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FIXED POINT THEOREM FOR KHALIMSKY PLANE

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ABSTRACT. The aim of this paper is to prove fixed point theorem for the Khalimsky plane.

Key words: Digital geometry, Khalimsky interval, fixed point property

1. INTRODUCTION

Digital geometry is considered in Z^n where as continuous geometry can be used in R^n. To represent continuous geometrical objects in the computer we are limited to some sort of approximations. There are points in the Euclidean plane that can be described exactly on a computer. By introducing notions as connectedness and continuity on discrete sets we can represent discrete objects with the same accuracy as Euclid had in his geometry. Herman [2] gives a general definition of a digital space.

2. PRELIMINARIES

A topological space X has the fixed point property if every continuous mapping $f : X \to X$ possesses a fixed point, that is there exist a point p such that $f(p) = p$. $C(Z, Z)$ denote the set of all continuous functions from Z to Z $C_{\#}(Z, Z)$ be the subset of $C(Z, Z)$ such that

$$C_{\#}(Z, Z) = \{f \in C(Z, Z),\ \exists\, s \in Z, f(s) > s \text{ and } \exists\, t \in Z, f(t) < t\}$$

then $f \in C(Z, Z)$ has a fixed point if and only if $f \in C_{\#}(Z, Z)$.

3. FIXED POINT THEOREM

In this section we discuss fixed point property of certain subsets of the Khalimsky plane.

Definition 3.1. A topological space X has the fixed point property if every continuous mapping $f : X \to X$ possesses a fixed point; that is, there exists a point p such that $f(p) = p$.

Remark 3.2. By $C(Z, Z)$ we denote the set of all continuous functions from Z to Z, $C_{\#}(Z, Z)$ be the subset of $C(Z, Z)$ such that

$$C_{\#}(Z, Z) = \{f \in C(Z, Z);\ \exists s \in Z, f(s) \geq s \text{ and } \exists\, t \in Z, f(t) \leq t\}$$

then $f \in C(Z, Z)$ has a fixed point if and only if $f \in C_{\#}(Z, Z)$.

Note 3.3. In a finite set X with N points there are N^N self mappings $X \to X$. Out of these $(N-1)^N$ do not have fixed points; thus there are $N^N - (N-1)^N$ mappings which have a fixed point. By introducing a topology on X, the number of continuous mappings C can be evaluated.

	Continuous	Discontinuous	Sum
Fixed point	C	$N^N - (N-1)^N - C$	$N^N - (N-1)^N$
No fixed point	0	$(N-1)^N$	$(N-1)^N$
Sum	C	$N^N - C$	N^N

Example 3.4. For a Khalimsky interval $\{a, a+1\}$ consisting of two points, there are four mappings; the two constant mappings, the identity, and the one interchanging a and $a+1$. The first three have a fixed point; the fourth does not. But it is discontinuous.

	Continuous	Discontinuous	Sum
Fixed point	3	0	3
No fixed point	0	1	1
Sum	3	1	4

Example 3.5. For the Khalimsky square $\{0, 1\}^2 \subset Z^2$, there are $N^N = 4^4 = 256$ self mappings of which $(N-1)^N = 3^4 = 81$ do not have fixed points. The remaining $256 - 81 = 175$ have a fixed point. Of the 16 mappings $\{0, 1\}^2 \to \{0, 1\}$, 6 are continuous. There are therefore $6^2 = 36$ continuous mappings $\{0, 1\}^2 \to \{0, 1\}^2$ and they all have fixed points.

	Continuous	Discontinuous	Sum
Fixed point	36	139	175
No fixed point	0	81	81
Sum	36	220	256

Of the 6 continuous mappings $\{0,1\}^2 \to \{0,1\}$, five maps $\{0,0\}$ to 0; the remaining one is the constant 1. Therefore, of the 36 continuous mappings $\{0,1\}^2 \to \{0,1\}^2$, $\{0,0\}$ is a fixed point except when one of the component is the constant 1. Thus they all have a fixed point. We shall now prove that a continuous mapping of an interval into itself has a fixed point.

Theorem 3.6. *Every bounded Khalimsky interval has the fixed point property.*

Proof. Let $f : I \to I$ be a continuous mapping, where $I = [a, b]_Z$ is a bounded interval. Extend f to a mapping $g : Z \to Z$ by defining $g(x) = f(a)$ for $x < a$ and $g(x) = f(b)$ for $x > b$. Then g is continuous and g belongs to $C_{\#}(Z, Z)$. Thus it has a fixed point $p \in Z$, but $p \in \operatorname{im} g \subset I$, p is a fixed point of f also. $\square$

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