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On general multiple Eulerian integrals involving the multivariable A-function , a general class of polynomials and the generalized multiple-index Mittag-Leffler function function

$F.Y.\ AYANT^1$

1 Teacher in High School , France
E-mail : fredericayant@gmail.com

ABSTRACT
The object of this paper is first,to evaluate a general multiple Eulerian integrals with general integrands involving the product of a multivariable A-function defined by Gautam et al [3], a general Class of polynomials, the generalized multiple-index Mittag-Leffler function and generalized hypergeometric function with general arguments. The second multiple Eulerian integral contain a general class of polynomials, the general polynomial set, the generalized multiple-index Mittag-Leffler function and multivariable A-function defined by Gautam et al [3] with general arguments. Our integral formulas are interesting and unified nature.

Keywords:A-function of several variables, general class of polynomials, sequence of function, multivariable H-function, Srivastava-Daoust function, multiple Eulerian integral.

2010 Mathematics Subject Classification :33C05, 33C60

1. Introduction

In this paper, we evaluate two multiple Eulerian integral involving the multivariable A-function , the generalized multiple-index Mittag-Leffler function and multivariable class of polynomials with general arguments. The multivariable A-function is a extension of the multivariable H-function defined by Srivastava et al [12]. We will give the contracted form.

The A-function is defined and represented in the following manner.

$$A(z_1,\cdots,z_r) = A^{m,n:m_1,n_1;\cdots;m_r,n_r}_{p,q:p_1,q_1;\cdots;p_r,q_r}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_j;A_j^{(1)},\cdots,A_j^{(r)})_{1,p} : \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_j;B_j^{(1)},\cdots,B_j^{(r)})_{1,q} : \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}_j^{(1)},C_j^{(1)})_{1,p_1};\cdots;(c_j^{(r)},C_j^{(r)})_{1,p_r} \\ \\ (\mathrm{d}_j^{(1)},D_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)},D_j^{(r)})_{1,q_r} \end{array}\right) \tag{1.1}$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.2}$$

where $\phi(s_1,\cdots,s_r),\theta_i(s_i), i=1,\cdots,r$ are given by :

$$\phi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{m}\Gamma(b_j-\sum_{i=1}^{r}B_j^{(i)}s_i)\prod_{j=1}^{n}\Gamma(1-a_j+\sum_{i=1}^{r}A_j^{(i)}s_j)}{\prod_{j=n+1}^{p}\Gamma(a_j-\sum_{i=1}^{r}A_j^{(i)}s_j)\prod_{j=m+1}^{q}\Gamma\left(1-b_j+\sum_{i=1}^{r}B_j^{(i)}s_j\right)} \tag{1.3}$$

$$\theta_i(s_i)=\frac{\prod_{j=1}^{n_i}\Gamma\left(1-c_j^{(i)}+C_j^{(i)}s_i\right)\prod_{j=1}^{m_i}\Gamma(d_j^{(i)}-D_j^{(i)}s_i)}{\prod_{j=n_i+1}^{p_i}\Gamma(c_j^{(i)}-C_j^{(i)}s_i)\prod_{j=m_i+1}^{q_i}\Gamma(1-d_j^{(i)}-D_j^{(i)}s_i)} \tag{1.4}$$

Here $m,n,p,m_i,n_i,p_i,c_i\in\mathbb{N}^*; i=1,\cdots,r$; $a_j,b_j,c_j^{(i)},d_j^{(i)},A_j^{(i)},B_j^{(i)},C_j^{(i)},D_j^{(i)}\in\mathbb{C}$
The multiple integral defining the A-function of r variables converges absolutely if :

$$|arg(\Omega_i)z_k|<\frac{1}{2}\eta_k\pi,\xi^*=0,\eta_i>0 \tag{1.5}$$

$$\Omega_i=\prod_{j=1}^{p}\{A_j^{(i)}\}^{A_j^{(i)}}\prod_{j=1}^{q}\{B_j^{(i)}\}^{-B_j^{(i)}}\prod_{j=1}^{q_i}\{D_j^{(i)}\}^{D_j^{(i)}}\prod_{j=1}^{p_i}\{C_j^{(i)}\}^{-C_j^{(i)}};i=1,\cdots,r \tag{1.6}$$

$$\xi_i^*=Im(\sum_{j=1}^{p}A_j^{(i)}-\sum_{j=1}^{q}B_j^{(i)}+\sum_{j=1}^{q_i}D_j^{(i)}-\sum_{j=1}^{p_i}C_j^{(i)});i=1,\cdots,r \tag{1.7}$$

$$\eta_i=Re\left(\sum_{j=1}^{n}A_j^{(i)}-\sum_{j=n+1}^{p}A_j^{(i)}+\sum_{j=1}^{m}B_j^{(i)}-\sum_{j=m+1}^{q}B_j^{(i)}+\sum_{j=1}^{m_i}D_j^{(i)}-\sum_{j=m_i+1}^{q_i}D_j^{(i)}+\sum_{j=1}^{n_i}C_j^{(i)}-\sum_{j=n_i+1}^{p_i}C_j^{(i)}\right)\text{ with}$$

$$i=1,\cdots,r \tag{1.8}$$

Let

$$X=m_1,n_1;\cdots;m_r,n_r;Y=p_1,q_1;\cdots;p_s,q_s \tag{1.9}$$

$$A=(a_j;A_j^{(1)},\cdots,A_j^{(r)})_{1,p}\quad;B=(b_j;B_j^{(1)},\cdots,B_j^{(r)})_{1,q} \tag{1.10}$$

$$\mathrm{C}=(\mathrm{c}_j^{(1)},C_j^{(1)})_{1,p_1};\cdots;(c_j^{(s)},C_j^{(r)})_{1,p_r};D=(\mathrm{d}_j^{(1)},D_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)},D_j^{(r)})_{1,q_r} \tag{1.11}$$

the contracted form is

$$A(z_1,\cdots,z_r)=A_{p,q:Y}^{m,n:X}\left(\begin{array}{c|c}\mathrm{z}_1 & \mathrm{A:C}\\ \cdot & \\ \cdot & \cdots\\ \cdot & \\ \mathrm{z}_r & \mathrm{B:D}\end{array}\right) \tag{1.12}$$

Srivastava and Garg [9] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1,\cdots,h_u}[z_1,\cdots,z_u]=\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}(-L)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)\frac{z_1^{R_1}\cdots z_u^{R_u}}{R_1!\cdots R_u!} \tag{1.13}$$

The coefficients $B(E;R_1,\cdots,R_u)$ are arbitrary constants, real or complex.

We will note

$$B_u = \frac{(-L)_{h_1R_1+\cdots+h_uR_u} B(E; R_1, \cdots, R_u)}{R_1! \cdots R_u!} \tag{1.14}$$

2. Generalized multiple-index Mittag-Leffler function

A further generalization of the Mittag-Leffler functions is proposed recently in Paneva-Konovska [4]. These are 3m-parametric Mittag-Leffler type functions generalizing the 3-parametric function , see Prabhakar [5] ,defined as:

$$E_{(\alpha_i),(\beta_i)}^{(\gamma_i),m}(z) = \sum_{n'=0}^{\infty} \frac{(\gamma_1)_{n'} \cdots (\gamma_m)_{n'}}{\Gamma(\alpha_1 n' + \beta_1) \cdots \Gamma(\alpha_m n' + \beta_m)} \frac{z^{n'}}{n'!} \tag{2.1}$$

where $\alpha_i, \beta_i, \gamma_i \in \mathbb{C}, i = 1, \cdots, m, Re(\alpha_i) > 0$

We will note $A_{n'} = \dfrac{(\gamma_1)_{n'} \cdots (\gamma_m)_{n'}}{\Gamma(\alpha_1 n' + \beta_1) \cdots \Gamma(\alpha_m n' + \beta_m)}$ (2.2)

3. Sequence of function

Agarwal and Chaubey [1], Salim [7] and several others have studied a general sequence of functions. In the present document we shall study the following useful series formula for a general sequence of functions.

$$R_n^{\alpha,\beta}[x; E, F, g, h; p, q; \gamma; \delta; e^{-sx^r}] = \sum_{w,v,u,t,e,k_1,k_2,} \psi(w, v, u, t, e, k_1, k_2) x^R \tag{3.1}$$

where $$\sum_{w,v,u,t,e,k_1,k_2} = \sum_{w=0}^{\infty} \sum_{v=0}^{n} \sum_{u=0}^{v} \sum_{t=0}^{n} \sum_{c=0}^{t} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \tag{3.2}$$

and the infinite series on the right side (2.1) is absolutely convergent, $R = ln + qv + pt + rw + k_1 r + k_2 q$ (3.3)

and $$\psi(w, v, u, t, e, k_1, k_2) = \frac{(-)^{t+w+k_2} (-v)_u (-t)_e (\alpha)_t l^n}{w! v! u! t! e! K_n k_1! k_2!} \frac{s^{w+k_1} F^{\gamma n - t}}{(1 - \alpha - t)_e} (\alpha - \gamma n)_e$$

$$(-\beta - \delta n)_v g^{v+k_2} h^{\delta n - v - k_2} (v - \delta n)_{k_2} E^t \left(\frac{pe + rw + \lambda + qn}{l} \right)_n \tag{3.4}$$

where K_n is a sequence of constants. This function will note $R_n^{\alpha,\beta}[x]$

By suitably specializing the parameters involving in (2.1), a general sequence of function reduced to generalized polynomials set studied by Raizada [6], a class of polynomials introduced by Fujiwara [2] and several others authors.

4. Integral representation of generalized hypergeometric function

The following generalized hypergeometric function in terms of multiple contour integrals is also required [11 ,page 39 eq .30]

$$\frac{\prod_{j=1}^{P}\Gamma(A_j)}{\prod_{j=1}^{Q}\Gamma(B_j)}\,{}_PF_Q\left[(A_P);(B_Q);-(x_1+\cdots+x_r)\right]$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\frac{\prod_{j=1}^{P}\Gamma(A_j+s_1+\cdots+s_r)}{\prod_{j=1}^{Q}\Gamma(B_j+s_1+\cdots+s_r)}\Gamma(-s_1)\cdots\Gamma(-s_r)x_1^{s_1}\cdots x_r^{s_r}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{4.1}$$

where the contours are of Barnes type with indentations, if necessary, to ensure that the poles of $\Gamma(A_j+s_1+\cdots+s_r)$ are separated from those of $\Gamma(-s_j), j=1,\cdots,r$. The above result (1.23) can be easily established by an appeal to the calculus of residues by calculating the residues at the poles of $\Gamma(-s_j), j=1,\cdots,r$

The equivalent form of Eulerian beta integral is :

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\mathrm{d}t=(b-a)^{\alpha+\beta}B(\alpha,\beta)(Re(\alpha)>0,Re(\beta)>0,a\neq b) \tag{4.2}$$

5. First integral

We note :

$$X=m_1,n_1;\cdots;m_r,n_r;1,0;\cdots;1,0;1,0;\cdots;1,0 \tag{5.1}$$

$$Y=p_1,q_1;\cdots;p_r,q_r;0,1;\cdots;0,1;0,1;\cdots;0,1 \tag{5.2}$$

$$A=(a_j;A_j^{(1)},\cdots,A_j^{(r)})_{1,p}\quad;B=(b_j;B_j^{(1)},\cdots,B_j^{(r)})_{1,q} \tag{5.3}$$

$$C=(c_j^{(1)},\gamma_j^{(1)})_{1,p_1};\cdots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r};(1,0);\cdots;(1,0);(1,0);\cdots;(1,0) \tag{5.4}$$

$$D=(d_j^{(1)},\bar{\delta}_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)},\bar{\delta}_j^{(r)})_{1,q_r};(0,1);\cdots;(0,1);(0,1);\cdots;(0,1) \tag{5.5}$$

$$A^*=[1+\sigma_i'-n'c_i'-\sum_{k=1}^{u}R_k\xi_i^{(1,k)};\rho_i^{(1,1)},\cdots,\rho_i^{(1,r)},\tau_i^{(1,1)},\cdots,\tau_i^{(1,l)},1,0,\cdots,0]_{1,s},\cdots,$$

$$[1+\sigma_i^{(T)}-n'c_i^{(T)}-\sum_{k=1}^{u}R_k\xi_i^{(T,k)};\rho_i^{(T,1)},\cdots,\rho_i^{(T,r)},\tau_i^{(T,1)},\cdots,\tau_i^{(T,l)},0,\cdots,0,1]_{1,s},$$

$$[1-A_j;0,\cdots,0,1,\cdots,1,0,\cdots,0]_{1,P},$$

$$[1-\alpha_i-n'a_i-\sum_{k=1}^{u}R_k\alpha_i^{(k)};\delta_i',\cdots,\delta_i^{(r)},\mu_i',\cdots,\mu_i^{(l)},\underbrace{1,\cdots,1}_{\text{W-items}},\underbrace{0,\cdots,0}_{\text{(T-W)-items}}]_{1,s},$$

$$[1-\beta_i-n'b_i-\sum_{k=1}^{u}R_k\beta_i^{(k)};\eta_i',\cdots,\eta_i^{(r)},\theta_i',\cdots,\theta_i^{(l)},\underbrace{0,\cdots,0}_{\text{W-items}},\underbrace{1,\cdots,1}_{\text{(T-W)-items}}]_{1,s} \tag{5.6}$$

$$B^*=[1+\sigma_i'-n'c_i'-\sum_{k=1}^{u}R_k\xi_i^{(1,k)};\rho_i^{(1,1)},\cdots,\rho_i^{(1,r)},\tau_i^{(1,1)},\cdots,\tau_i^{(1,l)},0,\cdots,0]_{1,s},\cdots,$$

$$[1+\sigma_i^{(T)}-n'c_i^{(T)}-\sum_{k=1}^{u}R_k\xi_i^{(T,k)};\rho_i^{(T,1)},\cdots,\rho_i^{(T,r)},\tau_i^{(T,1)},\cdots,\tau_i^{(T,l)},0,\cdots,0]_{1,s},$$

$$[1-B_j;0,\cdots,0,1,\cdots,1,0,\cdots,0]_{1,Q},$$

$$[1-\alpha_i-\beta_i-n'(a_i+b_i)\;-\sum_{k=1}^{u}R_k(\alpha_i^{(k)}+\beta_i^{(k)});(\delta_i'+\eta_i'),\cdots,(\delta_i^{(r)}+\eta_i^{(r)})$$

$$(\mu_i'+\theta_i'),\cdots,(\mu_i^{(l)}+\theta_i^{(l)}),1,\cdots,1]_{1,s} \tag{5.7}$$

We have the following multiple Eulerian integral and we obtain the A-function of $(r+l+T)-$variables

$$\int_{u_1}^{v_1}\cdots\int_{u_s}^{v_s}\prod_{i=1}^{s}\left[(x_i-u_i)^{\alpha_i-1}(v_i-x_i)^{\beta_i-1}\prod_{j=1}^{T}(u_i^{(j)}x_i+v_i^{(j)})^{\sigma_i^{(j)}}\right]$$

$$E_{(\alpha_i'),(\beta_i')}^{(\gamma_i),m}\left[z\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{a_i}(v_i-x_i)^{b_i}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{c_i^{(j)}}}\right]\right]$$

$$S_L^{h_1,\cdots,h_u}\left(\begin{array}{c}\mathrm{y}_1\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{\alpha_i'}(v_i-x_i)^{\beta_i'}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\xi_i^{(j,1)}}}\right]\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{y}_u\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{\alpha_i^{(u)}}(v_i-x_i)^{\beta_i^{(u)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\xi_i^{(j,u)}}}\right]\end{array}\right)$$

$$A_{p,q:Y}^{m,n:X}\left(\begin{array}{c}\mathrm{z}_1\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{\delta_i'}(v_i-x_i)^{\eta_i'}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i^{(j,1)}}}\right]\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{\delta_i^{(r)}}(v_i-x_i)^{\eta_i^{(r)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i^{(j,r)}}}\right]\end{array}\right)$$

$$
{}_PF_Q\left[(A_P);(B_Q);-\sum_{k=1}^{l}g_k\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{\mu_i^{(k)}}(v_i-x_i)^{\theta_i^{(r)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\tau_i^{(j,k)}}}\right]\right]\mathrm{d}x_1\cdots\mathrm{d}x_s
$$

$$
=\frac{\prod_{j=1}^{Q}\Gamma(B_j)}{\prod_{j=1}^{P}\Gamma(A_j)}\prod_{i=1}^{s}\left[(v_i-u_i)^{\alpha_i+\beta_i-1}\prod_{j=1}^{W}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{\sigma_i^{(j)}}\prod_{j=W+1}^{T}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{\sigma_i^{(j)}}\right]
$$

$$
\sum_{n'=0}^{\infty}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}B_u\,A_{n'}\,\frac{z^{n'}}{n'!}\,y_1^{R_1}\cdots y_u^{R_u}\quad\prod_{i=1}^{s}\left[(v_i-u_i)^{n'(a_i+b_i)+\sum_{k=1}^{u}R_k(\alpha_i^{(k)}+\beta_i^{(k)})}\right]
$$

$$
\prod_{i=1}^{s}\left[\prod_{j=1}^{W}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{-n'c_i^{(j)}-\sum_{k=1}^{u}R_k\xi_i^{(j,k)}}\prod_{j=W+1}^{T}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{-n'c_i^{(j)}-\sum_{k=1}^{u}R_k\xi_i^{(j,k)}}\right]
$$

$$
A_{p+sT+P+2s;q+sT+Q+s;Y}^{m,n+sT+P+2s;X}\left(\begin{array}{c|c}
\mathrm{z}_1w_1 & \mathrm{A}\ ;\mathrm{A}^*:C\\
\cdots & \cdot\\
\cdots & \cdot\\
\mathrm{z}_rw_r & \cdot\\
\mathrm{g}_1W_1 & \cdot\\
\cdots & \cdot\\
\cdots & \cdot\\
\mathrm{g}_lW_l & \cdot\\
\mathrm{G}_1 & \cdot\\
\cdots & \cdot\\
\cdots & \cdot\\
\mathrm{G}_T & \mathrm{B}\ ;\mathrm{B}^*:D
\end{array}\right) \tag{5.8}
$$

Where

$$
w_m=\prod_{i=1}^{s}\left[(v_i-u_i)^{\delta_i^{(m)}+\eta_i^{(m)}}\prod_{j=1}^{W}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{-\rho_i^{(j,m)}}\prod_{j=W+1}^{T}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{-\rho_i^{(j,m)}}\right],m=1,\cdots,r \tag{5.9}
$$

$$
W_k=\prod_{i=1}^{s}\left[(v_i-u_i)^{\mu_i^{(k)}+\theta_i^{(k)}}\prod_{j=1}^{W}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{-\tau_i^{(j,k)}}\prod_{j=W+1}^{T}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{-\tau_i^{(j,k)}}\right],k=1,\cdots,l \tag{5.10}
$$

$$
G_j=\prod_{i=1}^{s}\left[\frac{(v_i-u_i)U_i^{(j)}}{u_iU_i^{(j)}+V_i^{(j)}}\right],j=1,\cdots,W \tag{5.11}
$$

$$G_j = -\prod_{i=1}^{s}\left[\frac{(v_i-u_i)U_i^{(j)}}{u_iU_i^{(j)}+V_i^{(j)}}\right], j = W+1,\cdots,T \tag{5.12}$$

Provided that :

(A) $0 \leqslant W \leqslant T; u_i, v_i \in \mathbb{R}; i = 1,\cdots,r min\{a_i, b_i, c_i^{(j)}, \alpha_i^{(k)}, \beta_i^{(k)}, \xi_i^{(j,k)}\} > 0$, $i = 1,\cdots,s; k = 1,\cdots,u; j = 1,\cdots,T$

(B) $min\{\delta_i^{(t)}, \eta_i^{(t)}, \rho_i^{(j,t)}, \mu_i^{(k)}, \theta_i^{(k)}, \tau_i^{(j,k)}\} \geqslant 0; j = 1,\cdots,T; i = 1,\cdots,s; k = 1,\cdots,l; t = 1,\cdots,r$

(C) $\sigma_i^{(j)} \in \mathbb{R}, U_i^{(j)}, V_i^{(j)} \in \mathbb{C}, z_t, g_k \in \mathbb{C}; j = 1,\cdots,s; t = 1,\cdots,r; k = 1,\cdots,l; t = 1,\cdots,r$

(D) $max\left[\frac{(v_i-u_i)U_i^{(j)}}{u_iU_i^{(j)}+V_i^{(j)}}\right] < 1, i = 1,\cdots,s; j = 1,\cdots,W$ and

$max\left[\frac{(v_i-u_i)U_i^{(j)}}{u_iU_i^{(j)}+V_i^{(j)}}\right] < 1, i = 1,\cdots,s; j = W+1,\cdots,T$

(E) $|arg(\Omega_i)z_k| < \frac{1}{2}\eta_i\pi, \xi^* = 0, \eta_i > 0$

$$\Omega_i = \prod_{j=1}^{p}\{A_j^{(i)}\}^{A_j^{(i)}}\prod_{j=1}^{q}\{B_j^{(i)}\}^{-B_j^{(i)}}\prod_{j=1}^{q_i}\{D_j^{(i)}\}^{D_j^{(i)}}\prod_{j=1}^{p_i}\{C_j^{(i)}\}^{-C_j^{(i)}}; i = 1,\cdots,r$$

$$\xi_i^* = Im(\sum_{j=1}^{p}A_j^{(i)} - \sum_{j=1}^{q}B_j^{(i)} + \sum_{j=1}^{q_i}D_j^{(i)} - \sum_{j=1}^{p_i}C_j^{(i)}); i = 1,\cdots,r$$

$$\eta_i = Re\left(\sum_{j=1}^{n}A_j^{(i)} - \sum_{j=n+1}^{p}A_j^{(i)} + \sum_{j=1}^{m}B_j^{(i)} - \sum_{j=m+1}^{q}B_j^{(i)} + \sum_{j=1}^{m_i}D_j^{(i)} - \sum_{j=m_i+1}^{q_i}D_j^{(i)} + \sum_{j=1}^{n_i}C_j^{(i)} - \sum_{j=n_i+1}^{p_i}C_j^{(i)}\right)$$

$$-\delta_i^{(k)} - \eta_i^{(k)} - \sum_{j=1}^{T}\rho_i^{(j,k)} > 0$$

(F) $\left|arg\left(z_i\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{-\rho_i^{(j,k)}}\right)\right| < \frac{1}{2}\eta_i\pi$

(G) $Re\big[\alpha_i + n'a_i + \sum_{t=1}^{r}\delta_i^{(t)}\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(t)}}{D_j^{(t)}}\big] > 0; Re\big[\beta_i + n'b_i + \sum_{t=1}^{r}\eta_i^{(t)}\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(t)}}{D_j^{(t)}}\big] > 0; i = 1,\cdots,s$

(H) $\alpha_i', \beta_i', \gamma_i \in \mathbb{C}, i = 1,\cdots,m, Re(\alpha_i') > 0$

(I) The series occuring on the right-hand side of (4.13) are absolutely and uniformly convergent

(J) $P \leqslant Q+1$. The equality holds, when , in addition,

either $P > Q$ and $\sum_{k=1}^{l}\left|g_k\left(\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{-\tau_i^{(j,k)}}\right)\right|^{\frac{1}{Q-P}} < 1 \qquad (u_i \leqslant x_i \leqslant v_i; i = 1,\cdots,s)$

or $P \leqslant Q$ and $\max_{1\leqslant k\leqslant l}\left[\left|\left(g_k\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{-\tau_i^{(j,k)}}\right)\right|\right]<1 \qquad (u_i\leqslant x_i\leqslant v_i; i=1,\cdots,s)$

Proof

To establish the formula (5.8), we first use series representation (2.1) and (1.13) for $E_{(\alpha'_i),(\beta'_i)}^{(\gamma_i),m}[.]$ and $S_L^{h_1,\cdots,h_u}[.]$ respectively, we use contour integral representation with the help of (1.1) for the multivariable A-function occuring in its left-hand side and use the contour integral representation with the help of (4.1) for the generalized hypergeometric function ${}_PF_Q(.)$.
Changing the order of integration ans summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process) . Now we write :

$$\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}}=\prod_{j=1}^{W}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}}\prod_{j=W+1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}}$$

where $K_i^{(j)}=\eta_i^{(j)}-n'c_i^{(j)}-\sum_{k=1}^{u}R_k\xi_i^{(j,k)}-\sum_{t=1}^{r}\rho_i^{(j,t)}\zeta_t-\sum_{k=1}^{l}\tau_i^{(j,k)}\zeta_k \qquad i=1,\cdots,s;j=1,\cdots,T$ (5.13)

and express the factor occuring in R.H.S. Of (5.8) in terms of following Mellin-Barnes contour integral with the help of the result [10, page 18, eq.(2.6.4)]

$$\frac{1}{(2\pi\omega)^W}\int_{L'_1}\cdots\int_{L'_W}\prod_{j=1}^{W}\Big[\Gamma(-\zeta'_j)\Gamma(-K_i^{(j)}+\zeta'_j)\prod_{j=1}^{W}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}}=\prod_{j=1}^{W}\left[\frac{(U_i^{(j)}u_i+V_i^{(j)})^{K'^{(j)}_i}}{\Gamma(-K_i^{(j)})}\right]$$

$$\prod_{j=1}^{W}\left[\frac{(U_i^{(j)}(x_i-u_i)}{(u_iU_i^{(j)}+V_i^{(j)})}\right]^{\zeta'_j}\Big]\,\mathrm{d}\zeta'_1\cdots\mathrm{d}\zeta'_W \tag{5.14}$$

and

$$\frac{1}{(2\pi\omega)^{T-W}}\int_{L'_{W+1}}\cdots\int_{L'_T}\prod_{j=W+1}^{T}\Big[\Gamma(-\zeta'_j)\Gamma(-K_i^{(j)}+\zeta'_j)\prod_{j=W+1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}}=\prod_{j=W+1}^{T}\left[\frac{(U_i^{(j)}v_i+V_i^{(j)})^{K_i^{(j)}}}{\Gamma(-K_i^{(j)})}\right]$$

$$\prod_{j=W+1}^{T}\left[-\frac{(U_i^{(j)}(v_i-x_i)}{(v_iU_i^{(j)}+V_i^{(j)})}\right]^{\zeta'_j}\Big]\,\mathrm{d}\zeta'_{W+1}\cdots\mathrm{d}\zeta'_T \tag{5.15}$$

We apply the Fubini's theorem for multiple integral. Finally evaluating the innermost **x**-integral with the help of (4.2) and reinterpreting the multiple Mellin-Barnes contour integral in terms of multivariable A-function of $(r+l+T)-$ variables, we obtain the formula (5.8).

6. Second integral

We note :

$$X=m_1,n_1;\cdots;m_r,n_r;1,0;\cdots;1,0 \tag{6.1}$$

$$Y=p_1,q_1;\cdots;p_r,q_r;0,1;\cdots;0,1 \tag{6.2}$$

$$A=(a_j;A_j^{(1)},\cdots,A_j^{(r)})_{1,p} \quad ;B=(b_j;B_j^{(1)},\cdots,B_j^{(r)})_{1,q} \tag{6.3}$$

$$C=(c_j^{(1)},\gamma_j^{(1)})_{1,p_1};\cdots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r};(1,0);\cdots;(1,0) \tag{6.4}$$

$$D=(d_j^{(1)},\bar{\delta}_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)},\bar{\delta}_j^{(r)})_{1,q_r};(0,1);\cdots;(0,1) \tag{6.5}$$

$$A^{*}=[1+\sigma_i'-\theta_i'R-n'c_i'-\sum_{k=1}^{u}R_k\xi_i^{(1,k)};\rho_i^{(1,1)},\cdots,\rho_i^{(1,r)},1,0,\cdots,0]_{1,s},\cdots,$$

$$[1+\sigma_i^{(T)}-\theta_i^{(T)}R-n'c_i^{(T)}-\sum_{k=1}^{u}R_k\xi_i^{(T,k)};\rho_i^{(T,1)},\cdots,\rho_i^{(T,r)},0,\cdots,0,1]_{1,s},$$

$$[1-\alpha_i-\zeta_iR-n'a_i-\sum_{k=1}^{u}R_k\alpha_i^{(k)};\delta_i',\cdots,\delta_i^{(r)},1,\cdots,1,0,\cdots,0]_{1,s},$$

W-items (T-W)-items

$$[1-\beta_i-\lambda_iR-n'b_i-\sum_{k=1}^{u}R_k\beta_i^{(k)};\eta_i',\cdots,\eta_i^{(r)},0,\cdots,0,1,\cdots,1]_{1,s} \tag{6.6}$$

W-items (T-W)-items

$$B^{*}=[1+\sigma_i'-\theta_i'R-n'c_i'-\sum_{k=1}^{u}R_k\xi_i^{(1,k)};\rho_i^{(1,1)},\cdots,\rho_i^{(1,r)},0,\cdots,0]_{1,s},\cdots,$$

$$[1+\sigma_i^{(T)}-\theta_i^{(T)}R-n'c_i^{(T)}-\sum_{k=1}^{u}R_k\xi_i^{(T,k)};\rho_i^{(T,1)},\cdots,\rho_i^{(T,r)},0,\cdots,0]_{1,s},$$

$$[1-\alpha_i-\beta_i\,(\zeta_i+\lambda_i)R-n'(a_i+b_i)\ -\sum_{k=1}^{u}R_k(\alpha_i^{(k)}+\beta_i^{(k)});(\delta_i'+\eta_i'),\cdots,(\delta_i^{(r)}+\eta_i^{(r)})$$

$$(\mu_i'+\theta_i'),\cdots,(\mu_i^{(l)}+\theta_i^{(l)}),1,\cdots,1]_{1,s} \tag{6.7}$$

We have the following multiple Eulerian integral

$$\int_{u_1}^{v_1}\cdots\int_{u_s}^{v_s}\prod_{i=1}^{s}\left[(x_i-u_i)^{\alpha_i-1}(v_i-x_i)^{\beta_i-1}\prod_{j=1}^{T}(u_i^{(j)}x_i+v_i^{(j)})^{\sigma_i^{(j)}}\right]$$

$$E_{(\alpha_i'),(\beta_i')}^{(\gamma_i),m}\left[z\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{a_i}(v_i-x_i)^{b_i}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{c_i^{(j)}}}\right]\right]$$

$$S_L^{h_1,\cdots,h_u}\begin{pmatrix}\mathrm{y}_1\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{\alpha_i'}(v_i-x_i)^{\beta_i'}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\xi_i^{(j,1)}}}\right]\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{y}_u\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{\alpha_i^{(u)}}(v_i-x_i)^{\beta_i^{(u)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\xi_i^{(j,u)}}}\right]\end{pmatrix}R_n^{\alpha,\beta}\left[Z\prod_{j=1}^{s}\left[\frac{(x_i-u_i)^{\zeta_i}(v_i-x_i)^{\lambda_i}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\theta_i^{(j)}}}\right]\right]$$

$$A\left(\begin{array}{c} \mathrm{z}_1 \prod_{i=1}^s \left[\frac{(x_i-u_i)^{\delta_i'}(v_i-x_i)^{\eta_i'}}{\prod_{j=1}^T\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i^{(j,1)}}}\right] \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r \prod_{i=1}^s \left[\frac{(x_i-u_i)^{\delta_i^{(r)}}(v_i-x_i)^{\eta_i^{(r)}}}{\prod_{j=1}^T\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i^{(j,r)}}}\right] \end{array}\right)\mathrm{d}x_1\cdots\mathrm{d}x_s$$

$$=\prod_{i=1}^s\left[(v_i-u_i)^{\alpha_i+\beta_i-1}\prod_{j=1}^W\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{\sigma_i^{(j)}}\prod_{j=W+1}^T\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{\sigma_i^{(j)}}\right]\sum_{n'=0}^{\infty}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}$$

$$\sum_{w,v,u,t,e,k_1,k_2} B_uA_{n'}\frac{z^{n'}}{n'!}y_1^{R_1}\cdots y_u^{R_u}\psi'(w,v,u,t,e,k_1,k_2)Z^R\prod_{i=1}^s\left[(v_i-u_i)^{n'(a_i+b_i)+\sum_{k=1}^u R_k(\alpha_i^{(k)}+\beta_i^{(k)})}\right]$$

$$\prod_{i=1}^s\left[\prod_{j=1}^W\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{-n'c_i^{(j)}-\sum_{k=1}^u R_k\xi_i^{(j,k)}}\prod_{j=W+1}^T\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{-n'c_i^{(j)}-\sum_{k=1}^u R_k\xi_i^{(j,k)}}\right]$$

$$\psi'(w,v,u,t,e,k_1,k_2)A_{p+sT+2s;q+sT+s;Y}^{m,n+sT+2s;X}\left(\begin{array}{c|c} \mathrm{z}_1w_1 & \mathrm{A}\ ;\mathrm{A}^*:C \\ \cdots & \cdot \\ \cdots & \cdot \\ \mathrm{z}_rw_r & \cdot \\ & \cdot \\ \mathrm{G}_1 & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \mathrm{G}_T & \mathrm{B}\ ;\mathrm{B}^*:D \end{array}\right) \tag{6.8}$$

where

$$\psi'(w,v,u,t,e,k_1,k_2)=\frac{\psi(w,v,u,t,e,k_1,k_2,)\prod_{i=1}^s(v_i-u_i)^{(\zeta_i+\lambda_i)R}}{\prod_{i=1}^s\left[\prod_{j=1}^W(u_iU_i^{(j)}+V_i^{(j)})^{\theta_i^{(j)}R}\prod_{j=W+1}^T(u_iU_i^{(j)}+V_i^{(j)})^{\theta_i^{(j)}R}\right]} \tag{6.9}$$

$$w_l=\prod_{i=1}^s\left[(v_i-u_i)^{\delta_i^{(l)}+\eta_i^{(l)}}\prod_{j=1}^W\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{-\rho_i^{(j,l)}}\prod_{j=W+1}^T\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{-\rho_i^{(j,l)}}\right], l=1,\cdots,r \tag{6.10}$$

$$G_j=\prod_{i=1}^s\left[\frac{(v_i-u_i)U_i^{(j)}}{u_iU_i^{(j)}+V_i^{(j)}}\right], j=1,\cdots,W \tag{6.11}$$

$$G_j=-\prod_{i=1}^s\left[\frac{(v_i-u_i)U_i^{(j)}}{u_iU_i^{(j)}+V_i^{(j)}}\right], j=W+1,\cdots,T \tag{6.12}$$

Provided that :

(A)$min\{a_i,b_i,c_i^{(j)},\alpha_i^{(k)},\beta_i^{(k)},\xi_i^{(j,k)}\}>0$, $i=1,\cdots,s;k=1,\cdots,u;j=1,\cdots,T;0\leqslant W\leqslant T;u_i,v_i\in\mathbb{R}$

(B) $min\{\zeta_i,\lambda_i,\theta_i^{(j)},\delta_i^{(l)},\eta_i^{(l)},\rho_i^{(j,l)}\}\geqslant 0;j=1,\cdots,T;i=1,\cdots,s;l=1,\cdots,r$

(C) $Re(\alpha_i,\beta_i,v_i^{(j)})>0(i=1,\cdots,s;j=1,\cdots,T);|\tau|<1$

(D) $max\left[\frac{(v_i-u_i)U_i^{(j)}}{u_iU_i^{(j)}+V_i^{(j)}}\right]<1,i=1,\cdots,s;j=1,\cdots,W$ and

$$max\left[\frac{(v_i-u_i)U_i^{(j)}}{u_iU_i^{(j)}+V_i^{(j)}}\right]<1,i=1,\cdots,s;j=W+1,\cdots,T$$

(E) $|arg(\Omega_i)z_k|<\frac{1}{2}\eta_i\pi,\xi^*=0,\eta_i>0$

(F) $Re\big[\alpha_i+n'a_i+R\zeta_i+\sum_{t=1}^r\delta_i^{(t)}\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(t)}}{D_j^{(t)}}\big]>0;$

$$Re\big[\beta_i+n'b_i+R\lambda_i+\sum_{t=1}^r\eta_i^{(t)}\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(t)}}{D_j^{(t)}}\big]>0;i=1,\cdots,s$$

(G) $\Omega_i=\prod_{j=1}^p\{A_j^{(i)}\}^{A_j^{(i)}}\prod_{j=1}^q\{B_j^{(i)}\}^{-B_j^{(i)}}\prod_{j=1}^{q_i}\{D_j^{(i)}\}^{D_j^{(i)}}\prod_{j=1}^{p_i}\{C_j^{(i)}\}^{-C_j^{(i)}};i=1,\cdots,r$

$$\xi_i^*=Im(\sum_{j=1}^pA_j^{(i)}-\sum_{j=1}^qB_j^{(i)}+\sum_{j=1}^{q_i}D_j^{(i)}-\sum_{j=1}^{p_i}C_j^{(i)});i=1,\cdots,r$$

$$\eta_i=Re\left(\sum_{j=1}^nA_j^{(i)}-\sum_{j=n+1}^pA_j^{(i)}+\sum_{j=1}^mB_j^{(i)}-\sum_{j=m+1}^qB_j^{(i)}+\sum_{j=1}^{m_i}D_j^{(i)}-\sum_{j=m_i+1}^{q_i}D_j^{(i)}+\sum_{j=1}^{n_i}C_j^{(i)}-\sum_{j=n_i+1}^{p_i}C_j^{(i)}\right)$$

$$-\delta_i^{(k)}-\eta_i^{(k)}-\sum_{j=1}^T\rho_i^{(j,k)}>0$$

(H) $\left|arg\left(z_i\prod_{j=1}^T(U_i^{(j)}x_i+V_i^{(j)})^{-\rho_i^{(j,k)}}\right)\right|<\frac{1}{2}\eta_i\pi$

(I) The series occuring on the right-hand side of (5.13) are absolutely and uniformly convergent

(J) $\alpha_i',\beta_i',\gamma_i\in\mathbb{C},i=1,\cdots,m,Re(\alpha_i')>0$

Proof

To establish the formula (6.8), we first use series representation (2.1), (1.13) and (3.1) for , $E_{(\alpha_i'),(\beta_i')}^{(\gamma_i),m}[.]$, $S_L^{h_1,\cdots,h_u}[.]$ and $R_n^{\alpha,\beta}[.]$ respectively and the contour integral representation with the help of (1.1) for the multivariable A-function occuring in its left-hand side. Changing the order of integration ans summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process) .

Now,we write:

$$\prod_{j=1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}}=\prod_{j=1}^{W}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}}\prod_{j=W+1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}} \tag{6.10}$$

where $K_i^{(j)}=\eta_i^{(j)}-n'c_i^{(j)}-R\theta_i^{(j)}-\sum_{k=1}^{u}R_k\xi_i^{(j,k)}-\sum_{t=1}^{r}\rho_i^{(j,t)}\zeta_t\,;i=1,\cdots,s;j=1,\cdots,T$ (6.11)

and express the factors occuring in R.H.S. Of (6.8) in terms of following Mellin-Barnes contour integral , we obtain :

$$\prod_{j=1}^{W}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}}=\prod_{j=1}^{W}\left[\frac{(U_i^{(j)}u_i+V_i^{(j)})^{K_i^{(j)}}}{\Gamma(-K_i^{(j)})}\right]\frac{1}{(2\pi\omega)^W}\int_{L'_1}\cdots\int_{L'_W}\prod_{j=1}^{W}\Big[\Gamma(-\zeta'_j)\Gamma(-K_i^{(j)}+\zeta'_j)$$

$$\prod_{j=1}^{W}\left[\frac{(U_i^{(j)}(x_i-u_i)}{(u_iU_i^{(j)}+V_i^{(j)})}\right]^{\zeta'_j}\Bigg]\mathrm{d}\zeta'_1\cdots\mathrm{d}\zeta'_W \tag{6.12}$$

and

$$\prod_{j=W+1}^{T}(U_i^{(j)}x_i+V_i^{(j)})^{K_i^{(j)}}=\prod_{j=W+1}^{T}\left[\frac{(U_i^{(j)}v_i+V_i^{(j)})^{K_i^{(j)}}}{\Gamma(-K_i^{(j)})}\right]\frac{1}{(2\pi\omega)^{T-W}}\int_{L'_{W+1}}\cdots\int_{L'_T}\prod_{j=W+1}^{T}\Big[\Gamma(-\zeta'_j)\Gamma(-K_i^{(j)}+\zeta'_j)$$

$$\prod_{j=W+1}^{T}\left[\frac{(U_i^{(j)}(x_i-v_i)}{(v_iU_i^{(j)}+V_i^{(j)})}\right]^{\zeta'_j}\Bigg]\mathrm{d}\zeta'_{W+1}\cdots\mathrm{d}\zeta'_T \tag{6.13}$$

We apply the Fubini's theorem for multiple integral. Finally evaluating the innermost **x-**integral with the help of (4.2) and reinterpreting the multiple Mellin-Barnes contour integral in terms of multivariable A-function of $(r+T)-$ variables, we obtain the formula (6.8).

7. Multivarriable H-function

If $A_j^{(i)},B_j^{(i)},C_j^{(i)},D_j^{(i)}\in\mathbb{R}$ and $m=0$, the multivariable A-function reduces to multivariable H-function defined by Srivastava et al [12], we obtain the two following formulas.

Formula 1

$$\int_{u_1}^{v_1}\cdots\int_{u_s}^{v_s}\prod_{i=1}^{s}\left[(x_i-u_i)^{\alpha_i-1}(v_i-x_i)^{\beta_i-1}\prod_{j=1}^{T}(u_i^{(j)}x_i+v_i^{(j)})^{\sigma_i^{(j)}}\right]$$

$$E_{(\alpha'_i),(\beta'_i)}^{(\gamma_i),m}\left[z\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{a_i}(v_i-x_i)^{b_i}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{c_i^{(j)}}}\right]\right]$$

$$S_L^{h_1,\cdots,h_u}\left(\begin{array}{c} \mathrm{y}_1\prod_{i=1}^s\left[\frac{(x_i-u_i)^{\alpha'_i}(v_i-x_i)^{\beta'_i}}{\prod_{j=1}^T\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\xi_i^{(j,1)}}}\right] \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{y}_u\prod_{i=1}^s\left[\frac{(x_i-u_i)^{\alpha_i^{(u)}}(v_i-x_i)^{\beta_i^{(u)}}}{\prod_{j=1}^T\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\xi_i^{(j,u)}}}\right] \end{array}\right)$$

$$H_{p,q:Y}^{0,n:X}\left(\begin{array}{c} \mathrm{z}_1\prod_{i=1}^s\left[\frac{(x_i-u_i)^{\delta'_i}(v_i-x_i)^{\eta'_i}}{\prod_{j=1}^T\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i^{(j,1)}}}\right] \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r\prod_{i=1}^s\left[\frac{(x_i-u_i)^{\delta_i^{(r)}}(v_i-x_i)^{\eta_i^{(r)}}}{\prod_{j=1}^T\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i^{(j,r)}}}\right] \end{array}\right)$$

$${}_PF_Q\left[(A_P);(B_Q);-\sum_{k=1}^l g_k\prod_{i=1}^s\left[\frac{(x_i-u_i)^{\mu_i^{(k)}}(v_i-x_i)^{\theta_i^{(r)}}}{\prod_{j=1}^T\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\tau_i^{(j,k)}}}\right]\right]\mathrm{d}x_1\cdots\mathrm{d}x_s$$

$$=\frac{\prod_{j=1}^Q\Gamma(B_j)}{\prod_{j=1}^P\Gamma(A_j)}\prod_{i=1}^s\left[(v_i-u_i)^{\alpha_i+\beta_i-1}\prod_{j=1}^W\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{\sigma_i^{(j)}}\prod_{j=W+1}^T\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{\sigma_i^{(j)}}\right]$$

$$\sum_{n'=0}^\infty\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}B_u\,A_{n'}\,\frac{z^{n'}}{n'!}\,y_1^{R_1}\cdots y_u^{R_u}\quad\prod_{i=1}^s\left[(v_i-u_i)^{n'(a_i+b_i)+\sum_{k=1}^u R_k(\alpha_i^{(k)}+\beta_i^{(k)})}\right]$$

$$\prod_{i=1}^s\left[\prod_{j=1}^W\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{-n'c_i^{(j)}-\sum_{k=1}^u R_k\xi_i^{(j,k)}}\prod_{j=W+1}^T\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{-n'c_i^{(j)}-\sum_{k=1}^u R_k\xi_i^{(j,k)}}\right]$$

$$H^{0,n+sT+P+2s;X}_{p+sT+P+2s;q+sT+Q+s;Y}\left(\begin{array}{c|c} \mathrm{z}_1 w_1 & \mathrm{A}\ ;\mathrm{A}^*:C \\ \cdots & \cdot \\ \cdots & \cdot \\ \mathrm{z}_r w_r & \cdot \\ \mathrm{g}_1 W_1 & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \mathrm{g}_l W_l & \cdot \\ \mathrm{G}_1 & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \mathrm{G}_T & \mathrm{B}\ ;\mathrm{B}^*:D \end{array}\right) \tag{7.1}$$

under the same notations and conditions thar (5.3) with $A_j^{(i)}, B_j^{(i)}, C_j^{(i)}, D_j^{(i)} \in \mathbb{R}$ and $m=0$

Formula 2

$$\int_{u_1}^{v_1}\cdots\int_{u_s}^{v_s}\prod_{i=1}^{s}\left[(x_i-u_i)^{\alpha_i-1}(v_i-x_i)^{\beta_i-1}\prod_{j=1}^{T}(u_i^{(j)}x_i+v_i^{(j)})^{\sigma_i^{(j)}}\right]$$

$$E^{(\gamma_i),m}_{(\alpha'_i),(\beta'_i)}\left[z\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{a_i}(v_i-x_i)^{b_i}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{c_i^{(j)}}}\right]\right]$$

$$S_L^{h_1,\cdots,h_u}\left(\begin{array}{c} \mathrm{y}_1\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{\alpha'_i}(v_i-x_i)^{\beta'_i}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\xi_i^{(j,1)}}}\right] \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{y}_u\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{\alpha_i^{(u)}}(v_i-x_i)^{\beta_i^{(u)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\xi_i^{(j,u)}}}\right] \end{array}\right) R_n^{\alpha,\beta}\left[Z\prod_{j=1}^{s}\left[\frac{(x_i-u_i)^{\zeta_i}(v_i-x_i)^{\lambda_i}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\theta_i^{(j)}}}\right]\right]$$

$$H\left(\begin{array}{c} \mathrm{z}_1\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{\delta'_i}(v_i-x_i)^{\eta'_i}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i^{(j,1)}}}\right] \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{\delta_i^{(r)}}(v_i-x_i)^{\eta_i^{(r)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i^{(j,r)}}}\right] \end{array}\right)\mathrm{d}x_1\cdots\mathrm{d}x_s$$

$$= \prod_{i=1}^{s} \left[(v_i - u_i)^{\alpha_i+\beta_i-1} \prod_{j=1}^{W} \left(u_i U_i^{(j)} + V_i^{(j)} \right)^{\sigma_i^{(j)}} \prod_{j=W+1}^{T} \left(u_i U_i^{(j)} + V_i^{(j)} \right)^{\sigma_i^{(j)}} \right] \sum_{n'=0}^{\infty} \sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u \leqslant L}$$

$$\sum_{w,v,u,t,e,k_1,k_2} B_u A_{n'} \frac{z^{n'}}{n'!} y_1^{R_1} \cdots y_u^{R_u} \psi'(w,v,u,t,e,k_1,k_2) Z^R \prod_{i=1}^{s} \left[(v_i - u_i)^{n'(a_i+b_i)+\sum_{k=1}^{u} R_k(\alpha_i^{(k)}+\beta_i^{(k)})} \right]$$

$$\prod_{i=1}^{s} \left[\prod_{j=1}^{W} \left(u_i U_i^{(j)} + V_i^{(j)} \right)^{-n' c_i^{(j)} - \sum_{k=1}^{u} R_k \xi_i^{(j,k)}} \prod_{j=W+1}^{T} \left(u_i U_i^{(j)} + V_i^{(j)} \right)^{-n' c_i^{(j)} - \sum_{k=1}^{u} R_k \xi_i^{(j,k)}} \right]$$

$$\psi'(w,v,u,t,e,k_1,k_2) H_{p+sT+2s;q+sT+s;Y}^{0,n+sT+2s;X} \left(\begin{array}{c|c} \mathrm{z}_1 w_1 & \mathrm{A}\ ;\mathrm{A}^* : C \\ \cdots & \cdot \\ \cdots & \cdot \\ \mathrm{z}_r w_r & \cdot \\ & \cdot \\ \mathrm{G}_1 & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \mathrm{G}_T & \mathrm{B}\ ;\mathrm{B}^* : D \end{array} \right) \tag{7.2}$$

under the same notations and conditions thar (6.3) with $A_j^{(i)}, B_j^{(i)}, C_j^{(i)}, D_j^{(i)} \in \mathbb{R}$ and $m = 0$.

8. Srivastava-daoust function

If $B(L;R_1,\cdots,R_u) = \dfrac{\prod_{j=1}^{\bar{A}}(a_j)_{R_1\theta'_j+\cdots+R_u\theta_j^{(u)}} \prod_{j=1}^{B'}(b'_j)_{R_1\phi'_j} \cdots \prod_{j=1}^{B^{(u)}}(b_j^{(u)})_{R_u\phi_j^{(u)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{m_1\psi'_j+\cdots+m_u\psi_j^{(u)}} \prod_{j=1}^{D'}(d'_j)_{R_1\delta'_j} \cdots \prod_{j=1}^{D^{(u)}}(d_j^{(u)})_{R_u\delta_j^{(u)}}}$ **(8.1)**

then the general class of multivariable polynomial $S_L^{h_1,\cdots,h_u}[z_1,\cdots,z_u]$ reduces to generalized Lauricella function defined by Srivastava et al [8].

$$F_{\bar{C}:D';\cdots;D^{(u)}}^{1+\bar{A}:B';\cdots;B^{(u)}} \left(\begin{array}{c|c} \mathrm{y}_1 & [(\text{-L});\mathrm{R}_1,\cdots,R_u][(a);\theta',\cdots,\theta^{(u)}] : [(b');\phi'];\cdots;[(b^{(u)});\phi^{(u)}] \\ \cdots & \\ \cdots & [(\mathrm{c});\psi',\cdots,\psi^{(u)}] : [(d');\delta'];\cdots;[(d^{(u)});\delta^{(u)}] \\ \mathrm{y}_u & \end{array} \right) \tag{8.2}$$

and we have the two following formulas

Formula 1

$$\int_{u_1}^{v_1} \cdots \int_{u_s}^{v_s} \prod_{i=1}^{s} \left[(x_i - u_i)^{\alpha_i - 1} (v_i - x_i)^{\beta_i - 1} \prod_{j=1}^{T} (u_i^{(j)} x_i + v_i^{(j)})^{\sigma_i^{(j)}} \right]$$

$$E_{(\alpha_i'),(\beta_i')}^{(\gamma_i),m}\left[a,b,c;z\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{a_i}(v_i-x_i)^{b_i}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{c_i^{(j)}}}\right]\right]$$

$$F_{\bar{C}:D';\cdots;D^{(u)}}^{1+\bar{A}:B';\cdots;B^{(u)}}\left(\begin{array}{c}\mathrm{y}_1\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{\alpha_i'}(v_i-x_i)^{\beta_i'}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\xi_i^{(j,1)}}}\right]\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{y}_u\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{\alpha_i^{(u)}}(v_i-x_i)^{\beta_i^{(u)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\xi_i^{(j,u)}}}\right]\end{array}\right|$$

$$\left.\begin{array}{c}[(\text{-L});\mathrm{R}_1,\cdots,R_u][(a);\theta',\cdots,\theta^{(u)}]:[(b');\phi'];\cdots;[(b^{(u)});\phi^{(u)}]\\ [(\mathrm{c});\psi',\cdots,\psi^{(u)}]:[(d');\delta'];\cdots;[(d^{(u)});\delta^{(u)}]\end{array}\right)$$

$$A_{p,q:Y}^{m,n:X}\left(\begin{array}{c}\mathrm{z}_1\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{\delta_i'}(v_i-x_i)^{\eta_i'}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i^{(j,1)}}}\right]\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{\delta_i^{(r)}}(v_i-x_i)^{\eta_i^{(r)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i^{(j,r)}}}\right]\end{array}\right)$$

$${}_PF_Q\left[(A_P);(B_Q);-\sum_{k=1}^{l}g_k\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{\mu_i^{(k)}}(v_i-x_i)^{\theta_i^{(r)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\tau_i^{(j,k)}}}\right]\right]\mathrm{d}x_1\cdots\mathrm{d}x_s$$

$$=\frac{\prod_{j=1}^{Q}\Gamma(B_j)}{\prod_{j=1}^{P}\Gamma(A_j)}\prod_{i=1}^{s}\left[(v_i-u_i)^{\alpha_i+\beta_i-1}\prod_{j=1}^{W}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{\sigma_i^{(j)}}\prod_{j=W+1}^{T}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{\sigma_i^{(j)}}\right]$$

$$\sum_{n'=0}^{\infty}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}B_u\,A_{n'}\,\frac{z^{n'}}{n'!}\,y_1^{R_1}\cdots y_u^{R_u}\quad\prod_{i=1}^{s}\left[(v_i-u_i)^{n'(a_i+b_i)+\sum_{k=1}^{u}R_k(\alpha_i^{(k)}+\beta_i^{(k)})}\right]$$

$$\prod_{i=1}^{s}\left[\prod_{j=1}^{W}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{-n'c_i^{(j)}-\sum_{k=1}^{u}R_k\xi_i^{(j,k)}}\prod_{j=W+1}^{T}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{-n'c_i^{(j)}-\sum_{k=1}^{u}R_k\xi_i^{(j,k)}}\right]$$

$$A_{p+sT+P+2s;q+sT+Q+s;Y}^{m,n+sT+P+2s;X}\left(\begin{array}{c|c} \mathrm{z}_1 w_1 & \mathrm{A}\ ;\mathrm{A}^*:C \\ \cdots & \cdot \\ \cdots & \cdot \\ \mathrm{z}_r w_r & \cdot \\ \mathrm{g}_1 W_1 & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \mathrm{g}_l W_l & \cdot \\ \mathrm{G}_1 & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \mathrm{G}_T & \mathrm{B}\ ;\mathrm{B}^*:D \end{array}\right) \tag{8.3}$$

under the same notations and conditions thar (5.3)

and $B'_u = \dfrac{(-L)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)}{R_1!\cdots R_u!}$; $B(L;R_1,\cdots,R_u)$ **is defined by (8.1)**

Formula 2

$$\int_{u_1}^{v_1}\cdots\int_{u_s}^{v_s}\prod_{i=1}^{s}\left[(x_i-u_i)^{\alpha_i-1}(v_i-x_i)^{\beta_i-1}\prod_{j=1}^{T}(u_i^{(j)}x_i+v_i^{(j)})^{\sigma_i^{(j)}}\right]$$

$$E_{(\alpha_i'),(\beta_i')}^{(\gamma_i),m}\left[a,b,c;z\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{a_i}(v_i-x_i)^{b_i}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{c_i^{(j)}}}\right]\right]R_n^{\alpha,\beta}\left[Z\prod_{j=1}^{s}\left[\frac{(x_i-u_i)^{\zeta_i}(v_i-x_i)^{\lambda_i}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\theta_i^{(j)}}}\right]\right]$$

$$F_{\bar{C}:D';\cdots;D^{(u)}}^{1+\bar{A}:B';\cdots;B^{(u)}}\left(\begin{array}{c|} \mathrm{y}_1\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{\alpha_i'}(v_i-x_i)^{\beta_i'}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\xi_i^{(j,1)}}}\right] \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{y}_u\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{\alpha_i^{(u)}}(v_i-x_i)^{\beta_i^{(u)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\xi_i^{(j,u)}}}\right] \end{array}\right.$$

$$\left.\begin{matrix}[(\text{-L});\mathrm{R}_1,\cdots,R_u][(a);\theta',\cdots,\theta^{(u)}]:[(b');\phi'];\cdots;[(b^{(u)});\phi^{(u)}]\\ [(\mathrm{c});\psi',\cdots,\psi^{(u)}]:[(d');\delta'];\cdots;[(d^{(u)});\delta^{(u)}]\end{matrix}\right)$$

$$R_n^{\alpha,\beta}\left[Z\prod_{j=1}^{s}\left[\frac{(x_i-u_i)^{\zeta_i}(v_i-x_i)^{\lambda_i}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\theta_i^{(j)}}}\right]\right]$$

$$A\left(\begin{matrix}\mathrm{z}_1\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{\delta_i'}(v_i-x_i)^{\eta_i'}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i^{(j,1)}}}\right]\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\prod_{i=1}^{s}\left[\frac{(x_i-u_i)^{\delta_i^{(r)}}(v_i-x_i)^{\eta_i^{(r)}}}{\prod_{j=1}^{T}\left(U_i^{(j)}x_i+V_i^{(j)}\right)^{\rho_i^{(j,r)}}}\right]\end{matrix}\right)\mathrm{d}x_1\cdots\mathrm{d}x_s$$

$$=\prod_{i=1}^{s}\left[(v_i-u_i)^{\alpha_i+\beta_i-1}\prod_{j=1}^{W}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{\sigma_i^{(j)}}\prod_{j=W+1}^{T}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{\sigma_i^{(j)}}\right]\sum_{n'=0}^{\infty}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}$$

$$\sum_{w,v,u,t,e,k_1,k_2}B_uA_{n'}\frac{z^{n'}}{n'!}y_1^{R_1}\cdots y_u^{R_u}\psi'(w,v,u,t,e,k_1,k_2)Z^R\prod_{i=1}^{s}\left[(v_i-u_i)^{n'(a_i+b_i)+\sum_{k=1}^{u}R_k(\alpha_i^{(k)}+\beta_i^{(k)})}\right]$$

$$\prod_{i=1}^{s}\left[\prod_{j=1}^{W}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{-n'c_i^{(j)}-\sum_{k=1}^{u}R_k\xi_i^{(j,k)}}\prod_{j=W+1}^{T}\left(u_iU_i^{(j)}+V_i^{(j)}\right)^{-n'c_i^{(j)}-\sum_{k=1}^{u}R_k\xi_i^{(j,k)}}\right]$$

$$\psi'(w,v,u,t,e,k_1,k_2)A_{p+sT+2s;q+sT+s;Y}^{m,n+sT+2s;X}\left(\begin{matrix}\mathrm{z}_1w_1\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ \mathrm{z}_rw_r\\ \\ \mathrm{G}_1\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ \mathrm{G}_T\end{matrix}\left|\begin{matrix}\mathrm{A}\ ;\mathrm{A}^*:C\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{B}\ ;\mathrm{B}^*:D\end{matrix}\right.\right) \quad \textbf{(8.3)}$$

under the same notations and conditions thar (6.3)

and $B'_u = \dfrac{(-L)_{h_1R_1+\cdots+h_uR_u} B(E; R_1, \cdots, R_u)}{R_1! \cdots R_u!}$; $B(L; R_1, \cdots, R_u)$ is defined by (8.1)

9. Conclusion

In this paper we have evaluated two generalized multiple Eulerian integrals involving the multivariable A-functions defined by Gautam et al [4] with general arguments. The formulae established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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