

Application of special functions in one dimensional advective diffusion problem

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ABSTRACT

In the present paper, we construct an one dimensional advective diffusion model problem to evaluate the substance concentration distribution along x -axis between $x = -1$ and $x = 1$. Here, the diffusivity of the substance and the velocity of the solvent flow are both considered as a variable. Kumar et al [5] use the product of the class of multivariable polynomials and the multivariable H-function defined by Srivastava et al [13,14] to obtain the analytic solution. Then, we employ the product of the generalized hypergeometric function, class of multivariable polynomials, Srivastava-Daoust function [12], the multivariable A-function defined by Gautam et al [2] and the multivariable Aleph-function to obtain an analytic formula of our problem. Finally, some particular cases will be given.

Keywords :Multivariable Aleph-function, , classes of multivariable polynomials, generalized hypergeometric function, advective-diffusion model problem, expansion formula, multivariable H-function., Aleph-function of two variables, Srivastava-Daoust function, multivariable A-function.

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1.Introduction and preliminaries.

An advective diffusion is a process where solute particles in a solvent are diffused and transported with the flow of solvent. If $C(x, t)$ represents be the concentration of the diffusing substance, D denotes the diffusivity of the substance, \mathbf{u} is the velocity of the solvent flow and $F(x, t)$ is the external source function, then the governing advective-diffusion equation is given by (see Harrison and Perry [4], Hanna et al [3], Lyons et al [6])

$$\frac{\partial C}{\partial t} + \text{div}(C\mathbf{u}) = \text{div}(D\nabla C) + F \tag{1.1}$$

Here, we construct an one dimensional advective-diffusion model problem in which the diffusivity of the substance is considered as a variable of position x and the velocity of the solvent flow is supposed to be function of x . There is no generation or absorption of the solute in the solvent.

The class of multivariable polynomials defined by Srivastava [11], is given in the following manner :

$$S_{N_1, \dots, N_u}^{M_1, \dots, M_u} [y_1, \dots, y_u] = \sum_{K_1=0}^{[N_1/M_1]} \dots \sum_{K_u=0}^{[N_u/M_u]} \frac{(-N_1)_{M_1 K_1}}{K_1!} \dots \frac{(-N_u)_{M_u K_u}}{K_u!} A[N_1, K_1; \dots; N_u, K_u] y_1^{K_1} \dots y_u^{K_u} \tag{1.2}$$

The Srivastava-Daoust function is defined by (see [12]):

$$F_{\bar{C}:D^{(1)}; \dots; D^{(v)}}^{\bar{A}:B^{(1)}; \dots; B^{(v)}} \left(\begin{matrix} z_1 \\ \cdot \\ \cdot \\ z_v \end{matrix} \middle| \begin{matrix} [(a); \theta', \dots, \theta^{(v)}] : [(b'); \phi']; \dots; [(b^{(v)}); \phi^{(v)}] \\ \cdot \\ \cdot \\ [(c); \psi', \dots, \psi^{(v)}] : [(d'); \delta']; \dots; [(d^{(v)}); \delta^{(v)}] \end{matrix} \right) = \sum_{r_1, \dots, r_v=0}^{\infty} \frac{A' z_1^{r_1} \dots z_v^{r_v}}{r_1! \dots r_v!} \tag{1.3}$$

where

$$A' = \frac{\prod_{j=1}^{\bar{A}} (a_j)_{r_1 \theta'_j + \dots + r_v \theta_j^{(v)}} \prod_{j=1}^{B^{(1)}} (b'_j)_{r_1 \phi'_j} \dots \prod_{j=1}^{B^{(v)}} (b_j^{(v)})_{r_v \phi_j^{(v)}}}{\prod_{j=1}^{\bar{C}} (c_j)_{r_1 \psi'_j + \dots + r_v \psi_j^{(v)}} \prod_{j=1}^{D^{(1)}} (d'_j)_{r_1 \delta'_j} \dots \prod_{j=1}^{D^{(v)}} (d_j^{(v)})_{r_v \delta_j^{(v)}}} \tag{1.4}$$

The series given by (1.7) converges absolutely if

$$1 + \sum_{j=1}^{\bar{C}} \psi_j^{(i)} + \sum_{j=1}^{D^{(i)}} \delta_j^{(i)} - \sum_{j=1}^{\bar{A}} \theta_j^{(i)} - \sum_{j=1}^{B^{(i)}} \phi_j^{(i)} > 0; i = 1, \dots, v \quad (1.5)$$

The serie representation of the multivariable A-function is given by Gautam [2] as

$$A[u_1, \dots, u_s] = A_{A,C:(M',N'); \dots; (M^{(s)}, N^{(s)})}^{0,\lambda:(\alpha',\beta'); \dots; (\alpha^{(s)}, \beta^{(s)})} \left(\begin{matrix} u_1 \\ \vdots \\ u_s \end{matrix} \middle| \begin{matrix} [(\mathbf{g}_j); \gamma', \dots, \gamma^{(s)}]_{1,A} : \\ \vdots \\ [(\mathbf{f}_j); \xi', \dots, \xi^{(s)}]_{1,C} : \end{matrix} \right)$$

$$\left(\begin{matrix} (q^{(1)}, \eta^{(1)})_{1,M^{(1)}}; \dots; (q^{(s)}, \eta^{(s)})_{1,M^{(s)}} \\ \vdots \\ (p^{(1)}, \epsilon^{(1)})_{1,N^{(1)}}; \dots; (p^{(s)}, \epsilon^{(s)})_{1,N^{(s)}} \end{matrix} \right) = \sum_{G_i=1}^{\alpha^{(i)}} \sum_{g_i=1}^{\infty} \phi_1 \frac{\prod_{i=1}^s \phi_i u_i^{\eta_{G_i, g_i}} (-)^{\sum_{i=1}^s g_i}}{\prod_{i=1}^s \epsilon_{G_i}^{(i)} g_i!} \quad (1.6)$$

where

$$\phi_1 = \frac{\prod_{j=1}^{\lambda} \Gamma(1 - g_j + \sum_{i=1}^s \gamma_j^{(i)} \eta_{G_i, g_i})}{\prod_{j=\lambda'+1}^A \Gamma(g_j - \sum_{i=1}^s \gamma_j^{(i)} \eta_{G_i, g_i}) \prod_{j=1}^C \Gamma(1 - f_j + \sum_{i=1}^s \xi_j^{(i)} \eta_{G_i, g_i})} \quad (1.7)$$

$$\phi_i = \frac{\prod_{j=1, j \neq m_i}^{\alpha^{(i)}} \Gamma(p_j^{(i)} - \epsilon_j^{(i)} \eta_{G_i, g_i}) \prod_{j=1}^{\beta^{(i)}} \Gamma(1 - q_j^{(i)} + \eta_j^{(i)} \eta_{G_i, g_i})}{\prod_{j=\alpha^{(i)}+1}^{N^{(i)}} \Gamma(1 - p_j^{(i)} + \epsilon_j^{(i)} \eta_{G_i, g_i}) \prod_{j=\beta^{(i)}+1}^{M^{(i)}} \Gamma(q_j^{(i)} - \eta_j^{(i)} \eta_{G_i, g_i})}, i = 1, \dots, s \quad (1.8)$$

and

$$\eta_{G_i, g_i} = \frac{p_{G_i}^{(i)} + g_i}{\epsilon_{G_i}^{(i)}}, i = 1, \dots, s \quad (1.9)$$

which is valid under the following conditions :

$$\epsilon_{G_i}^{(i)} [p_j^{(i)} + p_i'] \neq \epsilon_j^{(i)} [p_{G_i} + g_i] \quad (1.10)$$

and

$$u_i \neq 0, \sum_{j=1}^A \gamma_j^{(i)} - \sum_{j=1}^C \xi_j^{(i)} + \sum_{j=1}^{M^{(i)}} \eta_j^{(i)} - \sum_{j=1}^{N^{(i)}} \epsilon_j^{(i)} < 0, i = 1, \dots, s \quad (1.11)$$

Here $\lambda, A, C, \alpha_i, \beta_i, m_i, n_i \in \mathbb{N}^*; i = 1, \dots, s; f_j, g_j, p_j^{(i)}, q_j^{(i)}, \gamma_j^{(i)}, \xi_j^{(i)}, \eta_j^{(i)}, \epsilon_j^{(i)} \in \mathbb{C}$

The multivariable Aleph-function is an extension of the multivariable I-function recently defined by C.K. Sharma and Ahmad [8] , itself is a generalization of the multivariable H-function defined by Srivastava et al [13, 14]. The multivariable Aleph-function is defined by means of the multiple contour integral :

$$\text{We have : } \aleph(z_1, \dots, z_r) = \aleph_{p_i, q_i, \tau_i; R: p_{i(1)}, q_{i(1)}, \tau_{i(1)}; R^{(1)}; \dots; p_{i(r)}, q_{i(r)}, \tau_{i(r)}; R^{(r)}}^{0, n: m_1, n_1, \dots, m_r, n_r} \left(\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \middle| \begin{matrix} [(a_j; \alpha_j^{(1)}, \dots, \alpha_j^{(r)})_{1, n}] \\ \vdots \\ [(\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \dots, \alpha_{ji}^{(r)})_{n+1, p_i}) \\ \vdots \\ [(\tau_i(b_{ji}; \beta_{ji}^{(1)}, \dots, \beta_{ji}^{(r)})_{m+1, q_i}) \end{matrix} \right)$$

$$V = m_1, n_1; \dots; m_r, n_r \quad (1.16)$$

$$W = p_{i(1)}, q_{i(1)}, \tau_{i(1)}; R^{(1)}; \dots; p_{i(r)}, q_{i(r)}, \tau_{i(r)}; R^{(r)} \quad (1.17)$$

$$A = \{(a_j; \alpha_j^{(1)}, \dots, \alpha_j^{(r)})_{1,n}\}, \{\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \dots, \alpha_{ji}^{(r)})_{n+1,p_i}\} : \{(c_j^{(1)}; \gamma_j^{(1)})_{1,n_1}\} \\ \{\tau_{i(1)}(c_{ji(1)}^{(1)}; \gamma_{ji(1)}^{(1)})_{n_1+1,p_{i(1)}}\}; \dots; \{(c_j^{(r)}; \gamma_j^{(r)})_{1,n_r}\}, \{\tau_{i(r)}(c_{ji(r)}^{(r)}; \gamma_{ji(r)}^{(r)})_{n_r+1,p_{i(r)}}\} \quad (1.18)$$

$$B = \{\tau_i(b_{ji}; \beta_{ji}^{(1)}, \dots, \beta_{ji}^{(r)})_{m+1,q_i}\} : \{(d_j^{(1)}; \delta_j^{(1)})_{1,m_1}\}, \{\tau_{i(1)}(d_{ji(1)}^{(1)}; \delta_{ji(1)}^{(1)})_{m_1+1,q_{i(1)}}\}; \dots; \\ \{(d_j^{(r)}; \delta_j^{(r)})_{1,m_r}\}, \{\tau_{i(r)}(d_{ji(r)}^{(r)}; \delta_{ji(r)}^{(r)})_{m_r+1,q_{i(r)}}\} \quad (1.19)$$

$$B' = \frac{(-N_1)_{M_1 K_1}}{K_1!} \dots \frac{(-N_u)_{M_u K_u}}{K_u!} A[N_1, K_1; \dots; N_u, K_u] \quad (1.20)$$

$$F = F_{\bar{C}:D^{(1)}; \dots; D^{(v)}}^{\bar{A}:B^{(1)}; \dots; B^{(v)}} \quad (1.21)$$

2. Statement of the problem and governing equation

Let us consider an advective-diffusion process in which the solute particles in a solvent are diffused and transported with the variable velocity u of the solvent flow in the direction of the x -axis between the limits $x = -1$ to $x = 1$ and that is supposed to be a function of position and proportional to $[1 + \alpha - \gamma - \beta + (\alpha + \beta - \gamma + 1)x]$, $Re(\alpha - \gamma) > -2$ $Re(\beta) > -1$, the diffusivity D of the substance (solute) is a function of position and is proportional to $(1 - x^2)$. The external source function is absent in the process that is there is no generation or absorption of the solute in the solvent. The desired function $C(x, t)$ represents the concentration distribution of the diffusing substance at the position x and at the time t . Then due to (1.1), we get an one dimensional advective-diffusion equation in the form given below :

$$\frac{\partial C}{\partial t} + [\lambda_1(\alpha - \gamma - \beta + 1) + (\alpha + \beta - \gamma + 1)x] + 2x\lambda_2 \frac{\partial C}{\partial x} + \lambda_1(\alpha + \beta - \gamma + 1)C - \lambda_2(1 - x^2) \frac{\partial^2 C}{\partial x^2} = 0 \quad (2.1)$$

where λ_1 and λ_2 are arbitrary non zero proportionality constants, $Re(\alpha - \gamma) > -2$ and $Re(\beta) > -1, t > 0$ and $-1 \leq x \leq 1$.

Let us suppose that the initial concentration of the solute in the solvent is given by

$$C(x, 0) = f(x), -1 \leq x \leq 1 \quad (2.2)$$

In (2.1), for simplicity, letting λ_1 and λ_2 , we get

$$\frac{1}{\lambda_1} \frac{\partial C}{\partial t} = (1 - x^2) \frac{\partial^2 C}{\partial x^2} + [\beta + \gamma - \alpha - 1 - (\alpha + \beta - \gamma + 3)x] \frac{\partial C}{\partial x} - (\alpha + \beta - \gamma + 1)C \quad (2.3)$$

where $\lambda_1 \neq 0, Re(\alpha - \gamma) > -2, Re(\beta) > -1, t > 0$ where $-1 \leq x \leq 1$

3. Main integral

In our work, we also require the following integral

$$\int_{-1}^1 (1-x)^\rho (1+x)^\sigma P_Q^{(\mu, \nu)}(x) {}_M F_N \left[\begin{matrix} (e_M) \\ \cdot \\ (f_N) \end{matrix} \middle| y(1-x)^g (1+x)^w \right]$$

$$S_{N_1, \dots, N_u}^{M_1, \dots, M_u} \left(\begin{matrix} y_1(1-x)^{f_1}(1+x)^{w_1} \\ \vdots \\ y_u(1-x)^{f_u}(1+x)^{w_u} \end{matrix} \right) F \left(\begin{matrix} t_1(1-x)^{a_1}(1+x)^{b_1} \\ \vdots \\ t_v(1-x)^{a_v}(1+x)^{b_v} \end{matrix} \right)$$

$$A \left(\begin{matrix} z_1(1-x)^{c_1}(1+x)^{d_1} \\ \vdots \\ z_s(1-x)^{c_s}(1+x)^{d_s} \end{matrix} \right) \aleph \left(\begin{matrix} Z_1(1-x)^{h_1}(1+x)^{k_1} \\ \vdots \\ Z_r(1-x)^{h_r}(1+x)^{k_r} \end{matrix} \right) dx = \frac{\Gamma(1+\mu+Q)2^{\rho+\sigma+1}}{\Gamma(1+\mu)\Gamma(1+Q)}$$

$$\sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_u=0}^{[N_u/M_u]} \sum_{r_1, \dots, r_v=0}^{\infty} \sum_{q=0}^{\infty} \sum_{k=0}^Q \sum_{G_i=1}^{\alpha^{(i)}} \sum_{g_i=1}^{\infty} \phi_1 \frac{\prod_{i=1}^s \phi_i u_i^{\eta_{G_i, g_i}} (-)^{\sum_{i=1}^s g_i}}{\prod_{i=1}^s \epsilon_{G_i}^i g_i!} A' B' \frac{[(e_M)]_q y^q}{[(f_N)]_q q!}$$

$$\frac{(-Q)_k (1+\mu+\nu+Q)_k y_1^{K_1} \cdots y_u^{K_u} t_1^{r_1} \cdots t_v^{r_v} 2^{gq+wq+\sum_{i=1}^u (f_i+w_i)K_i + \sum_{i=1}^v (a_i+b_i)r_i + \sum_{i=1}^s (c_i+d_i)\eta_{G_i, g_i}}{(1+\mu)_k k! r_1! \cdots r_v!}$$

$$\aleph_{p_i+2, q_i+1, \tau_i; R; W}^{0, \mathbf{n}+2; V} \left(\begin{matrix} Z_1 2^{h_1+k_1} \\ \cdot \\ \cdot \\ Z_r 2^{h_r+k_r} \end{matrix} \middle| \begin{matrix} (-\rho-k-gq-\sum_{i=1}^u f_i K_i - \sum_{i=1}^v a_i r_i - \sum_{i=1}^s c_i \eta_{G_i, g_i} : h_1, \dots, h_r), \\ \cdot \\ \cdot \\ \cdot \end{matrix} \right)$$

$$\left(\begin{matrix} (-\sigma-wq-\sum_{i=1}^u w_i K_i - \sum_{i=1}^v b_i r_i - \sum_{i=1}^s d_i \eta_{G_i, g_i} : k_1, \dots, k_r), A \\ \cdot \\ \cdot \\ (-1-k-\sigma-\rho-(g+w)q-\sum_{i=1}^u (f_i+w_i)K_i - \sum_{i=1}^v (a_i+b_i)r_i - \sum_{i=1}^s (c_i+d_i)\eta_{G_i, g_i} : h_1+k_1, \dots, h_r+k_r), B \end{matrix} \right) \quad (3.1)$$

Provided that

$$\min\{g, w, f_i, w_i, a_j, b_j, c_K, d_K, h_l, k_l\} > 0 \text{ for } i = 1, \dots, v; j = 1, \dots, u; K = 1, \dots, s; l = 1, \dots, r$$

$$\text{and } \operatorname{Re} \left[\rho + \sum_{i=1}^u K_i f_i + \sum_{i=1}^v r_i a_i + \sum_{i=1}^s c_i \eta_{G_i, g_i} + \sum_{i=1}^r h_i \min_{1 \leq j \leq m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} \right] > -1$$

$$\operatorname{Re} \left[\sigma + \sum_{i=1}^u K_i w_i + \sum_{i=1}^v r_i b_i + \sum_{i=1}^s d_i \eta_{G_i, g_i} + \sum_{i=1}^r k_i \min_{1 \leq j \leq m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} \right] > -1; \operatorname{Re}(\mu) > -1, \operatorname{Re}(\nu) > -1$$

$|arg Z_k| < \frac{1}{2^{h_i+k_i+1}} A_i^{(k)} \pi, i = 1, \dots, r$, where $A_i^{(k)}$ is defined by (1.15). $|y| < 1$

$$1 + \sum_{j=1}^{\bar{C}} \psi_j^{(i)} + \sum_{j=1}^{D^{(i)}} \delta_j^{(i)} - \sum_{j=1}^{\bar{A}} \theta_j^{(i)} - \sum_{j=1}^{B^{(i)}} \phi_j^{(i)} > 0; i = 1, \dots, v$$

and $z_i \neq 0, \sum_{j=1}^A \gamma_j^{(i)} - \sum_{j=1}^C \xi_j^{(i)} + \sum_{j=1}^{M^{(i)}} \eta_j^{(i)} - \sum_{j=1}^{N^{(i)}} \epsilon_j^{(i)} < 0, i = 1, \dots, s$

Proof

To prove (3.1), first expressing the Jacobi polynomial $P_Q^{(\mu, \nu)}(\cdot)$ in series, a class of multivariable polynomials defined by Srivastava [11] $S_{N_1, \dots, N_s}^{M_1, \dots, M_s}[\cdot]$, the Srivastava-Daoust function [12] $F[\cdot]$ and the multivariable A-function defined by Gautam et al [2] in series with the help of (1.2), (1.3) and (1.6) respectively, expressing the generalized hypergeometric in series and we interchange the order of summations and x -integral (which is permissible under the conditions stated). Expressing the \aleph -function of r -variables in Mellin-Barnes contour integral with the help of (1.12) and interchange the order of integrations which is justifiable due to absolute convergence of the integrals involved in the process. Now collecting the powers of $(1-x)$ and $(1+x)$ and evaluating the inner x -integral. Now, interpreting the Mellin-Barnes contour integral in multivariable \aleph -function, we obtain the desired result (3.1).

4. Solution of the problem and analysis

To solve the differential equation (2.3), we suppose that $C(x, t) = X(x)T(t)$ in it, we get

$$\begin{aligned} \frac{1}{\lambda_1 T} \frac{dC}{dt} &= \frac{1}{X} (1-x^2) \frac{d^2 X}{dx^2} + [\beta + \gamma - \alpha - 1 - (\alpha + \beta - \gamma + 3)x] \frac{dX}{dx} - (\alpha + \beta - \gamma + 1)X \\ &= -(v+1)(1 + \alpha + \beta - \gamma + v) \end{aligned} \tag{4.1}$$

and

$$\frac{dT}{dt} = -\lambda_1 (v+1)(1 + \alpha + \beta - \gamma + v)T \tag{4.2}$$

The solution of the equation (4.1) is $P_v^{(\alpha+\gamma+1, \beta)}(x)$ (see 7 page 258). Thus by (4.1) and (4.2), we obtain

$$X(x) = A_1 P_v^{(\alpha+\gamma+1, \beta)}(x), -1 \leq x \leq 1 \tag{4.3}$$

$$T(t) = A_2 \exp[\lambda_1 (v+1)(1 + \alpha + \beta - \gamma + v)t], t \geq 0 \tag{4.4}$$

The general solution of the equation (2.3) is :

$$C(x, t) = \sum_{v=0}^{\infty} A_v P_v^{(\alpha+\gamma+1, \beta)}(x) \exp[\lambda_1 (v+1)(1 + \alpha + \beta - \gamma + v)t], t \geq 0 \text{ and } -1 \leq x \leq 1 \tag{4.5}$$

Now making an appeal to (2.2) and taking the expression of (4.5) at $t = 0$, we have

$$f(x) = \sum_{v=0}^{\infty} A_v P_v^{(\alpha+\gamma+1, \beta)}(x) \tag{4.6}$$

Then to evaluate A_v ; multiply both sides of (4.6) by $(1-x)^{\alpha-\gamma+1}(1+x)^\beta P_\mu^{(\alpha-\gamma+1,\beta)}(x)$ and then integrate the two sides of (4.6) with respect x from -1 to 1 and use the orthogonality of Jacobi polynomials (see, [7, page 258]) therein, we get

$$A_v = \frac{2^{\gamma-\alpha-\beta-2}\Gamma(v+1)\Gamma(\alpha+\beta-\gamma+2v+2)\Gamma(\alpha+\beta-\gamma+v+2)}{\Gamma(\alpha+\beta-\gamma+2v+2)\Gamma(\alpha+\gamma+v+2)\Gamma(\beta+v+1)} \int_{-1}^1 (1-x)^{\alpha-\gamma+1}(1+x)^\beta P_v^{(\alpha+\gamma+1,\beta)}(x)f(x)dx \quad (4.7)$$

$Re(\alpha-\gamma) > -2, Re(\beta) > -1, \forall v \in \mathbb{N}$.

Finally, with the aid of (4.5) and (4.7), we obtain the concentration distribution of the solute in the form

$$C(x,t) = 2^{\gamma-\alpha-\beta-2} \sum_{v=0}^{\infty} \frac{\Gamma(v+1)\Gamma(\alpha+\beta-\gamma+2v+2)\Gamma(\alpha+\beta-\gamma+v+2)}{\Gamma(\alpha+\beta-\gamma+2v+2)\Gamma(\alpha+\gamma+v+2)\Gamma(\beta+v+1)} P_v^{(\alpha+\gamma+1,\beta)}(x) \exp[\lambda_1(v+1)(1+\alpha+\beta-\gamma+v)t] \int_{-1}^1 (1-x)^{\alpha-\gamma+1}(1+x)^\beta P_v^{(\alpha+\gamma+1,\beta)}(x)f(x)dx \quad (4.8)$$

provided $Re(\alpha-\gamma) > -2, Re(\beta) > -1, t > 0$ and $-1 \leq x \leq 1$

5.Example

Let the initial concentration of the solute in the solvent is given by

$$f(x) = (1-x)^\rho(1+x)^\sigma {}_M F_N \left[\begin{matrix} (e_M) \\ \cdot \\ (f_N) \end{matrix} \middle| y(1-x)^g(1+x)^w \right]$$

$$S_{N_1, \dots, N_u}^{M_1, \dots, M_u} \left(\begin{matrix} y_1(1-x)^{f_1}(1+x)^{w_1} \\ \vdots \\ y_u(1-x)^{f_u}(1+x)^{w_u} \end{matrix} \right) F \left(\begin{matrix} t_1(1-x)^{a_1}(1+x)^{b_1} \\ \vdots \\ t_v(1-x)^{a_v}(1+x)^{b_v} \end{matrix} \right)$$

$$A \left(\begin{matrix} z_1(1-x)^{c_1}(1+x)^{d_1} \\ \vdots \\ z_s(1-x)^{c_s}(1+x)^{d_s} \end{matrix} \right) \aleph \left(\begin{matrix} Z_1(1-x)^{h_1}(1+x)^{k_1} \\ \vdots \\ Z_r(1-x)^{h_r}(1+x)^{k_r} \end{matrix} \right) \quad (5.1)$$

Substituting the value of $f(x)$ in (4.8), we obtain

$$C(x, t) = 2^{\rho+\sigma} \sum_{v=0}^{\infty} \sum_{k=0}^v \frac{\Gamma(\alpha + \beta - \gamma + 2v + 3) \Gamma(\alpha + \beta - \gamma + v + 2 + k)}{\Gamma(\alpha + \beta - \gamma + 2v + 2) \Gamma(\alpha - \gamma + k + 2)} \frac{(-v)_k}{\Gamma(\beta + v + 1) k!}$$

$$P_v^{(\alpha+\gamma+1, \beta)}(x) \exp[\lambda_1(v+1)(1+\alpha+\beta-\gamma+v)t] F^{\alpha, \beta, \gamma, \rho, \sigma}(k; N_1, M_1; \dots; N_u, M_u) \quad (5.2)$$

where

$$F^{\alpha, \beta, \gamma, \rho, \sigma}(k; N_1, M_1; \dots; N_u, M_u) = \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_u=0}^{[N_u/M_u]} \sum_{r_1, \dots, r_u=0}^{\infty} \sum_{q=0}^{\infty}$$

$$\sum_{G_i=1}^{\alpha^{(i)}} \sum_{g_i=1}^{\infty} \phi_1 \frac{\prod_{i=1}^s \phi_i u_i^{\eta_{G_i, g_i}} (-)^{\sum_{i=1}^s g_i}}{\prod_{i=1}^s \epsilon_{G_i}^i g_i!} A' B' \frac{[(e_M)]_q y^q}{[(f_N)]_q q!}$$

$$\frac{y_1^{K_1} \dots y_u^{K_u} t_1^{r_1} \dots t_v^{r_v} 2^{gq+wq+\sum_{i=1}^u (f_i+w_i)K_i+\sum_{i=1}^v (a_i+b_i)r_i+\sum_{i=1}^s (c_i+d_i)\eta_{G_i, g_i}}}{r_1! \dots r_v!}$$

$$\mathcal{N}_{p_i+2, q_i+1, \tau_i; R: W}^{0, \mathbf{n}+2: V} \left(\begin{array}{c} Z_1 2^{h_1+k_1} \\ \vdots \\ Z_r 2^{h_r+k_r} \end{array} \middle| \begin{array}{c} (\gamma - \rho - \alpha - 1 - k - gq - \sum_{i=1}^u f_i K_i - \sum_{i=1}^v a_i r_i - \sum_{i=1}^s c_i \eta_{G_i, g_i} : h_1, \dots, h_r), \\ \vdots \\ \vdots \end{array} \right)$$

$$\left(\begin{array}{c} (-\sigma - \beta - wq - \sum_{i=1}^u w_i K_i - \sum_{i=1}^v b_i r_i - \sum_{i=1}^s d_i \eta_{G_i, g_i} : k_1, \dots, k_r), A \\ \vdots \\ (\gamma - 2 - k - \sigma - \rho - \alpha - \beta - (g+w)q - \sum_{i=1}^u (f_i + w_i) K_i - \sum_{i=1}^v (a_i + b_i) r_i - \sum_{i=1}^s (c_i + d_i) \eta_{G_i, g_i} : h_1 + k_1, \dots, h_r + k_r), B \end{array} \right) \quad (5.3)$$

Provided that

$$\min\{g, w, f_i, w_i, a_j, b_j, c_K, d_K, h_i, k_l\} > 0 \text{ for } i = 1, \dots, v; j = 1, \dots, u; K = 1, \dots, s; l = 1, \dots, r$$

$$\text{and } \operatorname{Re} \left[\rho + \alpha - \gamma + \sum_{i=1}^u K_i f_i + \sum_{i=1}^v r_i a_i + \sum_{i=1}^s c_i \eta_{G_i, g_i} + \sum_{i=1}^r h_i \min_{1 \leq j \leq m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} \right] > -1$$

$$\operatorname{Re} \left[\sigma + \beta + \sum_{i=1}^u K_i w_i + \sum_{i=1}^v r_i b_i + \sum_{i=1}^s d_i \eta_{G_i, g_i} + \sum_{i=1}^r k_i \min_{1 \leq j \leq m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} \right] > -1; \operatorname{Re}(\beta) > -1$$

$$|\arg Z_k| < \frac{1}{2^{h_i+k_i+1}} A_i^{(k)} \pi, i = 1, \dots, r, \text{ where } A_i^{(k)} \text{ is defined by (1.15). } |y| < 1$$

$$1 + \sum_{j=1}^{\bar{C}} \psi_j^{(i)} + \sum_{j=1}^{D^{(i)}} \delta_j^{(i)} - \sum_{j=1}^{\bar{A}} \theta_j^{(i)} - \sum_{j=1}^{B^{(i)}} \phi_j^{(i)} > 0; i = 1, \dots, v$$

and $z_i \neq 0, \sum_{j=1}^A \gamma_j^{(i)} - \sum_{j=1}^C \xi_j^{(i)} + \sum_{j=1}^{M^{(i)}} \eta_j^{(i)} - \sum_{j=1}^{N^{(i)}} \epsilon_j^{(i)} < 0, i = 1, \dots, s$

6. Particular cases

(ii) If $\gamma = \alpha + \beta + 1$ in (2.3), then the solvent moves with the constant velocity in the direction of \mathbf{u} and the the equation (2.3) reduces to

$$\frac{1}{\lambda_1} \frac{\partial C}{\partial t} = (1 - x^2) \frac{\partial^2 C}{\partial x^2} + 2(\beta - x) \frac{\partial C}{\partial x} \quad (6.1)$$

and the solution is

$$C(x, t) = 2^{\rho+\sigma} \sum_{v=0}^{\infty} \sum_{k=0}^v \frac{\Gamma(2v+2)\Gamma(v+1+k)}{\Gamma(2v+1)\Gamma(1-\beta+k)} \frac{(-v)_k}{\Gamma(\beta+v+1)k!}$$

$$P_v^{(-\beta, \beta)}(x) \exp[\lambda_1(v+1)vt] F^{\alpha, \beta, \alpha+1, \rho, \sigma}(k; N_1, M_1; \dots; N_u, M_u) \quad (6.2)$$

where

$$F^{\alpha, \beta, \alpha+1, \rho, \sigma}(k; N_1, M_1; \dots; N_u, M_u) = \sum_{K_1=0}^{[N_1/M_1]} \dots \sum_{K_u=0}^{[N_u/M_u]} \sum_{r_1, \dots, r_v=0}^{\infty} \sum_{q=0}^{\infty}$$

$$\sum_{G_i=1}^{\alpha^{(i)}} \sum_{g_i=1}^{\infty} \phi_1 \frac{\prod_{i=1}^s \phi_i u_i^{\eta_{G_i, g_i}} (-)^{\sum_{i=1}^s g_i}}{\prod_{i=1}^s \epsilon_{G_i}^i g_i!} A' B' \frac{[(e_M)]_q y^q}{[(f_N)]_q q!}$$

$$\frac{y_1^{K_1} \dots y_u^{K_u} t_1^{r_1} \dots t_v^{r_v} 2^{gq+wq+\sum_{i=1}^u (f_i+w_i)K_i+\sum_{i=1}^v (a_i+b_i)r_i+\sum_{i=1}^s (c_i+d_i)\eta_{G_i, g_i}}}{r_1! \dots r_v!}$$

$$\mathcal{N}_{p_i+2, q_i+1, \tau_i; R: W}^{0, \mathbf{n}+2: V} \left(\begin{array}{c} Z_1 2^{h_1+k_1} \\ \vdots \\ Z_r 2^{h_r+k_r} \end{array} \middle| \begin{array}{l} (\beta - \rho - k - gq - \sum_{i=1}^u f_i K_i - \sum_{i=1}^v a_i r_i - \sum_{i=1}^s c_i \eta_{G_i, g_i} : h_1, \dots, h_r), \\ \vdots \\ \vdots \end{array} \right) \quad (6.3)$$

$$\left(\begin{array}{l} (-\sigma - \beta - wq - \sum_{i=1}^u w_i K_i - \sum_{i=1}^v b_i r_i - \sum_{i=1}^s d_i \eta_{G_i, g_i} : k_1, \dots, k_r), A \\ \vdots \\ (-k-1-\sigma - \rho - (g+w)q - \sum_{i=1}^u (f_i + w_i)K_i - \sum_{i=1}^v (a_i + b_i)r_i - \sum_{i=1}^s (c_i + d_i)\eta_{G_i, g_i} : h_1 + k_1, \dots, h_r + k_r), B \end{array} \right)$$

Provided that

$$\min\{g, w, f_i, w_i, a_j, b_j, c_K, d_K, h_l, k_l\} > 0 \text{ for } i = 1, \dots, v; j = 1, \dots, u; K = 1, \dots, s; l = 1, \dots, r$$

and

$$\operatorname{Re} \left[\rho - \beta + \sum_{i=1}^u K_i f_i + \sum_{i=1}^v r_i a_i + \sum_{i=1}^s c_i \eta_{G_i, g_i} + \sum_{i=1}^r h_i \min_{1 \leq j \leq m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} \right] > -1$$

$$\operatorname{Re} \left[\sigma + \beta + \sum_{i=1}^u K_i w_i + \sum_{i=1}^v r_i b_i + \sum_{i=1}^s d_i \eta_{G_i, g_i} + \sum_{i=1}^r k_i \min_{1 \leq j \leq m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} \right] > -1$$

$$|\arg Z_k| < \frac{1}{2^{h_i+k_i+1}} A_i^{(k)} \pi, i = 1, \dots, r, \text{ where } A_i^{(k)} \text{ is defined by (1.15). } |y| < 1$$

$$1 + \sum_{j=1}^{\bar{C}} \psi_j^{(i)} + \sum_{j=1}^{D^{(i)}} \delta_j^{(i)} - \sum_{j=1}^{\bar{A}} \theta_j^{(i)} - \sum_{j=1}^{B^{(i)}} \phi_j^{(i)} > 0; i = 1, \dots, v$$

$$\text{and } z_i \neq 0, \sum_{j=1}^A \gamma_j^{(i)} - \sum_{j=1}^C \xi_j^{(i)} + \sum_{j=1}^{M^{(i)}} \eta_j^{(i)} - \sum_{j=1}^{N^{(i)}} \epsilon_j^{(i)} < 0, i = 1, \dots, s$$

(ii) If we put $\gamma = \alpha + 1$ and $\beta = 0$ in (2.3), the medium is stationary that is $\mathbf{u} = 0$ and the equation (2.3) reduces to the following equation

$$\frac{1}{\lambda_1} \frac{\partial C}{\partial t} = (1 - x^2) \frac{\partial^2 C}{\partial x^2} - 2x \frac{\partial C}{\partial x} \quad (6.4)$$

and the solution is

$$C(x, t) = 2^{\rho+\sigma} \sum_{v=0}^{\infty} \sum_{k=0}^v \frac{\Gamma(2v+2)\Gamma(v+1+k)}{\Gamma(2v+1)\Gamma(1-\beta+k)} \frac{(-v)_k}{\Gamma(v+1)k!}$$

$$P_v(x) \exp[\lambda_1(v+1)vt] F^{\alpha, 0, \alpha+1, \rho, \sigma}(k; N_1, M_1; \dots; N_u, M_u) \quad (6.5)$$

where

$$F^{\alpha, 0, \alpha+1, \rho, \sigma}(k; N_1, M_1; \dots; N_u, M_u) = \sum_{K_1=0}^{[N_1/M_1]} \dots \sum_{K_u=0}^{[N_u/M_u]} \sum_{r_1, \dots, r_v=0}^{\infty} \sum_{q=0}^{\infty}$$

$$\sum_{G_i=1}^{\alpha^{(i)}} \sum_{g_i=1}^{\infty} \phi_1 \frac{\prod_{i=1}^s \phi_i u_i^{\eta_{G_i, g_i}} (-)^{\sum_{i=1}^s g_i}}{\prod_{i=1}^s \epsilon_{G_i}^i g_i!} A' B' \frac{[(e_M)]_q y^q}{[(f_N)]_q q!}$$

$$\frac{y_1^{K_1} \dots y_u^{K_u} t_1^{r_1} \dots t_v^{r_v} 2^{gq+wq+\sum_{i=1}^u (f_i+w_i)K_i+\sum_{i=1}^v (a_i+b_i)r_i+\sum_{i=1}^s (c_i+d_i)\eta_{G_i, g_i}}}{r_1! \dots r_v!}$$

$$\begin{aligned}
& \mathcal{N}_{p_i+2, q_i+1, \tau_i; R: W}^{0, \mathbf{n}+2: V} \left(\begin{array}{c} Z_1 2^{h_1+k_1} \\ \vdots \\ \vdots \\ Z_r 2^{h_r+k_r} \end{array} \left| \begin{array}{l} (-\rho - k - gq - \sum_{i=1}^u f_i K_i - \sum_{i=1}^v a_i r_i - \sum_{i=1}^s c_i \eta_{G_i, g_i} : h_1, \dots, h_r), \\ \vdots \\ \vdots \\ \vdots \end{array} \right. \right) \\
& \left(\begin{array}{c} (-\sigma - wq - \sum_{i=1}^u w_i K_i - \sum_{i=1}^v b_i r_i - \sum_{i=1}^s d_i \eta_{G_i, g_i} : k_1, \dots, k_r), A \\ \vdots \\ \vdots \\ (-k-1-\sigma - \rho - (g+w)q - \sum_{i=1}^u (f_i + w_i) K_i - \sum_{i=1}^v (a_i + b_i) r_i - \sum_{i=1}^s (c_i + d_i) \eta_{G_i, g_i} : h_1 + k_1, \dots, h_r + k_r), B \end{array} \right) \quad (6.6)
\end{aligned}$$

Provided that

$$\min\{g, w, f_i, w_i, a_j, b_j, c_K, d_K, h_l, k_l\} > 0 \text{ for } i = 1, \dots, v; j = 1, \dots, u; K = 1, \dots, s; l = 1, \dots, r$$

$$\text{and } \operatorname{Re} \left[\rho + \sum_{i=1}^u K_i f_i + \sum_{i=1}^v r_i a_i + \sum_{i=1}^s c_i \eta_{G_i, g_i} + \sum_{i=1}^r h_i \min_{1 \leq j \leq m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} \right] > -1$$

$$\operatorname{Re} \left[\sigma + \sum_{i=1}^u K_i w_i + \sum_{i=1}^v r_i b_i + \sum_{i=1}^s d_i \eta_{G_i, g_i} + \sum_{i=1}^r k_i \min_{1 \leq j \leq m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} \right] > -1$$

$$|\arg Z_k| < \frac{1}{2^{h_i+k_i+1}} A_i^{(k)} \pi, i = 1, \dots, r, \text{ where } A_i^{(k)} \text{ is defined by (1.15). } |y| < 1$$

$$1 + \sum_{j=1}^{\bar{C}} \psi_j^{(i)} + \sum_{j=1}^{D^{(i)}} \delta_j^{(i)} - \sum_{j=1}^{\bar{A}} \theta_j^{(i)} - \sum_{j=1}^{B^{(i)}} \phi_j^{(i)} > 0; i = 1, \dots, v$$

$$\text{and } z_i \neq 0, \sum_{j=1}^A \gamma_j^{(i)} - \sum_{j=1}^C \xi_j^{(i)} + \sum_{j=1}^M \eta_j^{(i)} - \sum_{j=1}^N \epsilon_j^{(i)} < 0, i = 1, \dots, s$$

7. Multivariable H-function

If the multivariable Aleph-function reduces to multivariable H-function defined by Srivastava et al [13,14], the solution of our problem is :

$$C(x, t) = 2^{\rho+\sigma} \sum_{v=0}^{\infty} \sum_{k=0}^v \frac{\Gamma(\alpha + \beta - \gamma + 2v + 3) \Gamma(\alpha + \beta - \gamma + v + 2 + k)}{\Gamma(\alpha + \beta - \gamma + 2v + 2) \Gamma(\alpha - \gamma + k + 2)} \frac{(-v)_k}{\Gamma(\beta + v + 1) k!}$$

$$P_v^{(\alpha+\gamma+1, \beta)}(x) \exp[\lambda_1(v+1)(1 + \alpha + \beta - \gamma + v)t] F_1^{\alpha, \beta, \gamma, \rho, \sigma}(k; N_1, M_1; \dots; N_u, M_u) \quad (7.1)$$

where

$$F^{\alpha, \beta, \gamma, \rho, \sigma}(k; N_1, M_1; \dots; N_u, M_u) = \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_u=0}^{[N_u/M_u]} \sum_{r_1, \dots, r_u=0}^{\infty} \sum_{q=0}^{\infty}$$

$$\sum_{G_i=1}^{\alpha^{(i)}} \sum_{g_i=1}^{\infty} \phi_1 \frac{\prod_{i=1}^s \phi_i u_i^{\eta_{G_i, g_i}} (-)^{\sum_{i=1}^s g_i}}{\prod_{i=1}^s \epsilon_{G_i, g_i}^i} A' B' \frac{[(e_M)]_q y^q}{[(f_N)]_q q!}$$

$$\frac{y_1^{K_1} \dots y_u^{K_u} t_1^{r_1} \dots t_v^{r_v} 2^{gq+wq+\sum_{i=1}^u (f_i+w_i)K_i+\sum_{i=1}^v (a_i+b_i)r_i+\sum_{i=1}^s (c_i+d_i)\eta_{G_i, g_i}}}{r_1! \dots r_v!}$$

$$H_{p+2, q+1; W}^{0, \mathbf{n}+2; V} \left(\begin{array}{c} Z_1 2^{h_1+k_1} \\ \vdots \\ Z_r 2^{h_r+k_r} \end{array} \middle| \begin{array}{l} (\gamma - \rho - \alpha - 1 - k - gq - \sum_{i=1}^u f_i K_i - \sum_{i=1}^v a_i r_i - \sum_{i=1}^s c_i \eta_{G_i, g_i} : h_1, \dots, h_r), \\ \vdots \\ (\gamma - 2 - k - \sigma - \rho - \alpha - \beta - (g+w)q - \sum_{i=1}^u (f_i + w_i) K_i - \sum_{i=1}^v (a_i + b_i) r_i - \sum_{i=1}^s (c_i + d_i) \eta_{G_i, g_i} : h_1 + k_1, \dots, h_r + k_r), B \end{array} \right) \quad (7.2)$$

under the same conditions that (5.2)

8. Aleph-function of two variables

If $r = 2$, the multivariable Aleph-function reduces to Aleph-function of two variables defined by Sharma [10]. The solution of the problem is :

$$C(x, t) = 2^{\rho+\sigma} \sum_{v=0}^{\infty} \sum_{k=0}^v \frac{\Gamma(\alpha + \beta - \gamma + 2v + 3) \Gamma(\alpha + \beta - \gamma + v + 2 + k)}{\Gamma(\alpha + \beta - \gamma + 2v + 2) \Gamma(\alpha - \gamma + k + 2)} \frac{(-v)_k}{\Gamma(\beta + v + 1) k!}$$

$$P_v^{(\alpha+\gamma+1, \beta)}(x) \exp[\lambda_1(v+1)(1 + \alpha + \beta - \gamma + v)t] F^{\alpha, \beta, \gamma, \rho, \sigma}(k; N_1, M_1; \dots; N_u, M_u) \quad (8.1)$$

where

$$F^{\alpha, \beta, \gamma, \rho, \sigma}(k; N_1, M_1; \dots; N_u, M_u) = \sum_{K_1=0}^{[N_1/M_1]} \dots \sum_{K_u=0}^{[N_u/M_u]} \sum_{r_1, \dots, r_v=0}^{\infty} \sum_{q=0}^{\infty}$$

$$\sum_{G_i=1}^{\alpha^{(i)}} \sum_{g_i=1}^{\infty} \phi_1 \frac{\prod_{i=1}^s \phi_i u_i^{\eta_{G_i, g_i}} (-)^{\sum_{i=1}^s g_i}}{\prod_{i=1}^s \epsilon_{G_i, g_i}^i} A' B' \frac{[(e_M)]_q y^q}{[(f_N)]_q q!}$$

$$\frac{y_1^{K_1} \dots y_u^{K_u} t_1^{r_1} \dots t_v^{r_v} 2^{gq+wq+\sum_{i=1}^u (f_i+w_i)K_i+\sum_{i=1}^v (a_i+b_i)r_i+\sum_{i=1}^s (c_i+d_i)\eta_{G_i, g_i}}}{r_1! \dots r_v!}$$

$$N_{p_i+2, q_i+1, \tau_i; R}^{0, \mathbf{n}+2; V} \left(\begin{array}{c} Z_1 2^{h_1+k_1} \\ \vdots \\ Z_2 2^{h_2+k_2} \end{array} \middle| \begin{array}{l} (\gamma - \rho - \alpha - 1 - k - gq - \sum_{i=1}^u f_i K_i - \sum_{i=1}^v a_i r_i - \sum_{i=1}^s c_i \eta_{G_i, g_i} : h_1, h_2), \\ \vdots \\ \dots \end{array} \right)$$

$$\left. \begin{aligned} &(-\sigma - \beta - wq - \sum_{i=1}^u w_i K_i - \sum_{i=1}^v b_i r_i - \sum_{i=1}^s d_i \eta_{G_i, g_i} : k_1, k_2), A \\ &\quad \vdots \\ &(\gamma - 2 - k - \sigma - \rho - \alpha - \beta - (g + w)q - \sum_{i=1}^u (f_i + w_i) K_i - \sum_{i=1}^v (a_i + b_i) r_i - \sum_{i=1}^s (c_i + d_i) \eta_{G_i, g_i} : h_1 + k_1, h_2 + k_2), B \end{aligned} \right) \quad (8.2)$$

under the same conditions that (5.2) with $r = 2$.

9. I-function of two variables

If $r = 2$ and $\tau_i, \tau_{i'}, \tau_{i''} \rightarrow 1$, the multivariable Aleph-function reduces to I-funtion of two variables defined by Sharma et al [9]. The solution of the problem is :

$$C(x, t) = 2^{\rho+\sigma} \sum_{v=0}^{\infty} \sum_{k=0}^v \frac{\Gamma(\alpha + \beta - \gamma + 2v + 3) \Gamma(\alpha + \beta - \gamma + v + 2 + k)}{\Gamma(\alpha + \beta - \gamma + 2v + 2) \Gamma(\alpha - \gamma + k + 2)} \frac{(-v)_k}{\Gamma(\beta + v + 1) k!}$$

$$P_v^{(\alpha+\gamma+1, \beta)}(x) \exp[\lambda_1(v+1)(1+\alpha+\beta-\gamma+v)t] F^{\alpha, \beta, \gamma, \rho, \sigma}(k; N_1, M_1; \dots; N_u, M_u) \quad (9.1)$$

where

$$F^{\alpha, \beta, \gamma, \rho, \sigma}(k; N_1, M_1; \dots; N_u, M_u) = \sum_{K_1=0}^{[N_1/M_1]} \dots \sum_{K_u=0}^{[N_u/M_u]} \sum_{r_1, \dots, r_v=0}^{\infty} \sum_{q=0}^{\infty}$$

$$\sum_{G_i=1}^{\alpha^{(i)}} \sum_{g_i=1}^{\infty} \phi_1 \frac{\prod_{i=1}^s \phi_i u_i^{\eta_{G_i, g_i}} (-)^{\sum_{i=1}^s g_i}}{\prod_{i=1}^s \epsilon_{G_i}^i g_i!} A' B' \frac{[(e_M)]_q y^q}{[(f_N)]_q q!}$$

$$\frac{y_1^{K_1} \dots y_u^{K_u} t_1^{r_1} \dots t_v^{r_v} 2^{2gq+wq+\sum_{i=1}^u (f_i+w_i)K_i+\sum_{i=1}^v (a_i+b_i)r_i+\sum_{i=1}^s (c_i+d_i)\eta_{G_i, g_i}}}{r_1! \dots r_v!}$$

$$I_{p_i+2, q_i+1; R; W}^{0, \mathbf{n}+2; V} \left(\begin{array}{c} Z_1 2^{h_1+k_1} \\ \cdot \\ \cdot \\ Z_2 2^{h_2+k_2} \end{array} \middle| \begin{array}{c} (\gamma - \rho - \alpha - 1 - k - gq - \sum_{i=1}^u f_i K_i - \sum_{i=1}^v a_i r_i - \sum_{i=1}^s c_i \eta_{G_i, g_i} : h_1, h_2), \\ \cdot \\ \cdot \\ \dots \end{array} \right)$$

$$\left. \begin{aligned} &(-\sigma - \beta - wq - \sum_{i=1}^u w_i K_i - \sum_{i=1}^v b_i r_i - \sum_{i=1}^s d_i \eta_{G_i, g_i} : k_1, k_2), A \\ &\quad \vdots \\ &(\gamma - 2 - k - \sigma - \rho - \alpha - \beta - (g + w)q - \sum_{i=1}^u (f_i + w_i) K_i - \sum_{i=1}^v (a_i + b_i) r_i - \sum_{i=1}^s (c_i + d_i) \eta_{G_i, g_i} : h_1 + k_1, h_2 + k_2), B \end{aligned} \right) \quad (9.2)$$

under the same conditions that (5.2) with $r = 2$ and $\tau_i, \tau_{i'}, \tau_{i''} \rightarrow 1$.

6. Conclusion

Specializing the parameters of the multivariable Aleph-function [1], the generalized hypergeometric function, the multivariable polynomials [11], the Srivastava-Daoust function [12] and the multivariable A-function defined by Gautam et al [2], we can obtain a large number of results of problem of advective-diffusion process involving various special functions of one and several variables useful in Mathematics analysis, Applied Mathematics, Physics and

Mechanics. The result derived in this paper is of general character and may prove to be useful in several interesting situations appearing in the literature of sciences.

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