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A novel exponential stability analysis of hybrid dynamical systems

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***Abstract*— This paper is concerned with exponential stability of switched linear systems with interval time-varying delays. The time delay is any continuous function belonging to a given interval, in which the lower bound of delay is not restricted to zero. By constructing a suitable augmented Lyapunov-Krasovskii functional combined with Leibniz-Newton's formula, a switching rule for the exponential stability of switched linear systems with interval time-varying delays and new delay-dependent sufficient conditions for the exponential stability of the systems are first established in terms of LMIs. Finally, some examples are exploited to illustrate the effectiveness of the proposed schemes.**

Keywords– Exponential stability; Hybrid systems; Time-varying delays; Lyapunov-Krasovskii functional; Leibniz-Newton's formula

I. Introduction

Switched time-delay systems have been attracting considerable attention during the recent years [1-4], due to the significance both in theory development and practical applications. However, it is worth noting that only the state time delay is considered, and the time delay in the state derivatives is largely ignored in the existing literature. If each subsystem of a switched system has time delay in the state derivatives, then the switched system is called switched neutral system [5–7]. Switched neutral systems exist widely in engineering and social systems, many physical plants can be modelled as switched neutral systems, such as distributed networks and heat exchanges. For example, in [8–12], a switched neutral type delay equation with nonlinear perturbations was exploited to model the drilling system. Unlike other systems, the neutral has time-delay in both the state and derivative. However, it is well-known that time-delay in the system may be a source of instability or bad system performance. Thus many researchers try to study them to find stability criteria for such system with time-delay to be stable.Most of the known results on this problem are derived assuming only that the time-varying delay $h(t)$ is a continuously differentiable function, satisfying some boundedness condition on its derivative: $\dot{h}(t) \leq \delta < 1$. This paper gives the improved results for the exponential stability of switched linear systems with interval time-varying delay. The time delay is assumed to be a time-varying continuous function belonging to a given interval, but not necessary to be differentiable. Specifically, our goal is to develop a constructive way to design switching rule to the exponential stability of switched linear systems with interval time-varying delay. By constructing argument Lyapunov functional combined with LMI technique, we propose new criteria for the exponential stability of the switched linear system. The delay-dependent stability conditions are formulated in terms of LMIs.

The paper is organized as follows: Section II presents definitions and some well-known technical propositions needed for the proof of the main results. Delay-dependent exponential stability conditions of the switched linear system are presented in Section III.

II. Preliminaries

The following notations will be used in this paper. R^+ denotes the set of all real non-negative numbers; R^n denotes the $n-$dimensional space with the scalar product $\langle .,. \rangle$ and the vector norm $\| \ . \ \|$; $M^{n\times r}$ denotes the space of all matrices of $(n \times r)-$dimensions; A^T denotes the transpose of matrix A; A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\text{min/max}}(A) = \text{min/max}\{\text{Re}\lambda; \lambda \in \lambda(A)\}$; $x_t := \{x(t+s) : s \in [-h,0]\}, \|x_t\| = \sup_{s\in[-h,0]} \| x(t+s) \|$; $C([0,t], R^n)$ denotes the set of all R^n-valued continuous functions on $[0,t]$; Matrix A is called semi-positive definite $(A \geq 0)$ if $\langle Ax, x\rangle \geq 0$, for all $x \in R^n$; A is positive definite $(A > 0)$ if $\langle Ax, x\rangle > 0$ for all $x \neq 0$; $A > B$ means $A - B > 0$. $*$ denotes the symmetric term in a matrix.

Consider a linear system with interval time-varying delay of the form

$$\begin{aligned} &\dot{x}(t) = A_\gamma x(t) + D_\gamma x(t-h(t)), \quad t \in R^+, \\ &x(t) = \phi(t), t \in [-h_2, 0], \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state; $\gamma(.) : R^n \to \mathcal{N} := \{1, 2, \ldots, N\}$ is the switching rule, which is a function depending on the state at each time and will be designed. A switching function is a rule which determines a switching sequence for a given switching system. Moreover, $\gamma(x(t)) = i$ implies that the system realization is chosen as the i^{th} system, $i = 1, 2, ..., N$. It is seen that the system (1) can be viewed as an autonomous switched system in which the effective subsystem changes when the state $x(t)$ hits predefined boundaries. $A_i, D_i \in M^{n\times n}, i = 1, 2, ..., N$ are given constant matrices,

and $\phi(t) \in C([-h_2,0],R^n)$ is the initial function with the norm
$\| \phi \| = \sup_{s\in[-h_2,0]} \| \phi(s) \|$; The time-varying delay function $h(t)$ satisfies

$$0 \le h_1 \le h(t) \le h_2, \quad t \in R^+.$$

The stability problem for switched system (1) is to construct a switching rule that makes the system exponentially stable.

Remark 2.1. It is worth noting that the time delay is a time-varying function belonging to a given interval, in which the lower bound of delay is not restricted to zero.

Definition 2.1. Given $\alpha > 0$. The switched linear system (1) is $\alpha-$exponentially stable if there exists a switching rule $\gamma(.)$ such that every solution $x(t,\phi)$ of the system satisfies the following condition

$$\exists N > 0: \quad \| x(t,\phi) \| \le Ne^{-\alpha t} \| \phi \|, \quad \forall t \in R^+.$$

We end this section with the following technical well-known propositions, which will be used in the proof of the main results.

Definition 2.2. The system of matrices $\{J_i\}, i = 1,2,\dots,N,$ is said to be strictly complete if for every $x \in R^n \backslash \{0\}$ there is $i \in \{1,2,\dots,N\}$ such that $x^T J_i x < 0$.

It is easy to see that the system $\{J_i\}$ is strictly complete if and only if

$$\bigcup_{i=1}^{N} \alpha_i = R^n \backslash \{0\},$$

where

$$\alpha_i = \{x \in R^n: \quad x^T J_i x < 0\}, i = 1,2,...,N.$$

We end this section with the following technical well-known propositions, which will be used in the proof of the main results.

Proposition 2.1. [13] *The system $\{J_i\}, i = 1,2,\dots,N,$ is strictly complete if there exist $\delta_i \ge 0, i = 1,2,\dots,N, \sum_{i=1}^N \delta_i > 0$ such that*

$$\sum_{i=1}^{N} \delta_i J_i < 0.$$

If $N = 2$ then the above condition is also necessary for the strict completeness.

Proposition 2.2. (Cauchy inequality) *For any symmetric positive definite marix $N \in M^{n\times n}$ and $a,b \in R^n$ we have*

$$\pm a^T b \le a^T N a + b^T N^{-1} b.$$

Proposition 2.3. [13] *For any symmetric positive definite matrix $M \in M^{n\times n}$, scalar $\gamma > 0$ and vector function $\omega : [0,\gamma] \to R^n$ such that the integrations concerned are well defined, the following inequality holds*

$$\Big(\int_0^\gamma \omega(s)\,ds\Big)^T M\Big(\int_0^\gamma \omega(s)\,ds\Big) \le \gamma\Big(\int_0^\gamma \omega^T(s) M \omega(s)\,ds\Big).$$

Proposition 2.4. [14] *Let E,H and F be any constant matrices of appropriate dimensions and $F^TF \le I$. For any $\epsilon > 0$, we have*

$$EFH + H^TF^TE^T \le \epsilon EE^T + \epsilon^{-1}H^TH.$$

Proposition 2.5. (Schur complement lemma [14]). *Given constant matrices X,Y,Z with appropriate dimensions satisfying $X = X^T, Y = Y^T > 0$. Then $X + Z^TY^{-1}Z < 0$ if and only if*

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0 \quad \text{or} \quad \begin{pmatrix} -Y & Z \\ Z^T & X \end{pmatrix} < 0.$$

III. Main Results

Let us set

$$\mathcal{M}_i = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} \\ * & M_{22} & 0 & M_{24} & S_2 \\ * & * & M_{33} & M_{34} & S_3 \\ * & * & * & M_{44} & M_{45} \\ * & * & * & * & M_{55} \end{bmatrix},$$

$$\begin{aligned} & J_i = Q - S_1A_i - A_i^TS_1^T, \\ & \alpha_i = \{x \in R^n: \quad x^TJ_ix < 0\}, \quad i = 1,2,...,N, \\ & \bar{\alpha}_1 = \alpha_1, \quad \bar{\alpha}_i = \alpha_i \setminus \bigcup_{j=1}^{i-1} \bar{\alpha}_j, \quad i = 2,3,\dots,N, \\ & \lambda_1 = \lambda_{\min}(P), \\ & \lambda_2 = \lambda_{\max}(P) + 2h_2\lambda_{\max}(Q), \end{aligned} \tag{2}$$

$$\begin{aligned} & M_{11} = A_i^TP + PA_i + 2\alpha P + Q, \\ & M_{12} = -S_2A_i, \quad M_{13} = -S_3A_i, \\ & M_{14} = PD_i - S_1D_i - S_4A_i, \quad M_{15} = S_1 - S_5A_i, \\ & M_{22} = -e^{-2\alpha h_1}Q, \quad M_{24} = -S_2D_i, \\ & M_{33} = -e^{-2\alpha h_2}Q, \quad M_{34} = -S_3D_i, \\ & M_{44} = -S_4D_i, \quad M_{45} = S_4 - S_5D_i, \\ & M_{55} = S_5 + S_5^T. \end{aligned}$$

The main result of this paper is summarized in the following theorem.

Theorem 1. *Given $\alpha > 0$. The zero solution of the switched linear system (1) is $\alpha-$exponentially stable if there* *exist symmetric positive definite matrices P, Q, and matrices $S_i, i = 1, 2, ..., 5$ such that satisfying the following conditions*

(i) $\exists \delta_i \geq 0, i = 1, 2, \dots, N, \quad \sum_{i=1}^{N} \delta_i > 0 : \sum_{i=1}^{N} \delta_i J_i < 0.$

(ii) $\mathcal{M}_i < 0, \quad i = 1, 2, ..., N.$

Moreover, the solution $x(t, \phi)$ of the system satisfies

$$\| x(t, \phi) \| \leq \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\alpha t} \| \phi \|, \quad \forall t \in R^+.$$

Proof. We consider the following Lyapunov-Krasovskii functional for the system (1)

$$V(t, x_t) = \sum_{i=1}^{3} V_i,$$

where

$$\begin{aligned} V_1 &= x^T(t)Px(t), \\ V_2 &= \int_{t-h_1}^{t} e^{2\alpha(s-t)} x^T(s)Qx(s)\, ds, \\ V_3 &= \int_{t-h_2}^{t} e^{2\alpha(s-t)} x^T(s)Qx(s)\, ds. \end{aligned}$$

It easy to check that

$$\lambda_1 \| x(t) \|^2 \leq V(t, x_t) \leq \lambda_2 \| x_t \|^2, \quad \forall t \geq 0, \tag{3}$$

Taking the derivative of V_1 along the solution of system (1) we have

$$\begin{aligned} \dot{V}_1 =& 2x^T(t)P\dot{x}(t) \\ =& 2x^T(t)[A_i^T P + A_i P]x(t) + 2x^T(t)PD_i x(t - h(t)); \\ \dot{V}_2 =& x^T(t)Qx(t) - e^{-2\alpha h_1} x^T(t - h_1)Qx(t - h_1) - 2\alpha V_2; \\ \dot{V}_3 =& x^T(t)Qx(t) - e^{-2\alpha h_2} x^T(t - h_2)Qx(t - h_2) - 2\alpha V_3. \end{aligned}$$

Therefore, we have

$$\begin{aligned} \dot{V}(.) + 2\alpha V(.) \leq& 2x^T(t)[A_i^T P + A_i P + 2\alpha P + 2Q)]x(t) \\ &+ 2x^T(t)PD_i x(t - h(t)) \\ &- e^{-2\alpha h_1} x^T(t - h_1)Qx(t - h_1) \\ &- e^{-2\alpha h_2} x^T(t - h_2)Qx(t - h_2). \end{aligned} \tag{4}$$

By using the following identity relation

$$\dot{x}(t) - A_i x(t) - D_i x(t - h(t)) = 0,$$

we have

$$\begin{aligned} &2x^T(t)S_1\dot{x}(t) - 2x^T(t)S_1 A_i x(t) \\ &- 2x^T(t)S_1 D_i x(t - h(t)) = 0 \\ &2x^T(t - h_1)S_2\dot{x}(t) - 2x^T(t - h_1)S_2 A_i x(t) \\ &- 2x^T(t - h_1)S_2 D_i x(t - h(t)) = 0 \\ &2x^T(t - h_2)S_3\dot{x}(t) - 2x^T(t - h_2)S_3 A_i x(t) \\ &- 2x^T(t - h_2)S_3 D_i x(t - h(t)) = 0 \\ &2x^T(t - h(t))S_4\dot{x}(t) - 2x^T(t - h(t))S_4 A_i x(t) \\ &- 2x^T(t - h(t))S_4 D_i x(t - h(t)) = 0 \\ &2\dot{x}^T(t)S_5\dot{x}(t) - 2\dot{x}^T(t)S_5 A_i x(t) \\ &- 2\dot{x}^T(t)S_5 D_i x(t - h(t)) = 0 \end{aligned} \tag{5}$$

Adding all the zero items of (5) into (4), we obtain

$$\begin{aligned} \dot{V}(.) + 2\alpha V(.) \leq\, & x^T(t)[A_i^T P + PA_i + 2\alpha P - S_1 A_i \\ &- A_i^T S_1^T + 2Q]x(t) \\ &+ 2x^T(t)[e^{-2\alpha h_1} R - S_2 A_i]x(t - h_1) \\ &+ 2x^T(t)[-S_3 A_i]x(t - h_2) + 2x^T(t)[PD_i \\ &- S_1 D_i - S_4 A_i]x(t - h(t)) \\ &+ 2x^T(t)[S_1 - S_5 A_i]\dot{x}(t) \\ &+ x^T(t - h_1)[-e^{-2\alpha h_1} Q]x(t - h_1) \\ &+ 2x^T(t - h_1)[-S_2 D_i]x(t - h(t)) \\ &+ 2x^T(t - h_1)S_2\dot{x}(t) \\ &+ x^T(t - h_2)[-e^{-2\alpha h_2} Q]x(t - h_2) \\ &+ x^T(t - h_2)[-S_3 D_i]x(t - h(t)) \\ &+ 2x^T(t - h_2)S_3\dot{x}(t) \\ &+ x^T(t - h(t))[-S_4 D_i]x(t - h(t)) \\ &+ 2x^T(t - h(t))[S_4 - S_5 D_i]\dot{x}(t) \\ &+ \dot{x}^T(t)[S_5 + S_5^T]\dot{x}(t) \\ &= x^T(t)J_i x(t) + \zeta^T(t)\mathcal{M}_i\zeta(t), \end{aligned} \tag{6}$$

where

$$\begin{aligned} &\zeta(t) = [x(t), x(t - h_1), x(t - h_2), x(t - h(t)), \dot{x}(t)], \\ &J_i = Q - S_1 A_i - A_i^T S_1^T, \\ &\mathcal{M}_i = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} \\ * & M_{22} & 0 & M_{24} & S_2 \\ * & * & M_{33} & M_{34} & S_3 \\ * & * & * & M_{44} & M_{45} \\ * & * & * & * & M_{55} \end{bmatrix}, \\ &M_{11} = A_i^T P + PA_i + 2\alpha P + Q, \\ &M_{12} = -S_2 A_i, \quad M_{13} = -S_3 A_i, \\ &M_{14} = PD_i - S_1 D_i - S_4 A_i, \quad M_{15} = S_1 - S_5 A_i, \\ &M_{22} = -e^{-2\alpha h_1} Q, \quad M_{24} = -S_2 D_i, \\ &M_{33} = -e^{-2\alpha h_2} Q, \quad M_{34} = -S_3 D_i, \\ &M_{44} = -S_4 D_i, \quad M_{45} = S_4 - S_5 D_i, \\ &M_{55} = S_5 + S_5^T. \end{aligned}$$

Therefore, we finally obtain from (6) and the condition (ii) that

$$\dot{V}(.) + 2\alpha V(.) < x^T(t)J_i x(t), \quad \forall i = 1,2,....,N, \quad t \in R^+.$$

We now apply the condition (i) and Proposition 2.1., the system J_i is strictly complete, and the sets α_i and $\bar{\alpha}_i$ by (2) are well defined such that

$$\bigcup_{i=1}^{N} \alpha_i = R^n \backslash \{0\},$$

$$\bigcup_{i=1}^{N} \bar{\alpha}_i = R^n \backslash \{0\}, \quad \bar{\alpha}_i \cap \bar{\alpha}_j = \emptyset, i \neq j.$$

Therefore, for any $x(t) \in R^n, \quad t \in R^+$, there exists $i \in \{1, 2, \ldots, N\}$ such that $x(t) \in \bar{\alpha}_i$. By choosing switching rule as $\gamma(x(t)) = i$ whenever $\gamma(x(t)) \in \bar{\alpha}_i$, from (6) we have

$$\dot{V}(.) + 2\alpha V(.) \leq x^T(t)J_i x(t) < 0, \quad t \in R^+,$$

and hence

$$\dot{V}(t, x_t) \leq -2\alpha V(t, x_t), \quad \forall t \in R^+. \tag{7}$$

Integrating both sides of (7) from 0 to t, we obtain

$$V(t, x_t) \leq V(\phi)e^{-2\alpha t}, \quad \forall t \in R^+.$$

Furthermore, taking condition (3) into account, we have

$$\lambda_1 \parallel x(t, \phi) \parallel^2 \leq V(x_t) \leq V(\phi)e^{-2\alpha t} \leq \lambda_2 e^{-2\alpha t} \parallel \phi \parallel^2,$$

then

$$\parallel x(t, \phi) \parallel \leq \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\alpha t} \parallel \phi \parallel, \quad t \in R^+,$$

which concludes the proof by the Lyapunov stability theorem [14]. To illustrate the obtained result, let us give the following numerical example.

Remark 1. Note that the result proposed in [1–5, 7–11] for switching systems to be asymptotically stable under an arbitrary switching rule. The asymptotic stability for switching time-delay differential systems studied in [7–9] was limited to constant delays. In [5–8], a class of switching signals has been identified for the considered switched delay differential systems to be stable under the averaged well time scheme.

IV. Numerical Example

Example 1. Consider the following the switched systems with interval time-varying delay (1), where the delay function $h(t)$ is given by

$$h(t) = 0.2 + 1.5329 sin^2 t,$$

and

$$A_1 = \begin{pmatrix} -2 & 0.1 \\ 0.2 & -2.5 \end{pmatrix}, A_2 = \begin{pmatrix} -2.5 & 0.3 \\ 0.2 & -2.9 \end{pmatrix},$$

$$D_1 = \begin{pmatrix} -0.3 & 0.2 \\ 0.1 & -0.39 \end{pmatrix}, D_2 = \begin{pmatrix} -0.5 & 0.2 \\ 0.1 & -0.4 \end{pmatrix}.$$

It is worth noting that, the delay function $h(t)$ is non-differentiable and the exponent $\alpha \geq 1$. Therefore, the methods used is in $[3, 8, 9, 11 - 15, 17 - 23]$ are not applicable to this system. By LMI toolbox of Matlab, we find that the conditions (i), (ii) of Theorem 1 are satisfied with $h_1 = 0.1, h_2 = 1.7329, \delta_1 = 0.5, \delta_2 = 0.3, \alpha = 0.5, \rho_{11} = 0.1, \rho_{12} = 0.2, \rho_{21} = 0.1, \rho_{22} = 0.2$ and

$$P = \begin{pmatrix} 1.2397 & -0.3984 \\ -0.3984 & 1.3112 \end{pmatrix}, Q = \begin{pmatrix} 1.7931 & -0.0079 \\ -0.0079 & 0.2397 \end{pmatrix},$$

$$R = \begin{pmatrix} 2.3297 & -0.1121 \\ -0.1121 & 1.3397 \end{pmatrix}, U = \begin{pmatrix} 1.7394 & -0.0982 \\ -0.0982 & 0.6321 \end{pmatrix},$$

$$S_1 = \begin{pmatrix} -0.6210 & -0.0335 \\ 0.0499 & -0.3576 \end{pmatrix}, S_2 = \begin{pmatrix} -0.3602 & 0.0170 \\ 0.0298 & -0.3550 \end{pmatrix},$$

$$S_3 = \begin{pmatrix} -0.3602 & 0.0170 \\ 0.0298 & -0.3550 \end{pmatrix}, S_4 = \begin{pmatrix} 0.6968 & -0.0401 \\ -0.0525 & 0.7040 \end{pmatrix},$$

$$S_5 = \begin{pmatrix} -1.4043 & 0.0265 \\ -0.0028 & -0.9774 \end{pmatrix}.$$

In this case, we have

$$(J_1, J_2) = \left(\begin{bmatrix} -1.5667 & -0.0031 \\ -0.0031 & -1.9712 \end{bmatrix}, \begin{bmatrix} -1.5511 & 0.0029 \\ 0.0029 & -1.3297 \end{bmatrix} \right).$$

Moreover, the sum

$$\delta_1 J_1(R, Q) + \delta_2 J_2(R, Q) = \begin{bmatrix} -0.3269 & 0 \\ 0 & -0.7239 \end{bmatrix}$$

is negative definite; i.e. the first entry in the first row and the first column $-0.3269 < 0$ is negative and the determinant of the matrix is positive. The sets α_1 and α_2 are given as

$$\alpha_1 = \{(x_1, x_2) : -1.5667x_1^2 - 0.0062x_1x_2 - 1.9712x_2^2 < 0\},$$
$$\alpha_2 = \{(x_1, x_2) : 1.5511x_1^2 - 0.0058x_1x_2 + 1.3297x_2^2 > 0\}.$$

Obviously, the union of these sets is equal to $R^2 \setminus \{0\}$. The switching regions are defined as

$$\overline{\alpha}_1 = \{(x_1, x_2) : -1.5667x_1^2 - 0.0062x_1x_2 - 1.9712x_2^2 < 0\},$$
$$\overline{\alpha}_2 = \alpha_2 \setminus \overline{\alpha}_1.$$

By Theorem 1 the switched systems (1) is $0.5-$exponentially stable and the switching rule is chosen as $\gamma(x(t)) = i$ whenever $x(t) \in \bar{\alpha}_i$. Moreover, the solution $x(t, \phi)$ of the system satisfies

$$\parallel x(t, \phi) \parallel \leq \{1.0239 e^{-0.5t} \parallel \phi \parallel, \quad \forall t \in R^+.$$

V. Conclusion

This paper has proposed a switching design for the exponential stability of switched linear systems with interval time-varying delays. Based on the improved Lyapunov-Krasovskii functional, a switching rule for the exponential stability for the system is designed via linear matrix inequalities.

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