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Application on product of multivariable A-function and Generalized class of polynomials

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ABSTRACT
Yadav et al [10] have used the product of Kampe de Feriet function, multivariable H-function and generalized class of multivariable polynomials to find a solution of partial differential equations of heat conduction. In this paper we use the product of Srivastava-daoust function [8], multivariable I-function defined by Prathima et al [4], the general class of multivariable polynomials [7] and multivariable A-function [3] to find a solution of partial differential equations of heat conduction [1] at zero temperature with radiation at the ends in medium without source of thermal energy. It is interesting from mathematical point of view and also due to its application in various field of Mathematical Analysis, Physics and Mechanics. We shall see the two cases concerning the multivariable H-function [9] and the I-function of two variables defined by Rathie et al [5].

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1. Introduction

In this section, we consider a problem on outer heat conduction in a rod under certain boundary conditions. If the thermal coefficients are constants and there is no source of thermal energy, then the temperature $u(x,t)$ in one-dimensional rod $0 \leqslant x \leqslant L$ satisfies the following heat equation.

$$\frac{\partial U}{\partial t} = k\frac{\partial^2 U}{\partial x^2}\,, t \geqslant 0 \tag{1.1}$$

If we consider the following boundary conditions

$$\frac{\partial U(0,t)}{\partial x} - hU(0,t) = 0 \text{ and } \frac{\partial U(L,t)}{\partial x} + hU(L,t) = 0, h > 0 \tag{1.2}$$

$$U(x,t) \text{ is finite as } t \to \infty \tag{1.3}$$

and initial condition

$$U(x,0) = f(x) \tag{1.4}$$

The expression

$$e^{\mu E_q^2 t}(m \cos E_q x + n_q x) \tag{1.5}$$

satisfies the equation (1.1) ,equation (1.5) also satisfies (1.2) on the condition that

$$E_q n - hm = 0 \tag{1.6}$$

and

$$E_q(n_q L - m_q L) + h(m_q L + n_q L) = 0 \tag{1.7}$$

From (1.5) and (1.6), we get

$$A/B = E_q/h \tag{1.8}$$

and

$$\tan E_q L = \frac{2E_q h}{E_q^2 - h^2)} \tag{1.9}$$

Then the solution of our problem takes the following form

$$U(x,t)=\sum_{q=0}^{\infty}R_q(\cos E_qx+\frac{h}{E_q}\sin E_q)e^{-\mu E_q^2 t} \tag{1.10}$$

We shall take

$$U(x,0)=f(x)=\left(\sin\frac{\pi x}{L}\right)^{W-1}S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}\left(\begin{array}{c}\mathrm{y}_1\left(\sin\frac{\pi x}{L}\right)^{2\rho_1}\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ \mathrm{y}_u\left(\sin\frac{\pi x}{L}\right)^{2\rho_u}\end{array}\right)F\left(\begin{array}{c}\mathrm{t}_1\left(\sin\frac{\pi x}{L}\right)^{2\sigma_1}\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ \mathrm{t}_v\left(\sin\frac{\pi x}{L}\right)^{2\sigma_v}\end{array}\right)$$

$$I\left(\begin{array}{c}\mathrm{z}_1\left(\sin\frac{\pi x}{L}\right)^{2\xi_1}\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ \mathrm{z}_r\left(\sin\frac{\pi x}{L}\right)^{2\xi_r}\end{array}\right)A\left(\begin{array}{c}\mathrm{Z}_1\left(\sin\frac{\pi x}{L}\right)^{2\eta_1}\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ \mathrm{Z}_s\left(\sin\frac{\pi x}{L}\right)^{2\eta_s}\end{array}\right) \tag{1.11}$$

The generalized polynomials defined by Srivastava [7], is given in the following manner :

$$S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}[y_1,\cdots,y_u]\;=\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_u)_{M_uK_u}}{K_u!}$$

$$A[N_1,K_1;\cdots;N_u,K_u]y_1^{K_1}\cdots y_u^{K_u} \tag{1.12}$$

Where $M_1,\cdots,M_u$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_u,K_u]$ are arbitrary constants, real or complex.

The Srivastava-Daoust function is defined by (see [8]):

$$F_{\bar{C}:D^{(1)};\cdots;D^{(v)}}^{\bar{A}:B^{(1)};\cdots;B^{(v)}}\left(\begin{array}{c}\mathrm{z}_1\\ \cdot\\ \cdot\\ \mathrm{z}_v\end{array}\left|\begin{array}{c}[(\mathrm{a});\theta',\cdots,\theta^{(v)}]:[(b');\phi'];\cdots;[(b^{(v)});\phi^{(v)}]\\ \cdot\\ [(\mathrm{c});\psi',\cdots,\psi^{(v)}]:[(d');\delta'];\cdots;[(d^{(v)});\delta^{(v)}]\end{array}\right.\right)$$

$$=\sum_{r_1,\cdots,r_v=0}^{\infty}A'\frac{z_1^{r_1}\cdots z_v^{r_v}}{r_1!\cdots r_v!} \tag{1.13}$$

where

$$A'=\frac{\prod_{j=1}^{\bar{A}}(a_j)_{r_1\theta'_j+\cdots+r_v\theta_j^{(v)}}\prod_{j=1}^{B^{(1)}}(b'_j)_{r_1\phi'_j}\cdots\prod_{j=1}^{B^{(v)}}(b_j^{(v)})_{r_v\phi_j^{(v)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{r_1\psi'_j+\cdots+r_v\psi_j^{(v)}}\prod_{j=1}^{D^{(1)}}(d'_j)_{r_1\delta'_j}\cdots\prod_{j=1}^{D^{(v)}}(d_j^{(v)})_{r_v\delta_j^{(v)}}} \tag{1.14}$$

The series given by (1.13) converges absolutely if

$$1+\sum_{j=1}^{\bar{C}}\psi_j^{(i)}+\sum_{j=1}^{D^{(i)}}\delta_j^{(i)}-\sum_{j=1}^{\bar{A}}\theta_j^{(i)}-\sum_{j=1}^{B^{(i)}}\phi_j^{(i)}>0; i=1,\cdots,v \tag{1.15}$$

The multivariable I-function defined by Prathima et al [4] is defined in term of multiple Mellin-Barnes type integral :

$$\bar{I}(z_1,\cdots,z_r) = I_{P,Q:P_1,Q_1;\cdots;P_r,Q_r}^{0,N:M_1,N_1;\cdots;M_r,N_r}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_{1,P}: \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_{1,Q}: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,N_1},(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{N_1+1,P_1};\cdots;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,N_r},(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{N_r+1,P_r} \\ \\ (\mathrm{d}_j^{(1)},\delta_j^{(1)};1)_{1,M_1},(d_j^{(1)},\delta_j^{(1)};D_1)_{M_1+1,Q_1};\cdots;(d_j^{(r)},\delta_j^{(r)};1)_{1,M_r},(d_j^{(r)},\delta_j^{(r)};D_r)_{M_r+1,Q_r} \end{array}\right) \tag{1.16}$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.17}$$

where $\phi(s_1,\cdots,s_r),\theta_i(s_i), i=1,\cdots,r$ are given by :

$$\phi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{N}\Gamma^{A_j}\left(1-aj+\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)}{\prod_{j=N+1}^{P}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)\prod_{j=1}^{Q}\Gamma^{B_j}\left(1-bj+\sum_{i=1}^{r}\beta_j^{(i)}s_j\right)} \tag{1.18}$$

$$\phi_i(s_i)=\frac{\prod_{j=1}^{N_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)\prod_{j=1}^{M_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)}{\prod_{j=N_i+1}^{P_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)\prod_{j=M_i+1}^{Q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}s_i\right)} \tag{1.19}$$

For more details, see Prathima et al [4].

We can obtain the series representation and behaviour for small values for the function $\bar{I}(z_1,\cdots,z_r)$ defined and represented by (1.16). The series representation may be given as follows :

which is valid under the following conditions :

$\delta_i^{(h)}[d_i^{(j)}+r]\neq\delta_i^{(j)}[d_i^{(h)}+\mu]$ for $j\neq h, j,h=1,\cdots,M_i, r,\mu=0,1,2,\cdots$

$$U_i=\sum_{j=1}^{P}A_j\alpha_j^{(i)}-\sum_{j=1}^{Q}B_j\beta_j^{(i)}+\sum_{j=1}^{P_i}C_j^{(i)}\gamma_j^{(i)}-\sum_{j=M_i+1}^{Q_i}D_j^{(i)}\delta_j^{(i)}\leqslant 0, i=1,\cdots,r \quad \text{and} \quad z_i\neq 0$$

and if all the poles of (1.16) are simple ,then the integral (1.16) can be evaluated with the help of the Residue theorem to give

$$\bar{I}(z_1,\cdots,z_r)=\sum_{h_i=1}^{M_i}\sum_{g_i=1}^{\infty}\phi_1\frac{\prod_{i=1}^{r}\phi_i z_i^{\eta_{h_i,g_i}}(-)^{\sum_{i=1}^{r}g_i}}{\prod_{i=1}^{r}\delta_{h^{(i)}}^{(i)}\prod_{i=1}^{r}g_i!} \tag{1.20}$$

where ϕ_1 and ϕ_i are defined by

$$\phi_1 = \frac{\prod_{j=1}^{N} \Gamma^{A_j}\left(1-aj+\sum_{i=1}^{r}\alpha_j^{(i)}\eta_{h_i,g_i}\right)}{\prod_{j=N+1}^{P}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}\eta_{h_i,g_i}\right)\prod_{j=1}^{Q}\Gamma^{B_j}\left(1-bj+\sum_{i=1}^{r}\beta_j^{(i)}\eta_{h_i,g_i}\right)} \tag{1.21}$$

and

$$\phi_i = \frac{\prod_{j=1}^{N_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}\eta_{h_i,g_i}\right)\prod_{j=1}^{M_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}\eta_{h_i,g_i}\right)}{\prod_{j=N_i+1}^{P_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}\eta_{h_i,g_i}\right)\prod_{j=M_i+1}^{Q_i}\Gamma^{(D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}\eta_{h_i,g_i}\right)}, i=1,\cdots,r \tag{1.22}$$

where $\eta_{h_i,g_i} = \dfrac{d_{h^{(i)}}^{(i)}+g_i}{\delta_{h^{(i)}}^{(i)}}, i=1,\cdots,r$

Now, we consider the multivariable A-function defined by Gautam et al [3].

$$A(Z_1,\cdots,Z_s) = A_{p,q:p_1,q_1;\cdots;p_r,q_r}^{m,n:m_1,n_1;\cdots;m_r,n_r}\left(\begin{array}{c|l} \mathrm{Z}_1 & (\mathrm{a}_j;A_j^{(1)},\cdots,A_j^{(s)})_{1,p}: \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{Z}_s & (\mathrm{b}_j;B_j^{(1)},\cdots,B_j^{(s)})_{1,q}: \end{array}\right.$$

$$\left.\begin{array}{l}(\mathrm{c}_j^{(1)},C_j^{(1)})_{1,p_1};\cdots;(c_j^{(s)},C_j^{(s)})_{1,p_s} \\ \\ (\mathrm{d}_j^{(1)},D_j^{(1)})_{1,q_1};\cdots;(d_j^{(s)},D_j^{(s)})_{1,q_s}\end{array}\right) \tag{1.23}$$

$$= \frac{1}{(2\pi\omega)^s}\int_{L_1'}\cdots\int_{L_s'}\phi'(t_1,\cdots,t_s)\prod_{i=1}^{s}\theta_i'(t_i)Z_i^{t_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.24}$$

where $\phi'(t_1,\cdots,t_s),\theta_i'(t_i), i=1,\cdots,s$ are given by :

$$\phi'(t_1,\cdots,t_s) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\sum_{i=1}^{s}B_j^{(i)}t_i)\prod_{j=1}^{n}\Gamma(1-a_j+\sum_{i=1}^{s}A_j^{(i)}t_j)}{\prod_{j=n+1}^{p}\Gamma(a_j-\sum_{i=1}^{s}A_j^{(i)}t_j)\prod_{j=m+1}^{q}\Gamma\left(1-b_j+\sum_{i=1}^{s}B_j^{(i)}t_j\right)} \tag{1.25}$$

$$\theta_i'(t_i) = \frac{\prod_{j=1}^{n_i}\Gamma(1-c_j^{(i)}+C_j^{(i)}t_i)\prod_{j=1}^{m_i}\Gamma(d_j^{(i)}-D_j^{(i)}t_i)}{\prod_{j=n_i+1}^{p_i}\Gamma(c_j^{(i)}-C_j^{(i)}t_i)\prod_{j=m_i+1}^{q_i}\Gamma(1-d_j^{(i)}+D_j^{(i)}t_i)} \tag{1.26}$$

Here $m,n,p,m_i,n_i,p_i,c_i \in \mathbb{N}^*; i=1,\cdots,r$; $a_j,b_j,c_j^{(i)},d_j^{(i)},A_j^{(i)},B_j^{(i)},C_j^{(i)},D_j^{(i)} \in \mathbb{C}$

The multiple integral defining the A-function of r variables converges absolutely if :

$$|arg(\Omega_i')Z_k| < \frac{1}{2}\eta_k'\pi, \xi'^* = 0, \eta_i' > 0 \tag{1.27}$$

$$\Omega_i' = \prod_{j=1}^{p}\{A_j^{(i)}\}^{A_j^{(i)}}\prod_{j=1}^{q}\{B_j^{(i)}\}^{-B_j^{(i)}}\prod_{j=1}^{q_i}\{D_j^{(i)}\}^{D_j^{(i)}}\prod_{j=1}^{p_i}\{C_j^{(i)}\}^{-C_j^{(i)}};i=1,\cdots,s \tag{1.28}$$

$$\xi_i'^{*} = Im(\sum_{j=1}^{p}A_j^{(i)}-\sum_{j=1}^{q}B_j^{(i)}+\sum_{j=1}^{q_i}D_j^{(i)}-\sum_{j=1}^{p_i}C_j^{(i)});i=1,\cdots,s \tag{1.29}$$

$$\eta_i' = Re\left(\sum_{j=1}^{n}A_j^{(i)}-\sum_{j=n+1}^{p}A_j^{(i)}+\sum_{j=1}^{m}B_j^{(i)}-\sum_{j=m+1}^{q}B_j^{(i)}+\sum_{j=1}^{m_i}D_j^{(i)}-\sum_{j=m_i+1}^{q_i}D_j^{(i)}+\sum_{j=1}^{n_i}C_j^{(i)}-\sum_{j=n_i+1}^{p_i}C_j^{(i)}\right)$$

$$i=1,\cdots,s \tag{1.30}$$

In this paper, we shall note.

$$X = m_1,n_1;\cdots;m_s,n_s \quad : Y = p_1,q_1;\cdots;p_s,q_s \tag{1.31}$$

$$\mathbb{A} = (a_j;A_j^{(1)},\cdots,A_j^{(s)})_{1,p}:\ (\mathrm{c}_j^{(1)},C_j^{(1)})_{1,p_1};\cdots;(c_j^{(s)},C_j^{(s)})_{1,p_s} \tag{1.32}$$

$$\mathbb{B} = (b_j;B_j^{(1)},\cdots,B_j^{(s)})_{1,q}:\ (\mathrm{d}_j^{(1)},D_j^{(1)})_{1,q_1};\cdots;(d_j^{(s)},D_j^{(s)})_{1,q_s} \tag{1.33}$$

$$B' = \frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}A[N_1,K_1;\cdots;N_s,K_s] \tag{1.34}$$

$$F = F^{\bar{A}:B^{(1)};\cdots;B^{(v)}}_{\bar{C}:D^{(1)};\cdots;D^{(v)}} \tag{1.35}$$

2. Required integral

Lemma 1

$$\int_0^L\left(\sin\frac{\pi x}{L}\right)^{W-1}\sin\frac{\pi x E_p}{L}\mathrm{d}x = \frac{\pi\Gamma(W)L\sin\frac{\pi E_p}{2}}{2^{W-1}\Gamma\left(\frac{W\pm E_p+1}{2}\right)}, Re(W)>0 \tag{2.1}$$

Lemma 2

$$\int_0^L\left(\sin\frac{\pi x}{L}\right)^{W-1}\cos\frac{\pi x E_p}{L}\mathrm{d}x = \frac{\pi\Gamma(W)L\cos\frac{\pi E_p}{2}}{2^{W-1}\Gamma\left(\frac{W\pm E_p+1}{2}\right)}, Re(W)>0 \tag{2.2}$$

Lemme 3

$$\int_0^L(\cos E_q x+\frac{h}{E_q}\sin E_q x)(\cos E_q x+hE_q\sin E_q x)\mathrm{d}x = 2E_q^{-2}\left[E_q^2+h^2)L+2l\right]\delta_{pq} \tag{2.3}$$

where $\delta_{p,q}=1$ if $p=q$,0 else and E_q is positive root of the following transcendental equation

$$\tan E_q L = \frac{2E_q h}{E_q^2 - h^2} \tag{2.4}$$

3. Main integral

We evaluate the following integrals very useful throughout the paper :

Theorem 1

$$\int_0^L \left(\sin\frac{\pi x}{L}\right)^{W-1} \sin\frac{E_p \pi x}{L} S^{M_1,\cdots,M_u}_{N_1,\cdots,N_u}\left(\begin{array}{c} \mathrm{y}_1\left(\sin\frac{\pi x}{L}\right)^{2\rho_1} \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{y}_u\left(\sin\frac{\pi x}{L}\right)^{2\rho_u} \end{array}\right) F\left(\begin{array}{c} \mathrm{t}_1\left(\sin\frac{\pi x}{L}\right)^{2\sigma_1} \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{t}_v\left(\sin\frac{\pi x}{L}\right)^{2\sigma_v} \end{array}\right)$$

$$I\left(\begin{array}{c} \mathrm{z}_1\left(\sin\frac{\pi x}{L}\right)^{2\xi_1} \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{z}_r\left(\sin\frac{\pi x}{L}\right)^{2\xi_r} \end{array}\right) A\left(\begin{array}{c} \mathrm{Z}_1\left(\sin\frac{\pi x}{L}\right)^{2\eta_1} \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{Z}_s\left(\sin\frac{\pi x}{L}\right)^{2\eta_s} \end{array}\right) \mathrm{d}x = \frac{L\sin\left(\frac{E_p\pi}{2}\right)}{2^{W-1}} \sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]} \sum_{r_1,\cdots,r_v=0}^{\infty}$$

$$\sum_{h_i=1}^{M_i}\sum_{g_i=1}^{\infty} \phi_1 \frac{\prod_{i=1}^r \phi_i z_i^{\eta_{h_i,g_i}} (-)^{\sum_{i=1}^r g_i}}{\prod_{i=1}^r \delta^{(i)}_{h^{(i)}} \prod_{i=1}^r g_i!} A'B' \frac{y_1^{K_1}\cdots y_u^{K_u} t_1^{r_1}\cdots t_v^{r_v}}{r_1!\cdots r_v! 4^{\sum_{i=1}^u \rho_i K_i + \sum_{i=1}^v \sigma_i r_i + \sum_{i=1}^r \xi_i \eta_{D_i,\delta_i}}}$$

$$A^{m,n+1;X}_{p+1,q+2;Y}\left(\begin{array}{c|c} \mathrm{Z}_1 4^{-\eta_1} & (\text{1-W-2}(\sum_{i=1}^u \rho_i K_i + \sum_{i=1}^v \sigma_i r_i + \sum_{i=1}^r \xi_i \eta_{h_i,g_i}) : 2\eta_1,\cdots,2\eta_s), \mathbb{A} \\ \cdot\cdot\cdot & \cdot\cdot\cdot \\ \cdot\cdot\cdot & \cdot\cdot\cdot \\ \mathrm{Z}_s 4^{-\eta_s} & \left(\frac{1-W-2(\sum_{i=1}^u \rho_i K_i + \sum_{i=1}^v \sigma_i r_i + \sum_{i=1}^r \xi_i \eta_{h_i,g_i}) \pm E_p}{2} : \eta_1,\cdots,\eta_s\right), \mathbb{B} \end{array}\right) \tag{3.1}$$

Provided that

$Re(W) > 0, min\{\rho_i,\sigma_j,\xi_k,\eta_l\} > 0, i = 1,\cdots,s, j = 1,\cdots,v, k = 1,\cdots,r, l = 1,\cdots,s$

$1 + \sum_{j=1}^{\bar{C}} \psi_j^{(i)} + \sum_{j=1}^{D^{(i)}} \delta_j^{(i)} - \sum_{j=1}^{\bar{A}} \theta_j^{(i)} - \sum_{j=1}^{B^{(i)}} \phi_j^{(i)} > 0; i = 1,\cdots,v$

$U_i = \sum_{j=1}^{P} A_j\alpha_j^{(i)} - \sum_{j=1}^{Q} B_j\beta_j^{(i)} + \sum_{j=1}^{P_i} C_j^{(i)}\gamma_j^{(i)} - \sum_{j=M_i+1}^{Q_i} D_j^{(i)}\delta_j^{(i)} \leqslant 0, i = 1,\cdots,r$

$\left|arg(\Omega_i')zZ_k sin\left(\frac{\pi x}{L}\right)^{2\eta_i}\right| < \frac{1}{2}\eta_k'\pi, \xi'^{*} = 0, \eta_i' > 0, i = 1,\cdots,s$ and

$0Re\left[W + \sum_{i=1}^u K_i\rho_i + \sum_{i=1}^v r_i\sigma_i + \sum_{i=1}^r \xi_i\eta_{h_i,g_i} + \sum_{i=1}^s \eta_i \min_{1\leqslant j\leqslant m_i} \frac{d_j'^{(i)}}{D_j'^{(i)}}\right] >$

Proof

To prove (3.1), first expressing a class of multivariable polynomials defined by Srivastava [2] $S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[.]$ in serie with the help of (1.12), the Srivastava-Daoust function $F[.]$ in serie with the help of (1.13) ,the multivariable $I[.]$ defined by Prathima et al [4] in serie with the help of (1.20) and we interchange the order of summations and x-integral (which is permissible under the conditions stated). Expressing the A-function defined by Gautam et al [3] of s-variables in Mellin-contour integral and interchange the order of integrations which is justifiable due to absolute convergence of the integral involved in the process. Now collecting the power of $sin\left(\frac{\pi x}{L}\right)$ and evaluating the inner x-integral with the help of Lemma 1. Interpreting the Mellin-Barnes contour integral in multivariable A-function, we obtain the desired result (3.1).

Theorem 2

$$\int_0^L\left(\sin\frac{\pi x}{L}\right)^{W-1}\cos\frac{E_p\pi x}{L}S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}\begin{pmatrix}\mathrm{y}_1\left(\sin\frac{\pi x}{L}\right)^{2\rho_1}\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ \mathrm{y}_u\left(\sin\frac{\pi x}{L}\right)^{2\rho_u}\end{pmatrix}F\begin{pmatrix}\mathrm{t}_1\left(\sin\frac{\pi x}{L}\right)^{2\sigma_1}\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ \mathrm{t}_v\left(\sin\frac{\pi x}{L}\right)^{2\sigma_v}\end{pmatrix}$$

$$I\begin{pmatrix}\mathrm{z}_1\left(\sin\frac{\pi x}{L}\right)^{2\xi_1}\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ \mathrm{z}_r\left(\sin\frac{\pi x}{L}\right)^{2\xi_r}\end{pmatrix}A\begin{pmatrix}\mathrm{Z}_1\left(\sin\frac{\pi x}{L}\right)^{2\eta_1}\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ \mathrm{Z}_s\left(\sin\frac{\pi x}{L}\right)^{2\eta_s}\end{pmatrix}\mathrm{d}x=\frac{L\cos\left(\frac{E_p\pi}{2}\right)}{2^{W-1}}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{r_1,\cdots,r_v=0}^{\infty}$$

$$\sum_{h_i=1}^{M_i}\sum_{g_i=1}^{\infty}\phi_1\frac{\prod_{i=1}^r\phi_i z_i^{\eta_{h_i,g_i}}(-)^{\sum_{i=1}^r g_i}}{\prod_{i=1}^r\delta_{h^{(i)}}^{(i)}\prod_{i=1}^r g_i!}\;A'B'\;\frac{y_1^{K_1}\cdots y_u^{K_u}t_1^{r_1}\cdots t_v^{r_v}}{r_1!\cdots r_v!4^{\sum_{i=1}^u\rho_iK_i+\sum_{i=1}^v\sigma_ir_i+\sum_{i=1}^r\xi_i\eta_{D_i,\delta_i}}}$$

$$A_{p+1,q+2;Y}^{m,n+1;X}\left(\begin{matrix}\mathrm{Z}_14^{-\eta_1}\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ \mathrm{Z}_s4^{-\eta_s}\end{matrix}\left|\begin{matrix}(\text{1-W-2}(\sum_{i=1}^u\rho_iK_i+\sum_{i=1}^v\sigma_ir_i+\sum_{i=1}^r\xi_i\eta_{h_i,g_i}):2\eta_1,\cdots,2\eta_s),\mathbb{A}\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ \left(\frac{1-W-2(\sum_{i=1}^u\rho_iK_i+\sum_{i=1}^v\sigma_ir_i+\sum_{i=1}^r\xi_i\eta_{h_i,g_i})\pm E_p}{2}:\eta_1,\cdots,\eta_s\right),\mathbb{B}\end{matrix}\right.\right)\quad(3.2)$$

Provided that

$Re(W)>0,min\{\rho_i,\sigma_j,\xi_k,\eta_l\}>0,i=1,\cdots,s,j=1,\cdots,v,k=1,\cdots,r,l=1,\cdots,s$

$$1+\sum_{j=1}^{C}\psi_j^{(i)}+\sum_{j=1}^{D^{(i)}}\delta_j^{(i)}-\sum_{j=1}^{A}\theta_j^{(i)}-\sum_{j=1}^{B^{(i)}}\phi_j^{(i)}>0;i=1,\cdots,v$$

$$U_i=\sum_{j=1}^{P}A_j\alpha_j^{(i)}-\sum_{j=1}^{Q}B_j\beta_j^{(i)}+\sum_{j=1}^{P_i}C_j^{(i)}\gamma_j^{(i)}-\sum_{j=M_i+1}^{Q_i}D_j^{(i)}\delta_j^{(i)}\leqslant 0,i=1,\cdots,r$$

$\left|arg(\Omega_i')zZ_ksin\left(\frac{\pi x}{L}\right)^{2\eta_i}\right|<\frac{1}{2}\eta_k'\pi,\xi'^*=0,\eta_i'>0,i=1,\cdots,s$ and

$$0Re\left[W+\sum_{i=1}^{u}K_i\rho_i+\sum_{i=1}^{v}r_i\sigma_i+\sum_{i=1}^{r}\xi_i\eta_{h_i,g_i}+\sum_{i=1}^{s}\eta_i\min_{1\leqslant j\leqslant m_i}\frac{d_j'^{(i)}}{D_j'^{(i)}}\right]>$$

Proof

To prove (3.2), first expressing a class of multivariable polynomials defined by Srivastava [2] $S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[.]$ in serie with the help of (1.12), the Srivastava-Daoust function $F[.]$ in serie with the help of (1.13) ,the multivariable $\bar{I}[.]$ defined by Prathima et al [4] in serie with the help of (1.20) and we interchange the order of summations and x-integral (which is permissible under the conditions stated). Expressing the A-function defined by Gautam et al [3] of s-variables in Mellin-contour integral and interchange the order of integrations which is justifiable due to absolute convergence of the integral involved in the process. Now collecting the power of $sin\left(\frac{\pi x}{L}\right)$ and evaluating the inner x-integral with the help of Lemma 2. Interpreting the Mellin-Barnes contour integral in multivariable A-function, we obtain the desired result (3.2).

4. Application to heat conduction

The solution of problem is

$$U(x,t)=\frac{L}{2^{W-1}}\sum_{p=1}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{r_1,\cdots,r_v=0}^{\infty}\sum_{h_i=1}^{M_i}\sum_{g_i=1}^{\infty}\phi_1\frac{\prod_{i=1}^{r}\phi_i z_i^{\eta_{h_i,g_i}}(-)^{\sum_{i=1}^{r}g_i}}{\prod_{i=1}^{r}\delta_{h^{(i)}}^{(i)}\prod_{i=1}^{r}g_i!}$$

$$\frac{y_1^{K_1}\cdots y_u^{K_u}t_1^{r_1}\cdots t_v^{r_v}}{r_1!\cdots r_v!4^{\sum_{i=1}^{u}\rho_iK_i+\sum_{i=1}^{v}\sigma_ir_i+\sum_{i=1}^{r}\xi_i\eta_{D_i,\delta_i}}}\frac{E_p^2}{\left[E_p^2+h^2)L+2h\right]}\left(\cos E_px+\frac{h}{E_p}\sin E_px\right)\left(\cos E_p\pi+\frac{h}{E_p}\sin E_p\pi\right)$$

$$e^{-kE_p^2t}A_{p+1,q+2;Y}^{m,n+1;X}\left(\begin{array}{c}\mathrm{Z}_14^{-\eta_1}\\ \cdot\;\cdot\;\cdot\\ \cdot\;\cdot\;\cdot\\ \mathrm{Z}_s4^{-\eta_s}\end{array}\left|\begin{array}{c}(\text{1-W-2}(\sum_{i=1}^{u}\rho_iK_i+\sum_{i=1}^{v}\sigma_ir_i+\sum_{i=1}^{r}\xi_i\eta_{h_i,g_i}):2\eta_1,\cdots,2\eta_s),\mathbb{A}\\ \cdot\;\cdot\;\cdot\\ \cdot\;\cdot\;\cdot\\ \left(\frac{1-W-2(\sum_{i=1}^{u}\rho_iK_i+\sum_{i=1}^{v}\sigma_ir_i+\sum_{i=1}^{r}\xi_i\eta_{h_i,g_i})\pm E_p}{2}:\eta_1,\cdots,\eta_s\right),\mathbb{B}\end{array}\right.\right) \quad (4.1)$$

under the same notations and condition that (3.1)

Proof

If $t=0$, then from (1.9) and (1.10), we have

$$\left(\sin\frac{\pi x}{L}\right)^{W-1}S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}\left(\begin{array}{c}\mathrm{y}_1\left(\sin\frac{\pi x}{L}\right)^{2\rho_1}\\ \cdot\;\cdot\;\cdot\\ \cdot\;\cdot\;\cdot\\ \mathrm{y}_u\left(\sin\frac{\pi x}{L}\right)^{2\rho_u}\end{array}\right)F\left(\begin{array}{c}\mathrm{t}_1\left(\sin\frac{\pi x}{L}\right)^{2\sigma_1}\\ \cdot\;\cdot\;\cdot\\ \cdot\;\cdot\;\cdot\\ \mathrm{t}_v\left(\sin\frac{\pi x}{L}\right)^{2\sigma_v}\end{array}\right)I\left(\begin{array}{c}\mathrm{z}_1\left(\sin\frac{\pi x}{L}\right)^{2\xi_1}\\ \cdot\;\cdot\;\cdot\\ \cdot\;\cdot\;\cdot\\ \mathrm{z}_r\left(\sin\frac{\pi x}{L}\right)^{2\xi_r}\end{array}\right)$$

$$A\left(\begin{array}{c}\mathrm{Z}_1\left(\sin\frac{\pi x}{L}\right)^{2\eta_1}\\ \cdot\;\cdot\;\cdot\\ \cdot\;\cdot\;\cdot\\ \mathrm{Z}_s\left(\sin\frac{\pi x}{L}\right)^{2\eta_s}\end{array}\right)=\sum_{p=1}^{\infty}R_p\left(\cos E_px+\frac{h}{E_p}sinE_px\right) \quad (4.2)$$

Multiply both sides of (4.2) by $\cos E_q x + \frac{h}{E_q} sin E_q x$ then integrate with respect to x from 0 to L, and use the Lemme 3, (3.1) and (3.2) to obtain

$$R_q = \frac{1}{2^{W-1}} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_u=0}^{[N_u/M_u]} \sum_{r_1,\cdots,r_v=0}^{\infty} \sum_{h_i=1}^{M_i} \sum_{g_i=1}^{\infty} \phi_1 \frac{\prod_{i=1}^r \phi_i z_i^{\eta_{h_i,g_i}} (-)^{\sum_{i=1}^r g_i}}{\prod_{i=1}^r \delta_{h^{(i)}}^{(i)} \prod_{i=1}^r g_i!}$$

$$\frac{y_1^{K_1} \cdots y_u^{K_u} t_1^{r_1} \cdots t_v^{r_v}}{r_1! \cdots r_v! 4^{\sum_{i=1}^u \rho_i K_i + \sum_{i=1}^v \sigma_i r_i + \sum_{i=1}^r \xi_i \eta_{D_i,\delta_i}}} \frac{2LE_q^2}{[E_q^2 + h^2)L + 2h]} \left(\cos E_q \pi + \frac{h}{E_q} \sin E_q \pi \right)$$

$$A_{p+1,q+2;Y}^{m,n+1;X} \left(\begin{array}{c} \mathrm{Z}_1 4^{-\eta_1} \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{Z}_s 4^{-\eta_s} \end{array} \middle| \begin{array}{c} (\text{1-W-2}(\sum_{i=1}^u \rho_i K_i + \sum_{i=1}^v \sigma_i r_i + \sum_{i=1}^r \xi_i \eta_{h_i,g_i}) : 2\eta_1, \cdots, 2\eta_s), \mathbb{A} \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \left(\frac{1-W-2(\sum_{i=1}^u \rho_i K_i + \sum_{i=1}^v \sigma_i r_i + \sum_{i=1}^r \xi_i \eta_{h_i,g_i}) \pm E_p}{2} : \eta_1, \cdots, \eta_s \right), \mathbb{B} \end{array} \right) \tag{4.3}$$

Substitute the value of R_q from (4.3) in (1.9) the desired solution (4.1) is established.

5. Particular cases.

a) If $A_j'^{(i)}, B_j'^{(i)}, C_j'^{(i)}, D_j'^{(i)} \in \mathbb{R}$ and $m' = 0$, the multivariable A-function reduces to multivariable H-function defined by Srivastava et al [9], we obtain the following result. The solution of problem is

$$U(x,t) = \frac{L}{2^{W-1}} \sum_{p=1}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_u=0}^{[N_u/M_u]} \sum_{r_1,\cdots,r_v=0}^{\infty} \sum_{h_i=1}^{M_i} \sum_{g_i=1}^{\infty} \phi_1 \frac{\prod_{i=1}^r \phi_i z_i^{\eta_{h_i,g_i}} (-)^{\sum_{i=1}^r g_i}}{\prod_{i=1}^r \delta_{h^{(i)}}^{(i)} \prod_{i=1}^r g_i!}$$

$$\frac{y_1^{K_1} \cdots y_u^{K_u} t_1^{r_1} \cdots t_v^{r_v}}{r_1! \cdots r_v! 4^{\sum_{i=1}^u \rho_i K_i + \sum_{i=1}^v \sigma_i r_i + \sum_{i=1}^r \xi_i \eta_{D_i,\delta_i}}} \frac{E_p^2}{[E_p^2 + h^2)L + 2h]} \left(\cos E_p x + \frac{h}{E_p} \sin E_p x \right) \left(\cos E_p \pi + \frac{h}{E_p} \sin E_p \pi \right)$$

$$e^{-kE_p^2 t} H_{p+1,q+2;Y}^{0,n+1;X} \left(\begin{array}{c} \mathrm{Z}_1 4^{-\eta_1} \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{Z}_s 4^{-\eta_s} \end{array} \middle| \begin{array}{c} (\text{1-W-2}(\sum_{i=1}^u \rho_i K_i + \sum_{i=1}^v \sigma_i r_i + \sum_{i=1}^r \xi_i \eta_{h_i,g_i}) : 2\eta_1, \cdots, 2\eta_s), \mathbb{A} \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \left(\frac{1-W-2(\sum_{i=1}^u \rho_i K_i + \sum_{i=1}^v \sigma_i r_i + \sum_{i=1}^r \xi_i \eta_{h_i,g_i}) \pm E_p}{2} : \eta_1, \cdots, \eta_s \right), \mathbb{B} \end{array} \right) \tag{5.1}$$

under the same notations and conditions that (4.1) with $A_j'^{(i)}, B_j'^{(i)}, C_j'^{(i)}, D_j'^{(i)} \in \mathbb{R}$ and $m' = 0$

b) If $r = 2$ then the multivariable $\bar{I}$-function defined by Prathima reduces to I-function of two variables defined by Rathie et al [5] and we have.

$$U(x,t) = \frac{L}{2^{W-1}} \sum_{p=1}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_u=0}^{[N_u/M_u]} \sum_{r_1,\cdots,r_v=0}^{\infty} \sum_{h_i=1}^{M_i} \sum_{g_i=1}^{\infty} \phi_1 \frac{\prod_{i=1}^2 \phi_i z_i^{\eta_{h_i,g_i}} (-)^{\sum_{i=1}^2 g_i}}{\prod_{i=1}^2 \delta_{h^{(i)}}^{(i)} \prod_{i=1}^2 g_i!}$$

$$\frac{y_1^{K_1}\cdots y_u^{K_u}t_1^{r_1}\cdots t_v^{r_v}}{r_1!\cdots r_v!4^{\sum_{i=1}^u \rho_i K_i+\sum_{i=1}^v \sigma_i r_i+\sum_{i=1}^r \xi_i\eta_{D_i,\delta_i}}}\frac{E_p^2}{[E_p^2+h^2)L+2h]}\left(\cos E_p x+\frac{h}{E_p}\sin E_p x\right)\left(\cos E_p\pi+\frac{h}{E_p}\sin E_p\pi\right)$$

$$e^{-kE_p^2 t}A_{p+1,q+2;Y}^{m,n+1;X}\left(\begin{array}{c|c} \mathrm{Z}_1 4^{-\eta_1} & (1\text{-W-}2(\sum_{i=1}^u \rho_i K_i+\sum_{i=1}^v \sigma_i r_i+\sum_{i=1}^2 \xi_i\eta_{h_i,g_i}):2\eta_1,\cdots,2\eta_s),\mathbb{A} \\ \cdots & \cdots \\ \cdots & \cdots \\ \mathrm{Z}_s 4^{-\eta_s} & \left(\frac{1-W-2(\sum_{i=1}^u \rho_i K_i+\sum_{i=1}^v \sigma_i r_i+\sum_{i=1}^2 \xi_i\eta_{h_i,g_i})\pm E_p}{2}:\eta_1,\cdots,\eta_s\right),\mathbb{B} \end{array}\right) \tag{5.2}$$

under the same notations and conditions that (4.1) with $r=2$.

Remark

If the Srivastava-Daoust function and multivariable I-function vanish and the multivariable A-function reduces to multivariable H-function, we obtain the result of Chandel et al [2].

6. Conclusion

Similarly, specializing the coefficient $A[N_1,K_1;\cdots;N_u,K_u]$ and parameters of multivariable I-function defined by Prathima et al [2], multivariable A-function defined by Gautam et al [3] and Srivastava-Daoust function, we can obtain large number of results involving various special functions of one and several variables useful in Mathematics analysis, Applied Mathematics, Physics and Mechanics.

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