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# Finite triple integrals involving the general class of multivariable polynomials and special functions

$F.Y.\ AYANT^1$

1 Teacher in High School , France
E-mail : fredericayant@gmail.com

ABSTRACT
In this paper a triple finite integrals involving a general class og multivariable polynomials, multivariable A-function and modified multivariable H-function with general arguments have been evaluated. The result is believed to be new and is capable of giving a very large number of triple, double or simpler integrals involving a large number of special functions and polynomials as its special cases. We shall see several corollaries at the end.

**Keywords:** General class of multivariable polynomials, finite triple integral, modified multivariable H-function, multivariable H-function, multivariable A-function, A-function, H-function.

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## 1. Introduction

Recently Chaurasia and Tak [2] have studied some double integrals concerning the product of general class of polynomials defined by Srivastava [10] and the multivariable H-function defined by Srivastava and Panda [11,12], Al-Jarrah [1] has established a triple finite integral involving the $\bar{H}$-function. In this present paper, we evaluate a triple finite integral involving general class of multivariable polynomials, multivariable A-function and modified multivariable H-function. At the end of this document, we shall study several corollaries.

The generalized polynomials of multivariables defined by Srivastava [10], is given in the following manner :

$$S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[y_1,\cdots,y_v]=\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\frac{(-N_1)_{\mathfrak{M}_1K_1}}{K_1!}\cdots\frac{(-N_v)_{\mathfrak{M}_vK_v}}{K_v!}A[N_1,K_1;\cdots;N_v,K_v]y_1^{K_1}\cdots y_v^{K_v} \quad (1.1)$$

where $\mathfrak{M}_1,\cdots,\mathfrak{M}_v$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_v,K_v]$ are arbitrary constants, real or complex.

We shall note

$$a_v'=\frac{(-N_1)_{\mathfrak{M}_1K_1}}{K_1!}\cdots\frac{(-N_v)_{\mathfrak{M}_vK_v}}{K_v!}A[N_1,K_1;\cdots;N_v,K_v] \quad (1.2)$$

The A-function of several variables is an extension of multivariable H-function .The serie representation of the multivariable A-function is given by Gautam and Asgar [5] as

$$A[z_u,\cdots,z_u]=A_{A,C:(M',N');\cdots;(M^{(u)},N^{(u)})}^{0,\lambda:(\alpha',\beta');\cdots;(\alpha^{(u)},\beta^{(u)})}\left(\begin{array}{c|c} \mathrm{z}_u & [(\mathrm{g}_j);\gamma',\cdots,\gamma^{(u)}]_{1,A}: \\ \cdot & \cdots \\ \cdot & \\ \mathrm{z}_u & [(\mathrm{f}_j);\xi',\cdots,\xi^{(u)}]_{1,C}: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{q}^{(1)},\eta^{(1)})_{1,M^{(1)}};\cdots;(q^{(u)},\eta^{(u)})_{1,M^{(u)}} \\ \cdots \\ \cdots \\ (\mathrm{p}^{(1)},\epsilon^{(1)})_{1,N^{(1)}};\cdots;(p^{(u)},\epsilon^{(u)})_{1,N^{(u)}} \end{array}\right)=\sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=0}^{\infty}\phi\frac{\prod_{i=1}^{u}\phi_i u_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^{u}g_i}}{\prod_{i=1}^{u}\epsilon_{G_i}^{(i)}g_i!} \quad (1.3)$$

where

$$\phi = \frac{\prod_{j=1}^{\lambda}\Gamma\left(1-g_j+\sum_{i=1}^{u}\gamma_j^{(i)}\eta_{G_i,g_i}\right)}{\prod_{j=\lambda'+1}^{A}\Gamma\left(g_j-\sum_{i=1}^{u}\gamma_j^{(i)}U_i\right)\prod_{j=1}^{C}\Gamma\left(1-f_j+\sum_{i=1}^{u}\xi_j^{(i)}\eta_{G_i,g_i}\right)} \tag{1.4}$$

$$\phi_i = \frac{\prod_{j=1,j\neq m_i}^{\alpha^{(i)}}\Gamma\left(p_j^{(i)}-\epsilon_j^{(i)}\eta_{G_i,g_i}\right)\prod_{j=1}^{\beta^{(i)}}\Gamma\left(1-q_j^{(i)}+\eta_j^{(i)}\eta_{G_i,g_i}\right)}{\prod_{j=\alpha^{(i)}+1}^{N^{(i)}}\Gamma\left(1-p_j^{(i)}+\epsilon_j^{(i)}\eta_{G_i,g_i}\right)\prod_{j=\beta^{(i)}+1}^{M^{(i)}}\Gamma\left(q_j^{(i)}-\eta_j^{(i)}\eta_{G_i,g_i}\right)}, i=1,\cdots,u \tag{1.5}$$

$$\eta_{G_i,g_i} = \frac{p_{G_i}^{(i)}+g_i}{\epsilon_{G_i}^{(i)}}, i=1,\cdots,u \tag{1.6}$$

$$\sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=0}^{\infty} = \sum_{G_1,\cdots,G_u=1}^{\alpha^{(1)},\cdots,\alpha^{(u)}}\sum_{g_1,\cdots,g_u=0}^{\infty} \tag{1.7}$$

and

$$z_i \neq 0, \sum_{j=1}^{A}\gamma_j^{(i)}-\sum_{j=1}^{C}\xi_j^{(i)}+\sum_{j=1}^{M^{(i)}}\eta_j^{(i)}-\sum_{j=1}^{N^{(i)}}\epsilon_j^{(i)} < 0, i=1,\cdots,u \tag{1.8}$$

Here $\lambda, A, C, \alpha_i, \beta_i, M_i, N_i \in \mathbb{N}^*; i=1,\cdots,u\, ; f_j, g_j, p_j^{(i)}, q_j^{(i)}, \gamma_j^{(i)}, \xi_j^{(i)}, \eta_j^{(i)}, \epsilon_j^{(i)} \in \mathbb{C}$

The modified H-function defined by Prasad and Singh [8] generalizes the multivariable H-function defined by Srivastava and Panda [11,12]. It is defined in term of multiple Mellin-Barnes type integral :

$$H(z_1,\cdots,z_r) = H_{\mathbf{p},\mathbf{q}:|R:p_1,q_1;\cdots;p_r,q_r}^{\mathbf{m},\mathbf{n}:|\mathbf{R}:m_1,n_1;\cdots;m_r,n_r}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathbf{p}} : \\ \cdot & \\ \cdot & \\ \cdot & (\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(r)})_{1,\mathbf{q}} : \\ \mathrm{z}_r & \end{array}\right.$$

$$\left.\begin{array}{l} (\mathrm{e}_j;u_j^{(1)}g_j^{(1)},\cdots,u_j^{(r)}g_j^{(r)})_{1,\mathbf{R}} : (c_j^{(1)};\gamma_j^{(1)})_{1,p_1},\cdots,(c_j^{(r)};\gamma_j^{(r)})_{1,p_r} \\ \\ (\mathrm{l}_j;U_j^{(1)}f_j^{(1)},\cdots,U_j^{(r)}f_j^{(r)})_{1,R} : (d_j^{(1)};\delta_j^{(1)})_{1,q_1},\cdots,(d_j^{(r)};\delta_j^{(r)})_{1,q_r} \end{array}\right) \tag{1.9}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.10}$$

where $\phi(s_1,\cdots,s_r), \theta_i(s_i), i=1,\cdots,r$ are given by :

$$\phi(s_1,\cdots,s_r) = \frac{\prod_{j=1}^{\mathbf{m}}\Gamma(b_j-\sum_{i=1}^{r}\beta_j^{(i)}s_i)\prod_{j=1}^{\mathbf{n}}\Gamma\left(1-a_j+\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)}{\prod_{j=\mathbf{n}+1}^{\mathbf{p}}\Gamma\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)\prod_{j=\mathbf{m}+1}^{\mathbf{q}}\Gamma\left(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}s_j\right)}$$

$$\frac{\prod_{j=1}^{\mathbf{R}}\Gamma(e_j+\sum_{i=1}^{r}u_j^{(i)}g_j^{(i)}s_i)}{\prod_{j=1}^{R}\Gamma(l_j+\sum_{i=1}^{r}U_j^{(i)}f_j^{(i)}s_i)} \tag{1.11}$$

$$\phi_i(s_i) = \frac{\prod_{j=1}^{n_i}\Gamma\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)\prod_{j=1}^{m_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)}{\prod_{j=n_i+1}^{p_i}\Gamma\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)\prod_{j=m_i+1}^{q_i}\Gamma\left(1-d_j^{(i)}+\delta_j^{(i)}s_i\right)} \tag{1.12}$$

The multiple integral (1.17) converges absolutely if

$$|argz_k| < \frac{1}{2}U_i\pi \quad (i=1,\cdots,r) \tag{1.13}$$

with $$U_i=\sum_{j=1}^{\mathbf{m}}\beta_j^{(i)}-\sum_{j=\mathbf{m}+1}^{\mathbf{q}}\beta_j^{(i)}+\sum_{j=1}^{\mathbf{n}}\alpha_j^{(i)}-\sum_{j=\mathbf{n}+1}^{\mathbf{p}}\alpha_j^{(i)}+\sum_{j=1}^{m_i}\delta_j^{(i)}-\sum_{j=1+m_i}^{q_i}\delta_j^{(i)}$$

$$+\sum_{j=1}^{n_i}\gamma_j^{(i)}-\sum_{j=n_i+1}^{p_i}\gamma_j^{(i)}+\sum_{j=1}^{\mathbf{R}}g_j^{(i)}-\sum_{j=1}^{R}f_j^{(i)}>0(i=1,\cdots,r) \tag{1.14}$$

## 2. Required integrals

In this section, we give the following integrals with the help of results ([6] and [7])

**Lemma 1**

$$\int_0^{\frac{\pi}{2}} e^{\omega(\beta+\gamma)\theta}\sin^{\beta-1}\theta\cos^{\gamma-1}\theta\,\mathrm{d}\theta = e^{\omega\pi\beta/2}\frac{\Gamma(\beta)\Gamma(\gamma)}{\Gamma(\beta+\gamma)} \tag{2.1}$$

where $Re(\beta)>0, Re(\gamma)>0$

**Lemma 2**

$$\int_0^1 x^{\beta'-1}(1-x)^{\gamma'-1}\left[ax+b(1-x)\right]^{-\beta'-\gamma'}{}_2F_1\left[c,d;\beta';\frac{ax}{ax+b(1-x)}\right]\mathrm{d}x = \frac{\Gamma(\beta')\Gamma(\gamma')\Gamma(\beta'+\gamma'-c-d)}{a^{\beta'}b^{\gamma'}\Gamma(\beta'+\gamma'-c)\Gamma(\beta'+\gamma'-d)} \tag{2.2}$$

where $Re(\beta')>0, Re(\gamma')>0, Re(\beta'+\gamma'-c-d)>0$, $a$ and $b$ are such that the expression$[ax+b(1-x)]\neq 0$ for $0\leqslant x\leqslant 1$ and

**Lemma 3**

$$\int_0^1 y^{t-1}(1-y)^{q-1}\mathrm{d}y = \frac{\Gamma(t)\Gamma(q)}{\Gamma(t+q)} \tag{2.3}$$

where $Re(t)>0, Re(q)>0$

## 3. Main integral

In this section, we establish a triple finite integral with general arguments.

Let

$$X(\theta,x,y;c,d,e,g,h) = e^{\omega(c+d)\theta}\sin^c\theta\cos^d\theta\left[b(1-x)\right]^e\left[ax+b(1-x)\right]^{-e}y^g(1-y)^h \tag{3.1}$$

$$X = m_1,n_1;\cdots;m_r,n_r \tag{3.2}$$

$$Y = p_1,q_1;\cdots;p_r,q_r \tag{3.3}$$

$$\mathbb{A} = (a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathbf{p}};(\mathrm{e}_j;u_j^{(1)}g_j^{(1)},\cdots,u_j^{(r)}g_j^{(r)})_{1,\mathbf{R}}:(\mathrm{c}_j^{(1)};\gamma_j^{(1)})_{1,p_1},\cdots,(c_j^{(r)};\gamma_j^{(r)})_{1,p_r} \tag{3.4}$$

$$\mathbb{B} = (b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)})_{1,\mathbf{q}};(\mathrm{l}_j;U_j^{(1)}f_j^{(1)},\cdots,U_j^{(r)}f_j^{(r)})_{1,R}:(d_j^{(1)};\delta_j^{(1)})_{1,q_1},\cdots,(d_j^{(r)};\delta_j^{(r)})_{1,q_r} \tag{3.5}$$

**Theorem 1**

$$\int_0^{\frac{\pi}{2}}\int_0^1\int_0^1 e^{\omega(\beta+\gamma)\theta}\sin^{\beta-1}\theta\cos^{\gamma-1}\theta x^{\beta'-1}(1-x)^{\gamma'-1}\left[ax+b(1-x)\right]^{-\beta'-\gamma'}{}_2F_1\left[c,d;\beta';\frac{ax}{ax+b(1-x)}\right]$$

$$y^{t-1}(1-y)^{q-1}S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\begin{pmatrix} \mathrm{y}_1X(\theta,x,y;\upsilon_1'',\mu_1'',\sigma_1'',f_1'',h_1'') \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{y}_vX(\theta,x,y;\upsilon_v'',\mu_v'',\sigma_v'',f_v'',h_v'') \end{pmatrix}$$

$$A\begin{pmatrix} \mathrm{z}_1'X(\theta,x,y;\upsilon_1',\mu_1',\sigma_1',f_1',h_1') \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_u'X(\theta,x,y;\upsilon_u',\mu_u',\sigma_u',f_u',h_u') \end{pmatrix} H_{\mathbf{p},\mathbf{q}:|R:Y}^{\mathbf{m},\mathbf{n}:|\mathbf{R}:X}\left(\begin{matrix} \mathrm{z}_1X(\theta,x,y;\upsilon_1,\mu_1,\sigma_1,f_1,h_1) \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_rX(\theta,x,y;\upsilon_r,\mu_r,\sigma_r,f_r,h_r) \end{matrix}\;\middle|\;\begin{matrix} \mathbb{A} \\ \cdot \\ \cdot \\ \mathbb{B} \end{matrix}\right)\mathrm{d}\theta\mathrm{d}x\mathrm{d}y =$$

$$\frac{\Gamma(\beta')e^{\omega\pi\beta/2}}{a^{\beta'}b^{\gamma'}}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=0}^{\infty}\phi\frac{\prod_{i=1}^u\phi_k z_i'^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^u g_i}}{\prod_{i=1}^u\delta_{G^{(i)}}^{(i)}\prod_{i=1}^u g_i!}a_v'y_1^{K_1}\cdots y_v^{K_v}$$

$$e^{(\omega(\sum_{i=1}^v\upsilon_i''K_i+\sum_{j=1}^u\upsilon_j'\eta_{G_j,g_j}))/2}\,H_{\mathbf{p}+6,\mathbf{q}+4:|R:Y}^{\mathbf{m},\mathbf{n}+6:|\mathbf{R}:X}\left(\begin{matrix} \mathrm{z}_1e^{\pi\omega\upsilon_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_re^{\pi\omega\upsilon_r} \end{matrix}\;\middle|\;\begin{matrix} \mathrm{A} \\ \cdot \\ \cdot \\ \mathrm{B} \end{matrix}\right) \tag{3.6}$$

where

$$A = \left(1-\beta-\sum_{i=1}^v K_i\upsilon_i''-\sum_{j=1}^u\upsilon_j'\eta_{G_j,g_j};\upsilon_1,\cdots,\upsilon_r\right),\left(1-\gamma-\sum_{i=1}^v K_i\mu_i''-\sum_{j=1}^u\mu_j'\eta_{G_j,g_j};\mu_1,\cdots,\mu_r\right),$$

$$\left(1-\beta'-\gamma'-\sum_{i=1}^v K_i\sigma_i''-\sum_{j=1}^u\sigma_j'\eta_{G_j,g_j};\sigma_1,\cdots,\sigma_r\right),\left(1-\gamma'-\sum_{i=1}^v K_i\sigma_i''-\sum_{j=1}^u\sigma_j'\eta_{G_j,g_j};\sigma_1,\cdots,\sigma_r\right),$$

$$\left(1-t-\sum_{i=1}^v K_i f_i''-\sum_{j=1}^u f_j'\eta_{G_j,g_j};f_1,\cdots,f_r\right),\left(1-q-\sum_{i=1}^v K_i h_i''-\sum_{j=1}^u h_j'\eta_{G_j,g_j};h_1,\cdots,h_r\right),\mathbb{A} \tag{3.7}$$

$$B = \left(1-\beta-\gamma-\sum_{i=1}^v K_i(\mu_i''+\upsilon_i'')-\sum_{j=1}^u(\mu_j'+\upsilon_j')\eta_{G_j,g_j};\mu_1+\upsilon_1,\cdots,\mu_r+\upsilon_r\right),$$

$$\left(1-\beta'-\gamma'+c-\sum_{i=1}^{v}K_i\sigma''_i-\sum_{j=1}^{u}\sigma'_j\eta_{G_j,g_j};\sigma_1,\cdots,\sigma_r\right),\left(1-\beta'-\gamma'+d-\sum_{i=1}^{v}K_i\sigma''_i-\sum_{j=1}^{u}\sigma'_j\eta_{G_j,g_j};\sigma_1,\cdots,\sigma_r\right),$$

$$\left(1-t-q-\sum_{i=1}^{v}K_i(f''_i+h''_i)-\sum_{j=1}^{u}(f'_j+h'_j)\eta_{G_j,g_j};f_1+h_1,\cdots,f_r+h_r\right),\mathbb{B}$$

Provided that

$min\{\upsilon''_i,\mu''_i,\sigma''_i,f''_i,h''_i,\upsilon'_j,\mu'_j,\sigma'_j,f'_i,h'_i,\upsilon_k,\mu_k,\sigma_k,f_k,h_k\}>0$ ; $i=1,\cdots,v;j=1,\cdots,u;k=1,\cdots,s$

$$Re\left(\beta+\sum_{j=1}^{u}\upsilon'_j\eta_{G_j,g_j}\right)+\sum_{k=1}^{r}\upsilon_k\min_{1\leqslant K\leqslant m_k}Re\left(\frac{d_K^{(k)}}{\delta_K^{(k)}}\right)>0$$

$$Re\left(\gamma+\sum_{j=1}^{u}\mu'_j\eta_{G_j,g_j}\right)+\sum_{k=1}^{r}\mu_k\min_{1\leqslant K\leqslant m_k}Re\left(\frac{d_K^{(k)}}{\delta_K^{(k)}}\right)>0\,;Re(\beta')>0\,;Re(\beta'+\gamma'-c-d)>0$$

$$Re\left(\gamma'+\sum_{j=1}^{u}\sigma'_j\eta_{G_j,g_j}\right)+\sum_{k=1}^{r}\sigma_k\min_{1\leqslant K\leqslant m_k}Re\left(\frac{d_K^{(k)}}{\delta_K^{(k)}}\right)>0$$

$$Re\left(t+\sum_{j=1}^{u}f'_j\eta_{G_j,g_j}\right)+\sum_{k=1}^{r}f_k\min_{1\leqslant K\leqslant m_k}Re\left(\frac{d_K^{(k)}}{\delta_K^{(k)}}\right)>0$$

$$Re\left(q+\sum_{j=1}^{u}h'_j\eta_{G_j,g_j}\right)+\sum_{k=1}^{r}h_k\min_{1\leqslant K\leqslant m_k}Re\left(\frac{d_K^{(k)}}{\delta_K^{(k)}}\right)>0$$

$$z'_i\neq 0,\sum_{j=1}^{A}\gamma_j^{(i)}-\sum_{j=1}^{C}\xi_j^{(i)}+\sum_{j=1}^{M^{(i)}}\eta_j^{(i)}-\sum_{j=1}^{N^{(i)}}\epsilon_j^{(i)}<0,i=1,\cdots,u$$

$|arg(z_k)|<\frac{1}{2}U_k\pi,k=1,\cdots,r$ where $U_k$ is defined by (1.14) and the multiple series in the left-hand side of (3.6) converges absolutely, the constant $a$ and $b$ are such none the expression $a,b,$ $[ax+b(1-x)]$ is zero, $0\leqslant x\leqslant 1$.

Proof

To establish the main integral (3.6), first we replace the class of multivariable polynomials $S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[.]$, the multivariable A-function, occurring in the left-hand side of the main integral in term of series with the help of equations (1.1) and (1.3) respectively. Now we express the modified multivariable H-function in Mellin-Barnes integrals contour with the help of (1.10). Nest, we change the order of summations, $(s_1,\cdots,s_r)$-integrals and $(\theta,x,y)$-integrals ( which is justified due to the absolute convergence of the integrals involved in the process reviewing the conditions stated with ( the conditions stated with (3.6)), and we obtain the following result (say L.H.S.) :

$$b^{-\gamma'}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=0}^{\infty}\phi\frac{\prod_{i=1}^{u}\phi_k z_i'^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^{u}g_i}}{\prod_{i=1}^{u}\delta_{G^{(i)}}^{(i)}\prod_{i=1}^{u}g_i!}a'_v y_1^{K_1}\cdots y_v^{K_v}$$

$$\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}\int_0^{\frac{\pi}{2}}e^{(\omega(\beta+\gamma+\sum_{i=1}^{v}(\upsilon_i''+\mu_i'')K_i+\sum_{j=1}^{u}(\upsilon_j'+\mu_j')\eta_{G_j,g_j})+\sum_{k=1}^{r}(\upsilon_k+\mu_k)s_k)/2}$$

$$(\sin\theta)^{\beta+\sum_{i=1}^{v}\upsilon_i''K_i+\sum_{j=1}^{u}\upsilon_j'\eta_{G_j,g_j}+\sum_{k=1}^{r}\upsilon_k s_k-1}(\cos\theta)^{\gamma+\sum_{i=1}^{v}mu_i''K_i+\sum_{j=1}^{u}\mu_j'\eta_{G_j,g_j}+\sum_{k=1}^{r}\mu_k s_k-1}\mathrm{d}\theta\Bigg]$$

$$\left[\int_0^1 x^{\beta'-1}\left[b(1-x)\right]^{\gamma'+\sum_{i=1}^{v}K_i\sigma_i''+\sum_{j=1}^{u}\sigma_j'\eta_{G_j,g_j}+\sum_{k=1}^{r}\sigma_k t_k-1}\right.$$

$$\left.[ax+b(1-x)]^{-\gamma'-\beta'-\sum_{i=1}^{v}K_i\sigma_i'-\sum_{j=1}^{u}\sigma_j'\eta_{G_j,g_j}-\sum_{k=1}^{r}\sigma_k t_k}\mathrm{d}x\right]$$

$$\left[\int_0^1 y^{t+\sum_{i=1}^{v}K_i f_i''+\sum_{j=1}^{u}f_j'\eta_{G_j,g_j}+\sum_{k=1}^{r}f_k t_k-1}(1-y)^{q+\sum_{i=1}^{v}K_i h_i''+\sum_{j=1}^{u}h_j'\eta_{G_j,g_j}+\sum_{k=1}^{r}h_k t_k-1}\mathrm{d}y\right]\mathrm{d}s_1\cdots\mathrm{d}s_r \qquad (3.9)$$

Now evaluating the inner integrals followed in (3.9) with the help of the lemma 1, 2 et 3, finally reinterpreting the result thus obtained in terms of modified multivariable H-function. We obtain the right side of (3.6) after algebric manipulations.

## 4. Corollaries

The main double finite integral (3.6) is general characters. In this section, we shall see the particular cases. Concerning the following corollary, the class of multivariable polynomials vanish, the multivariable A-function and modified multivariable H-function reduce respectively to A-function of one variable defined by Gautam and Asgar [4] and H-function of one variable defined by Fox [3]. We get

**Corollary 1**

$$\int_0^{\frac{\pi}{2}}\int_0^1\int_0^1 e^{\omega(\beta+\gamma)\theta}\sin^{\beta-1}\theta\cos^{\gamma-1}\theta x^{\beta'-1}(1-x)^{\gamma'-1}\left[ax+b(1-x)\right]^{-\beta'-\gamma'}{}_2F_1\left[c,d;\beta';\frac{ax}{ax+b(1-x)}\right]$$

$$y^{t-1}(1-y)^{q-1}A\left(\ \mathrm{z}_1'X(\theta,x,y;\upsilon_1',\mu_1',\sigma_1',f_1',h_1')\ \right)H_{p_1,q_1}^{m_1,n_1}\left(\ \mathrm{z}_1X(\theta,x,y;\upsilon_1,\mu_1,\sigma_1,f_1,h_1)\ \right)$$

$$\mathrm{d}\theta\mathrm{d}x\mathrm{d}y=\frac{\Gamma(\beta')e^{\omega\pi\beta/2}}{a^{\beta'}b^{\gamma'}}\sum_{G_1=1}^{\alpha^{(1)}}\sum_{g_1=0}^{\infty}\frac{\phi_1 z_1'^{\eta_{G_1,g_1}}(-)^{g_1}}{\delta_{G^{(1)}}^{(1)}g_1!}e^{(\omega(\upsilon_1'\eta_{G_1,g_1}))/2}H_{p_1,q_1}^{m_1,n_1}\left(\begin{array}{c|c} & \mathrm{A}\\ \mathrm{z}_1e^{\pi\omega\upsilon_1} & \cdot\\ & \cdot\\ & \mathrm{B}\end{array}\right) \qquad (4.1)$$

where

$$A=(1-\beta-\upsilon_1'\eta_{G_1,g_1};\upsilon_1),(1-\gamma-\mu_1'\eta_{G_1,g_1};\mu_1),(1-\beta'-\gamma'-\rho_1'\eta_{G_1,g_1};\rho_1),$$

$$(1-\gamma'-\sigma_1'\eta_{G_1,g_1};\sigma_1),(1-t-f_1'\eta_{G_1,g_1};f_1),(1-q-h_1'\eta_{G_1,g_1};h_1),(\mathrm{c}_j^{(1)};\gamma_j^{(1)})_{1,p_1} \qquad (4.2)$$

$$B=(1-\beta-\gamma-(\mu_1'+\upsilon_1')\eta_{G_1,g_1};\mu_1+\upsilon_1),(1-\beta'-\gamma'+c-\sigma_1'\eta_{G_1,g_1};\sigma_1),$$

$$(1-\beta'-\gamma'+d-\sigma_1'\eta_{G_1,g_1};\sigma_1)\,,\,(1-t-q-(f_1'+h_1')\eta_{G_1,g_1};f_1+h_1,)\,,\,(d_j^{(1)};\delta_j^{(1)})_{1,q_1} \tag{4.3}$$

Provided that

$min\{\upsilon_1',\mu_1',\rho_1',\sigma_1',f_1',h_1',\upsilon_1,\mu_1,\rho_1,\sigma_1,f_1,h_1\}>0$

$$Re\,(\beta+\upsilon_1'\eta_{G_1,g_1})+\upsilon_1\min_{1\leqslant K\leqslant m_1}Re\left(\frac{d_K^{(1)}}{\delta_K^{(1)}}\right)>0\,;Re\,(\gamma+\mu_1'\eta_{G_1,g_1})+\mu_1\min_{1\leqslant K\leqslant m_1}Re\left(\frac{d_K^{(1)}}{\delta_K^{(1)}}\right)>0$$

$$;\,Re(\beta')>0\,;\,Re(\beta'+\gamma'-c-d)>0\,;Re\,(\gamma'+\sigma_1'\eta_{G_1,g_1})+\sigma_1\min_{1\leqslant K\leqslant m_1}Re\left(\frac{d_K^{(1)}}{\delta_K^{(1)}}\right)>0$$

$$Re\,(t+f_1'\eta_{G_1,g_1})+\sum_{k=1}^{r}f_1\min_{1\leqslant K\leqslant m_1}Re\left(\frac{d_K^{(1)}}{\delta_K^{(1)}}\right)>0\,;Re\,(q+h_1'\eta_{G_1,g_1})+h_1\min_{1\leqslant K\leqslant m_1}Re\left(\frac{d_K^{(1)}}{\delta_K^{(1)}}\right)>0$$

$$z_1'\neq 0,\sum_{j=1}^{A}\gamma_j^{(1)}-\sum_{j=1}^{C}\xi_j^{(1)}+\sum_{j=1}^{M^{(i)}}\eta_j^{(1)}-\sum_{j=1}^{N^{(1)}}\epsilon_j^{(1)}<0$$

$|arg(z_1)|<\frac{1}{2}U_1'\pi$, where $U_1'$ is defined by (1.14) and the multiple series in the left-hand side of (3.6) converges absolutely, $[ax+b(1-x)]\neq 0, 0\leqslant x\leqslant 1$.

Concerning the following corollary, the class of multivariable polynomials reduces to class of polynomials of one variable [8] , the multivariable A-function and multivariable I-function reduce respectively to A-function of one variable defined by Gautam and Asgar [4] and H-function of one variable defined by Fox [3]. We get

**Corollary 2**

$$\int_0^{\frac{\pi}{2}}\int_0^1\int_0^1 e^{\omega(\beta+\gamma)\theta}\sin^{\beta-1}\theta\cos^{\gamma-1}\theta x^{\beta'-1}(1-x)^{\gamma'-1}\left[ax+b(1-x)\right]^{-\beta'-\gamma'}{}_2F_1\left[c,d;\beta';\frac{ax}{ax+b(1-x)}\right]$$

$$y^{t-1}(1-y)^{q-1}\,S_{N_1}^{\mathfrak{M}_1}\left(\ \mathrm{y}_1X(\theta,x,y;\upsilon_1'',\mu_1'',\sigma_1'',f_1'',h_1'')\ \right)A\left(\ \mathrm{z}_1'X(\theta,x,y;\upsilon_1',\mu_1',\sigma_1',f_1';h_1')\ \right)$$

$$H_{p_1,q_1}^{m_1,n_1}\left(\ \mathrm{z}_1X(\theta,x,y;\upsilon_1,\mu_1,\sigma_1,f_1,h_1)\ \right)\mathrm{d}\theta\mathrm{d}x\mathrm{d}y=\frac{\Gamma(\beta')e^{\omega\pi\beta/2}}{a^{\beta'}b^{\gamma'}}$$

$$\sum_{K=0}^{[N_1/\mathfrak{M}_1]}\sum_{G_1=1}^{\alpha^{(1)}}\sum_{g_1=0}^{\infty}\frac{\phi_1 z_1'^{\eta_{G_1,g_1}}(-)^{g_1}}{\delta_{G^{(1)}}^{(1)}g_1!}\frac{(-N_1)_{\mathfrak{M}_1K}A_{N_1,K}}{K!}y_1^K\,e^{(\omega(K\upsilon_1''+\upsilon_1'\eta_{G_1,g_1}))/2}\,H_{p_1,q_1}^{m_1,n_1}\left(\begin{array}{c}\\ \mathrm{z}_1e^{\pi\omega\upsilon_1}\\ \\ \end{array}\middle|\begin{array}{c}\mathrm{A}\\ \cdot\\ \cdot\\ \mathrm{B}\end{array}\right) \tag{4.4}$$

where

$$A=(1-\beta-K\upsilon_1''-\upsilon_1'\eta_{G_1,g_1};\upsilon_1)\,,(1-\gamma-K\mu_1''-\mu_1'\eta_{G_1,g_1};\mu_1)\,,(1-\beta'-\gamma'-K\sigma_1''-\rho_1'\eta_{G_1,g_1};\rho_1)\,,$$

$$(1-\gamma'-K\sigma_1''-\sigma_1'\eta_{G_1,g_1};\sigma_1)\,,(1-t-Kf_1''-f_1'\eta_{G_1,g_1};f_1)\,,(1-q-Kh_1''-h_1'\eta_{G_1,g_1};h_1)\,,(\mathrm{c}_j^{(1)};\gamma_j^{(1)})_{1,p_1} \tag{4.5}$$

$$B=(1-\beta-\gamma-K(\mu_1''+\upsilon_1'')-(\mu_1'+\upsilon_1')\eta_{G_1,g_1};\mu_1+\upsilon_1)\,,(1-\beta'-\gamma'+c-\sigma_1''K-\sigma_1'\eta_{G_1,g_1};\sigma_1)\,,$$

$$(1-\beta'-\gamma'+d-\sigma_1''K-\sigma_1'\eta_{G_1,g_1};\sigma_1),(1-t-q-(f_1''+g_1'')K-(f_1'+h_1')\eta_{G_1,g_1};f_1+h_1,),(d_j^{(1)};\delta_j^{(1)})_{1,q_1} \quad (4.6)$$

Provided that

$min\{\upsilon_1'',\mu_1'',\rho_1'',\sigma_1'',f_1'',h_1'',\upsilon_1',\mu_1',\rho_1',\sigma_1',f_1',h_1',\upsilon_1,\mu_1,\rho_1,\sigma_1,f_1,h_1\}>0$, the other conditions are the same that (4.1).

Consider the above corollary , by applying our result given in (4.4) to the case the Laguerre polynomials ([14], page 101, eq.(15.1.6)) and ([13], page 159) and by setting

$$S_N^1(x)\rightarrow L_N^{\alpha}(x)$$

In which case $\mathfrak{M}=1, A_{N,K}=\binom{N+\alpha}{N}\frac{1}{(\alpha+1)_K}$ we have the following interesting consequencies of the main results.

$$\int_0^{\frac{\pi}{2}}\int_0^1\int_0^1 e^{\omega(\beta+\gamma)\theta}\sin^{\beta-1}\theta\cos^{\gamma-1}\theta x^{\beta'-1}(1-x)^{\gamma'-1}\left[ax+b(1-x)\right]^{-\beta'-\gamma'}{}_2F_1\left[c,d;\beta';\frac{ax}{ax+b(1-x)}\right]$$

$$y^{t-1}(1-y)^{q-1}L_N^{\alpha}\left(\frac{y_1}{2}\mathrm{X}(\theta,x,y;\upsilon_1'',\mu_1'',\sigma_1'',f_1'',h_1'')\right)A\left(\mathrm{z}_1'X(\theta,x,y;\upsilon_1',\mu_1',\sigma_1',f_1';h_1')\right)$$

$$H_{p_1,q_1}^{m_1,n_1}\left(\mathrm{z}_1X(\theta,x,y;\upsilon_1,\mu_1,\sigma_1,f_1,h_1)\right)\mathrm{d}\theta\mathrm{d}x\mathrm{d}y=\frac{\Gamma(\beta')e^{\omega\pi\beta/2}}{a^{\beta'}b^{\gamma'}}$$

$$\sum_{K=0}^{N}\sum_{G_1=1}^{\alpha^{(1)}}\sum_{g_1=0}^{\infty}\frac{\phi_1 {z_1'}^{\eta_{G_1,g_1}}(-)^{g_1}}{\delta_{G^{(1)}}^{(1)}g_1!}\binom{N+\alpha}{N-K}(-y_1)^K e^{(\omega(K\upsilon_1''+\upsilon_1'\eta_{G_1,g_1}))/2}H_{p_1,q_1}^{m_1,n_1}\left(\mathrm{z}_1e^{\pi\omega\upsilon_1}\,\middle|\,\begin{matrix}\mathrm{A}\\ \cdot\\ \cdot\\ \mathrm{B}\end{matrix}\right) \quad (4.4)$$

with the same notations and conditions that (4.4).

## 5. Conclusion

In this paper we have evaluated an unified triple finite integrals involving the product of modified multivariable H-function defined by Prasad and Singh. [5], a expansion of multivariable A-function defined by Gautam and Asgar [5] and class of multivariable polynomials defined by Srivastava [10] with general arguments. The formulae established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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