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An application of the multivariable I-function in solving the general partial differential equation governing the forced vibrations of a circular membrane

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ABSTRACT
Singh [4] have given an application of the multivariable H-function in solving the general differential equation governing the forced vibrations of a circular membrane. The present paper deals with the application of the multivariable I-function defined by Prasad [2], the Srivastava-Daoust function and a class of multivariable polynomials in solving the partial differential equation governing the forced vibrations of a circular membrane. The results obtained are very general in nature and are the generalization of the previous results due to Singh [5], Prasad et al [3], Sneddon [5], etc. We shall see the particular cases concerning the multivariable H-function, when the membrane is at rest its equilibrium position at $t=0$ and the class of polynomials defined by Srivastava [7].

Keywords : multivariable I-function, class of polynomials, forced vibration, circular membrane, H-function of several variables, class of multivariable polynomials, Hankel transform, Srivastava-Daoust function.

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1.Introduction and preliminaries.

In the present paper an attempt has been made to apply the multivariable I-function defined by Prasad [2], the Srivastava-Daoust function and a class of multivariable polynomials in solving the partial differential equation governing the forced vibrations of a circular membrane.

The generalized polynomials defined by Srivastava [7], is given in the following manner :

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1,\cdots,y_s] = \sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]} \frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}A[N_1,K_1;\cdots;N_s,K_s]y_1^{K_1}\cdots y_s^{K_s} \tag{1.1}$$

Where $M_1,\cdots,M_s$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_s,K_s]$ are arbitrary constants, real or complex.

The multivariable I-function defined by Prasad [1] is an extension of the multivariable H-function defined by Srivastava and Panda [6,7]. This function is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,\cdots,z_r) = I_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p',q';\cdots;p^{(r)},q^{(r)}}^{0,n_2;0,n_3;\cdots;0,n_r:m',n';\cdots;m^{(r)},n^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots;(\mathrm{a}_{rj};\alpha'_{rj},\cdots,\alpha_{rj}^{(r)})_{1,p_r}:(a'_j,\alpha'_j)_{1,p'};\cdots;(a_j^{(r)},\alpha_j^{(r)})_{1,p^{(r)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r & (\mathrm{b}_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots;(\mathrm{b}_{rj};\beta'_{rj},\cdots,\beta_{rj}^{(r)})_{1,q_r}:(b'_j,\beta'_j)_{1,q'};\cdots;(b_j^{(r)},\beta_j^{(r)})_{1,q^{(r)}} \end{array}\right) \tag{1.2}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\xi(s_1,\cdots,s_r)\prod_{i=1}^{s}\phi_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.3}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [1]. The condition for absolute convergence of multiple Mellin-Barnes type contour (1.6) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_i| < \frac{1}{2}\Omega_i\pi$, where

$$\Omega_i = \sum_{k=1}^{n^{(i)}}\alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}}\alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}}\beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}}\beta_k^{(i)} + \left(\sum_{k=1}^{n_2}\alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2}\alpha_{2k}^{(i)}\right) +$$

$$+ \left(\sum_{k=1}^{n_r}\alpha_{rk}^{(i)} - \sum_{k=n_r+1}^{p_r}\alpha_{rk}^{(i)}\right) - \left(\sum_{k=1}^{q_2}\beta_{2k}^{(i)} + \sum_{k=1}^{q_3}\beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_r}\beta_{rk}^{(i)}\right) \tag{1.4}$$

where $i = 1,\cdots,r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.
We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1,\cdots,z_r) = 0(\,|z_1|^{\alpha_1'},\cdots,|z_r|^{\alpha_r'}\,)\,, max(\,|z_1|,\cdots,|z_r|\,) \rightarrow 0$$

$$I(z_1,\cdots,z_r)\ = 0(\,|z_1|^{\beta_r'},\cdots,|z_r|^{\beta_s'}\,)\,, min(\,|z_1|\,,\cdots,|z_r|\,) \rightarrow \infty$$

where $k = 1,\cdots,z : \alpha_k' = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1,\cdots,m_k$ and

$$\beta_k' = max[Re((a_j^{(k)}-1)/\alpha_j^{(k)})], j = 1,\cdots,n_k$$

The Srivastava-Daoust function is defined by (see [8]):

$$F_{\bar{C}:D^{(1)};\cdots;D^{(v)}}^{\bar{A}:B^{(1)};\cdots;B^{(v)}}\left(\begin{array}{c|c} \mathrm{z}_1 & [(\mathrm{a});\theta',\cdots,\theta^{(v)}] : [(b');\phi'];\cdots;[(b^{(v)});\phi^{(v)}] \\ \cdot & \cdot \\ \cdot & [(\mathrm{c});\psi',\cdots,\psi^{(v)}] : [(d');\delta'];\cdots;[(d^{(v)});\delta^{(v)}] \\ \mathrm{z}_v & \end{array}\right)$$

$$= \sum_{r_1,\cdots,r_v=0}^{\infty} A'\frac{z_1^{r_1}\cdots z_v^{r_v}}{r_1!\cdots r_v!} \tag{1.5}$$

where

$$A' = \frac{\prod_{j=1}^{\bar{A}}(a_j)_{r_1\theta_j'+\cdots+r_v\theta_j^{(v)}}\prod_{j=1}^{B^{(1)}}(b_j')_{r_1\phi_j'}\cdots\prod_{j=1}^{B^{(v)}}(b_j^{(v)})_{r_v\phi_j^{(v)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{r_1\psi_j'+\cdots+r_v\psi_j^{(v)}}\prod_{j=1}^{D^{(1)}}(d_j')_{r_1\delta_j'}\cdots\prod_{j=1}^{D^{(v)}}(d_j^{(v)})_{r_v\delta_j^{(v)}}} \tag{1.6}$$

The series given by (1.5) converges absolutely if

$$1 + \sum_{j=1}^{\bar{C}}\psi_j^{(i)} + \sum_{j=1}^{D^{(i)}}\delta_j^{(i)} - \sum_{j=1}^{\bar{A}}\theta_j^{(i)} - \sum_{j=1}^{B^{(i)}}\phi_j^{(i)} > 0; i = 1,\cdots,v \tag{1.7}$$

We shall note

$$U = p_2, q_2; p_3, q_3; \cdots ; p_{r-1}, q_{r-1}; V = 0, n_2; 0, n_3; \cdots ; 0, n_{s-1} \tag{1.8}$$

$$W = (p', q'); \cdots ; (p^{(r)}, q^{(r)}); X = (m', n'); \cdots ; (m^{(r)}, n^{(r)}) \tag{1.9}$$

$$A = (a_{2k}, \alpha'_{2k}, \alpha''_{2k})_{1,p_2}; \cdots ; (a_{(r-1)k}, \alpha'_{(r-1)k}, \alpha''_{(r-1)k}, \cdots , \alpha^{(r-1)}_{(r-1)k})_{1,p_{r-1}} \tag{1.10}$$

$$B = (b_{2k}, \beta'_{2k}, \beta''_{2k})_{1,q_2}; \cdots ; (b_{(r-1)k}, \beta'_{(r-1)k}, \beta''_{(r-1)k}, \cdots , \beta^{(r-1)}_{(r-1)k})_{1,q_{r-1}} \tag{1.11}$$

$$\mathbb{A} = (a_{rk}; \alpha'_{rk}, \alpha''_{rk}, \cdots , \alpha^{(r)}_{rk})_{1,p_r} :(a'_k, \alpha'_k)_{1,p'}; \cdots ; (a^{(r)}_k, \alpha^{(r)}_k)_{1,p^{(r)}} \tag{1.12}$$

$$\mathbb{B} = (b_{rk}; \beta'_{rk}, \beta''_{rk}, \cdots , \beta^{(r)}_{rk})_{1,q_r}: (b'_k, \beta'_k)_{1,p'}; \cdots ; (b^{(r)}_k, \beta^{(r)}_k)_{1,q^{(r)}} \tag{1.13}$$

$$B_1 = \frac{(-N_1)_{M_1K_1}}{K_1!} \cdots \frac{(-N_s)_{M_sK_s}}{K_s!} A[N_1, K_1; \cdots ; N_s, K_s] \tag{1.14}$$

$$F = F^{\bar{A}:B^{(1)};\cdots;B^{(v)}}_{\bar{C}:D^{(1)};\cdots;D^{(v)}} \tag{1.15}$$

2. Main integral

We shall use the following result in the subsequent discussion

$$\int_0^y x^{\rho-1}(y-x)^{\mu-1} J_0(\omega_i y) S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\left(\begin{array}{c} \mathrm{y}_1 x^{\rho_1}(y-x)^{\zeta_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{y}_s x^{\rho_s}(y-x)^{\zeta_s} \end{array}\right) F \left(\begin{array}{c} \mathrm{t}_1 x^{\sigma_1}(y-x)^{\xi_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{t}_v x^{\sigma_v}(y-x)^{\xi_v} \end{array}\right) I \left(\begin{array}{c} \mathrm{Z}_1 x^{h_1}(y-x)^{k_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{Z}_r x^{h_r}(y-x)^{k_r} \end{array}\right) \mathrm{d}x$$

$$= y^{\rho+\mu-1} \sum_{R=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_s=0}^{[N_s/M_s]} \sum_{m_1,\cdots,m_v=0}^{\infty} A_1 B_1 \frac{y_1^{K_1} \cdots y_s^{K_s} t_1^{m_1} \cdots t_v^{m_v}}{m_1! \cdots m_v!} y^{\sum_{i=1}^s K_i(\rho_i+\zeta_i)+\sum_{i=1}^u m_i(\sigma_i+\xi_i)} \frac{(-1)^R}{R!^2} \left(\frac{w_i y}{2}\right)^{2R}$$

$$I^{V;0,n_r+2:X}_{U;p_r+2,q_r+1:W}\left(\begin{array}{c} \mathrm{Z}_1 y^{h_1+k_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{Z}_r y^{h_r+k_r} \end{array}\middle| \begin{array}{c} \mathrm{A}\ ;\ (1\text{-}\rho - \sum_{i=1}^s K_i\rho_i - \sum_{i=1}^v m_i\sigma_i - 2R : h_1, \cdots , h_r), \\ \cdot \\ \cdot \\ \mathrm{B};\ .\ .\ . \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-}\mu - \sum_{i=1}^s K_i\zeta_i - \sum_{i=1}^v m_i\xi_i : k_1, \cdots , k_r), \mathbb{A} \\ \cdot \\ \cdot \\ (1\text{-}\rho - \mu - \sum_{i=1}^s K_i(\rho_i+\zeta_i) - \sum_{i=1}^v m_i(\sigma_i+\xi_i) - 2R : h_1+k_1, \cdots , h_r+k_r), \mathbb{B} \end{array}\right) \tag{2.1}$$

Provided that

$Re(\rho) > 0, min\{\rho_i, \zeta_i, \sigma_j, \xi_j, h_l, k_l\} > 0, i = 1, \cdots, s, j = 1, \cdots, v, l = 1, \cdots, r$

$|argZ_i| < \dfrac{1}{2}\Omega_i \pi$, where Ω_i is defined by (1.7)

$Re\left[\rho + \sum_{i=1}^{s} K_i\rho_i + \sum_{i=1}^{v} m_i\sigma_i + \sum_{i=1}^{r} h_i \min_{1\leqslant j\leqslant m^{(i)}} \dfrac{b_j^{(i)}}{\beta_j^{(i)}}\right] > 0$ and $Re\left[\mu + \sum_{i=1}^{s} K_i\zeta_i + \sum_{i=1}^{v} m_i\xi_i + \sum_{i=1}^{r} k_i \min_{1\leqslant j\leqslant m^{(i)}} \dfrac{b_j^{(i)}}{\beta_j^{(i)}}\right] > 0$

$1 + \sum_{j=1}^{C} \psi_j^{(i)} + \sum_{j=1}^{D^{(i)}} \delta_j^{(i)} - \sum_{j=1}^{A} \theta_j^{(i)} - \sum_{j=1}^{B^{(i)}} \phi_j^{(i)} > 0; i = 1, \cdots, v$

Proof

To prove (2.1), first expressing a class of multivariable polynomials defined by Srivastava [7] $S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[.]$ in serie with the help of (1.1), the Srivastava-Daoust function $F[.]$ in serie with the help of (1.5) and the Bessel function $J_0(w_i y)$ in serie and we interchange the order of summations and x-integral (which is permissible under the conditions stated). Expressing the I-function of r-variables in Mellin-Barnes contour integral and interchange the order of integrations which is justifiable due to absolute convergence of the integral involved in the process. Now collecting the power of x and $(y-x)$ and use the result ([3], (2.2.4), page 296, Eq.8) and interpreting the Mellin-Barnes contour integral in multivariable I-function, we obtain the desired result (2.1).

3. Physics model

The forced symmetrical vibrations of a circular membrane of radius a where z is the transverse displacement of the point whose polar corrdinates are (x,θ) of the membrane from the plane $z=0; c^2 = T/\sigma, T$ being the uniform tension and σ, the mass per unit area and $P(x,t)$ is the external force per unit area acting on the membrane normal to the plane $z=0$ producing vibrations. This problem satisfies the following heat equation, see (Sneddon [5], Eq (86), page 125).

$$\frac{\partial^2 z}{\partial x^2} + \frac{1}{x}\frac{\partial z}{\partial x} = \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2} - \frac{P(x,t)}{T} \tag{3.1}$$

We shall solve (3.1) under the following boundary conditions :

(i) initially , at $t=0$, $z=f(x)$ and $\dfrac{\partial z}{\partial x} = g(x)$ and

(ii) $z=0$ when $x=a$ for all $t>0$

From (I), it is clear that the membrane is set in motion from the position $z=f(x)$ with a velocity $\dfrac{\partial z}{\partial x} = g(x)$. We shall, in addition, suppose that the external force $P(x,t)$ producing vibrations is of general character given by

$$P(x,t) = F(x)G(t) \tag{3.2}$$

where $F(x)$ is a function of x alone and $G(t)$ is a function of t alone. We shall solve (3.1) by taking $f(x), g(x), F(x)$, $G(t)$ to be the generalized functions of several complex variables defined in the section 1. We shall use the result (2.1) in subsequent results.

4.Finite Hankel transform

Multiplying (3.1) throughout $xJ_0(\alpha_i x)$, integrating with respect to x from 0 to a and making use of the boundary conditions (ii), we find that the finite Hankel transform of order 0, see (Sneddon [5], page 127).

$$\bar{z}_i(\alpha_i,t)=\int_0^a xz(x,t)J_0(\alpha_i x)\mathrm{d}x \tag{4.1}$$

satisfies the ordinary linear differential equation

$$\left(\frac{d^2}{dt^2}+c^2\alpha_i\right)\bar{z}_i(\alpha_i,t)=\frac{1}{\sigma}\bar{P}(\alpha_i,t) \tag{4.2}$$

where α_i is a root of the transcendental equation

$$J_0(\alpha_i a)=0 \tag{4.3}$$

and $\bar{P}(\alpha_i,t)$ denotes the Hankel transform of $P(x,t)$.

5. Solution of the problem

Let $f(x)$ and $g(x)$ can be defined in terms of product of the multivariable I-function defined by Prasad [2], the Srivastava-Daoust function [8] and a class of multivariable polynomials [7] as follows :

$$f(x)=\left(\frac{x}{a}\right)^{\rho-2}\left(1-\frac{x}{a}\right)^{\mu-1}S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix}\mathrm{y}_1\left(\frac{x}{a}\right)^{\rho_1}\left(1-\frac{x}{a}\right)^{\zeta_1}\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ \mathrm{y}_s\left(\frac{x}{a}\right)^{\rho_s}\left(1-\frac{x}{a}\right)^{\zeta_s}\end{pmatrix}$$

$$F\begin{pmatrix}\mathrm{t}_1\left(\frac{x}{a}\right)^{\sigma_1}\left(1-\frac{x}{a}\right)^{\xi_1}\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ \mathrm{t}_v\left(\frac{x}{a}\right)^{\sigma_1}\left(1-\frac{x}{a}\right)^{\xi_1}\end{pmatrix}I\begin{pmatrix}\mathrm{z}_1\left(\frac{x}{a}\right)^{h_1}\left(1-\frac{x}{a}\right)^{k_1}\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ \mathrm{z}_r\left(\frac{x}{a}\right)^{h_r}\left(1-\frac{x}{a}\right)^{k_r}\end{pmatrix} \tag{5.1}$$

and

$$g(x)=\left(\frac{x}{a}\right)^{\rho'-2}\left(1-\frac{x}{a}\right)^{\mu'-1}S_{N'_1,\cdots,N'_s}^{M'_1,\cdots,M'_s}\begin{pmatrix}\mathrm{y}_1^*\left(\frac{x}{a}\right)^{\rho'_1}\left(1-\frac{x}{a}\right)^{\zeta'_1}\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ \mathrm{y}_s^*\left(\frac{x}{a}\right)^{\rho'_s}\left(1-\frac{x}{a}\right)^{\zeta'_s}\end{pmatrix}$$

$$F'\begin{pmatrix}\mathrm{t}_1^*\left(\frac{x}{a}\right)^{\sigma'_1}\left(1-\frac{x}{a}\right)^{\xi'_1}\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ \mathrm{t}_v^*\left(\frac{x}{a}\right)^{\sigma'_1}\left(1-\frac{x}{a}\right)^{\xi'_1}\end{pmatrix}I'\begin{pmatrix}\mathrm{z}_1^*\left(\frac{x}{a}\right)^{h'_1}\left(1-\frac{x}{a}\right)^{k'_1}\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ \mathrm{z}_r^*\left(\frac{x}{a}\right)^{h'_r}\left(1-\frac{x}{a}\right)^{k'_r}\end{pmatrix} \tag{5.2}$$

Similarly, $F(x)$ and $G(t)$ can be defined in terms of product of the multivariable I-function defined by Prasad [2], the Srivastava-Daoust function [8] and a class of multivariable polynomials [7] as follows :

$$F(x)=\left(\frac{x}{a}\right)^{\rho''-2}\left(1-\frac{x}{a}\right)^{\mu''-1}S_{N''_1,\cdots,N''_s}^{M''_1,\cdots,M''_s}\begin{pmatrix}\mathrm{Y}_1\left(\frac{x}{a}\right)^{\rho''_1}\left(1-\frac{x}{a}\right)^{\zeta''_1}\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ \mathrm{Y}_s\left(\frac{x}{a}\right)^{\rho''_s}\left(1-\frac{x}{a}\right)^{\zeta''_s}\end{pmatrix}$$

$$F''\begin{pmatrix} \mathrm{T}_1\left(\frac{x}{a}\right)^{\sigma_1''}\left(1-\frac{x}{a}\right)^{\xi_1''} \\ \cdots \\ \cdots \\ \mathrm{T}_v\left(\frac{x}{a}\right)^{\sigma_1''}\left(1-\frac{x}{a}\right)^{\xi_1''} \end{pmatrix} I''\begin{pmatrix} \mathrm{Z}_1\left(\frac{x}{a}\right)^{h_1''}\left(1-\frac{x}{a}\right)^{k_1''} \\ \cdots \\ \cdots \\ \mathrm{Z}_r\left(\frac{x}{a}\right)^{h_r''}\left(1-\frac{x}{a}\right)^{k_r''} \end{pmatrix} \tag{5.3}$$

and

$$G(t)=\left(\frac{t}{a'}\right)^{\rho'''-2}\left(1-\frac{t}{a'}\right)^{\mu'''-1} S^{M_1''',\cdots,M_s'''}_{N_1''',\cdots,N_s'''}\begin{pmatrix} \mathrm{Y}_1^*\left(\frac{t}{a'}\right)^{\rho_1'''}\left(1-\frac{t}{a'}\right)^{\zeta_1'''} \\ \cdots \\ \cdots \\ \mathrm{Y}_s^*\left(\frac{t}{a'}\right)^{\rho_s'''}\left(1-\frac{t}{a'}\right)^{\zeta_s'''} \end{pmatrix}$$

$$F'''\begin{pmatrix} \mathrm{T}_1^*\left(\frac{t}{a'}\right)^{\sigma_1'''}\left(1-\frac{t}{a'}\right)^{\xi_1'''} \\ \cdots \\ \cdots \\ \mathrm{T}_v^*\left(\frac{t}{a'}\right)^{\sigma_1'''}\left(1-\frac{t}{a'}\right)^{\xi_1'''} \end{pmatrix} I'''\begin{pmatrix} \mathrm{Z}_1^*\left(\frac{t}{a'}\right)^{h_1'''}\left(1-\frac{t}{a'}\right)^{k_1'''} \\ \cdots \\ \cdots \\ \mathrm{Z}_r^*\left(\frac{t}{a'}\right)^{h_r'''}\left(1-\frac{t}{a'}\right)^{k_r'''} \end{pmatrix} \tag{5.4}$$

where $a'=\dfrac{a}{c}$ and the parameters and integers are related in the same way as in (5.1). From (2.2), we have

$$\bar{P}(\alpha_i,t)=G(t)\sum_{N=0}^{\infty}\sum_{K_1''=0}^{[N_1''/M_1'']}\cdots\sum_{K_s''=0}^{[N_s''/M_s'']}\sum_{m_1'',\cdots,m_v''=0}^{\infty} A_1''B_1''\frac{Y_1^{K_1''}\cdots Y_s^{K_s''}T_1^{m_1''}\cdots T_v^{m_v''}}{m_1''!\cdots m_v''!}\frac{(-1)^N}{N!^2}\left(\frac{\alpha_i a}{2}\right)^{2N}$$

$$I^{V'';0,n_r''+2:X''}_{U'';p_r''+2,q_r''+1:W''}\left(\begin{array}{c} \mathrm{Z}_1 \\ \cdots \\ \cdots \\ \mathrm{Z}_r \end{array}\middle|\begin{array}{c} \mathrm{A}"\ ;\ (1\text{-}\rho''-\sum_{j=1}^{s}K_j''\rho_j''-\sum_{i=1}^{v}m_j''\sigma_j''-2N:h_1'',\cdots,h_r''), \\ \cdot \\ \cdot \\ \mathrm{B}";\ \cdots \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-}\mu''-\sum_{j=1}^{s}K_j''\zeta_j''-\sum_{j=1}^{v}m_j''\xi_j'':k_1'',\cdots,k_r''),\mathbb{A}'' \\ \cdot \\ \cdot \\ (1\text{-}\rho''-\mu''-\sum_{j=1}^{s}K_j''(\rho_j''+\zeta_j'')-\sum_{j=1}^{v}m_j''(\sigma_j''+\xi_j'')-2N:h_1''+k_1'',\cdots,h_r''+k_r''),\mathbb{B}'' \end{array}\right) \tag{5.5}$$

provided that

$Re(\rho'')>0, min\{\rho_i'',\zeta_i''\sigma_j'',\xi_j'',h_l'',k_l''\}>0, i=1,\cdots,s, j=1,\cdots,v, l=1,\cdots,r$

$|argZ_i|<\dfrac{1}{2}\Omega_i''\pi$

$Re\left[\rho''+\sum_{i=1}^{s}K_i\rho_i''+\sum_{i=1}^{v}m_i\sigma_i''+\sum_{i=1}^{r}h_i''\min_{1\leqslant j\leqslant m''^{(i)}}\dfrac{b_j''^{(i)}}{\beta_j''^{(i)}}\right]>0$ and

$Re\left[\mu''+\sum_{i=1}^{s}K_i\zeta_i''+\sum_{i=1}^{v}m_i\xi_i''+\sum_{i=1}^{r}k_i''\min_{1\leqslant j\leqslant m''^{(i)}}\dfrac{b_j''^{(i)}}{\beta_j''^{(i)}}\right]>0$

$$1+\sum_{j=1}^{C''}\psi_j''^{(i)}+\sum_{j=1}^{D''^{(i)}}\delta_j''^{(i)}-\sum_{j=1}^{A''}\theta_j''^{(i)}-\sum_{j=1}^{B''^{(i)}}\phi_j''^{(i)}>0; i=1,\cdots,v$$

From the boundary (ii), we find that the complementary function of (4.2) is found to be (Sneddon [5], page 129)

$$\bar{z}_i(\alpha_i,t)=\cos(c\alpha_j t)\int_0^a xf(x)J_0(\alpha_i x)\mathrm{d}x+\frac{\sin(c\alpha_j t)}{c\alpha_i}\int_0^a xg(x)J_0(\alpha_i x)\mathrm{d}x \tag{5.6}$$

Inverting (5.6) by (Sneddon [5], Eq.(44), page 83), we obtain finally for the displacement of the membrane

$$z(x,t)=\frac{2}{a^2}\sum_{i=1}^{\infty}\frac{J_0(\alpha_i x)}{\left[J_1(\alpha_i a)\right]^2}\cos(c\alpha_j t)\int_0^a xf(x)J_0(\alpha_i x)\mathrm{d}x+$$

$$\frac{2}{ca^2}\sum_{i=1}^{\infty}\frac{J_0(\alpha_i x)}{\left[J_1(\alpha_i a)\right]^2}\frac{\sin(c\alpha_j t)}{c\alpha_i}\int_0^a xg(x)J_0(\alpha_i x)\mathrm{d}x+\frac{1}{c\alpha_i}\sum_{i,N=0}^{\infty}\sum_{K_1''=0}^{[N_1''/M_1'']}\cdots\sum_{K_s''=0}^{[N_s''/M_s'']}\sum_{m_1'',\cdots,m_v''=0}^{\infty}$$

$$A_1''B_1''\frac{Y_1^{K_1''}\cdots Y_s^{K_s''}T_1^{m_1''}\cdots T_v^{m_v''}}{m_1''!\cdots m_v''!}\frac{(-1)^N}{N!^2}\left(\frac{\alpha_i a}{2}\right)^{2N}\frac{J_0(\alpha_i x)}{\left[J_1(\alpha_i a)\right]^2}$$

$$I_{U'';p_r''+2,q_r''+1:W''}^{V'';0,n_r''+2:X''}\left(\begin{array}{c}\mathrm{Z}_1\\ \cdot\,\cdot\,\cdot\\ \cdot\,\cdot\,\cdot\\ \mathrm{Z}_r\end{array}\middle|\begin{array}{l}\mathrm{A''};\ (1\text{-}\rho''-\sum_{j=1}^{s}K_j''\rho_j''-\sum_{j=1}^{v}m_j''\sigma_j''-2N:h_1'',\cdots,h_r''),\\ \cdot\\ \cdot\\ \mathrm{B''};\ \cdot\,\cdot\,\cdot\end{array}\right.$$

$$\left.\begin{array}{c}(1\text{-}\mu''-\sum_{j=1}^{s}K_j''\zeta_j''-\sum_{j=1}^{v}m_j''\xi_j'':k_1'',\cdots,k_r''),\mathbb{A}''\\ \cdot\\ \cdot\\ (1\text{-}\rho''-\mu''-\sum_{j=1}^{s}K_j''(\rho_j''+\zeta_j'')-\sum_{j=1}^{v}m_j''(\sigma_j''+\xi_j'')-2N:h_1''+k_1'',\cdots,h_r''+k_r''),\mathbb{B}''\end{array}\right)\int_0^t G(u)\sin[c\alpha_i(t-u)]\mathrm{d}u \tag{5.7}$$

The sum is taken over all the positive roots of (4.3), under the same conditions that (5.5).

Subtituting the values of the integrals in (5.6) from (2.1) and evaluate the integral $\int_0^t G(u)\sin[c\alpha_i(t-u)]\mathrm{d}u$ by

expanding $\sin\left[c\alpha_i(t-u)\right]$ in power of $\left[c\alpha_i(t-u)\right]$ with help of (2.1), we obtain the complete solution of (3.1) as

$$z(x,t)=a\sum_{N,i=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{m_1,\cdots,m_v=0}^{\infty}A_1B_1\frac{y_1^{K_1}\cdots y_s^{K_s}t_1^{m_1}\cdots t_v^{m_v}}{m_1!\cdots m_v!}\frac{(-1)^N}{N!^2}\left(\frac{\alpha_i a}{2}\right)^{2N}\frac{J_0(\alpha_i x)}{\left[J_1(\alpha_i a)\right]^2}$$

$$\cos(c\alpha_i t)\, I_{U;p_r+2,q_r+1:W}^{V;0,n_r+2:X}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A}\ ;\ (1\text{-}\rho-\sum_{j=1}^{s}K_j\rho_j-\sum_{j=1}^{v}m_j\sigma_i-2N:h_1,\cdots,h_r), \\ \cdots & \cdot \\ \cdots & \cdot \\ \mathrm{z}_r & \mathrm{B};\ \cdots \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-}\mu-\sum_{j=1}^{s}K_j\zeta_j-\sum_{j=1}^{v}m_j\xi_j:k_1,\cdots,k_r),\mathbb{A} \\ \cdot \\ \cdot \\ (1\text{-}\rho-\mu-\sum_{j=1}^{s}K_j(\rho_j+\zeta_j)-\sum_{j=1}^{v}m_j(\sigma_j+\xi_j)-2N:h_1+k_1,\cdots,h_r+k_r),\mathbb{B} \end{array}\right) \quad +$$

$$\frac{2a}{c}\sum_{N,i=0}^{\infty}\ \sum_{K_1''=0}^{[N_1''/M_1'']}\cdots\sum_{K_s''=0}^{[N_s''/M_s'']}\ \sum_{m_1',\cdots,m_v'=0}^{\infty}A_1'B_1'\frac{y_1^{*K_1'}\cdots y_s^{*K_s'}t_1^{*m_1'}\cdots t_v^{*m_v'}}{m_1'!\cdots m_v'!}\ \frac{(-1)^N}{N!^2}\left(\frac{\alpha_i a}{2}\right)^{2N}\frac{J_0(\alpha_i x)}{\left[J_1(\alpha_i a)\right]^2}\cos(c\alpha_i t)$$

$$I_{U';p_r'+2,q_r'+1:W'}^{V';0,n_r'+2:X'}\left(\begin{array}{c|c} \mathrm{z}_1^* & \mathrm{A'}\ ;\ (1\text{-}\rho'-\sum_{j=1}^{s}K_j'\rho_j'-\sum_{j=1}^{v}m_j'\sigma_j'-2N:h_1',\cdots,h_r'), \\ \cdots & \cdot \\ \cdots & \cdot \\ \mathrm{z}_r^* & \mathrm{B'};\ \cdots \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-}\mu'-\sum_{j=1}^{s}K_j'\zeta_j'-\sum_{j=1}^{v}m_j'\xi_j':k_1',\cdots,k_r'),\mathbb{A}' \\ \cdot \\ \cdot \\ (1\text{-}\rho'-\mu'-\sum_{j=1}^{s}K_j'(\rho_j'+\zeta_j')-\sum_{j=1}^{v}m_j'(\sigma_j'+\xi_j')-2N:h_1'+k_1',\cdots,h_r'+k_r'),\mathbb{B}' \end{array}\right) \quad +$$

$$\frac{2a}{c}\sum_{N,R,i=0}^{\infty}\ \sum_{K_1''=0}^{[N_1''/M_1'']}\cdots\sum_{K_s''=0}^{[N_s''/M_s'']}\ \sum_{m_1'',\cdots,m_v''=0}^{\infty}\ \sum_{K_1'''=0}^{[N_1'''/M_1''']}\cdots\sum_{K_s'''=0}^{[N_s'''/M_s''']}\ \sum_{m_1''',\cdots,m_v'''=0}^{\infty}A_1''B_1''A_1'''B_1'''$$

$$\frac{Y_1^{K_1''}\cdots Y_s^{K_s''}T_1^{m_1''}\cdots T_v^{m_v''}}{m_1''!\cdots m_v''!}\,\frac{Y_1^{*K_1'''}\cdots Y_s^{*K_s'''}T_1^{*m_1'''}\cdots T_v^{*m_v'''}}{m_1'''!\cdots m_v'''!}\,\frac{(-)^{R+N+1}}{N!^2(2R-1)!}\left(\frac{1}{2}\alpha_i a\right)^{2N}(c\alpha_i)^{2R-2}\,\frac{J_0(\alpha_i x)}{\left[J_1(\alpha_i a)\right]^2}$$

$$I_{U'';p_r''+2,q_r''+1:W''}^{V'';0,n_r''+2:X''}\left(\begin{array}{c|c} \mathrm{Z}_1 & \mathrm{A''}\ ;\ (1\text{-}\rho''-\sum_{j=1}^{s}K_j''\rho_j''-\sum_{j=1}^{v}m_j''\sigma_j''-2N:h_1'',\cdots,h_r''), \\ \cdots & \cdot \\ \cdots & \cdot \\ \mathrm{Z}_r & \mathrm{B''};\ \cdots \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-}\mu''-\sum_{j=1}^{s}K_j''\zeta_j''-\sum_{j=1}^{v}m_j''\xi_j'':k_1'',\cdots,k_r''),\mathbb{A}'' \\ \cdot \\ \cdot \\ (1\text{-}\rho''-\mu''-\sum_{j=1}^{s}K_j''(\rho_j''+\zeta_j'')-\sum_{j=1}^{v}m_j''(\sigma_j''+\xi_j'')-2N:h_1''+k_1'',\cdots,h_r''+k_r''),\mathbb{B}'' \end{array}\right)$$

$$I^{V''';0,n_r'''+2:X'''}_{U''';p_r'''+2,q_r'''+1:W'''}\left(\begin{array}{c|c} Z_1^* & A"' ; (1-\rho''' - \sum_{j=1}^s K_j'''\rho_j''' - \sum_{j=1}^v m_j'''\sigma_j''' - 2N : h_1''', \cdots, h_r'''), \\ \cdots & \cdot \\ \cdots & \cdot \\ Z_r^* & B"'; \cdots \end{array}\right.$$

$$\left.\begin{array}{c} (1-\mu''' - \sum_{j=1}^s K_j'''\zeta_j''' - \sum_{j=1}^v m_j'''\xi_j''' : k_1''', \cdots, k_r'''), \mathbb{A}''' \\ \cdot \\ \cdot \\ (1-\rho''' - \mu''' - \sum_{j=1}^s K_j'''(\rho_j''' + \zeta_j''') - \sum_{j=1}^v m_j'''(\sigma_j''' + \xi_j''') - 2N : h_1''' + k_1''', \cdots, h_r''' + k_r'''), \mathbb{B}''' \end{array}\right) \tag{5.8}$$

where $a' = a/c$. The initial conditions $z(x,o) = f(x)$ and $\left(\dfrac{\partial z(x,t)}{\partial t}\right)_{t=0} = g(x)$ are satisfied due to the inversion theorem 30 (Sneddon [5], page 83).

6. Particular case

When the membrane is at rest in its equilibrium position at $t = 0$, then $f(x) = g(x) = 0$ and hence from (4.8), the solution of (3.1) is found to be

$$z(x,t) = \frac{2a}{c}\sum_{N,R,i=0}^{\infty}\sum_{K_1''=0}^{[N_1''/M_1'']}\cdots\sum_{K_s''=0}^{[N_s''/M_s'']}\sum_{m_1'',\cdots,m_v''=0}^{\infty}\sum_{K_1'''=0}^{[N_1'''/M_1''']}\cdots\sum_{K_s'''=0}^{[N_s'''/M_s''']}\sum_{m_1''',\cdots,m_v'''=0}^{\infty} A_1''B_1''A_1'''B_1'''$$

$$\frac{Y_1^{K_1''}\cdots Y_s^{K_s''}T_1^{m_1''}\cdots T_v^{m_v''}}{m_1''!\cdots m_v''!}\frac{Y_1^{*K_1'''}\cdots Y_s^{*K_s'''}T_1^{*m_1'''}\cdots T_v^{*m_v'''}}{m_1'''!\cdots m_v'''!}\frac{(-)^{R+N+1}}{N!^2(2R-1)!}\left(\frac{1}{2}\alpha_i a\right)^{2N}(c\alpha_i)^{2R-2}\frac{J_0(\alpha_i x)}{\left[J_1(\alpha_i a)\right]^2}$$

$$I^{V'';0,n_r''+2:X''}_{U'';p_r''+2,q_r''+1:W''}\left(\begin{array}{c|c} Z_1 & A" ; (1-\rho'' - \sum_{j=1}^s K_j''\rho_j'' - \sum_{j=1}^v m_j''\sigma_j'' - 2N : h_1'', \cdots, h_r''), \\ \cdots & \cdot \\ \cdots & \cdot \\ Z_r & B"; \cdots \end{array}\right.$$

$$\left.\begin{array}{c} (1-\mu'' - \sum_{j=1}^s K_j''\zeta_j'' - \sum_{j=1}^v m_j''\xi_j'' : k_1'', \cdots, k_r''), \mathbb{A}'' \\ \cdot \\ \cdot \\ (1-\rho'' - \mu'' - \sum_{j=1}^s K_j''(\rho_j'' + \zeta_j'') - \sum_{j=1}^v m_j''(\sigma_j'' + \xi_j'') - 2N : h_1'' + k_1'', \cdots, h_r'' + k_r''), \mathbb{B}'' \end{array}\right)$$

$$I^{V''';0,n_r'''+2:X'''}_{U''';p_r'''+2,q_r'''+1:W'''}\left(\begin{array}{c|c} Z_1^* & A"' ; (1-\rho''' - \sum_{j=1}^s K_j'''\rho_j''' - \sum_{j=1}^v m_j'''\sigma_j''' - 2N : h_1''', \cdots, h_r'''), \\ \cdots & \cdot \\ \cdots & \cdot \\ Z_r^* & B"'; \cdots \end{array}\right.$$

$$\left.\begin{matrix}(1\text{-}\mu''' - \sum_{j=1}^{s} K_j'''\zeta_j''' - \sum_{j=1}^{v} m_j'''\xi_j''' : k_1''', \cdots, k_r'''), \mathbb{A}''' \\ \cdot \\ \cdot \\ (1\text{-}\rho''' - \mu''' - \sum_{j=1}^{s} K_j'''(\rho_j''' + \zeta_j''') - \sum_{j=1}^{v} m_j'''(\sigma_j''' + \xi_j''') - 2N : h_1''' + k_1''', \cdots, h_r''' + k_r'''), \mathbb{B}'''\end{matrix}\right) \tag{6.1}$$

The conditions appropriate to (5.3) and (5.4) are satisfied.

7. Multivariable H-function

If $U = V = A = B = U' = V' = A' = B' = U'' = V'' = A'' = B'' = U''' = V''' = A''' = B''' = 0$, the I-functions of several variables defined above reduce to multivariable H-functions defined by Srivastava et al [9,10] and we have

$$z(x,t) = a \sum_{N,i=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_s=0}^{[N_s/M_s]} \sum_{m_1,\cdots,m_v=0}^{\infty} A_1 B_1 \frac{y_1^{K_1} \cdots y_s^{K_s} t_1^{m_1} \cdots t_v^{m_v}}{m_1! \cdots m_v!} \frac{(-1)^N}{N!^2} \left(\frac{\alpha_i a}{2}\right)^{2N} \frac{J_0(\alpha_i x)}{\left[J_1(\alpha_i a)\right]^2}$$

$$\cos(c\alpha_i t)\, H^{0,n_r+2:X}_{p_r+2,q_r+1:W}\left(\begin{matrix}\mathrm{z}_1 \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{z}_r\end{matrix}\left|\begin{matrix}(1\text{-}\rho - \sum_{j=1}^{s} K_j\rho_j - \sum_{j=1}^{v} m_j\sigma_i - 2N : h_1, \cdots, h_r), \\ \cdot \\ \cdot \\ \cdot\cdot\cdot\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(1\text{-}\mu - \sum_{j=1}^{s} K_j\zeta_j - \sum_{j=1}^{v} m_j\xi_j : k_1, \cdots, k_r), \mathbb{A} \\ \cdot \\ \cdot \\ (1\text{-}\rho - \mu - \sum_{j=1}^{s} K_j(\rho_j + \zeta_j) - \sum_{j=1}^{v} m_j(\sigma_j + \xi_j) - 2N : h_1 + k_1, \cdots, h_r + k_r), \mathbb{B}\end{matrix}\right) +$$

$$\frac{2a}{c} \sum_{N,i=0}^{\infty} \sum_{K_1'=0}^{[N_1'/M_1']} \cdots \sum_{K_s'=0}^{[N_s'/M_s']} \sum_{m_1',\cdots,m_v'=0}^{\infty} A_1' B_1' \frac{y_1^{*K_1'} \cdots y_s^{*K_s'} t_1^{*m_1'} \cdots t_v^{*m_v'}}{m_1'! \cdots m_v'!} \frac{(-1)^N}{N!^2} \left(\frac{\alpha_i a}{2}\right)^{2N} \frac{J_0(\alpha_i x)}{\left[J_1(\alpha_i a)\right]^2} \cos(c\alpha_i t)$$

$$H^{;0,n_r'+2:X'}_{p_r'+2,q_r'+1:W'}\left(\begin{matrix}\mathrm{z}_1^* \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{z}_r^*\end{matrix}\left|\begin{matrix}\text{A'} ;\ (1\text{-}\rho' - \sum_{j=1}^{s} K_j'\rho_j' - \sum_{j=1}^{v} m_j'\sigma_j' - 2N : h_1', \cdots, h_r'), \\ \cdot \\ \cdot \\ \text{B'};\ \cdot\cdot\cdot\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(1\text{-}\mu' - \sum_{j=1}^{s} K_j'\zeta_j' - \sum_{j=1}^{v} m_j'\xi_j' : k_1', \cdots, k_r'), \mathbb{A}' \\ \cdot \\ \cdot \\ (1\text{-}\rho' - \mu' - \sum_{j=1}^{s} K_j'(\rho_j' + \zeta_j') - \sum_{j=1}^{v} m_j'(\sigma_j' + \xi_j') - 2N : h_1' + k_1', \cdots, h_r' + k_r'), \mathbb{B}'\end{matrix}\right) +$$

$$\frac{2a}{c} \sum_{N,R,i=0}^{\infty} \sum_{K_1''=0}^{[N_1''/M_1'']} \cdots \sum_{K_s''=0}^{[N_s''/M_s'']} \sum_{m_1'',\cdots,m_v''=0}^{\infty} \sum_{K_1'''=0}^{[N_1'''/M_1''']} \cdots \sum_{K_s'''=0}^{[N_s'''/M_s''']} \sum_{m_1''',\cdots,m_v'''=0}^{\infty} A_1'' B_1'' A_1''' B_1'''$$

$$\frac{Y_1^{K_1''}\cdots Y_s^{K_s''}T_1^{m_1''}\cdots T_v^{m_v''}}{m_1''!\cdots m_v''!}\frac{Y_1^{*K_1'''}\cdots Y_s^{*K_s'''}T_1^{*m_1'''}\cdots T_v^{*m_v'''}}{m_1'''!\cdots m_v'''!}\frac{(-)^{R+N+1}}{N!^2(2R-1)!}\left(\frac{1}{2}\alpha_i a\right)^{2N}(c\alpha_i)^{2R-2}\frac{J_0(\alpha_i x)}{\left[J_1(\alpha_i a)\right]^2}$$

$$H_{p_r''+2,q_r''+1:W''}^{;0,n_r''+2:X''}\left(\begin{array}{c}\mathrm{Z}_1\\ \cdots\\ \cdots\\ \mathrm{Z}_r\end{array}\middle|\begin{array}{c}(1\text{-}\rho''-\sum_{j=1}^{s}K_j''\rho_j''-\sum_{j=1}^{v}m_j''\sigma_j''-2N:h_1'',\cdots,h_r''),\\ \cdot\\ \cdot\\ \cdots\end{array}\right.$$

$$\left.\begin{array}{c}(1\text{-}\mu''-\sum_{j=1}^{s}K_j''\zeta_j''-\sum_{j=1}^{v}m_j''\xi_j'':k_1'',\cdots,k_r''),\mathbb{A}''\\ \cdot\\ \cdot\\ (1\text{-}\rho''-\mu''-\sum_{j=1}^{s}K_j''(\rho_j''+\zeta_j'')-\sum_{j=1}^{v}m_j''(\sigma_j''+\xi_j'')-2N:h_1''+k_1'',\cdots,h_r''+k_r''),\mathbb{B}''\end{array}\right)$$

$$H_{p_r'''+2,q_r'''+1:W'''}^{0,n_r'''+2:X'''}\left(\begin{array}{c}\mathrm{Z}_1^*\\ \cdots\\ \cdots\\ \mathrm{Z}_r^*\end{array}\middle|\begin{array}{c}(1\text{-}\rho'''-\sum_{j=1}^{s}K_j'''\rho_j'''-\sum_{j=1}^{v}m_j'''\sigma_j'''-2N:h_1''',\cdots,h_r'''),\\ \cdot\\ \cdot\\ \cdots\end{array}\right.$$

$$\left.\begin{array}{c}(1\text{-}\mu'''-\sum_{j=1}^{s}K_j'''\zeta_j'''-\sum_{j=1}^{v}m_j'''\xi_j''':k_1''',\cdots,k_r'''),\mathbb{A}'''\\ \cdot\\ \cdot\\ (1\text{-}\rho'''-\mu'''-\sum_{j=1}^{s}K_j'''(\rho_j'''+\zeta_j''')-\sum_{j=1}^{v}m_j'''(\sigma_j'''+\xi_j''')-2N:h_1'''+k_1''',\cdots,h_r'''+k_r'''),\mathbb{B}'''\end{array}\right) \quad (6.1)$$

unser the same conditions that (5.8) with :

$U=V=A=B=U'=V'=A'=B'=U''=V''=A''=B''=U'''=V'''=A'''=B'''=0$

When the membrane is at rest in its equilibrium position at $t=0$, then $f(x)=g(x)=0$ and we have

$$z(x,t)=\frac{2t}{c}\sum_{N,R,i=0}^{\infty}\sum_{K_1''=0}^{[N_1''/M_1'']}\cdots\sum_{K_s''=0}^{[N_s''/M_s'']}\sum_{m_1'',\cdots,m_v''=0}^{\infty}\sum_{K_1'''=0}^{[N_1'''/M_1''']}\cdots\sum_{K_s'''=0}^{[N_s'''/M_s''']}\sum_{m_1''',\cdots,m_v'''=0}^{\infty}A_1''B_1''A_1'''B_1'''$$

$$\frac{Y_1^{K_1''}\cdots Y_s^{K_s''}T_1^{m_1''}\cdots T_v^{m_v''}}{m_1''!\cdots m_v''!}\frac{Y_1^{*K_1'''}\cdots Y_s^{*K_s'''}T_1^{*m_1'''}\cdots T_v^{*m_v'''}}{m_1'''!\cdots m_v'''!}\frac{(-)^{R+N+1}}{N!^2(2R-1)!}\left(\frac{1}{2}\alpha_i a\right)^{2N}(c\alpha_i)^{2R-2}\frac{J_0(\alpha_i x)}{\left[J_1(\alpha_i a)\right]^2}$$

$$H_{p_r''+2,q_r''+1:W''}^{;0,n_r''+2:X''}\left(\begin{array}{c}\mathrm{Z}_1\\ \cdots\\ \cdots\\ \mathrm{Z}_r\end{array}\middle|\begin{array}{c}(1\text{-}\rho''-\sum_{j=1}^{s}K_j''\rho_j''-\sum_{j=1}^{v}m_j''\sigma_j''-2N:h_1'',\cdots,h_r''),\\ \cdot\\ \cdot\\ \cdots\end{array}\right.$$

$$\left.\begin{matrix} (1\text{-}\mu'' - \sum_{j=1}^{s} K_j''\zeta_j'' - \sum_{j=1}^{v} m_j''\xi_j'' : k_1'', \cdots, k_r''), \mathbb{A}'' \\ \cdot \\ \cdot \\ (1\text{-}\rho'' - \mu'' - \sum_{j=1}^{s} K_j''(\rho_j'' + \zeta_j'') - \sum_{j=1}^{v} m_j''(\sigma_j'' + \xi_j'') - 2N : h_1'' + k_1'', \cdots, h_r'' + k_r''), \mathbb{B}'' \end{matrix}\right)$$

$$H_{p_r'''+2,q_r'''+1:W'''}^{0,n_r'''+2:X'''}\left(\begin{matrix} Z_1^* \\ \cdots \\ \cdots \\ Z_r^* \end{matrix}\middle|\begin{matrix} (1\text{-}\rho''' - \sum_{j=1}^{s} K_j'''\rho_j''' - \sum_{j=1}^{v} m_j'''\sigma_j''' - 2N : h_1''', \cdots, h_r'''), \\ \cdot \\ \cdot \\ \cdots \end{matrix}\right.$$

$$\left.\begin{matrix} (1\text{-}\mu''' - \sum_{j=1}^{s} K_j'''\zeta_j''' - \sum_{j=1}^{v} m_j'''\xi_j''' : k_1''', \cdots, k_r'''), \mathbb{A}''' \\ \cdot \\ \cdot \\ (1\text{-}\rho''' - \mu''' - \sum_{j=1}^{s} K_j'''(\rho_j''' + \zeta_j''') - \sum_{j=1}^{v} m_j'''(\sigma_j''' + \xi_j''') - 2N : h_1''' + k_1''', \cdots, h_r''' + k_r'''), \mathbb{B}''' \end{matrix}\right) \quad (6.2)$$

Remark

If the class of multivariable polynomials and the Srivastava-Daoust function vanish, we obtain the result of Singh [4].

7. Class of polynomials of one variable

The class of multivariable polynomials defined by Srivastava [7] reduces in the class of polynomials defined by Srivastava [6], we obtain :

$$z(x,t) = a \sum_{N,i=0}^{\infty} \sum_{K=0}^{[n/m]} \sum_{m_1,\cdots,m_v=0}^{\infty} A_1 B_2 \frac{y^K t_1^{m_1} \cdots t_v^{m_v}}{m_1! \cdots m_v!} \frac{(-1)^N}{N!^2} \left(\frac{\alpha_i a}{2}\right)^{2N} \frac{J_0(\alpha_i x)}{\left[J_1(\alpha_i a)\right]^2} \cos(c\alpha_i t)$$

$$I_{U;p_r+2,q_r+1:W}^{V;0,n_r+2:X}\left(\begin{matrix} z_1 \\ \cdots \\ \cdots \\ z_r \end{matrix}\middle|\begin{matrix} \mathrm{A}\ ;\ (1\text{-}\rho - K\epsilon - \sum_{j=1}^{v} m_j\sigma_i - 2N : h_1, \cdots, h_r), \\ \cdot \\ \cdot \\ \mathrm{B};\ \cdots \end{matrix}\right.$$

$$\left.\begin{matrix} (1\text{-}\mu - K\upsilon - \sum_{j=1}^{v} m_j\xi_j : k_1, \cdots, k_r), \mathbb{A} \\ \cdot \\ \cdot \\ (1\text{-}\rho - \mu - K(\epsilon + \upsilon) - \sum_{j=1}^{v} m_j(\sigma_j + \xi_j) - 2N : h_1 + k_1, \cdots, h_r + k_r), \mathbb{B} \end{matrix}\right) \quad +$$

$$\frac{2a}{c} \sum_{N,i=0}^{\infty} \sum_{K'=0}^{[n'/m']} \sum_{m_1',\cdots,m_v'=0}^{\infty} A_1' B_2' \frac{y^{*K'} t_1^{*m_1'} \cdots t_v^{*m_v'}}{m_1'! \cdots m_v'!} \frac{(-1)^N}{N!^2} \left(\frac{\alpha_i a}{2}\right)^{2N} \frac{J_0(\alpha_i x)}{\left[J_1(\alpha_i a)\right]^2} \cos(c\alpha_i t)$$

$$I_{U';p_r'+2,q_r'+1:W'}^{V';0,n_r'+2:X'}\left(\begin{matrix} z_1^* \\ \cdots \\ \cdots \\ z_r^* \end{matrix}\middle|\begin{matrix} \mathrm{A'}\ ;\ (1\text{-}\rho' - K'\epsilon' - \sum_{j=1}^{v} m_j'\sigma_j' - 2N : h_1', \cdots, h_r'), \\ \cdot \\ \cdot \\ \mathrm{B'};\ \cdots \end{matrix}\right.$$

$$
\left.\begin{array}{c}
(1\text{-}\mu' - K'\upsilon' - \sum_{j=1}^{v} m'_j\xi'_j : k'_1, \cdots, k'_r), \mathbb{A}' \\
. \\
. \\
(1\text{-}\rho' - \mu' - K(\epsilon' + \upsilon') - \sum_{j=1}^{v} m'_j(\sigma'_j + \xi'_j) - 2N : h'_1 + k'_1, \cdots, h'_r + k'_r), \mathbb{B}'
\end{array}\right) +
$$

$$
\frac{2at}{c} \sum_{N,R,i=0}^{\infty} \sum_{K''=0}^{[n''/m'']} \sum_{m''_1,\cdots,m''_v=0}^{\infty} \sum_{K'''=0}^{[n'''/m''']} \sum_{m'''_1,\cdots,m'''_v=0}^{\infty} A''_1 B''_2 A'''_1 B'''_2
$$

$$
\frac{Y^{K''} T_1^{m''_1} \cdots T_v^{m''_v}}{m''_1! \cdots m''_v!} \frac{Y^{*K'''} T_1^{*m'''_1} \cdots T_v^{*m'''_v}}{m'''_1! \cdots m'''_v!} \frac{(-)^{R+N+1}}{N!^2(2R-1)!} \left(\frac{1}{2}\alpha_i a\right)^{2N} (c\alpha_i)^{2R-2} \frac{J_0(\alpha_i x)}{\left[J_1(\alpha_i a)\right]^2}
$$

$$
I_{U'';p''_r+2,q''_r+1:W''}^{V'';0,n''_r+2:X''} \left(\begin{array}{c} \mathrm{Z}_1 \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{Z}_r \end{array} \right| \begin{array}{c} \mathrm{A}'' ; (1\text{-}\rho'' - K''\epsilon'' - \sum_{j=1}^{v} m''_j\sigma''_j - 2N : h''_1, \cdots, h''_r), \\ . \\ . \\ \mathrm{B}''; \ldots \end{array}
$$

$$
\left.\begin{array}{c}
(1\text{-}\mu'' - k''\upsilon'' - \sum_{j=1}^{v} m''_j\xi''_j : k''_1, \cdots, k''_r), \mathbb{A}'' \\
. \\
. \\
(1\text{-}\rho'' - \mu'' - K''(\epsilon'' + \upsilon'') - \sum_{j=1}^{v} m''_j(\sigma''_j + \xi''_j) - 2N : h''_1 + k''_1, \cdots, h''_r + k''_r), \mathbb{B}''
\end{array}\right)
$$

$$
I_{U''';p'''_r+2,q'''_r+1:W'''}^{V''';0,n'''_r+2:X'''} \left(\begin{array}{c} \mathrm{Z}^*_1 \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{Z}^*_r \end{array} \right| \begin{array}{c} \mathrm{A}''' ; (1\text{-}\rho''' - K'''\epsilon''' - \sum_{j=1}^{v} m'''_j\sigma'''_j - 2N : h'''_1, \cdots, h'''_r), \\ . \\ . \\ \mathrm{B}'''; \ldots \end{array}
$$

$$
\left.\begin{array}{c}
(1\text{-}\mu''' - K'''\upsilon''' - \sum_{j=1}^{v} m'''_j\xi'''_j : k'''_1, \cdots, k'''_r), \mathbb{A}''' \\
. \\
. \\
(1\text{-}\rho''' - \mu''' - K'''(\epsilon''' + \upsilon''') - \sum_{j=1}^{v} m'''_j(\sigma'''_j + \xi'''_j) - 2N : h'''_1 + k'''_1, \cdots, h'''_r + k'''_r), \mathbb{B}'''
\end{array}\right) \tag{7.1}
$$

When the membrane is at rest in its equilibrium position at $t = 0$, then $f(x) = g(x) = 0$

$$
z(x,t) = \frac{2at}{c} \sum_{N,R,i=0}^{\infty} \sum_{K''=0}^{[n''/m'']} \sum_{m''_1,\cdots,m''_v=0}^{\infty} \sum_{K'''=0}^{[n'''/m''']} \sum_{m'''_1,\cdots,m'''_v=0}^{\infty} A''_1 B''_2 A'''_1 B'''_2
$$

$$\frac{Y^{K''}T_1^{m_1''}\cdots T_v^{m_v''}}{m_1''!\cdots m_v''!}\frac{Y^{*K'''}T_1^{*m_1'''}\cdots T_v^{*m_v'''}}{m_1'''!\cdots m_v'''!}\frac{(-)^{R+N+1}}{N!^2(2R-1)!}\left(\frac{1}{2}\alpha_i a\right)^{2N}(c\alpha_i)^{2R-2}\frac{J_0(\alpha_i x)}{\left[J_1(\alpha_i a)\right]^2}$$

$$I_{U'';p_r''+2,q_r''+1:W''}^{V'';0,n_r''+2:X''}\left(\begin{array}{c|c} \mathrm{Z}_1 & \mathrm{A}'' ;\ (1\text{-}\rho''-K''\epsilon''-\sum_{j=1}^{v}m_j''\sigma_j''-2N:h_1'',\cdots,h_r''), \\ \cdot\cdot\cdot & \cdot \\ \cdot\cdot\cdot & \cdot \\ \mathrm{Z}_r & \mathrm{B}'';\ \cdot\cdot\cdot \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-}\mu''-k''\upsilon''-\sum_{j=1}^{v}m_j''\xi_j'':k_1'',\cdots,k_r''),\mathbb{A}'' \\ \cdot \\ \cdot \\ (1\text{-}\rho''-\mu''-K''(\epsilon''+\upsilon'')-\sum_{j=1}^{v}m_j''(\sigma_j''+\xi_j'')-2N:h_1''+k_1'',\cdots,h_r''+k_r''),\mathbb{B}'' \end{array}\right)$$

$$I_{U''';p_r'''+2,q_r'''+1:W'''}^{V''';0,n_r'''+2:X'''}\left(\begin{array}{c|c} \mathrm{Z}_1^* & \mathrm{A}''' ;\ (1\text{-}\rho'''-K'''\epsilon'''-\sum_{j=1}^{v}m_j'''\sigma_j'''-2N:h_1''',\cdots,h_r'''), \\ \cdot\cdot\cdot & \cdot \\ \cdot\cdot\cdot & \cdot \\ \mathrm{Z}_r^* & \mathrm{B}''';\ \cdot\cdot\cdot \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-}\mu'''-K'''\upsilon'''-\sum_{j=1}^{v}m_j'''\xi_j''':k_1''',\cdots,k_r'''),\mathbb{A}''' \\ \cdot \\ \cdot \\ (1\text{-}\rho'''-\mu'''-K'''(\epsilon'''+\upsilon''')-\sum_{j=1}^{v}m_j'''(\sigma_j'''+\xi_j''')-2N:h_1'''+k_1''',\cdots,h_r'''+k_r'''),\mathbb{B}''' \end{array}\right) \tag{7.2}$$

6. Conclusion

The importance of our results lies in its manifold generality. In view of the generality of the multivariable I-function and the Srivastava-Daoust function, on specializing the various parameters and variables, from our results, we can obtain a large number of special functions of one and several variables. Secondly, on suitably specializing the parameters of the classes of multivariable polynomials, our results can be reduced to a large number of the solution of the problem involving the products of Jacobi polynomials, Hermite polynomials, Laguerre polynomials, Brafman polynomials, etc. Thus, the result derived in this document would at once yield a very large number of results, involving a large variety of various specials functions and polynomials concerning the forced vibrations of a circular membrane .

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