Free oscillations of water in a circular lake and the I-function of several variables

with general class of polynomials and Srivastava-Daoust function

F.A. $AYANT^1$

1 Teacher in High School , France
E-mail : fredericayant@gmail.com

ABSTRACT
Chaurasia et al [1] have studied the free oscillations of water in a circular lake and the H-function of several variables, the Fox's H-function with a general class of polynomials. The object of this paper is to discuss the application of certain products involving the classes of polynomials and multivariable polynomials, the Srivastava-Daoust function, the multivariable A-function defined by Gautam et al [4] and the multivariable I-function defined by Prathima et al [5] in obtaining a solution of the partial differential equation concerning to free oscillations of water in a circular lake. We shall see the particular cases.

Keywords :Multivariable I-function, multivariable A-function, I-function of two variables, multivariable H-function, general classes of polynomials, Srivastava-Daoust function, oscillations of water.

2010 : Mathematics Subject Classification. 33C45, 33C60, 26D20

1. Introduction

The free oscillations of water in a lake is given by the following partial differential equation

$$x^2\frac{\partial^2\psi}{\partial x^2}+x\frac{\partial\psi}{\partial x}+\frac{\partial^2\psi}{\partial\theta^2}+\rho^2x^2\psi=0 \tag{1.1}$$

where ψ shows the depth of water surface from its position of equilibrium and $\rho=\frac{y}{\sqrt{\alpha_1\beta_1}}$ and with the following considerations that

(i) In any vibrational mode ψ varies harmonically with line and ψ is small enough for its square to be neglected .
(ii) The lake is stationary in space.
(iii) There is no loss of energy.

We put the solution of (1.1) as follows (McLachlan) [3], page 62)

$$\psi(x,\theta,t)=\sum_{\mu=0}^{\infty}R_\mu J_\mu(\beta x)\cos(\mu\theta-\mu\phi)cos(\mu\rho t-\mu\xi) \tag{1.2}$$

where $\theta=0=t$, let $\psi(x,0,0)=f(x)$

Letting

$$f(x)=x^\lambda\ \ S_N^M(zx^{2h})S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}\begin{pmatrix}\mathrm{y}_1x^{2\rho_1}\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ \mathrm{y}_ux^{2\rho_u}\end{pmatrix}F\begin{pmatrix}\mathrm{t}_1x^{2\sigma_1}\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ \mathrm{t}_vx^{2\sigma_v}\end{pmatrix}A\begin{pmatrix}\mathrm{z}_1x^{2\xi_1}\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ \mathrm{z}_rx^{2\xi_r}\end{pmatrix}I\begin{pmatrix}\mathrm{Z}_1x^{2\eta_1}\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ \mathrm{Z}_sx^{2\eta_s}\end{pmatrix} \tag{1.3}$$

The generalized polynomials defined by Srivastava [7], is given in the following manner :

$$S_N^M(x)=\sum_{K=0}^{[N/M]}\frac{(-N)_{MK}}{K!}A[N,K]x^K \tag{1.4}$$

The class of multivariable polynomials defined by Srivastava [8], is given in the following manner :

$$S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}[y_1,\cdots,y_u] = \sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_u)_{M_uK_u}}{K_u!}A[N_1,K_1;\cdots;N_u,K_u]y_1^{K_1}\cdots y_u^{K_u} \tag{1.5}$$

The Srivastava-Daoust function is defined by (see [9]):

$$F_{\bar{C}:D^{(1)};\cdots;D^{(v)}}^{\bar{A}:B^{(1)};\cdots;B^{(v)}}\left(\begin{array}{c|c} \mathrm{z}_1 & [(\mathrm{a});\theta',\cdots,\theta^{(v)}]:[(b');\phi'];\cdots;[(b^{(v)});\phi^{(v)}] \\ \cdot & \cdot \\ \cdot & \\ \mathrm{z}_v & [(\mathrm{c});\psi',\cdots,\psi^{(v)}]:[(d');\delta'];\cdots;[(d^{(v)});\delta^{(v)}] \end{array}\right) = \sum_{r_1,\cdots,r_v=0}^{\infty} A'\,\frac{z_1^{r_1}\cdots z_v^{r_v}}{r_1!\cdots r_v!} \tag{1.6}$$

where

$$A' = \frac{\prod_{j=1}^{\bar{A}}(a_j)_{m_1\theta_j'+\cdots+m_v\theta_j^{(v)}}\prod_{j=1}^{B^{(1)}}(b_j')_{m_1\phi_j'}\cdots\prod_{j=1}^{B^{(v)}}(b_j^{(v)})_{m_v\phi_j^{(v)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{m_1\psi_j'+\cdots+m_v\psi_j^{(v)}}\prod_{j=1}^{D^{(1)}}(d_j')_{m_1\delta_j'}\cdots\prod_{j=1}^{D^{(v)}}(d_j^{(v)})_{m_v\delta_j^{(v)}}} \tag{1.7}$$

The series given by (1.7) converges absolutely if

$$1+\sum_{j=1}^{\bar{C}}\psi_j^{(i)}+\sum_{j=1}^{D^{(i)}}\delta_j^{(i)}-\sum_{j=1}^{\bar{A}}\theta_j^{(i)}-\sum_{j=1}^{B^{(i)}}\phi_j^{(i)} > 0; i=1,\cdots,v \tag{1.8}$$

The serie representation of the multivariable A-function is given by Gautam [4] as

$$A[u_1,\cdots,u_r] = A_{A,C:(M',N');\cdots;(M^{(r)},N^{(r)})}^{0,\lambda:(\alpha',\beta');\cdots;(\alpha^{(r)},\beta^{(r)})}\left(\begin{array}{c|c} \mathrm{u}_1 & [(\mathrm{g}_j);\gamma',\cdots,\gamma^{(r)}]_{1,A}: \\ \cdot & \cdots \\ \cdot & \\ \mathrm{u}_r & [(\mathrm{f}_j);\xi',\cdots,\xi^{(r)}]_{1,C}: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{q}^{(1)},\eta^{(1)})_{1,M^{(1)}};\cdots;(q^{(r)},\eta^{(r)})_{1,M^{(r)}} \\ \cdots \\ \cdots \\ (\mathrm{p}^{(1)},\epsilon^{(1)})_{1,N^{(1)}};\cdots;(p^{(r)},\epsilon^{(r)})_{1,N^{(r)}} \end{array}\right) = \sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=1}^{\infty}\phi_1\frac{\prod_{i=1}^{r}\phi_i u_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^{r}g_i}}{\prod_{i=1}^{r}\epsilon_{G_i}^{(i)}g_i!} \tag{1.9}$$

where

$$\phi_1 = \frac{\prod_{j=1}^{\lambda}\Gamma\left(1-g_j+\sum_{i=1}^{r}\gamma_j^{(i)}\eta_{G_i,g_i}\right)}{\prod_{j=\lambda'+1}^{A}\Gamma\left(g_j-\sum_{i=1}^{r}\gamma_j^{(i)}U_i\right)\prod_{j=1}^{C}\Gamma\left(1-f_j+\sum_{i=1}^{r}\xi_j^{(i)}\eta_{G_i,g_i}\right)} \tag{1.10}$$

$$\phi_i = \frac{\prod_{j=1,j\neq m_i}^{\alpha^{(i)}}\Gamma\left(p_j^{(i)}-\epsilon_j^{(i)}\eta_{G_i,g_i}\right)\prod_{j=1}^{\beta^{(i)}}\Gamma\left(1-q_j^{(i)}+\eta_j^{(i)}\eta_{G_i,g_i}\right)}{\prod_{j=\alpha^{(i)}+1}^{N^{(i)}}\Gamma\left(1-p_j^{(i)}+\epsilon_j^{(i)}\eta_{G_i,g_i}\right)\prod_{j=\beta^{(i)}+1}^{M^{(i)}}\Gamma\left(q_j^{(i)}-\eta_j^{(i)}\eta_{G_i,g_i}\right)}, i=1,\cdots,r \tag{1.11}$$

and

$$\eta_{G_i,g_i} = \frac{p_{G_i}^{(i)}+g_i}{\epsilon_{G_i}^{(i)}}, i=1,\cdots,r \tag{1.12}$$

which is valid under the following conditions :

$$\epsilon_{G_i}^{(i)}[p_j^{(i)}+p_i'] \neq \epsilon_j^{(i)}[p_{G_i}+g_i] \tag{1.13}$$

and

$$u_i \neq 0, \sum_{j=1}^{A}\gamma_j^{(i)} - \sum_{j=1}^{C}\xi_j^{(i)} + \sum_{j=1}^{M^{(i)}}\eta_j^{(i)} - \sum_{j=1}^{N^{(i)}}\epsilon_j^{(i)} < 0, i=1,\cdots,r \tag{1.14}$$

Here $\lambda, A, C, \alpha_i, \beta_i, m_i, n_i \in \mathbb{N}^*; i=1,\cdots,r\,; f_j, g_j, p_j^{(i)}, q_j^{(i)}, \gamma_j^{(i)}, \xi_j^{(i)}, \eta_j^{(i)}, \epsilon_j^{(i)} \in \mathbb{C}$

The multivariable I-function defined by Prathima et al [5] is an extension of the multivariable H-function defined by Srivastava et al [11,12]. It is defined and represented in the following manner.

$$I(z_1,\cdots,z_s) = I_{p,q:p_1,q_1;\cdots;p_s,q_s}^{0,n:m_1,n_1;\cdots;m_s,n_s}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(s)};A_j)_{1,p}: \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_s & (\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(s)};B_j)_{1,q}: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,p_1};\cdots;(c_j^{(s)},\gamma_j^{(s)};C_j^{(s)})_{1,p_r} \\ \\ (\mathrm{d}_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,q_1};\cdots;(d_j^{(s)},\delta_j^{(s)};D_j^{(s)})_{1,q_r} \end{array}\right) \tag{1.15}$$

$$= \frac{1}{(2\pi\omega)^s}\int_{L_1'}\cdots\int_{L_s'}\phi(t_1,\cdots,t_s)\prod_{i=1}^{s}\zeta_i(t_i)z_i^{t_i}\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{1.16}$$

where $\phi(t_1,\cdots,t_s), \zeta_i(t_i), i=1,\cdots,s$ are given by :

$$\phi(t_1,\cdots,t_s) = \frac{\prod_{j=1}^{n}\Gamma^{A_j}\left(1-a_j+\sum_{i=1}^{s}\alpha_j^{(i)}t_j\right)}{\prod_{j=n+1}^{p}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{s}\alpha_j^{(i)}t_j\right)\prod_{j=m+1}^{q}\Gamma^{B_j}\left(1-b_j+\sum_{i=1}^{s}\beta_j^{(i)}t_j\right)} \tag{1.17}$$

$$\zeta_i(t_i) = \frac{\prod_{j=1}^{n_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}t_i\right)\prod_{j=1}^{m_i}\Gamma^{D_j^{(i)}}\left(d_j^{(i)}-\delta_j^{(i)}t_i\right)}{\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}t_i\right)\prod_{j=m_i+1}^{q_i'}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}t_i\right)} \tag{1.18}$$

For more details, see Prathima et al [5]. Following the result of Braaksma the I-function of r variables is analytic if

$$U_i = \sum_{j=1}^{p}A_j\alpha_j^{(i)} - \sum_{j=1}^{q}B_j\beta_j^{(i)} + \sum_{j=1}^{p_i}C_j^{(i)}\gamma_j^{(i)} - \sum_{j=1}^{q_i}D_j^{(i)}\delta_j^{(i)} \leqslant 0, i=1,\cdots,s \tag{1.19}$$

The integral (2.1) converges absolutely if

where $|arg(z_k)| < \frac{1}{2}\Delta_k\pi, k=1,\cdots,s$

$$\Delta_k = -\sum_{j=n+1}^{p} A_j\alpha_j^{(k)} - \sum_{j=1}^{q} B_j\beta_j^{(k)} + \sum_{j=1}^{m_k} D_j^{(k)}\delta_j^{(k)} - \sum_{j=m_k+1}^{q_k} D_j^{(k)}\delta_j'^{(k)} + \sum_{j=1}^{n_k} C_j^{(k)}\gamma_j^{(k)} - \sum_{j=n_k+1}^{p_k} C_j^{(k)}\gamma_j^{(k)} > 0 \quad (1.20)$$

In this paper, we shall note.

$$X = m_1, n_1; \cdots ; m_s, n_s \quad : Y = p_1, q_1; \cdots ; p_s, q_s \quad (1.21)$$

$$\mathbb{A} = (a_j; A_j^{(1)}, \cdots , A_j^{(s)}; A_j)_{1,p} : \ (\mathrm{c}_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{1,p_1}; \cdots ; (c_j^{(s)}, \gamma_j^{(s)}; C_j^{(s)})_{1,p_s} \quad (1.22)$$

$$\mathbb{B} = (b_j; B_j^{(1)}, \cdots , B_j^{(s)}; B_j)_{1,q} : (\mathrm{d}_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{1,q_1}; \cdots ; (d_j^{(s)}, \delta_j^{(s)}; D_j^{(s)})_{1,q_s} \quad (1.24)$$

$$B' = \frac{(-N_1)_{M_1K_1}}{K_1!} \cdots \frac{(-N_u)_{M_uK_u}}{K_u!} A[N_1, K_1; \cdots ; N_u, K_u] \quad (1.25)$$

$$F = F_{\bar{C}:D^{(1)};\cdots;D^{(v)}}^{\bar{A}:B^{(1)};\cdots;B^{(v)}} \quad (1.26)$$

2. Main integral

Lemma (Erdelyi et al ([2] page 49, Eq.(19))

$$\int_0^\infty x^{\lambda-1} J_\upsilon(\beta x)\mathrm{d}x = \frac{2^{\lambda-1}\beta^{-\lambda}\Gamma\left(\frac{\lambda+\upsilon}{2}\right)}{\Gamma\left(1+\frac{\upsilon-\lambda}{2}\right)} \quad (2.1)$$

where $-Re(\upsilon) < Re(\lambda) < \frac{3}{2}$

We have the following integral

Theorem

$$\int_0^\infty x^{\lambda-1} J_\upsilon(\beta x)\ S_N^M(zx^{2h}) S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}\begin{pmatrix} \mathrm{y}_1 x^{2\rho_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{y}_u x^{2\rho_u} \end{pmatrix} F \begin{pmatrix} \mathrm{t}_1 x^{2\sigma_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{t}_v x^{2\sigma_v} \end{pmatrix} A \begin{pmatrix} \mathrm{z}_1 x^{2\xi_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{z}_r x^{2\xi_r} \end{pmatrix} I \begin{pmatrix} \mathrm{Z}_1 x^{2\eta_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{Z}_s x^{2\eta_s} \end{pmatrix} \mathrm{d}x$$

$$= \frac{2^{\lambda-1}}{\beta^\lambda} \sum_{K=0}^{[N/M]} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_u=0}^{[N_u/M_u]} \sum_{r_1,\cdots,r_v=0}^{\infty} \sum_{G_i=1}^{\alpha^{(i)}} \sum_{g_i=1}^{\infty} \phi_1 \frac{\prod_{i=1}^r \phi_i u_i^{\eta_{G_i,g_i}} (-)^{\sum_{i=1}^r g_i}}{\prod_{i=1}^r \epsilon_{G_i}^{(i)} g_i!} \frac{(-N)_{MK}}{K!} A[N,K] z^K A' B'$$

$$\frac{y_1^{K_1} \cdots y_u^{K_u} t_1^{r_1} \cdots t_v^{r_v} 4^{Kh+\sum_{i=1}^u \rho_i K_i + \sum_{i=1}^v \sigma_i r_i + \sum_{i=1}^r \xi_i \eta_{G_i,g_i}}}{r_1! \cdots r_v! \beta^{2(Kh+\sum_{i=1}^u \rho_i K_i + \sum_{i=1}^v \sigma_i r_i + \sum_{i=1}^r \xi_i \eta_{G_i,g_i})}}$$

$$I_{p+1,q+1;Y}^{0,n+1;X} \left(\begin{matrix} \mathrm{Z}_1 \beta^{-2h_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{Z}_s \beta^{-2h_s} \end{matrix} \ \middle| \ \begin{matrix} (1\text{-}\frac{\upsilon}{2}\text{-}\frac{\lambda}{2}\text{-Kh-}\sum_{i=1}^u \rho_i K_i - \sum_{i=1}^v \sigma_i r_i - \sum_{i=1}^r \xi_i \eta_{G_i,g_i} : 2\eta_1, \cdots , 2\eta_s; 1), \mathbb{A} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ (\text{-}\frac{\upsilon}{2}+\frac{\lambda}{2}\text{-Kh-}\sum_{i=1}^u \rho_i K_i - \sum_{i=1}^v \sigma_i r_i - \sum_{i=1}^r \xi_i \eta_{G_i,g_i} : 2\eta_1, \cdots , 2\eta_s; 1), \mathbb{B} \end{matrix} \right) \quad (2.2)$$

provided

$$min\{h,\rho_i,\sigma_j,\xi_k,\eta_l\}>0, i=1,\cdots,s, j=1,\cdots,v, k=1,\cdots,r, l=1,\cdots,s$$

$$1+\sum_{j=1}^{\bar{C}}\psi_j^{(i)}+\sum_{j=1}^{D^{(i)}}\delta_j^{(i)}-\sum_{j=1}^{\bar{A}}\theta_j^{(i)}-\sum_{j=1}^{B^{(i)}}\phi_j^{(i)}>0; i=1,\cdots,v$$

$$z_i\neq 0, \sum_{j=1}^{A}\gamma_j^{(i)}-\sum_{j=1}^{C}\xi_j^{(i)}+\sum_{j=1}^{M^{(i)}}\eta_j^{(i)}-\sum_{j=1}^{N^{(i)}}\epsilon_j^{(i)}<0, i=1,\cdots,r$$

$$Re\left[\lambda+\upsilon+\sum_{i=1}^{u}K_i\rho_i+\sum_{i=1}^{v}\sigma_i r_i+\sum_{i=1}^{r}\xi_i\eta_{G_i,g_i}+\sum_{i=1}^{s}\eta_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}\right]>0$$

$|arg(z_k)|<\frac{1}{2}\Delta_k\pi, k=1,\cdots,s$ where Δ_k is defined by (1.20)

and $$Re\left[\lambda-\sum_{i=1}^{u}K_i\rho_i-\sum_{i=1}^{v}r_i\sigma_i-\sum_{i=1}^{r}\xi_i\eta_{G_i,g_i}-\sum_{i=1}^{s}\eta_i\max_{1\leqslant j\leqslant n_i}\frac{1-c_j^{(i)}}{\gamma_j^{(i)}}\right]<\frac{3}{2}$$

Proof

To prove (2.1), first expressing a class of polynomials $S_N^M(.)$ defined by Srivastava [7], a class of multivariable polynomials defined by Srivastava [8] $S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[.]$, the Srivastava-Daoust function $F[.]$ and the multivariable $A[.]$ function defined by Gautam et al [4] in series with the help of (1.4), (1.5), (1.6) and (1.9) respectively and we interchange the order of summations and x-integral (which is permissible under the conditions stated). Expressing the I-function of s-variables defined by Prathima et al [5] in Mellin-contour integral with the help of (1.16) and interchange the order of integrations which is justifiable due to absolute convergence of the integral involved in the process. Now collecting the power of x and evaluating the inner x-integral with the help of the Lemma. Interpreting the Mellin-Barnes contour integral in multivariable I-function, we obtain the desired result (2.1).

3. Application in Free oscillations of water in a circular lake

The solution of the problem posed is given by the following solution.

$$\psi(x,\theta,t)=\left(\frac{2}{\beta}\right)^{\lambda}\sum_{\mu=0}^{\infty}\sum_{K=0}^{[N/M]}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{r_1,\cdots,r_v=0}^{\infty}\sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=1}^{\infty}\phi_1\frac{\prod_{i=1}^{r}\phi_i u_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^{r}g_i}}{\prod_{i=1}^{r}\epsilon_{G_i}^{(i)}g_i!}\frac{(-N)_{MK}}{K!}$$

$$A[N,K]z^K A'B'\frac{y_1^{K_1}\cdots y_u^{K_u}t_1^{r_1}\cdots t_v^{r_v}4^{Kh+\sum_{i=1}^{u}\rho_iK_i+\sum_{i=1}^{v}\sigma_ir_i+\sum_{i=1}^{r}\xi_i\eta_{G_i,g_i}}}{r_1!\cdots r_v!\beta^{2(Kh+\sum_{i=1}^{u}\rho_iK_i+\sum_{i=1}^{v}\sigma_ir_i+\sum_{i=1}^{r}\xi_i\eta_{G_i,g_i})}}\mu J_\mu(\beta x)\frac{\cos(\mu\theta-\mu\phi)\cos(\mu\rho t-\mu\xi)}{\cos(\mu\theta)\cos(\mu\xi)}$$

$$I_{p+1,q+1;Y}^{0,n+1;X}\left(\begin{array}{c|c}\mathrm{Z}_1\beta^{-2h_1} & (1\text{-}\frac{\upsilon}{2}\text{-}\frac{\lambda}{2}\text{-Kh-}\sum_{i=1}^{u}\rho_iK_i-\sum_{i=1}^{v}\sigma_ir_i-\sum_{i=1}^{r}\xi_i\eta_{G_i,g_i}:2\eta_1,\cdots,2\eta_s;1),\mathbb{A}\\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\\ \mathrm{Z}_s\beta^{-2h_s} & (-\frac{\upsilon}{2}+\frac{\lambda}{2}\text{-Kh-}\sum_{i=1}^{u}\rho_iK_i-\sum_{i=1}^{v}\sigma_ir_i-\sum_{i=1}^{r}\xi_i\eta_{G_i,g_i}:2\eta_1,\cdots,2\eta_s;1),\mathbb{B}\end{array}\right) \quad (3.1)$$

Under the same conditions and notations needed for (2.1).

Proof

Let

$$f(x)=\sum_{\mu=0}^{\infty}R_\mu J_\mu(\beta x)\cos(\mu\phi)cos(\mu\xi) \tag{3.2}$$

Multiplying both sides of (3.1) by $x^{-1}J_\upsilon(\beta x)$, integrate with respect to x from 0 to ∞ and use (2.1) and the orthogonal property of the Bessel functions. We thus obtain

$$R_\upsilon=\left(\frac{2}{\beta}\right)^\lambda\sum_{K=0}^{[N/M]}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{r_1,\cdots,r_v=0}^{\infty}\sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=1}^{\infty}\phi_1\frac{\prod_{i=1}^r\phi_i u_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^r g_i}}{\prod_{i=1}^r\epsilon_{G_i}^{(i)}g_i!}\frac{(-N)_{MK}}{K!}\;A[N,K]z^KA'B'$$

$$\frac{y_1^{K_1}\cdots y_u^{K_u}t_1^{r_1}\cdots t_v^{r_v}4^{Kh+\sum_{i=1}^u\rho_iK_i+\sum_{i=1}^v\sigma_ir_i+\sum_{i=1}^r\xi_i\eta_{G_i,g_i}}}{r_1!\cdots r_v!\beta^{2(Kh+\sum_{i=1}^u\rho_iK_i+\sum_{i=1}^v\sigma_ir_i+\sum_{i=1}^r\xi_i\eta_{G_i,g_i})}}\frac{1}{\cos(\upsilon\theta)\cos(\upsilon\xi)}$$

$$I_{p+1,q+1;Y}^{0,n+1;X}\left(\begin{array}{c}\mathrm{Z}_1\beta^{-2h_1}\\ \cdot\;\cdot\;\cdot\\ \cdot\;\cdot\;\cdot\\ \mathrm{Z}_s\beta^{-2h_s}\end{array}\left|\begin{array}{c}(1\text{-}\frac{\upsilon}{2}\text{-}\frac{\lambda}{2}\text{-Kh-}\sum_{i=1}^u\rho_iK_i-\sum_{i=1}^v\sigma_ir_i-\sum_{i=1}^r\xi_i\eta_{G_i,g_i}:2\eta_1,\cdots,2\eta_s;1),\mathbb{A}\\ \cdot\;\cdot\;\cdot\\ \cdot\;\cdot\;\cdot\\ (\text{-}\frac{\upsilon}{2}+\frac{\lambda}{2}\text{-Kh-}\sum_{i=1}^u\rho_iK_i-\sum_{i=1}^v\sigma_ir_i-\sum_{i=1}^r\xi_i\eta_{G_i,g_i}:2\eta_1,\cdots,2\eta_s;1),\mathbb{B}\end{array}\right.\right) \tag{3.3}$$

Substituting the value of R_υ from (3.3) in (1.2), we obtain the desired solution (3.1).

4. Special cases

1) By applying our result given in (2.1) and (3.1) to the case of Hermite polynomials ([13], page 106, eq.(5.54)) and ([10] ,page 158) and by setting

$$S_N^2x)\rightarrow x^{n/2}H_n\left(\frac{1}{2\sqrt{x}}\right)$$

In which case $m=2, A_{N,K}=(-)^K$ we have the following interesting consequencies of the main results.

$$\int_0^\infty x^{\lambda-1}J_\upsilon(\beta x)\;H_N\left\{\frac{1}{2\sqrt{zx^{2h}}}\right\}S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}\left(\begin{array}{c}\mathrm{y}_1x^{2\rho_1}\\ \cdot\;\cdot\;\cdot\\ \cdot\;\cdot\;\cdot\\ \mathrm{y}_ux^{2\rho_u}\end{array}\right)F\left(\begin{array}{c}\mathrm{t}_1x^{2\sigma_1}\\ \cdot\;\cdot\;\cdot\\ \cdot\;\cdot\;\cdot\\ \mathrm{t}_vx^{2\sigma_v}\end{array}\right)A\left(\begin{array}{c}\mathrm{z}_1x^{2\xi_1}\\ \cdot\;\cdot\;\cdot\\ \cdot\;\cdot\;\cdot\\ \mathrm{z}_rx^{2\xi_r}\end{array}\right)I\left(\begin{array}{c}\mathrm{Z}_1x^{2\eta_1}\\ \cdot\;\cdot\;\cdot\\ \cdot\;\cdot\;\cdot\\ \mathrm{Z}_sx^{2\eta_s}\end{array}\right)$$

$$\mathrm{d}x=\frac{2^{\lambda-1}}{\beta^\lambda}\sum_{K=0}^{[N/2]}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{r_1,\cdots,r_v=0}^{\infty}\sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=1}^{\infty}\phi_1\frac{\prod_{i=1}^r\phi_i u_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^r g_i}}{\prod_{i=1}^r\epsilon_{G_i}^{(i)}g_i!}\frac{(-N)_{2K}}{K!}(-z)^KA'B'$$

$$\frac{y_1^{K_1}\cdots y_u^{K_u}t_1^{r_1}\cdots t_v^{r_v}4^{Kh+\sum_{i=1}^u\rho_iK_i+\sum_{i=1}^v\sigma_ir_i+\sum_{i=1}^r\xi_i\eta_{G_i,g_i}}}{r_1!\cdots r_v!\beta^{2(Kh+\sum_{i=1}^u\rho_iK_i+\sum_{i=1}^v\sigma_ir_i+\sum_{i=1}^r\xi_i\eta_{G_i,g_i})}}$$

$$I_{p+1,q+1;Y}^{0,n+1;X}\left(\begin{array}{c}\mathrm{Z}_1\beta^{-2h_1}\\ \cdots\\ \cdots\\ \mathrm{Z}_s\beta^{-2h_s}\end{array}\middle|\begin{array}{c}(1\text{-}\frac{v}{2}\text{-}\frac{\lambda}{2}\text{-Kh-}\sum_{i=1}^u\rho_iK_i-\sum_{i=1}^v\sigma_ir_i-\sum_{i=1}^r\xi_i\eta_{G_i,g_i}:2\eta_1,\cdots,2\eta_s;1),\mathbb{A}\\ \cdots\\ \cdots\\ (\text{-}\frac{v}{2}+\frac{\lambda}{2}\text{-Kh-}\sum_{i=1}^u\rho_iK_i-\sum_{i=1}^v\sigma_ir_i-\sum_{i=1}^r\xi_i\eta_{G_i,g_i}:2\eta_1,\cdots,2\eta_s;1),\mathbb{B}\end{array}\right) \tag{4.1}$$

under the same notations and conditions that (2.1) and

$$\psi(x,\theta,t)=\left(\frac{2}{\beta}\right)^{\lambda}\sum_{\mu=0}^{\infty}\sum_{K=0}^{[N/2]}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{r_1,\cdots,r_v=0}^{\infty}\sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=1}^{\infty}\phi_1\frac{\prod_{i=1}^r\phi_iu_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^r g_i}}{\prod_{i=1}^r\epsilon_{G_i}^{(i)}g_i!}\frac{(-N)_{2K}}{K!}(-z)^K$$

$$A'B'\frac{y_1^{K_1}\cdots y_u^{K_u}t_1^{r_1}\cdots t_v^{r_v}4^{Kh+\sum_{i=1}^u\rho_iK_i+\sum_{i=1}^v\sigma_ir_i+\sum_{i=1}^r\xi_i\eta_{G_i,g_i}}}{r_1!\cdots r_v!\beta^{2(Kh+\sum_{i=1}^u\rho_iK_i+\sum_{i=1}^v\sigma_ir_i+\sum_{i=1}^r\xi_i\eta_{G_i,g_i})}}\mu J_\mu(\beta x)\frac{\cos(\mu\theta-\mu\phi)\cos(\mu\rho t-\mu\xi)}{\cos(\mu\theta)\cos(\mu\xi)}$$

$$I_{p+1,q+1;Y}^{0,n+1;X}\left(\begin{array}{c}\mathrm{Z}_1\beta^{-2h_1}\\ \cdots\\ \cdots\\ \mathrm{Z}_s\beta^{-2h_s}\end{array}\middle|\begin{array}{c}(1\text{-}\frac{v}{2}\text{-}\frac{\lambda}{2}\text{-Kh-}\sum_{i=1}^u\rho_iK_i-\sum_{i=1}^v\sigma_ir_i-\sum_{i=1}^r\xi_i\eta_{G_i,g_i}:2\eta_1,\cdots,2\eta_s;1),\mathbb{A}\\ \cdots\\ \cdots\\ (\text{-}\frac{v}{2}+\frac{\lambda}{2}\text{-Kh-}\sum_{i=1}^u\rho_iK_i-\sum_{i=1}^v\sigma_ir_i-\sum_{i=1}^r\xi_i\eta_{G_i,g_i}:2\eta_1,\cdots,2\eta_s;1),\mathbb{B}\end{array}\right) \tag{4.2}$$

under the same notations and conditions that (2.1)

2) For the Laguerre polynomials ([13], page 101, eq.(15.1.6)) and ([10] ,page 159) and by setting

$$S_N^1(x)\rightarrow L_N^{\alpha'}(x)$$

In which case $m=1, A_{N,K}=\binom{N+\alpha'}{N}\frac{1}{(\alpha'+1)_K}$ we have the following interesting consequencies of the main results.

$$\int_0^\infty x^{\lambda-1}J_v(\beta x)\ L_N^{\alpha'}(zx^{2h})S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}\left(\begin{array}{c}\mathrm{y}_1x^{2\rho_1}\\ \cdots\\ \cdots\\ \mathrm{y}_ux^{2\rho_u}\end{array}\right)F\left(\begin{array}{c}\mathrm{t}_1x^{2\sigma_1}\\ \cdots\\ \cdots\\ \mathrm{t}_vx^{2\sigma_v}\end{array}\right)A\left(\begin{array}{c}\mathrm{z}_1x^{2\xi_1}\\ \cdots\\ \cdots\\ \mathrm{z}_rx^{2\xi_r}\end{array}\right)I\left(\begin{array}{c}\mathrm{Z}_1x^{2\eta_1}\\ \cdots\\ \cdots\\ \mathrm{Z}_sx^{2\eta_s}\end{array}\right)\mathrm{d}x$$

$$=\frac{2^{\lambda-1}}{\beta^\lambda}\sum_{K=0}^{N}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{r_1,\cdots,r_v=0}^{\infty}\sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=1}^{\infty}\phi_1\frac{\prod_{i=1}^r\phi_iu_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^r g_i}}{\prod_{i=1}^r\epsilon_{G_i}^{(i)}g_i!}\binom{N+\alpha'}{N-K}\frac{(-z)^K}{K!}A'B'$$

$$\frac{y_1^{K_1}\cdots y_u^{K_u}t_1^{r_1}\cdots t_v^{r_v}4^{Kh+\sum_{i=1}^u\rho_iK_i+\sum_{i=1}^v\sigma_ir_i+\sum_{i=1}^r\xi_i\eta_{G_i,g_i}}}{r_1!\cdots r_v!\beta^{2(Kh+\sum_{i=1}^u\rho_iK_i+\sum_{i=1}^v\sigma_ir_i+\sum_{i=1}^r\xi_i\eta_{G_i,g_i})}}$$

$$I_{p+1,q+1;Y}^{0,n+1;X}\left(\begin{array}{c} \mathrm{Z}_1\beta^{-2h_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{Z}_s\beta^{-2h_s} \end{array}\left|\begin{array}{c} (1\text{-}\frac{v}{2}\text{-}\frac{\lambda}{2}\text{-Kh-}\sum_{i=1}^{u}\rho_iK_i-\sum_{i=1}^{v}\sigma_ir_i-\sum_{i=1}^{r}\xi_i\eta_{G_i,g_i}:2\eta_1,\cdots,2\eta_s;1),\mathbb{A} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ (-\frac{v}{2}+\frac{\lambda}{2}\text{-Kh-}\sum_{i=1}^{u}\rho_iK_i-\sum_{i=1}^{v}\sigma_ir_i-\sum_{i=1}^{r}\xi_i\eta_{G_i,g_i}:2\eta_1,\cdots,2\eta_s;1),\mathbb{B} \end{array}\right.\right) \tag{4.3}$$

under the same notations and conditions that (2.1) and

$$\psi(x,\theta,t)=\left(\frac{2}{\beta}\right)^{\lambda}\sum_{\mu=0}^{\infty}\sum_{K=0}^{N}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{r_1,\cdots,r_v=0}^{\infty}\sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=1}^{\infty}\phi_1\frac{\prod_{i=1}^{r}\phi_iu_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^{r}g_i}}{\prod_{i=1}^{r}\epsilon_{G_i}^{(i)}g_i!}\binom{N+\alpha'}{N-K}\frac{(-z)^K}{K!}$$

$$A'B'\frac{y_1^{K_1}\cdots y_u^{K_u}t_1^{r_1}\cdots t_v^{r_v}4^{Kh+\sum_{i=1}^{u}\rho_iK_i+\sum_{i=1}^{v}\sigma_ir_i+\sum_{i=1}^{r}\xi_i\eta_{G_i,g_i}}}{r_1!\cdots r_v!\beta^{2(Kh+\sum_{i=1}^{u}\rho_iK_i+\sum_{i=1}^{v}\sigma_ir_i+\sum_{i=1}^{r}\xi_i\eta_{G_i,g_i})}}\mu J_\mu(\beta x)\frac{\cos(\mu\theta-\mu\phi)\cos(\mu\rho t-\mu\xi)}{\cos(\mu\theta)\cos(\mu\xi)}$$

$$I_{p+1,q+1;Y}^{0,n+1;X}\left(\begin{array}{c} \mathrm{Z}_1\beta^{-2h_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{Z}_s\beta^{-2h_s} \end{array}\left|\begin{array}{c} (1\text{-}\frac{v}{2}\text{-}\frac{\lambda}{2}\text{-Kh-}\sum_{i=1}^{u}\rho_iK_i-\sum_{i=1}^{v}\sigma_ir_i-\sum_{i=1}^{r}\xi_i\eta_{G_i,g_i}:2\eta_1,\cdots,2\eta_s;1),\mathbb{A} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ (-\frac{v}{2}+\frac{\lambda}{2}\text{-Kh-}\sum_{i=1}^{u}\rho_iK_i-\sum_{i=1}^{v}\sigma_ir_i-\sum_{i=1}^{r}\xi_i\eta_{G_i,g_i}:2\eta_1,\cdots,2\eta_s;1),\mathbb{B} \end{array}\right.\right) \tag{4.4}$$

under the same notations and conditions that (2.1)

5. Multivariable H-function

If $A_j=B_j=C_j^{(i)}=D_j^{(i)}=1$, the multivariable I-function defined by Prathima et al [5] reduces to multivariable H-function defined by Srivastava et al [11,12] and the solution of our problem is :

$$\psi(x,\theta,t)=\left(\frac{2}{\beta}\right)^{\lambda}\sum_{\mu=0}^{\infty}\sum_{K=0}^{[N/M]}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\sum_{r_1,\cdots,r_v=0}^{\infty}\sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=1}^{\infty}\phi_1\frac{\prod_{i=1}^{r}\phi_iu_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^{r}g_i}}{\prod_{i=1}^{r}\epsilon_{G_i}^{(i)}g_i!}\frac{(-N)_{MK}}{K!}$$

$$A[N,K]z^KA'B'\frac{y_1^{K_1}\cdots y_u^{K_u}t_1^{r_1}\cdots t_v^{r_v}4^{Kh+\sum_{i=1}^{u}\rho_iK_i+\sum_{i=1}^{v}\sigma_ir_i+\sum_{i=1}^{r}\xi_i\eta_{G_i,g_i}}}{r_1!\cdots r_v!\beta^{2(Kh+\sum_{i=1}^{u}\rho_iK_i+\sum_{i=1}^{v}\sigma_ir_i+\sum_{i=1}^{r}\xi_i\eta_{G_i,g_i})}}\mu J_\mu(\beta x)\frac{\cos(\mu\theta-\mu\phi)\cos(\mu\rho t-\mu\xi)}{\cos(\mu\theta)\cos(\mu\xi)}$$

$$H_{p+1,q+1;Y}^{0,n+1;X}\left(\begin{array}{c} \mathrm{Z}_1\beta^{-2h_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{Z}_s\beta^{-2h_s} \end{array}\left|\begin{array}{c} (1\text{-}\frac{v}{2}\text{-}\frac{\lambda}{2}\text{-Kh-}\sum_{i=1}^{u}\rho_iK_i-\sum_{i=1}^{v}\sigma_ir_i-\sum_{i=1}^{r}\xi_i\eta_{G_i,g_i}:2\eta_1,\cdots,2\eta_s;1),\mathbb{A} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ (-\frac{v}{2}+\frac{\lambda}{2}\text{-Kh-}\sum_{i=1}^{u}\rho_iK_i-\sum_{i=1}^{v}\sigma_ir_i-\sum_{i=1}^{r}\xi_i\eta_{G_i,g_i}:2\eta_1,\cdots,2\eta_s;1),\mathbb{B} \end{array}\right.\right) \tag{5.1}$$

under the same conditions and notations needed for (2.1) with $A_j=B_j=C_j^{(i)}=D_j^{(i)}=1$.

6. I-function of two variables

If $r = 2$, the multivariable I-function defined by Prathima et al [5] reduces to I-function of two variables defined by Rathie et al [6] and the solution of our problem is :

$$\psi(x,\theta,t) = \left(\frac{2}{\beta}\right)^{\lambda} \sum_{\mu=0}^{\infty} \sum_{K=0}^{[N/M]} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_u=0}^{[N_u/M_u]} \sum_{r_1,\cdots,r_v=0}^{\infty} \sum_{G_i=1}^{\alpha^{(i)}} \sum_{g_i=1}^{\infty} \phi_1 \frac{\prod_{i=1}^{r} \phi_i u_i^{\eta_{G_i,g_i}} (-)^{\sum_{i=1}^{r} g_i}}{\prod_{i=1}^{r} \epsilon_{G_i}^{(i)} g_i!} \frac{(-N)_{MK}}{K!}$$

$$A[N,K] z^K A' B' \frac{y_1^{K_1} \cdots y_u^{K_u} t_1^{r_1} \cdots t_v^{r_v} 4^{Kh+\sum_{i=1}^{u} \rho_i K_i + \sum_{i=1}^{v} \sigma_i r_i + \sum_{i=1}^{r} \xi_i \eta_{G_i,g_i}}}{r_1! \cdots r_v! \beta^{2(Kh+\sum_{i=1}^{u} \rho_i K_i + \sum_{i=1}^{v} \sigma_i r_i + \sum_{i=1}^{r} \xi_i \eta_{G_i,g_i})}} \mu J_\mu(\beta x) \frac{\cos(\mu\theta - \mu\phi)\cos(\mu\rho t - \mu\xi)}{\cos(\mu\theta)\cos(\mu\xi)}$$

$$I_{p+1,q+1;Y}^{0,n+1;X} \left(\begin{array}{c|c} \mathrm{Z}_1 \beta^{-2h_1} & (1\text{-}\frac{v}{2}\text{-}\frac{\lambda}{2}\text{-Kh-}\sum_{i=1}^{u} \rho_i K_i - \sum_{i=1}^{v} \sigma_i r_i - \sum_{i=1}^{r} \xi_i \eta_{G_i,g_i} : 2\eta_1, 2\eta_2), \mathbb{A} \\ \cdot\cdot\cdot & \cdot\cdot\cdot \\ \cdot\cdot\cdot & \cdot\cdot\cdot \\ \mathrm{Z}_2 \beta^{-2h_2} & (\text{-}\frac{v}{2}+\frac{\lambda}{2}\text{-Kh-}\sum_{i=1}^{u} \rho_i K_i - \sum_{i=1}^{v} \sigma_i r_i - \sum_{i=1}^{r} \xi_i \eta_{G_i,g_i} : 2\eta_1, 2\eta_2), \mathbb{B} \end{array} \right) \quad (6.1)$$

Under the same conditions and notations needed for (2.1) with $r = 2$.

7. Conclusion

Similarly, specializing the following coefficients $A[N,K]$, $A[N_1,K_1;\cdots;N_u,K_u]$ and parameters of multivariable I-function defined by Prathima et al [5], multivariable A-function defined by Gautam et al [4] and Srivastava-Daoust function, we can obtain large number of results involving various special functions of one and several variables useful in Mathematics analysis, Applied Mathematics, Physics and Mechanics.

References

[1] Chaurasia V.B.L. and Patni R. Free oscillations of water in a circular lake and the H-function of several variables with general class of polynomials. Acta. Ciencia. Indica. Math. Vol 24 (no 1), page 45-50.

[2] Erdelyi A., Magnus W. , Oberhettinger F. and Tricomi F.G. Higher tanscendental functions. Vol II. McGraw-Hill Book co., Inc., New York, Toronton and London (1953).

[3] McMachlan N.W. Bessel functions for engineers, Calrendon Press Oxford, 1961.

[4] Gautam B.P., Asgar A.S. and Goyal A.N. On the multivariable A-function. Vijnana Parishas Anusandhan Patrika Vol 29(4) 1986, page 67-81.

[5] Prathima J. Nambisan V. and Kurumujji S.K. A Study of I-function of Several Complex Variables, International Journalof Engineering Mathematics Vol (2014) , 2014 page 1-12.

[6] Rathie A.K. Kumari K.S. and Vasudevan Nambisan T.M. A study of I-functions of two variables. Le Matematiche Vol 64 (1), page 285-305.

[7] Srivastava H.M. A contour integral involving Fox's H-function. Indian. J. Math. Vol 14, 1972, page 1-6.

[8] Srivastava H.M. A multilinear generating function for the Konhauser set of biorthogonal polynomials suggested by Laguerre polynomial, Pacific. J. Math. 177(1985), page 183-191.

[9] Srivastava H.M. and Daoust M.C. Certain generalized Neuman expansions associated with the Kampé de Fériet function. Nederl. Akad. Wetensch. Indag. Math, 31 (1969), page 449-457.

[10] Srivastava H.M. and Singh N.P. The integration of certain products of the multivariable H-function with a general

class of polynomials. Rend. Circ. Mat. Palermo. Vol 32 (No 2), 1983, page 157-187.

[11] Srivastava H.M. and Panda R. Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24(1975), page.119-137.

[12] Srivastava H.M. and Panda R. Some expansion theorems and generating relations for the H-function of several complex variables II. Comment. Math. Univ. St. Paul. 25(1976), page.167-197.

[13] Szego C. Orthogonal polynomials. Amer. Math. Soc. Colloq. Publ. 23 fourth edition. Amer. Math. Soc. Providence. Rhodes Island, 1975.

Personal adress : 411 Avenue Joseph Raynaud

Le parc Fleuri , Bat B

83140 , Six-Fours les plages

Tel : 06-83-12-49-68

Department : VAR

Country : FRANCE