^{\phi_s}\end{pmatrix}$$

$$H\left(\begin{matrix}\text{z}_1x^{\sigma_1}y^{\rho_1}(1-y)^{\lambda_1} \\ \cdots \\ \text{z}_rx^{\sigma_r}y^{\rho_r}(1-y)^{\lambda_r}\end{matrix}\left|\begin{matrix}\mathfrak{A}:\text{ A'} \\ \mathfrak{B}:\text{ B'}\end{matrix}\right.\right)\text{d}x\text{d}y=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{m,m_1,\cdots,m_n=0}^{\infty}$$

$$\frac{((d),m_1+\cdots m_n)(\delta,m)((e'),m_1)\cdots((e^{(n)}),m_n)s'^mr_1^{m_1}\cdots r_m^{m_n}}{((g),m_1+\cdots m_n)(f,m)((h'),m_1)\cdots((h^{(n)}),m_n)m!m_1!\cdots m_n!}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L} B_sa^{R_1}\cdots a_s^{R_s}$$

$$H_{p_r+4,q_r+3;W}^{0,n_r+4;X}\left(\begin{matrix}\text{z}_1 \\ \cdots \\ \text{z}_r\end{matrix}\left|\begin{matrix}(1\text{-c -}\sum_{i=1}^{s} R_it_i;\sigma_1,\cdots,\sigma_r), & (1/2\text{-c +a+b -}\sum_{i=1}^{s} R_it_i;\sigma_1,\cdots,\sigma_r), \\ \cdots & \cdots \\ (1/2\text{-c+a-}\sum_{i=1}^{s} R_it_i;\sigma_1,\cdots,\sigma_r), & (1/2\text{-c+b-}\sum_{i=1}^{s} R_it_i;\sigma_1,\cdots,\sigma_r),\end{matrix}\right.\right.$$

$$(1 \text{ - } \beta-m_1-\sum_{i=1}^{s} R_i\phi_i;\lambda_1,\cdots,\lambda_r),$$
$$\cdots$$
$$(1\text{- } \alpha-\beta-\sum_{i=1}^{s} R_i(n_i+\phi_i)-m-\sum_{i=1}^{n} m_i;\rho_1+\lambda_1,\cdots,\rho_r+\lambda_r),$$

$$\left.\begin{matrix}(1 \text{ - } \alpha-m-\sum_{i=1}^{s} R_in_i;\rho_1,\cdots,\rho_r),\mathfrak{A}:A' \\ \cdots \\ \mathfrak{B}:\text{ B'}\end{matrix}\right) \tag{4.2}$$

under the same condition and notations that (3.2) with $U=V=A=B=0$

Corollary 3

$$\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}(1-y)^{\beta-1}{}_1F_1(\delta;f;s'y)$$

$$F_{G:H}^{D:E}\left[\begin{matrix}\text{(d)}:\text{(e')};\cdots;(e^{(n)});\\ \text{(g)}:\text{(h')};\cdots;(h^{(n)});\end{matrix} r_1x,\cdots,r_kx,r_{k+1}(1-x),\cdots,r_n(1-x)\right]$$

$$S_L^{h_1,\cdots,h_s}\begin{pmatrix}\text{a}_1x^{t_1}y^{n_1}(1-y)^{\phi_1}\\ \cdots\\ \text{a}_sx^{t_s}y^{n_s}(1-y)^{\phi_s}\end{pmatrix}H\left(\begin{matrix}\text{z}_1x^{\sigma_1}y^{\rho_1}(1-y)^{\lambda_1}\\ \cdots\\ \text{z}_rx^{\sigma_r}y^{\rho_r}(1-y)^{\lambda_r}\end{matrix}\ \middle|\ \begin{matrix}\mathfrak{A}:\text{A'}\\ \mathfrak{B}:\text{B'}\end{matrix}\right)\mathrm{d}x\mathrm{d}y$$

$$=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{m,m_1,\cdots,m_n=0}^{\infty}\frac{((d),m_1+\cdots m_n)(\delta,m)}{((g),m_1+\cdots m_n)(f,m)}$$

$$\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}B_sa^{R_1}\cdots a_s^{R_s}\frac{((e'),m_1)\cdots((e^{(n)}),m_n)s'^mr_1^{m_1}\cdots r_m^{m_n}}{((h'),m_1)\cdots((h^{(n)}),m_n)m!m_1!\cdots m_n!}$$

$$H_{p_r+4,q_r+3;W}^{0,n_r+4;X}\left(\begin{matrix}\text{z}_1\\ \cdots\\ \text{z}_r\end{matrix}\ \middle|\ \begin{matrix}(\text{1-c -}\sum_{i=1}^sR_it_i;\sigma_1,\cdots,\sigma_r),\quad(\text{1/2-c +a+b -}\sum_{i=1}^sR_it_i;\sigma_1,\cdots,\sigma_r),\\ \cdots\qquad\qquad\cdots\\ (\text{1/2-c+a-}\sum_{i=1}^sR_it_i;\sigma_1,\cdots,\sigma_r),\quad(\text{1/2-c+b-}\sum_{i=1}^sR_it_i;\sigma_1,\cdots,\sigma_r),\end{matrix}\right.$$

$$\begin{matrix}(1-\beta-\sum_{i=1}^{k+1}m_i-\sum_{i=1}^sR_i\phi_i;\lambda_1,\cdots,\lambda_r),\\ \cdots\\ (1-\alpha-\beta-\sum_{i=1}^sR_i(n_i+\phi_i)-m-\sum_{i=1}^nm_i;\rho_1+\lambda_1,\cdots,\rho_r+\lambda_r),\end{matrix}$$

$$\left.\begin{matrix}(1-\alpha-m-\sum_{i=1}^km_i-\sum_{i=1}^sR_in_i;\rho_1,\cdots,\rho_r),A:A'\\ \cdots\\ \mathfrak{B}:\text{B'}\end{matrix}\right)\tag{4.3}$$

under the same condition and notations that (3.3) with $U=V=A=B=0$

6. Srivastava-Daoust polynomial

If $B(L;R_1,\cdots,R_s)=\dfrac{\prod_{j=1}^{\bar{A}}(a_j)_{R_1\theta'_j+\cdots+R_s\theta_j^{(s)}}\prod_{j=1}^{B'}(b'_j)_{R_1\phi'_j}\cdots\prod_{j=1}^{B^{(s)}}(b_j^{(s)})_{R_s\phi_j^{(s)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{m_1\psi'_j+\cdots+m_s\psi_j^{(s)}}\prod_{j=1}^{D'}(d'_j)_{R_1\delta'_j}\cdots\prod_{j=1}^{D^{(s)}}(d_j^{(s)})_{R_s\delta_j^{(s)}}}$ **(5.1)**

then the general class of multivariable polynomial $S_L^{h_1,\cdots,h_s}[z_1,\cdots,z_s]$ reduces to generalized Lauricella function defined by Srivastava et al [3].

$$F^{1+\bar{A}:B';\cdots;B^{(s)}}_{\bar{C}:D';\cdots;D^{(s)}}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdots \\ \cdots \\ \mathrm{z}_s \end{array}\left|\begin{array}{c} [(\text{-L});\mathrm{R}_1,\cdots,R_s][(a);\theta',\cdots,\theta^{(s)}]:[(b');\phi'];\cdots;[(b^{(s)});\phi^{(s)}] \\ [(\mathrm{c});\psi',\cdots,\psi^{(s)}]:[(d');\delta'];\cdots;[(d^{(s)});\delta^{(s)}] \end{array}\right.\right) \tag{5.2}$$

and we have the following formulas

Corollary 4

$$\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}(1-y)^{\beta-1}{}_1F_1(\delta;f;s'y)$$

$$F^{D:E}_{G:H}\left[\begin{array}{l}(\mathrm{d}):(\mathrm{e'});\cdots;(e^{(n)}); \\ (\mathrm{g}):(\mathrm{h'});\cdots;(h^{(n)});\end{array} r_1y,\cdots,r_ny\right] F^{1+\bar{A}:B';\cdots;B^{(s)}}_{\bar{C}:D';\cdots;D^{(s)}}\left(\begin{array}{c} \mathrm{a}_1x^{t_1}y^{n_1}(1-y)^{\phi_1} \\ \cdots \\ \mathrm{a}_sx^{t_s}y^{n_s}(1-y)^{\phi_s} \end{array}\right|$$

$$\left.\begin{array}{c} [(\text{-L});\mathrm{R}_1,\cdots,R_s][(a);\theta',\cdots,\theta^{(s)}]:[(b');\phi'];\cdots;[(b^{(s)});\phi^{(s)}] \\ [(\mathrm{c});\psi',\cdots,\psi^{(s)}]:[(d');\delta'];\cdots;[(d^{(s)});\delta^{(s)}] \end{array}\right)$$

$$I\left(\begin{array}{c} \mathrm{z}_1x^{\sigma_1}y^{\rho_1}(1-y)^{\lambda_1} \\ \cdots \\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}(1-y)^{\lambda_r} \end{array}\left|\begin{array}{c} \mathrm{A}\ ;\mathfrak{A}:\ \mathrm{A'} \\ \mathrm{B}\ ;\ \mathfrak{B}:\ \mathrm{B'} \end{array}\right.\right)\mathrm{d}x\mathrm{d}y = \frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{m,m_1,\cdots,m_n=0}^{\infty}$$

$$\frac{((d),m_1+\cdots m_n)(\delta,m)((e'),m_1)\cdots((e^{(n)}),m_n)s'^m r_1^{m_1}\cdots r_m^{m_n}}{((g),m_1+\cdots m_n)(f,m)((h'),m_1)\cdots((h^{(n)}),m_n)m!m_1!\cdots m_n!}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L} B'_s a^{R_1}\cdots a_s^{R_s}$$

$$I^{V;0,n_r+4;X}_{U:p_r+4,q_r+3;W}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdots \\ \mathrm{z}_r \end{array}\left|\begin{array}{l} \mathrm{A};\ (1\text{-c}-\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r),\quad (1/2\text{-c}+\mathrm{a}+\mathrm{b}-\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r), \\ \qquad\cdots\qquad\qquad\qquad\qquad\cdots \\ \mathrm{B};\ (1/2\text{-c}+\mathrm{a}-\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r),\quad (1/2\text{-c}+\mathrm{b}-\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r), \end{array}\right.\right.$$

$$\begin{array}{c} (1-\beta-\sum_{i=1}^s R_i\phi_i;\lambda_1,\cdots,\lambda_r), \\ \cdots \\ (1-\alpha-\beta-\sum_{i=1}^s R_i(n_i+\phi_i)-m-\sum_{i=1}^n m_i;\rho_1+\lambda_1,\cdots,\rho_r+\lambda_r), \end{array}$$

$$\left.\begin{array}{c} (1-\alpha-m-\sum_{i=1}^n m_i-\sum_{i=1}^s R_in_i;\rho_1,\cdots,\rho_r),\mathfrak{A}:A' \\ \cdots \\ \mathfrak{B}:\mathrm{B'} \end{array}\right) \tag{5.3}$$

under the same condition and notations that (3.1)

where $B'_s = \dfrac{(-L)_{h_1R_1+\cdots+h_sR_s} B(E;R_1,\cdots,R_s)}{R_1!\cdots R_s!}$; $B(L;R_1,\cdots,R_s)$ is defined by (5.1).

Corollary 5

$$\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}(1-y)^{\beta-1}{}_1F_1(\delta;f;s'y)$$

$$F_{G:H}^{D:E}\left[\begin{matrix}(\mathrm{d}) : (\mathrm{e'}) ; \cdots ; (e^{(n)}); \\ (\mathrm{g}) : (\mathrm{h'}) ; \cdots ; (h^{(n)});\end{matrix} r_1y, r_2\cdots, r_n\right] F_{\bar{C}:D';\cdots;D^{(s)}}^{1+\bar{A}:B';\cdots;B^{(s)}}\left(\begin{matrix}\mathrm{a}_1x^{t_1}y^{n_1}(1-y)^{\phi_1} \\ \cdots \\ \mathrm{a}_sx^{t_s}y^{n_s}(1-y)^{\phi_s}\end{matrix}\right|$$

$$\left.\begin{matrix}[(\text{-L});\mathrm{R}_1,\cdots,R_s][(a);\theta',\cdots,\theta^{(s)}] : [(b');\phi'];\cdots;[(b^{(s)});\phi^{(s)}] \\ [(\mathrm{c});\psi',\cdots,\psi^{(s)}] : [(d');\delta'];\cdots;[(d^{(s)});\delta^{(s)}]\end{matrix}\right)$$

$$I\left(\begin{matrix}\mathrm{z}_1x^{\sigma_1}y^{\rho_1}(1-y)^{\lambda_1} \\ \cdots \\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}(1-y)^{\lambda_r}\end{matrix}\middle|\begin{matrix}\mathrm{A} ;\mathfrak{A} : \mathrm{A'} \\ \mathrm{B} ; \mathfrak{B} : \mathrm{B'}\end{matrix}\right)\mathrm{d}x\mathrm{d}y = \frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{m,m_1,\cdots,m_n=0}^{\infty}$$

$$\frac{((d),m_1+\cdots m_n)(\delta,m)((e'),m_1)\cdots((e^{(n)}),m_n)s'^m r_1^{m_1}\cdots r_m^{m_n}}{((g),m_1+\cdots m_n)(f,m)((h'),m_1)\cdots((h^{(n)}),m_n)m!m_1!\cdots m_n!}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L} B'_s a^{R_1}\cdots a_s^{R_s}$$

$$I_{U:p_r+4,q_r+3;W}^{V;0,n_r+4;X}\left(\begin{matrix}\mathrm{z}_1 \\ \cdots \\ \mathrm{z}_r\end{matrix}\middle|\begin{matrix}\mathrm{A} : (\text{1-c} -\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r), \quad (\text{1/2-c +a+b} -\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r), \\ \cdots \qquad \cdots \\ \mathrm{B} : (\text{1/2-c+a-}\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r), \quad (\text{1/2-c+b-}\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r),\end{matrix}\right.$$

$$\begin{matrix}(1 - \beta - m_1 - \sum_{i=1}^s R_i\phi_i;\lambda_1,\cdots,\lambda_r), \\ \cdots \\ (1\text{-} \alpha-\beta-\sum_{i=1}^s R_i(n_i+\phi_i)-m-\sum_{i=1}^n m_i;\rho_1+\lambda_1,\cdots,\rho_r+\lambda_r),\end{matrix}$$

$$\left.\begin{matrix}(1 - \alpha - m - \sum_{i=1}^s R_in_i;\rho_1,\cdots,\rho_r),\mathfrak{A} : A' \\ \cdots \\ \mathfrak{B} : \mathrm{B'}\end{matrix}\right) \tag{5.4}$$

under the same condition and notations that (3.1) where $B'_s = \dfrac{(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)}{R_1!\cdots R_s!}$; $B(L;R_1,\cdots,R_s)$ is defined by (5.1).

Corollary 6

$$\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}(1-y)^{\beta-1}{}_1F_1(\delta;f;s'y)$$

$$F_{G:H}^{D:E}\left[\begin{matrix}\text{(d) : (e') ; }\cdots;(e^{(n)});\\ \text{(g) : (h') ; }\cdots;(h^{(n)});\end{matrix} r_1x,\cdots,r_kx,r_{k+1}(1-x),\cdots,r_n(1-x)\right]$$

$$F_{\bar{C}:D';\cdots;D^{(s)}}^{1+\bar{A}:B';\cdots;B^{(s)}}\left(\begin{matrix}\mathrm{a}_1x^{t_1}y^{n_1}(1-y)^{\phi_1}\\ \cdots\\ \mathrm{a}_sx^{t_s}y^{n_s}(1-y)^{\phi_s}\end{matrix}\right|$$

$$\left.\begin{matrix}[\text{(-L)};\mathrm{R}_1,\cdots,R_s][(a);\theta',\cdots,\theta^{(s)}]:[(b');\phi'];\cdots;[(b^{(s)});\phi^{(s)}]\\ [\text{(c)};\psi',\cdots,\psi^{(s)}]:[(d');\delta'];\cdots;[(d^{(s)});\delta^{(s)}]\end{matrix}\right) I\left(\begin{matrix}\mathrm{z}_1x^{\sigma_1}y^{\rho_1}(1-y)^{\lambda_1}\\ \cdots\\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}(1-y)^{\lambda_r}\end{matrix}\right|$$

$$\left.\begin{matrix}\text{A };\mathfrak{A}:\text{ A'}\\ \text{B ; }\mathfrak{B}:\text{ B'}\end{matrix}\right)\quad \mathrm{d}x\mathrm{d}y$$

$$=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{m,m_1,\cdots,m_n=0}^{\infty}\frac{((d),m_1+\cdots m_n)(\delta,m)}{((g),m_1+\cdots m_n)(f,m)}$$

$$\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L} B'_sa^{R_1}\cdots a_s^{R_s}\frac{((e'),m_1)\cdots((e^{(n)}),m_n)s'^mr_1^{m_1}\cdots r_m^{m_n}}{((h'),m_1)\cdots((h^{(n)}),m_n)m!m_1!\cdots m_n!}$$

$$I_{U:p_r+4,q_r+3;W}^{V;0,n_r+4;X}\left(\begin{matrix}\mathrm{z}_1\\ \cdots\\ \mathrm{z}_r\end{matrix}\right|\begin{matrix}\text{A: (1-c -}\sum_{i=1}^sR_it_i;\sigma_1,\cdots,\sigma_r),\quad \text{(1/2-c +a+b -}\sum_{i=1}^sR_it_i;\sigma_1,\cdots,\sigma_r),\\ \cdots\qquad\qquad\qquad\cdots\\ \text{B: (1/2-c+a-}\sum_{i=1}^sR_it_i;\sigma_1,\cdots,\sigma_r),\quad \text{(1/2-c+b-}\sum_{i=1}^sR_it_i;\sigma_1,\cdots,\sigma_r),\end{matrix}$$

$$\begin{matrix}\text{(1 - }\beta-\sum_{i=1}^{k+1}m_i-\sum_{i=1}^sR_i\phi_i;\lambda_1,\cdots,\lambda_r),\\ \cdots\\ \text{(1- }\alpha-\beta-\sum_{i=1}^sR_i(n_i+\phi_i)-m-\sum_{i=1}^nm_i;\rho_1+\lambda_1,\cdots,\rho_r+\lambda_r),\end{matrix}$$

$$\left.\begin{matrix}\text{(1 - }\alpha-m-\sum_{i=1}^km_i-\sum_{i=1}^sR_in_i;\rho_1,\cdots,\rho_r),\mathfrak{A}:A'\\ \cdots\\ \mathfrak{B}\text{ :B'}\end{matrix}\right) \tag{5.5}$$

under the same condition and notations that (3.3)

where $B'_s = \dfrac{(-L)_{h_1R_1+\cdots+h_sR_s} B(E;R_1,\cdots,R_s)}{R_1!\cdots R_s!}$; $B(L;R_1,\cdots,R_s)$ is defined by (5.1).

6. Conclusion

The I-function of several variables defined by Prasad [2] is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions such as H-function of several variables defined by Srivastava et al [5]

References :

[1] Exton, H, Handbook of hypergeometric integrals, Ellis Horwood Ltd, Chichester (1978)

[2] Y.N. Prasad , Multivariable I-function , Vijnana Parishad Anusandhan Patrika 29 (1986) , page 231-237.

[3] Srivastava H.M. and Daoust M.C. Certain generalized Neumann expansions associated with Kampé de Fériet function. Nederl. Akad. Wetensch. Proc. Ser A72 = Indag Math 31(1969) page 449-457.

[4] Srivastava H.M. And Garg M. Some integral involving a general class of polynomials and multivariable H-function. Rev. Roumaine Phys. 32(1987), page 685-692.

[5] H.M. Srivastava And R.Panda. Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24(1975), p.119-137.

[6] Vyas V.M. and Rathie K., An integral involving hypergeometric function. The mathematics education 31(1997) page33