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Euler type double integrals involving Kampé de Férier function, general class of polynomials and multivariable Aleph-function

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ABSTRACT
The aim of the present document is to evaluate three double Euler type integrals involving Kampé de Fériet function, general class of polynomials and multivariable Aleph-function. Importance of our findings lies in the fact that they involve the multivariable Aleph-function, which are the sufficiently general in nature and are capable of yielding a large number of simpler and useful results merely by specializing the parameters in them.

KEYWORDS : Aleph-function of several variables, double Euler type integrals, Kampé de Fériet function, general class of polynomials.

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1. Introduction and preliminaries.

The object of this document is to evaluate three double Eulerian integrals involving Kampé de Fériet function,general class of polynomial and the Aleph-function of several variables. This function generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [2], itself is an a generalisation of G and H-functions of multiple variables defined by Srivastava et al [5]. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i(1)},q_{i(1)},\tau_{i(1)};R^{(1)};\cdots;p_{i(r)},q_{i(r)};\tau_{i(r)};R^{(r)}}\left(\begin{array}{c|} \mathrm{z}_1\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\end{array}\right.$

$$\begin{array}{ll}[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}] & ,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:\\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:\end{array}$$

$$\left.\begin{array}{l}[(\mathrm{c}_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;\ ;[(\mathrm{c}_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]\\ [(\mathrm{d}_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;;[(\mathrm{d}_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\zeta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{\sum_{i=1}^{R}[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k)]} \tag{1.2}$$

and $$\zeta_k(s_k)=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

where $j=1$ to r and $k=1$ to r

Suppose , as usual , that the parameters

$a_j, j=1,\cdots,p; b_j, j=1,\cdots,q;$

$c_j^{(k)}, j=1,\cdots,n_k; c_{ji^{(k)}}^{(k)}, j=n_k+1,\cdots,p_{i^{(k)}};$

$d_j^{(k)}, j=1,\cdots,m_k; d_{ji^{(k)}}^{(k)}, j=m_k+1,\cdots,q_{i^{(k)}};$

with $k=1\cdots,r, i=1,\cdots,R\ , i^{(k)}=1,\cdots,R^{(k)}$

are complex numbers , and the $\alpha's, \beta's, \gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)}=\sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)}+\tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)}+\sum_{j=1}^{n_k}\gamma_j^{(k)}+\tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}-\tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)}-\sum_{j=1}^{m_k}\delta_j^{(k)}$$

$$-\tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)}\leqslant 0 \tag{1.4}$$

The reals numbers τ_i are positives for $i=1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)}=1$ to $R^{(k)}$

The contour L_k is in the s_k-p lane and run from $\sigma-i\infty$ to $\sigma+i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)$ with $j=1$ to m_k are separated from those of $\Gamma(1-a_j+\sum_{i=1}^{r}\alpha_j^{(k)}s_k)$ with $j=1$ to n and $\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)$ with $j=1$ to n_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k|<\frac{1}{2}A_i^{(k)}\pi,$ where

$$A_i^{(k)}=\sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)}-\tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)}-\tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)}+\sum_{j=1}^{n_k}\gamma_j^{(k)}-\tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{m_k}\delta_j^{(k)}-\tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)}>0, \text{ with } k=1\cdots,r, i=1,\cdots,R\ , i^{(k)}=1,\cdots,R^{(k)} \tag{1.5}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\,)\,,max(\,|z_1|,\cdots,|z_r|\,)\rightarrow 0$

$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\,)\,,min(\,|z_1|,\cdots,|z_r|\,)\rightarrow\infty$
where, with $k=1,\cdots,r:\alpha_k=min[Re(d_j^{(k)}/\delta_j^{(k)})], j=1,\cdots,m_k$ and

$$\beta_k=max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})], j=1,\cdots,n_k$$

We will use these following notations in this paper

$$U = p_i, q_i, \tau_i; R \ ; \ V = m_1, n_1; \cdots ; m_r, n_r \tag{1.6}$$

$$\mathrm{W} = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}, \cdots, p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)} \tag{1.7}$$

$$A = \{(a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)})_{1,n}\}, \{\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \cdots, \alpha_{ji}^{(r)})_{n+1,p_i}\} \tag{1.8}$$

$$B = \{\tau_i(b_{ji}; \beta_{ji}^{(1)}, \cdots, \beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.9}$$

$$C = \{(c_j^{(1)}; \gamma_j^{(1)})_{1,n_1}\}, \tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)}; \gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\}, \cdots, \{(c_j^{(r)}; \gamma_j^{(r)})_{1,n_r}\}, \tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)}; \gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.10}$$

$$D = \{(d_j^{(1)}; \delta_j^{(1)})_{1,m_1}\}, \tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}; \delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\}, \cdots, \{(d_j^{(r)}; \delta_j^{(r)})_{1,m_r}\}, \tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}; \delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.11}$$

The multivariable Aleph-function write :

$$\aleph(z_1, \cdots, z_r) = \aleph_{U:W}^{0,\mathfrak{n}:V} \left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A : C} \\ \cdot & \\ \cdot & \cdot \ \cdot \ \cdot \\ \cdot & \\ \mathrm{z}_r & \mathrm{B : D} \end{array} \right) \tag{1.12}$$

Srivastava and Garg [4] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1, \cdots, h_s}[z_1, \cdots, z_s] = \sum_{R_1, \cdots, R_s = 0}^{h_1 R_1 + \cdots h_s R_s \leqslant L} (-L)_{h_1 R_1 + \cdots + h_s R_s} B(E; R_1, \cdots, R_s) \frac{z_1^{R_1} \cdots z_s^{R_s}}{R_1! \cdots R_s!} \tag{1.13}$$

The coefficients $B(E; R_1, \cdots, R_s)$ are arbitrary constants, real or complex.

We will note

$$B_s = \frac{(-L)_{h_1 R_1 + \cdots + h_s R_s} B(E; R_1, \cdots, R_s)}{R_1! \cdots R_s!} \tag{1.14}$$

2 . Results required :

Lemme 1

$$\int_0^1 x^{a-1}(1-x)^{b-1} {}_1F_1(c; f; sx) \, F_{G:H}^{D:E} \left[\begin{array}{l} \mathrm{(d)} : \mathrm{(e')} ; \cdots ; (e^{(n)}); \\ \mathrm{(g)} : \mathrm{(h')} ; \cdots ; (h^{(n)}); \end{array} r_1 x, \cdots, r_n x \right] \mathrm{d}x$$

$$= \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} \sum_{m, m_1, \cdots, m_n = 0}^{\infty} \frac{(a, m + m_1 + \cdots + m_n)((d), m_1 + \cdots m_n)(c, m)}{(a+b, m + m_1 + \cdots + m_n)((g), m_1 + \cdots m_n)(f, m)}$$

$$\frac{((e'), m_1) \cdots ((e^{(n)}), m_n) s^m r_1^{m_1} \cdots r_m^{m_n}}{((h'), m_1) \cdots ((h^{(n)}), m_n) m! m_1! \cdots m_n!} \tag{2.1}$$

Lemme 2

$$\int_0^1 x^{a-1}(1-x)^{b-1}{}_1F_1(c;f;sx)\ F_{G:H}^{D:E}\left[\begin{matrix}\text{(d)}:\ \text{(e')}\ ;\ \cdots;(e^{(n)});\\ \text{(g)}:\ \text{(h')}\ ;\ \cdots;(h^{(n)});\end{matrix} r_1x,r_2,\cdots,r_n\right]\mathrm{d}x$$

$$=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}\sum_{m,m_1,\cdots,m_n=0}^{\infty}\frac{((d),m_1+\cdots+m_n)(a,m)(c,m)}{((g),m_1+\cdots+m_n)(a+b,m+m_1)(f,m)}$$

$$\frac{((e'),m_1)\cdots((e^{(n)}),m_n)s^m r_1^{m_1}\cdots r_m^{m_n}}{((h'),m_1)\cdots((h^{(n)}),m_n)m!m_1!\cdots m_n!} \tag{2.2}$$

Lemme 3

$$\int_0^1 x^{a-1}(1-x)^{b-1}{}_1F_1(c;f;sx)\ F_{G:H}^{D:E}\left[\begin{matrix}\text{(d)}:\ \text{(e')}\ ;\ \cdots;(e^{(n)});\\ \text{(g)}:\ \text{(h')}\ ;\ \cdots;(h^{(n)});\end{matrix}\right.$$

$$\left. r_1x,\cdots,r_kx,r_{k+1}(1-x),\cdots,r_n(1-x)\right]\mathrm{d}x$$

$$=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}\sum_{m,m_1,\cdots,m_n=0}^{\infty}\frac{((d),m_1+\cdots+m_n)(a,m+m_1+\cdots+m_k)(c,m)(b;m_{k+1}+\cdots+m_n)}{(a+b,m+m_1\cdots+m_n)((g),m_1+\cdots+m_n)(f,m)}$$

$$\frac{((e'),m_1)\cdots((e^{(n)}),m_n)s^m r_1^{m_1}\cdots r_m^{m_n}}{((h'),m_1)\cdots((h^{(n)}),m_n)m!m_1!\cdots m_n!} \tag{2.3}$$

For more details , see Exton [1]

Lemme 4

$$\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)\mathrm{d}x=\frac{\pi\Gamma(c)\Gamma(a+b+1/2)\Gamma(c-a-b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)\Gamma(c-a+1/2)\Gamma(c-b+1/2)} \tag{2.4}$$

Where Re(c) > 0, Re(2c-a-b) > -1 , see Vyas and Rathie [6]

3. Main results

Theorem 1

$$\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}(1-y)^{\beta-1}{}_1F_1(\delta;f;s'y)$$

$$F_{G:H}^{D:E}\left[\begin{matrix}\text{(d)}:\ \text{(e')}\ ;\ \cdots;(e^{(n)});\\ \text{(g)}:\ \text{(h')}\ ;\ \cdots;(h^{(n)});\end{matrix} r_1y,\cdots,r_ny\right] S_L^{h_1,\cdots,h_s}\begin{pmatrix}\mathrm{a}_1x^{t_1}y^{n_1}(1-y)^{\phi_1}\\ \cdot\ \cdot\ \cdot\\ \mathrm{a}_sx^{t_s}y^{n_s}(1-y)^{\phi_s}\end{pmatrix}$$

$$\aleph\begin{pmatrix} z_1 x^{\sigma_1} y^{\rho_1}(1-y)^{\lambda_1} \\ \cdot\ \cdot\ \cdot \\ z_r x^{\sigma_r} y^{\rho_r}(1-y)^{\lambda_r} \end{pmatrix}\left|\begin{matrix} \mathrm{A}:\mathrm{C} \\ \mathrm{B}:\mathrm{D} \end{matrix}\right) \mathrm{d}x\mathrm{d}y = \frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)} \sum_{m,m_1,\cdots,m_n=0}^{\infty}$$

$$\frac{((d),m_1+\cdots m_n)(\delta,m)((e'),m_1)\cdots((e^{(n)}),m_n)s'^m r_1^{m_1}\cdots r_m^{m_n}}{((g),m_1+\cdots m_n)(f,m)((h'),m_1)\cdots((h^{(n)}),m_n)m!m_1!\cdots m_n!} \sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L} B_s a^{R_1}\cdots a_s^{R_s}$$

$$\aleph^{0,n+4:V}_{U_{43}:W}\left(\begin{matrix} z_1 \\ \cdot\ \cdot\ \cdot \\ z_r \end{matrix}\right|\begin{matrix} (1\text{-c -}\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r), & (1/2\text{-c +a+b -}\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r), \\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot \\ (1/2\text{-c+a-}\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r), & (1/2\text{-c+b-}\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r), \end{matrix}$$

$$\begin{matrix} (1\text{ - }\beta-\sum_{i=1}^s R_i\phi_i;\lambda_1,\cdots,\lambda_r), \\ \cdot\ \cdot\ \cdot \\ (1\text{- }\alpha-\beta-\sum_{i=1}^s R_i(n_i+\phi_i)-m-\sum_{i=1}^n m_i;\rho_1+\lambda_1,\cdots,\rho_r+\lambda_r), \end{matrix}$$

$$\left.\begin{matrix} (1\text{ - }\alpha-m-\sum_{i=1}^n m_i-\sum_{i=1}^s R_in_i;\rho_1,\cdots,\rho_r), A:C \\ \cdot\ \cdot\ \cdot \\ \mathrm{B}:\mathrm{D} \end{matrix}\right) \tag{3.1}$$

Where $U_{43} = p_i+4, q_i+3, \tau_i; R$

Provided that :

1) $min\{t_i, n_i, \phi_i, \sigma_j; \rho_j, \lambda_j\} > 0; i = 1,\cdots,s; j = 1,\cdots,r$

2) $Re(c+\sum_{i=1}^r \sigma_i s_i + \sum_{i=1}^s t_iR_i) > 0$; $Re(c-a-b+\sum_{i=1}^r \sigma_i s_i + \sum_{i=1}^s t_iR_i) > -1/2$

3) $Re(\beta+\sum_{i=1}^s \phi_iR_i + \sum_{i=1}^r \lambda_i s_i) > 0$; $Re(c-a+\sum_{i=1}^r \sigma_i s_i + \sum_{i=1}^s t_iR_i) > -1/2$

4) $Re(c-b+\sum_{i=1}^r \sigma_i s_i + \sum_{i=1}^s t_iR_i) > -1/2$; $Re(\alpha+m+\sum_{i=1}^n m_i + \sum_{i=1}^r \rho_i s_i + \sum_{i=1}^s R_in_i) > 0$

5) $Re(\alpha+\beta+m+\sum_{i=1}^n m_i + \sum_{i=1}^r(\rho_i+\lambda_i)s_i + \sum_{i=1}^s R_i(n_i+\phi_i)) > 0$

6) $|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.5)

Theorem 2

$$\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}(1-y)^{\beta-1}{}_1F_1(\delta;f;s'y)$$

$$F_{G:H}^{D:E}\left[\begin{matrix}(\mathrm{d}) : (\mathrm{e'}) ; \cdots ; (e^{(n)}); \\ (\mathrm{g}) : (\mathrm{h'}) ; \cdots ; (h^{(n)});\end{matrix} r_1 y, r_2 \cdots , r_n\right] S_L^{h_1,\cdots,h_s}\left(\begin{matrix}\mathrm{a}_1 x^{t_1} y^{n_1}(1-y)^{\phi_1} \\ \cdots \\ \mathrm{a}_s x^{t_s} y^{n_s}(1-y)^{\phi_s}\end{matrix}\right)$$

$$\aleph\left(\begin{matrix}\mathrm{z}_1 x^{\sigma_1} y^{\rho_1}(1-y)^{\lambda_1} \\ \cdots \\ \mathrm{z}_r x^{\sigma_r} y^{\rho_r}(1-y)^{\lambda_r}\end{matrix}\ \middle|\ \begin{matrix}\mathrm{A : C} \\ \mathrm{B : D}\end{matrix}\right) \mathrm{d}x\mathrm{d}y = \frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)} \sum_{m,m_1,\cdots,m_n=0}^{\infty}$$

$$\frac{((d), m_1+\cdots m_n)(\delta, m)((e'), m_1)\cdots((e^{(n)}), m_n) s'^m r_1^{m_1}\cdots r_m^{m_n}}{((g), m_1+\cdots m_n)(f, m)((h'), m_1)\cdots((h^{(n)}), m_n) m! m_1! \cdots m_n!} \sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L} B_s a^{R_1}\cdots a_s^{R_s}$$

$$\aleph_{U_{43}:W}^{0,n+4:V}\left(\begin{matrix}\mathrm{z}_1 \\ \cdots \\ \mathrm{z}_r\end{matrix}\ \middle|\ \begin{matrix}(1\text{-c -}\sum_{i=1}^s R_i t_i; \sigma_1, \cdots, \sigma_r), \quad (1/2\text{-c +a+b -}\sum_{i=1}^s R_i t_i; \sigma_1, \cdots, \sigma_r), \\ \cdots \qquad\qquad \cdots \\ (1/2\text{-c+a-}\sum_{i=1}^s R_i t_i; \sigma_1, \cdots, \sigma_r), \quad (1/2\text{-c+b-}\sum_{i=1}^s R_i t_i; \sigma_1, \cdots, \sigma_r),\end{matrix}\right.$$

$$\begin{matrix}(1 - \beta - m_1 - \sum_{i=1}^s R_i\phi_i; \lambda_1, \cdots, \lambda_r), \\ \cdots \\ (1\text{-}\ \alpha - \beta - \sum_{i=1}^s R_i(n_i+\phi_i) - m - \sum_{i=1}^n m_i; \rho_1+\lambda_1, \cdots, \rho_r+\lambda_r),\end{matrix}$$

$$\left.\begin{matrix}(1 - \alpha - m - \sum_{i=1}^s R_i n_i; \rho_1, \cdots, \rho_r), A : C \\ \cdots \\ \mathrm{B : D}\end{matrix}\right) \tag{3.2}$$

Where $U_{43} = p_i + 4, q_i + 3, \tau_i; R$

Provided that :

1) $min\{t_i, n_i, \phi_i, \sigma_j; \rho_j, \lambda_j\} > 0; i = 1, \cdots, s; j = 1, \cdots, r$

2) $Re(c + \sum_{i=1}^r \sigma_i s_i + \sum_{i=1}^s t_i R_i) > 0$; $Re(c - a - b + \sum_{i=1}^r \sigma_i s_i + \sum_{i=1}^s t_i R_i) > -1/2$

3) $Re(\beta + m_1 + \sum_{i=1}^s \phi_i R_i + \sum_{i=1}^r \lambda_i s_i) > 0$; $Re(c - a + \sum_{i=1}^r \sigma_i s_i + \sum_{i=1}^s t_i R_i) > -1/2$

4) $Re(c - b + \sum_{i=1}^r \sigma_i s_i + \sum_{i=1}^s t_i R_i) > -1/2$; $Re(\alpha + m + \sum_{i=1}^r \rho_i s_i + \sum_{i=1}^s R_i n_i) > 0$

5) $Re(\alpha + \beta + m + m_1 + \sum_{i=1}^r (\rho_i + \lambda_i) s_i + \sum_{i=1}^s R_i(n_i + \phi_i)) > 0$

6) $|arg z_k| < \frac{1}{2} A_i^{(k)} \pi$, where $A_i^{(k)}$ is given in (1.5)

Theorem 3

$$\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}(1-y)^{\beta-1}{}_1F_1(\delta;f;s'y)$$

$$F_{G:H}^{D:E}\left[\begin{matrix}(\mathrm{d}):(\mathrm{e'});\cdots;(e^{(n)});\\(\mathrm{g}):(\mathrm{h'});\cdots;(h^{(n)});\end{matrix} r_1x,\cdots,r_kx,r_{k+1}(1-x),\cdots,r_n(1-x)\right]$$

$$S_L^{h_1,\cdots,h_s}\begin{pmatrix}\mathrm{a}_1x^{t_1}y^{n_1}(1-y)^{\phi_1}\\ \cdots\\ \mathrm{a}_sx^{t_s}y^{n_s}(1-y)^{\phi_s}\end{pmatrix}\aleph\left(\begin{matrix}\mathrm{z}_1x^{\sigma_1}y^{\rho_1}(1-y)^{\lambda_1}\\ \cdots\\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}(1-y)^{\lambda_r}\end{matrix}\ \middle|\ \begin{matrix}\mathrm{A}:\mathrm{C}\\ \mathrm{B}:\mathrm{D}\end{matrix}\right)\mathrm{d}x\mathrm{d}y$$

$$=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{m,m_1,\cdots,m_n=0}^{\infty}\frac{((d),m_1+\cdots m_n)(\delta,m)}{((g),m_1+\cdots m_n)(f,m)}$$

$$\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L} B_sa^{R_1}\cdots a_s^{R_s}\frac{((e'),m_1)\cdots((e^{(n)}),m_n)s'^mr_1^{m_1}\cdots r_m^{m_n}}{((h'),m_1)\cdots((h^{(n)}),m_n)m!m_1!\cdots m_n!}$$

$$\aleph_{U_{43}:W}^{0,n+4:V}\left(\begin{matrix}\mathrm{z}_1\\ \cdots\\ \mathrm{z}_r\end{matrix}\ \middle|\ \begin{matrix}(1\text{-c }-\sum_{i=1}^sR_it_i;\sigma_1,\cdots,\sigma_r),\quad(1/2\text{-c }+\text{a}+\text{b }-\sum_{i=1}^sR_it_i;\sigma_1,\cdots,\sigma_r),\\ \cdots\qquad\qquad\cdots\\ (1/2\text{-c}+\text{a}-\sum_{i=1}^sR_it_i;\sigma_1,\cdots,\sigma_r),\quad(1/2\text{-c}+\text{b}-\sum_{i=1}^sR_it_i;\sigma_1,\cdots,\sigma_r),\end{matrix}\right.$$

$$\begin{matrix}(1\text{ - }\beta-\sum_{i=1}^{k+1}m_i-\sum_{i=1}^sR_i\phi_i;\lambda_1,\cdots,\lambda_r),\\ \cdots\\ (1\text{- }\alpha-\beta-\sum_{i=1}^sR_i(n_i+\phi_i)-m-\sum_{i=1}^nm_i;\rho_1+\lambda_1,\cdots,\rho_r+\lambda_r),\end{matrix}$$

$$\left.\begin{matrix}(1\text{ - }\alpha-m-\sum_{i=1}^km_i-\sum_{i=1}^sR_in_i;\rho_1,\cdots,\rho_r),A:C\\ \cdots\\ \mathrm{B}:\mathrm{D}\end{matrix}\right)\tag{3.3}$$

Where $U_{43}=p_i+4,q_i+3,\tau_i;R$

Provided that :

1) $min\{t_i,n_i,\phi_i,\sigma_j;\rho_j,\lambda_j\}>0;i=1,\cdots,s;j=1,\cdots,r$

2) $Re(c+\sum_{i=1}^r\sigma_is_i+\sum_{i=1}^st_iR_i)>0$; $Re(c-a-b+\sum_{i=1}^r\sigma_is_i+\sum_{i=1}^st_iR_i)>-1/2$

3) $Re(\alpha+m+\sum_{i=1}^km_i+\sum_{i=1}^s\phi_iR_i+\sum_{i=1}^r\lambda_is_i)>0$; $Re(c-a+\sum_{i=1}^r\sigma_is_i+\sum_{i=1}^st_iR_i)>-1/2$

4) $Re(c-b+\sum_{i=1}^{r}\sigma_i s_i+\sum_{i=1}^{s}t_i R_i)>-1/2$; $Re(\beta+\sum_{i=k+1}^{r}m_i+\sum_{i=1}^{r}\lambda_i s_i+\sum_{i=1}^{s}R_i n_i)>0$

5) $Re(\alpha+\beta+m+\sum_{i=1}^{n}m_i+\sum_{i=1}^{r}(\rho_i+\lambda_i)s_i+\sum_{i=1}^{s}R_i(n_i+\phi_i))>0$

6) $|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.5)

Proof of (3.1: The equation (3.1) writes

$$\int_0^1 x^c(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)\Bigg[\int_0^1 y^{\alpha-1}(1-y)^{\beta-1}\ {}_1F_1(\delta;f;s'y)$$

$$F_{G:H}^{D:E}\left[\begin{matrix}\text{(d) : (e') ; }\cdots\text{ ; }(e^{(n)});\\ \text{(g) : (h') ; }\cdots\text{ ; }(h^{(n)});\end{matrix} r_1y,\cdots,r_ny\right]\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L} B_s\left[a_1x^{t_1}y^{n_1}(1-y)^{\phi_1}\right]^{R_1}$$

$$\cdots\quad\left[a_sx^{t_s}y^{n_s}(1-y)^{\phi_s}\right]^{R_s}\aleph\left(\begin{matrix}\text{z}_1x^{\sigma_1}y^{\rho_1}(1-y)^{\lambda_1}\\ \cdot\ \cdot\ \cdot\\ \text{z}_rx^{\sigma_r}y^{\rho_r}(1-y)^{\lambda_r}\end{matrix}\ \middle|\ \begin{matrix}\text{A :C}\\ \text{B : D}\end{matrix}\right)\mathrm{d}y\Bigg]\mathrm{d}x \tag{3.4}$$

We first express the multivariable Aleph-function involving in L.H.S. of (3.4) in terms od Mellin-Barnes contour integral with the help of (1.1) and then interchange the order of integration. We find that L.H.S. of (3.4)

$$\frac{1}{(2\pi\omega)^r}\Bigg(\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\zeta_k(s_k)z_k^{s_k}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}B_sa^{R_1}\cdots a_s^{R_s}$$

$$\left(\int_0^1 x^{c+\sigma_1s_1+\cdots+\sigma_rs_r}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)\mathrm{d}x\right)$$

$$\Bigg[\int_0^1 y^{a+\rho_1s_1+\cdots+\rho_rs_r+n_1R_1+\cdots+n_sR_s-1}\ (1-y)^{\lambda_1s_1+\cdots+\lambda_rs_r+\phi_1R_1+\cdots+\phi_sR_s-1}{}_1F_1(\delta;f;s'y)$$

$$F_{G:H}^{D:E}\left[\begin{matrix}\text{(d) : (e') ; }\cdots\text{ ; }(e^{(n)});\\ \text{(g) : (h') ; }\cdots\text{ ; }(h^{(n)});\end{matrix} r_1y,\cdots,r_ny\right]\mathrm{d}y\Bigg]\mathrm{d}s_1\cdots\mathrm{d}s_r\Bigg) \tag{3.6}$$

Now using the result (2.1) to evaluate the y-integral and (2.4) to evaluate the x-integral and reinterpreting the multiple contour integral so obtained with the help of (3.1) , we arrive at the R.H.S. of (3.1).
The results (3.2) and (3.3) can be proved similarly with the help of results given by (2.2), (2.3) and (2.4).

4. Multivariable I-function

If $\tau,\tau_1,\cdots,\tau_r\rightarrow 1$, the multivariable Aleph-function reduces to multivariable I-function defined by Sharma et al [2]. We have the following results.

Corollary 1

$$\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}(1-y)^{\beta-1}{}_1F_1(\delta;f;s'y)$$

$$F_{G:H}^{D:E}\left[\begin{matrix}\text{(d) : (e') ; }\cdots;(e^{(n)});\\ \text{(g) : (h') ; }\cdots;(h^{(n)});\end{matrix} r_1y,\cdots,r_ny\right] S_L^{h_1,\cdots,h_s}\begin{pmatrix}\mathrm{a}_1x^{t_1}y^{n_1}(1-y)^{\phi_1}\\ \cdots\\ \mathrm{a}_sx^{t_s}y^{n_s}(1-y)^{\phi_s}\end{pmatrix}$$

$$I\left(\begin{matrix}\mathrm{z}_1x^{\sigma_1}y^{\rho_1}(1-y)^{\lambda_1}\\ \cdots\\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}(1-y)^{\lambda_r}\end{matrix}\middle|\begin{matrix}\text{A :C}\\ \text{B : D}\end{matrix}\right)\mathrm{d}x\mathrm{d}y=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{m,m_1,\cdots,m_n=0}^{\infty}$$

$$\frac{((d),m_1+\cdots m_n)(\delta,m)((e'),m_1)\cdots((e^{(n)}),m_n)s'^m r_1^{m_1}\cdots r_m^{m_n}}{((g),m_1+\cdots m_n)(f,m)((h'),m_1)\cdots((h^{(n)}),m_n)m!m_1!\cdots m_n!}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}B_s a^{R_1}\cdots a_s^{R_s}$$

$$I_{U_{43}:W}^{0,n+4:V}\left(\begin{matrix}\mathrm{z}_1\\ \cdots\\ \mathrm{z}_r\end{matrix}\middle|\begin{matrix}(\text{1-c -}\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r),\quad (\text{1/2-c +a+b -}\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r),\\ \cdots\qquad\qquad\cdots\\ (\text{1/2-c+a-}\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r),\quad (\text{1/2-c+b-}\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r),\end{matrix}\right.$$

$$\begin{matrix}(1 - \beta-\sum_{i=1}^s R_i\phi_i;\lambda_1,\cdots,\lambda_r),\\ \cdots\\ (1\text{-}\ \alpha-\beta-\sum_{i=1}^s R_i(n_i+\phi_i)-m-\sum_{i=1}^n m_i;\rho_1+\lambda_1,\cdots,\rho_r+\lambda_r),\end{matrix}$$

$$\left.\begin{matrix}(1 - \alpha-m-\sum_{i=1}^n m_i-\sum_{i=1}^s R_in_i;\rho_1,\cdots,\rho_r),A:C\\ \cdots\\ \text{B : D}\end{matrix}\right) \tag{4.1}$$

Where $U_{43}=p_i+4,q_i+3;R$

under the same condition and notations that (3.1) with $\tau,\tau_1,\cdots,\tau_r\rightarrow 1$,

Corollary 2

$$\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}(1-y)^{\beta-1}{}_1F_1(\delta;f;s'y)$$

$$F_{G:H}^{D:E}\left[\begin{matrix}\text{(d) : (e') ; }\cdots;(e^{(n)});\\ \text{(g) : (h') ; }\cdots;(h^{(n)});\end{matrix} r_1y,r_2\cdots,r_n\right] S_L^{h_1,\cdots,h_s}\begin{pmatrix}\mathrm{a}_1x^{t_1}y^{n_1}(1-y)^{\phi_1}\\ \cdots\\ \mathrm{a}_sx^{t_s}y^{n_s}(1-y)^{\phi_s}\end{pmatrix}$$

$$I\begin{pmatrix}\mathrm{z}_1x^{\sigma_1}y^{\rho_1}(1-y)^{\lambda_1}\\ \cdots\\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}(1-y)^{\lambda_r}\end{pmatrix}\left|\begin{matrix}\mathrm{A}:\mathrm{C}\\ \mathrm{B}:\mathrm{D}\end{matrix}\right)\mathrm{d}x\mathrm{d}y=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{m,m_1,\cdots,m_n=0}^{\infty}$$

$$\frac{((d),m_1+\cdots m_n)(\delta,m)((e'),m_1)\cdots((e^{(n)}),m_n)s'^m r_1^{m_1}\cdots r_m^{m_n}}{((g),m_1+\cdots m_n)(f,m)((h'),m_1)\cdots((h^{(n)}),m_n)m!m_1!\cdots m_n!}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}B_sa^{R_1}\cdots a_s^{R_s}$$

$$I_{U_{43}:W}^{0,n+4:V}\left(\begin{matrix}\mathrm{z}_1\\ \cdots\\ \mathrm{z}_r\end{matrix}\left|\begin{matrix}(1\text{-c }-\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r),\quad(1/2\text{-c }+\text{a+b }-\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r),\\ \cdots\qquad\qquad\cdots\\ (1/2\text{-c+a-}\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r),\quad(1/2\text{-c+b-}\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r),\end{matrix}\right.\right.$$

$$\begin{matrix}(1\text{ - }\beta-m_1-\sum_{i=1}^s R_i\phi_i;\lambda_1,\cdots,\lambda_r),\\ \cdots\\ (1\text{- }\alpha-\beta-\sum_{i=1}^s R_i(n_i+\phi_i)-m-\sum_{i=1}^n m_i;\rho_1+\lambda_1,\cdots,\rho_r+\lambda_r),\end{matrix}$$

$$\left.\begin{matrix}(1\text{ - }\alpha-m-\sum_{i=1}^s R_in_i;\rho_1,\cdots,\rho_r),A:C\\ \cdots\\ \mathrm{B}:\mathrm{D}\end{matrix}\right)\tag{4.2}$$

Where $U_{43}=p_i+4,q_i+3;R$

under the same condition and notations that (3.2) with $\tau,\tau_1,\cdots,\tau_r\to 1$,

Corollary 3

$$\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}(1-y)^{\beta-1}{}_1F_1(\delta;f;s'y)$$

$$F_{G:H}^{D:E}\left[\begin{matrix}\text{(d) : (e') ; }\cdots;(e^{(n)});\\ \text{(g) : (h') ; }\cdots;(h^{(n)});\end{matrix}r_1x,\cdots,r_kx,r_{k+1}(1-x),\cdots,r_n(1-x)\right]$$

$$S_L^{h_1,\cdots,h_s}\begin{pmatrix}\mathrm{a}_1x^{t_1}y^{n_1}(1-y)^{\phi_1}\\ \cdots\\ \mathrm{a}_sx^{t_s}y^{n_s}(1-y)^{\phi_s}\end{pmatrix}I\begin{pmatrix}\mathrm{z}_1x^{\sigma_1}y^{\rho_1}(1-y)^{\lambda_1}\\ \cdots\\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}(1-y)^{\lambda_r}\end{pmatrix}\left|\begin{matrix}\mathrm{A}:\mathrm{C}\\ \mathrm{B}:\mathrm{D}\end{matrix}\right)\mathrm{d}x\mathrm{d}y$$

$$=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{m,m_1,\cdots,m_n=0}^{\infty}\frac{((d),m_1+\cdots m_n)(\delta,m)}{((g),m_1+\cdots m_n)(f,m)}$$

$$\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}B_sa^{R_1}\cdots a_s^{R_s}\frac{((e'),m_1)\cdots((e^{(n)}),m_n)s'^m r_1^{m_1}\cdots r_m^{m_n}}{((h'),m_1)\cdots((h^{(n)}),m_n)m!m_1!\cdots m_n!}$$

$$I_{U_{43}:W}^{0,n+4:V}\left(\begin{array}{c} z_1 \\ \cdots \\ z_r \end{array}\middle| \begin{array}{c} (1\text{-c} -\sum_{i=1}^{s} R_i t_i;\sigma_1,\cdots,\sigma_r), \quad (1/2\text{-c} +\text{a+b} -\sum_{i=1}^{s} R_i t_i;\sigma_1,\cdots,\sigma_r), \\ \cdots \qquad\qquad\qquad \cdots \\ (1/2\text{-c+a-}\sum_{i=1}^{s} R_i t_i;\sigma_1,\cdots,\sigma_r), \quad (1/2\text{-c+b-}\sum_{i=1}^{s} R_i t_i;\sigma_1,\cdots,\sigma_r), \end{array}\right.$$

$$\begin{array}{c} (1 - \beta-\sum_{i=1}^{k+1} m_i-\sum_{i=1}^{s} R_i\phi_i;\lambda_1,\cdots,\lambda_r), \\ \cdots \\ (1- \alpha-\beta-\sum_{i=1}^{s} R_i(n_i+\phi_i)-m-\sum_{i=1}^{n} m_i;\rho_1+\lambda_1,\cdots,\rho_r+\lambda_r), \end{array}$$

$$\left.\begin{array}{c} (1 - \alpha-m-\sum_{i=1}^{k} m_i-\sum_{i=1}^{s} R_i n_i;\rho_1,\cdots,\rho_r), A:C \\ \cdots \\ \text{B :D} \end{array}\right) \tag{4.3}$$

Where $U_{43}=p_i+4,q_i+3;R$

under the same condition and notations that (3.3) with $\tau,\tau_1,\cdots,\tau_r\rightarrow 1$,

6. Srivastava-Daoust polynomial

If $$B(L;R_1,\cdots,R_s)=\frac{\prod_{j=1}^{\bar{A}}(a_j)_{R_1\theta_j'+\cdots+R_s\theta_j^{(s)}}\prod_{j=1}^{B'}(b_j')_{R_1\phi_j'}\cdots\prod_{j=1}^{B^{(s)}}(b_j^{(s)})_{R_s\phi_j^{(s)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{m_1\psi_j'+\cdots+m_s\psi_j^{(s)}}\prod_{j=1}^{D'}(d_j')_{R_1\delta_j'}\cdots\prod_{j=1}^{D^{(s)}}(d_j^{(s)})_{R_s\delta_j^{(s)}}} \tag{5.1}$$

then the general class of multivariable polynomial $S_L^{h_1,\cdots,h_s}[z_1,\cdots,z_s]$ reduces to generalized Lauricella function defined by Srivastava et al [3].

$$F_{\bar{C}:D';\cdots;D^{(s)}}^{1+\bar{A}:B';\cdots;B^{(s)}}\left(\begin{array}{c} z_1 \\ \cdots \\ \cdots \\ z_s \end{array}\middle| \begin{array}{c} [(\text{-L});\text{R}_1,\cdots,R_s][(a);\theta',\cdots,\theta^{(s)}]:[(b');\phi'];\cdots;[(b^{(s)});\phi^{(s)}] \\ [(\text{c});\psi',\cdots,\psi^{(s)}]:[(d');\delta'];\cdots;[(d^{(s)});\delta^{(s)}] \end{array}\right) \tag{5.2}$$

and we have the following formulas

Corollary 4

$$\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}(1-y)^{\beta-1}{}_1F_1(\delta;f;s'y)$$

$$F_{G:H}^{D:E}\left[\begin{array}{c} (\text{d}):(\text{e'});\cdots;(e^{(n)}); \\ (\text{g}):(\text{h'});\cdots;(h^{(n)}); \end{array} r_1y,\cdots,r_ny\right] F_{\bar{C}:D';\cdots;D^{(s)}}^{1+\bar{A}:B';\cdots;B^{(s)}}\left(\begin{array}{c} \text{a}_1x^{t_1}y^{n_1}(1-y)^{\phi_1} \\ \cdots \\ \text{a}_sx^{t_s}y^{n_s}(1-y)^{\phi_s} \end{array}\middle|\right.$$

$$[\text{(-L)};\text{R}_1,\cdots,R_s][(a);\theta',\cdots,\theta^{(s)}]:[(b');\phi'];\cdots;[(b^{(s)});\phi^{(s)}]$$
$$\left.[\text{(c)};\psi',\cdots,\psi^{(s)}]:[(d');\delta'];\cdots;[(d^{(s)});\delta^{(s)}]\right)$$

$$\aleph\left(\begin{array}{c}\text{z}_1x^{\sigma_1}y^{\rho_1}(1-y)^{\lambda_1}\\ \cdots\\ \text{z}_rx^{\sigma_r}y^{\rho_r}(1-y)^{\lambda_r}\end{array}\middle|\begin{array}{c}\text{A :C}\\ \text{B : D}\end{array}\right)\text{d}x\text{d}y=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{m,m_1,\cdots,m_n=0}^{\infty}$$

$$\frac{((d),m_1+\cdots m_n)(\delta,m)((e'),m_1)\cdots((e^{(n)}),m_n)s'^mr_1^{m_1}\cdots r_m^{m_n}}{((g),m_1+\cdots m_n)(f,m)((h'),m_1)\cdots((h^{(n)}),m_n)m!m_1!\cdots m_n!}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}B_s'a^{R_1}\cdots a_s^{R_s}$$

$$\aleph_{U_{43}:W}^{0,n+4:V}\left(\begin{array}{c}\text{z}_1\\ \cdots\\ \text{z}_r\end{array}\middle|\begin{array}{cc}(\text{1-c -}\sum_{i=1}^sR_it_i;\sigma_1,\cdots,\sigma_r), & (\text{1/2-c +a+b -}\sum_{i=1}^sR_it_i;\sigma_1,\cdots,\sigma_r),\\ \cdots & \cdots\\ (\text{1/2-c+a-}\sum_{i=1}^sR_it_i;\sigma_1,\cdots,\sigma_r), & (\text{1/2-c+b-}\sum_{i=1}^sR_it_i;\sigma_1,\cdots,\sigma_r),\end{array}\right.$$

$$(1\text{ - }\beta-\sum_{i=1}^sR_i\phi_i;\lambda_1,\cdots,\lambda_r),$$
$$\cdots$$
$$(1\text{- }\alpha-\beta-\sum_{i=1}^sR_i(n_i+\phi_i)-m-\sum_{i=1}^nm_i;\rho_1+\lambda_1,\cdots,\rho_r+\lambda_r),$$

$$\left.\begin{array}{c}(1\text{ - }\alpha-m-\sum_{i=1}^nm_i-\sum_{i=1}^sR_in_i;\rho_1,\cdots,\rho_r),A:C\\ \cdots\\ \text{B : D}\end{array}\right) \tag{5.3}$$

under the same condition and notations that (3.1)

where $B_s'=\dfrac{(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)}{R_1!\cdots R_s!}$; $B(L;R_1,\cdots,R_s)$ is defined by (5.1).

Corollary 5

$$\int_0^1\int_0^1x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}(1-y)^{\beta-1}{}_1F_1(\delta;f;s'y)$$

$$F_{G:H}^{D:E}\left[\begin{array}{l}\text{(d) : (e') ;}\cdots;(e^{(n)});\\ \text{(g) : (h') ;}\cdots;(h^{(n)});\end{array}r_1y,r_2\cdots,r_n\right]F_{\bar{C}:D';\cdots;D^{(s)}}^{1+\bar{A}:B';\cdots;B^{(s)}}\left(\begin{array}{c}\text{a}_1x^{t_1}y^{n_1}(1-y)^{\phi_1}\\ \cdots\\ \text{a}_sx^{t_s}y^{n_s}(1-y)^{\phi_s}\end{array}\right|$$

$$[\text{(-L)};\text{R}_1,\cdots,R_s][(a);\theta',\cdots,\theta^{(s)}]:[(b');\phi'];\cdots;[(b^{(s)});\phi^{(s)}]$$
$$\left.[\text{(c)};\psi',\cdots,\psi^{(s)}]:[(d');\delta'];\cdots;[(d^{(s)});\delta^{(s)}]\right)$$

$$\aleph\left(\begin{matrix}z_1x^{\sigma_1}y^{\rho_1}(1-y)^{\lambda_1}\\ \cdots\\ z_rx^{\sigma_r}y^{\rho_r}(1-y)^{\lambda_r}\end{matrix}\ \middle|\ \begin{matrix}\mathrm{A}:\mathrm{C}\\ \mathrm{B}:\mathrm{D}\end{matrix}\right)\mathrm{d}x\mathrm{d}y=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{m,m_1,\cdots,m_n=0}^{\infty}$$

$$\frac{((d),m_1+\cdots m_n)(\delta,m)((e'),m_1)\cdots((e^{(n)}),m_n)s'^m r_1^{m_1}\cdots r_m^{m_n}}{((g),m_1+\cdots m_n)(f,m)((h'),m_1)\cdots((h^{(n)}),m_n)m!m_1!\cdots m_n!}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}B'_s a^{R_1}\cdots a_s^{R_s}$$

$$\aleph_{U_{43}:W}^{0,n+4:V}\left(\begin{matrix}z_1\\ \cdots\\ z_r\end{matrix}\ \middle|\ \begin{matrix}(1\text{-c }-\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r),\quad (1/2\text{-c }+\text{a}+\text{b }-\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r),\\ \cdots\qquad\qquad\cdots\\ (1/2\text{-c}+\text{a}-\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r),\quad (1/2\text{-c}+\text{b}-\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r),\end{matrix}\right.$$

$$\begin{matrix}(1\text{ - }\beta-m_1-\sum_{i=1}^s R_i\phi_i;\lambda_1,\cdots,\lambda_r),\\ \cdots\\ (1\text{- }\alpha-\beta-\sum_{i=1}^s R_i(n_i+\phi_i)-m-\sum_{i=1}^n m_i;\rho_1+\lambda_1,\cdots,\rho_r+\lambda_r),\end{matrix}$$

$$\left.\begin{matrix}(1\text{ - }\alpha-m-\sum_{i=1}^s R_in_i;\rho_1,\cdots,\rho_r),A:C\\ \cdots\\ \mathrm{B}:\mathrm{D}\end{matrix}\right) \tag{5.4}$$

under the same condition and notations that (3.1)

where $B'_s=\dfrac{(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)}{R_1!\cdots R_s!}$; $B(L;R_1,\cdots,R_s)$ is defined by (5.1).

Corollary 6

$$\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}(1-y)^{\beta-1}{}_1F_1(\delta;f;s'y)$$

$$F_{G:H}^{D:E}\left[\begin{matrix}\text{(d) : (e') ; }\cdots;(e^{(n)});\\ \text{(g) : (h') ; }\cdots;(h^{(n)});\end{matrix}r_1x,\cdots,r_kx,r_{k+1}(1-x),\cdots,r_n(1-x)\right]$$

$$F_{\bar{C}:D';\cdots;D^{(s)}}^{1+\bar{A}:B';\cdots;B^{(s)}}\left(\begin{matrix}a_1x^{t_1}y^{n_1}(1-y)^{\phi_1}\\ \cdots\\ a_sx^{t_s}y^{n_s}(1-y)^{\phi_s}\end{matrix}\ \middle|\right.$$

$$\left.\begin{matrix}[(\text{-L});\text{R}_1,\cdots,R_s][(a);\theta',\cdots,\theta^{(s)}]:[(b');\phi'];\cdots;[(b^{(s)});\phi^{(s)}]\\ [(\text{c});\psi',\cdots,\psi^{(s)}]:[(d');\delta'];\cdots;[(d^{(s)});\delta^{(s)}]\end{matrix}\right)\aleph\left(\begin{matrix}z_1x^{\sigma_1}y^{\rho_1}(1-y)^{\lambda_1}\\ \cdots\\ z_rx^{\sigma_r}y^{\rho_r}(1-y)^{\lambda_r}\end{matrix}\ \middle|\right.$$

$$\left.\begin{matrix}\text{A :C}\\ \text{B : D}\end{matrix}\right) \mathrm{d}x\mathrm{d}y$$

$$= \frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)} \sum_{m,m_1,\cdots,m_n=0}^{\infty} \frac{((d),m_1+\cdots m_n)(\delta,m)}{((g),m_1+\cdots m_n)(f,m)}$$

$$\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L} B'_s a^{R_1}\cdots a_s^{R_s} \frac{((e'),m_1)\cdots((e^{(n)}),m_n)s'^m r_1^{m_1}\cdots r_m^{m_n}}{((h'),m_1)\cdots((h^{(n)}),m_n)m!m_1!\cdots m_n!}$$

$$\aleph_{U_{43}:W}^{0,n+4:V}\left(\begin{matrix}\text{z}_1\\ \cdot\cdot\cdot\\ \text{z}_r\end{matrix}\ \middle|\ \begin{matrix}(\text{1-c -}\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r), \quad (\text{1/2-c +a+b -}\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r),\\ \cdot\cdot\cdot \qquad\qquad \cdot\cdot\cdot\\ (\text{1/2-c+a-}\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r), \quad (\text{1/2-c+b-}\sum_{i=1}^s R_it_i;\sigma_1,\cdots,\sigma_r),\end{matrix}\right.$$

$$\begin{matrix}(1 - \beta - \sum_{i=1}^{k+1} m_i - \sum_{i=1}^s R_i\phi_i;\lambda_1,\cdots,\lambda_r),\\ \cdot\cdot\cdot\\ (1\text{-} \ \alpha-\beta-\sum_{i=1}^s R_i(n_i+\phi_i)-m-\sum_{i=1}^n m_i;\rho_1+\lambda_1,\cdots,\rho_r+\lambda_r),\end{matrix}$$

$$\left.\begin{matrix}(1 - \alpha - m - \sum_{i=1}^k m_i - \sum_{i=1}^s R_in_i;\rho_1,\cdots,\rho_r), A:C\\ \cdot\cdot\cdot\\ \text{B :D}\end{matrix}\right) \tag{5.5}$$

under the same condition and notations that (3.3)

where $B'_s = \dfrac{(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)}{R_1!\cdots R_s!}$; $B(L;R_1,\cdots,R_s)$ is defined by (5.1).

6. Conclusion

The aleph-function of several variables presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions such as I-function of several variables defined by Sharma and Ahmad [2], the multivariable H-function, see Srivastava et al [5].

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