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Some integrals involving special functions, generalized polynomials and multivariable A-function

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ABSTRACT
The aim of this paper is to derive some integrals pertaining to a product of multivariable A-function given by Gautam and Asgar, generalized multivariable polynomials defined by Srivastava, multivariable Aleph-function and $\bar{H}$-function introduced by Inayat-Hussain. The integrals thus obtained is believed to be one of the most general integral established so far. The findings of this paper are sufficiently general nature and are capable of yielding numerous (known and new) results involving classical orthogonal polynomials scattered in the literature.

Keywords: Serie expansion of the multivariable Aleph-function, multivariable A-function, class of multivariable polynomials, multivariable H-function, Aleph-function,A-function, $\bar{H}$-function.

2010 Mathematics Subject Classification : 33C05, 33C60

1.Introduction

Recently, Gupta and Jangid [6] have studied some integrals involving generalized multivariable polynomials and the generalized H-function. The aim of this paper is to establish some integrals pertaining the series expansion of multivariable Aleph-function, generalized multivariable polynomials, the multivariable A-function and the $\bar{H}$-function.

The $\bar{H}$-function occurring in the present paper was introduced by Inayat Hussain [7] and studied by Buschman and Srivastava [1] and others. The following series representation for the H-function can easily be obtained from a result given by Rathie [9].

$$\bar{H}(z)=\bar{H}_{P,Q}^{M,N}\left(\mathrm{z}\ \middle|\ \begin{array}{c}(\mathrm{e}_j,E_j;\epsilon_j)_{1,N},(E_j,E_j)_{N+1,P}\\(\mathrm{f}_j,F_j)_{1,M},(f_j,F_j;\mathfrak{F}_j)_{M+1,Q}\end{array}\right)=\frac{1}{2\pi\omega}\int_L\ \Omega_{P,Q}^{M,N}(s)z^s\mathrm{d}s \tag{1.1}$$

for all z different to 0 and

$$\Omega\,{}_{P,Q}^{M,N}(s)=\frac{\prod_{j=1,j\neq g}^{M}\Gamma(f_j-F_js)\prod_{j=1}^{N}\Gamma^{\epsilon_j}(1-e_j+E_js)}{\prod_{j=N+1}^{p}\Gamma(e_j-E_js)\prod_{j=M+1}^{q}\Gamma^{\mathfrak{F}_j}(1-f_j+F_js)} \tag{1.2}$$

The serie representation of $\bar{H}$-function is

$$\bar{H}(z)=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\Omega_{P,Q}^{M,N}(\eta_{G,g})\frac{z^{\eta_{G,g}}(-)^g}{\delta_G g!} \tag{1.3}$$

where

$$\eta_{G,g}=\frac{f_g+G}{F_G} \tag{1.4}$$

The sufficient conditions for the absolute convergence of the defining integral for $\bar{H}$-function given by (1.1) have been given by Gupta, Jain and Agrawal [5]. The behavior of the $\bar{H}_{P,Q}^{M,N}(z)$ function for small values of z is given by Saxena et al. [10, p.112, eq.(2.3)]. It is assumed that $\bar{H}$-function occurring at various places in the present paper satisfy the conditions of existence corresponding appropriately to those mentioned by Gupta, Jain and Agrawal [5].

The generalized polynomials of multivariables defined by Srivastava [13], is given in the following manner :

$$S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[y_1,\cdots,y_v] = \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\frac{(-N_1)_{\mathfrak{M}_1K_1}}{K_1!}\cdots\frac{(-N_v)_{\mathfrak{M}_vK_v}}{K_v!}A[N_1,K_1;\cdots;N_v,K_v]y_1^{K_1}\cdots y_v^{K_v} \quad (1.5)$$

where $\mathfrak{M}_1,\cdots,\mathfrak{M}_v$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_v,K_v]$ are arbitrary constants, real or complex. We shall note.

We shall note $a_v = \frac{(-N_1)_{\mathfrak{M}_1K_1}}{K_1!}\cdots\frac{(-N_v)_{\mathfrak{M}_vK_v}}{K_v!}A[N_1,K_1;\cdots;N_v,K_v]$ (1.6)

The Aleph-function of several variables is an extension the multivariable I-function defined by Sharma and Ahmad [11] , itself is a generalisation of G and H-functions of several variables defined by Srivastava et Panda [14,15]. The multiple Mellin-Barnes integral occurring in this paper will be referred to as the multivariables Aleph-function of $r-$ variables throughout our present study and will be defined and represented as follows.

We have : $\aleph(z_1,\cdots,z_r) = \aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & [(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}], \\ \cdot & \cdot \\ \cdot & \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots , \\ \cdot & \\ \mathrm{z}_r & \end{array}\right.$

$$[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}] : [(\mathrm{c}_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;$$
$$\cdot \qquad\qquad \cdot$$
$$[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}] : [(\mathrm{d}_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;$$

$$\left.\begin{array}{c} [(\mathrm{c}_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}] \\ \cdot \\ [(\mathrm{d}_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}] \end{array}\right) = \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,\cdots,s_r)\prod_{k=1}^{r}\phi_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \quad (1.7)$$

with $\omega = \sqrt{-1}$

$$\phi(s_1,\cdots,s_r) = \frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{\sum_{i=1}^{R}[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k)]} \quad (1.8)$$

and

$$\phi_k(s_k) = \frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \quad (1.9)$$

The condition for absolute convergence of multiple Mellin-Barnes type contour can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)} - \tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)} - \tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)} + \sum_{j=1}^{n_k}\gamma_j^{(k)} - \tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)} + \sum_{j=1}^{m_k}\delta_j^{(k)} - \tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)} > 0,$$

with $k = 1,\cdots,r, i = 1,\cdots,R \ , i^{(k)} = 1,\cdots,R^{(k)}$ (1.10)

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

If all the poles of (1.4) are simples ,then the integral (1.6) can be evaluated with the help of the residue theorem to give

$$\aleph(z_1,\cdots,z_r)=\sum_{G_k=1}^{m_k}\sum_{g_k=0}^{\infty}\phi\frac{\prod_{k=1}^r\phi_k z_k^{\eta_{G_k,g_k}}(-)^{\sum_{k=1}^r g_k}}{\prod_{k=1}^r\delta_{G^{(k)}}^{(k)}\prod_{k=1}^r g_k!} \tag{1.11}$$

where

$$\phi=\frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^r\alpha_j^{(i)}S_k)}{\sum_{i=1}^R[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^r\alpha_{ji}^{(k)}S_k)\prod_{j=1}^{q_k}\Gamma(1-b_{ji}+\sum_{k=1}^r\beta_{ji}^{(k)}S_k)]} \tag{1.12}$$

$$\phi_k=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}S_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}S_k)}{\sum_{i^{(i)}=1}^{R^{(i)}}[\tau_{i^{(i)}}\prod_{j=m_i+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}S_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}S_k)]} \tag{1.13}$$

$$\sum_{G_k=1}^{m_i}\sum_{g_k=0}^{\infty}=\sum_{G_1,\cdots,G_r=1}^{m_1,\cdots,m_r}\sum_{g_1,\cdots,g_r=0}^{\infty} \tag{1.14}$$

and

$$S_k=\eta_{G_k,g_k}=\frac{d_{g_k}^{(k)}+G_k}{\delta_{g_k}^{(k)}}\text{ for }k=1,\cdots,r \tag{1.15}$$

which is valid under the following conditions : $\epsilon_{M_k}^{(k)}[p_j^{(k)}+p_k']\neq\epsilon_j^{(k)}[p_{M_k}+g_k]$

The multivariable A-function defined by Gautam and Asgar [4] is an extension of the multivariable H-function defined by Srivastava et al [14,15]. The multivariable A-function is defined in term of multiple Mellin-Barnes type integral.

$$A(Z_1,\cdots,Z_s)=A_{\mathbf{p},\mathbf{q}:p_1,q_1;\cdots;p_s,q_s}^{\mathbf{m},\mathbf{n}:m_1,n_1;\cdots;m_s,n_s}\left(\begin{array}{c|c}\mathrm{z}_1 & (\mathrm{a}_j;A_j^{(1)},\cdots,A_j^{(s)})_{1,\mathbf{p}}:\\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_s & (\mathrm{b}_j;B_j^{(1)},\cdots,B_j^{(s)})_{1,\mathbf{q}}:\end{array}\right.$$

$$\left.\begin{array}{c}(\mathrm{c}_j^{(1)},C_j^{(1)})_{1,p_1};\cdots;(c_j^{(s)},C_j^{(s)})_{1,p_s}\\ \\ (\mathrm{d}_j^{(1)},D_j^{(1)})_{1,q_1};\cdots;(d_j^{(s)},D_j^{(s)})_{1,q_s}\end{array}\right) \tag{1.16}$$

$$=\frac{1}{(2\pi\omega)^s}\int_{L_1'}\cdots\int_{L_s'}\phi'(u_1,\cdots,u_s)\prod_{i=1}^s\theta_i'(u_i)Z_i^{u_i}\mathrm{d}u_1\cdots\mathrm{d}u_s \tag{1.17}$$

where $\phi'(u_1,\cdots,u_s)$, $\theta_i'(u_i)$ for $i=1,\cdots,s$ are given by :

$$\phi'(u_1,\cdots,u_s)=\frac{\prod_{j=1}^{\mathbf{m}}\Gamma(b_j-\sum_{i=1}^s B_j^{(i)}u_i)\prod_{j=1}^{\mathbf{n}}\Gamma(1-a_j+\sum_{i=1}^s A_j^{(i)}u_j)}{\prod_{j=\mathbf{n}+1}^{\mathbf{p}}\Gamma(a_j-\sum_{i=1}^s A_j^{(i)}u_j)\prod_{j=\mathbf{m}+1}^{\mathbf{q}}\Gamma\left(1-b_j+\sum_{i=1}^s B_j^{(i)}u_j\right)} \tag{1.18}$$

$$\theta_i'(u_i) = \frac{\prod_{j=1}^{n_i}\Gamma(1-c_j^{(i)}+C_j^{(i)}u_i)\prod_{j=1}^{m_i}\Gamma(d_j^{(i)}-D_j^{(i)}u_i)}{\prod_{j=n_i+1}^{p_i}\Gamma(c_j^{(i)}-C_j^{(i)}u_i)\prod_{j=m_i+1}^{q_i}\Gamma(1-d_j^{(i)}+D_j^{(i)}u_i)} \tag{1.19}$$

Here $\mathbf{m},\mathbf{n},\mathbf{p},\mathbf{q},m_i,n_i,p_i,c_i \in \mathbb{N}^*; i=1,\cdots,r$; $a_j,b_j,c_j^{(i)},d_j^{(i)},A_j^{(i)},B_j^{(i)},C_j^{(i)},D_j^{(i)} \in \mathbb{C}$

The multiple integral defining the A-function of r variables converges absolutely if :

$$|arg(\Omega_i')Z_k| < \frac{1}{2}\eta_k'\pi, \xi'^* = 0, \eta_i' > 0 \tag{1.20}$$

$$\Omega_i' = \prod_{j=1}^{\mathbf{p}}\{A_j^{(i)}\}^{A_j^{(i)}}\prod_{j=1}^{\mathbf{q}}\{B_j^{(i)}\}^{-B_j^{(i)}}\prod_{j=1}^{q_i}\{D_j^{(i)}\}^{D_j^{(i)}}\prod_{j=1}^{p_i}\{C_j^{(i)}\}^{-C_j^{(i)}}; i=1,\cdots,s \tag{1.21}$$

$$\xi_i'^* = Im\big(\sum_{j=1}^{\mathbf{p}}A_j^{(i)} - \sum_{j=1}^{\mathbf{q}}B_j^{(i)} + \sum_{j=1}^{q_i}D_j^{(i)} - \sum_{j=1}^{p_i}C_j^{(i)}\big); i=1,\cdots,s \tag{1.22}$$

$$\eta_i' = Re\left(\sum_{j=1}^{\mathbf{n}}A_j^{(i)} - \sum_{j=\mathbf{n}+1}^{\mathbf{p}}A_j^{(i)} + \sum_{j=1}^{\mathbf{m}}B_j^{(i)} - \sum_{j=\mathbf{m}+1}^{\mathbf{q}}B_j^{(i)} + \sum_{j=1}^{m_i}D_j^{(i)} - \sum_{j=m_i+1}^{q_i}D_j^{(i)} + \sum_{j=1}^{n_i}C_j^{(i)} - \sum_{j=n_i+1}^{p_i}C_j^{(i)}\right)$$
$$i=1,\cdots,s \tag{1.23}$$

For convenience, we shall note.

$$X = m_1,n_1;\cdots;m_s,n_s \quad : Y = p_1,q_1;\cdots;p_s,q_s \tag{1.24}$$

$$\mathbb{A} = (a_j;A_j^{(1)},\cdots,A_j^{(s)})_{1,\mathbf{p}} : (\mathrm{c}_j^{(1)},C_j^{(1)})_{1,p_1};\cdots;(c_j^{(s)},C_j^{(s)})_{1,p_s} \tag{1.25}$$

$$\mathbb{B} = (b_j;B_j^{(1)},\cdots,B_j^{(s)})_{1,\mathbf{q}} : \quad (\mathrm{d}_j^{(1)},D_j^{(1)})_{1,q_1};\cdots;(d_j^{(s)},D_j^{(s)})_{1,q_s} \tag{1.26}$$

2. Required integrals

We have the following form of the known result ([8], p.70, Eq.(3.1.2)).

Lemma 1

$$\int_a^b \frac{(t-a)^{\lambda-1}(b-t)^{\mu-1}}{[b-a+u(t-a)+w(b-t)]^{\lambda+\mu}}\mathrm{d}t = (b-a)^{-1}(1+u)^{-\lambda}(1+w)^{-\mu}\frac{\Gamma(\lambda)\Gamma(\mu)}{\Gamma(\lambda+\mu)} \tag{2.1}$$

Provided that $Re(\lambda)>0, Re(\mu)>0, b\neq a$ and the constants u and w are such that none of the expression $1+u, 1+w, b-a+u(t-a)+w(b-t)$, where $a\leqslant t\leqslant b$ is zero.

The following result ([8], p.72, Eq.(3.1.18)) and ([2], p.192, Eq. (46)) given below in a slightly modified form instead of (2.1)

$$\int_a^b \frac{(t-a)^{\lambda-1}(b-t)^{\mu-1}}{[b-a+u(t-a)+w(b-t)]^{\lambda+\mu}}\,{}_2F_1\left[\begin{array}{c}\alpha,\beta \\ \mu\end{array} ; \frac{(b-t)(1+w)}{\{b-a+u(t-a)+w(b-t)\}}\right]\mathrm{d}t =$$

$$(b-a)^{-1}(1+u)^{-\lambda}(1+w)^{-\mu}\frac{\Gamma(\lambda)\Gamma(\mu)\Gamma(\lambda+\mu-\alpha-\beta)}{\Gamma(\lambda+\mu-\alpha)\Gamma(\lambda+\mu-\beta)} \tag{2.2}$$

where $Re(\lambda+\mu-\alpha-\beta)>0,\ Re(\lambda)>0, Re(\mu)>0, b\neq a$ and the constants u and w are such that none of the expression$1+u, 1+w, b-a+u(t-a)+w(b-t)$, where $a\leqslant t\leqslant b$ is zero.

3. Main integrals

Let $X(t;\alpha,\beta)=\dfrac{(t-a)^{\alpha}(b-t)^{\beta}}{[b-a+u(t-a)+w(b-t)]^{\alpha+\beta}}$

We shall establish the following general integral formula :

Theorem 1

$$\int_a^b \frac{(t-a)^{\lambda-1}(b-t)^{\mu-1}}{[b-a+u(t-a)+w(b-t)]^{\lambda+\mu}}\bar{H}\left(xX(t;\lambda',\mu')\right)S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\begin{pmatrix}\mathrm{y}_1X(t;\lambda_1'',\mu_1'')\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{y}_vX(t;\lambda_v'',\mu_v'')\end{pmatrix}$$

$$\aleph\begin{pmatrix}\mathrm{z}_1X(t;\lambda_1',\mu_1')\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rX(z;\lambda_r',\mu_r')\end{pmatrix}A\begin{pmatrix}\mathrm{z}_1'X(t;\lambda_1,\mu_1)\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_s'X(t;\lambda_s,\mu_s)\end{pmatrix}\mathrm{d}t=(b-a)^{-1}(1+u)^{-\lambda}(1+w)^{-\mu}$$

$$\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{G_i=1}^{m_i}\sum_{g_i=0}^{\infty}\phi\frac{\prod_{i=1}^r\phi_i z_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^r g_i}}{\prod_{i=1}^r\delta_{G^{(i)}}^{(i)}\prod_{i=1}^r g_i!}\ \Omega_{P,Q}^{M,N}(\eta_{G,g})\frac{x^{\eta_{G,g}}(-)^g}{\delta_G g!}$$

$$a_v y_1^{K_1}\cdots y_v^{K_v}(1+u)^{-(\lambda+\lambda'\eta_{G,g}+\sum_{i=1}^v\lambda_i''K_i+\sum_{j=1}^s\lambda_j'\eta_{G_j,g_j})}\,(1+w)^{-(\mu+\mu'\eta_{G,g}+\sum_{i=1}^v\mu_i''K_i+\sum_{j=1}^s\mu_j'\eta_{G_j,g_j})}$$

$$A_{\mathbf{p}+2,\mathbf{q}+1;Y}^{\mathbf{m},\mathbf{n}+2;X}\left(\begin{matrix}\mathrm{z}_1'(1+u)^{-\lambda_1}(1+w)^{-\mu_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_s'(1+u)^{-\lambda_s}(1+w)^{-\mu_s}\end{matrix}\ \middle|\ \begin{matrix}\mathrm{A}\\ \cdot\\ \cdot\\ \mathrm{B}\end{matrix}\right) \tag{3.1}$$

Provided

$min\{\lambda',\mu',\lambda_i'',\mu_i'',\sigma_i'',\lambda_j',\mu_j',\lambda_k,\mu_k\}>0$; $i=1,\cdots,v; j=1,\cdots,r; k=1,\cdots,s$

$$Re\left(\lambda+\sum_{i=1}^r\lambda_i'\eta_{G_i,g_i}+\lambda'\eta_{G,g}\right)+\sum_{k=1}^s\lambda_k\min_{1\leqslant K\leqslant m_k}Re\left(\frac{d_K^{(k)}}{D_K^{(k)}}\right)>0$$

$$Re\left(\mu+\sum_{i=1}^r\mu_i'\eta_{G_i,g_i}+\mu'\eta_{G,g}\right)+\sum_{k=1}^s\mu_k\min_{1\leqslant K\leqslant m_k}Re\left(\frac{d_K^{(k)}}{D_K^{(k)}}\right)>0$$

$b\neq a$ and the constants u and w are such that none of the expression$1+u, 1+w, b-a+u(t-a)+w(b-t)$, where $a\leqslant t\leqslant b$ is zero.

$|arg(\Omega'_i)z'_k| < \dfrac{1}{2}\eta'_k\pi, \xi'^{*} = 0, \eta'_i > 0$ and the multiple series in the left-hand side of (3.1) converges absolutely, where

$$A = \left(1-\lambda-\lambda'\eta_{G,g}-\sum_{i=1}^{v}\lambda''_i K_i-\sum_{j=1}^{r}\lambda'_j\eta_{G_j,g_j};\lambda_1,\cdots,\lambda_s\right)$$

$$,\left(1-\mu-\mu'\eta_{G,g}-\sum_{i=1}^{v}\mu''_i K_i-\sum_{j=1}^{r}\mu'_j\eta_{G_j,g_j};\mu_1,\cdots,\mu_s\right),\mathbb{A} \tag{3.2}$$

$$B = \left(1-(\lambda+\mu)-(\lambda'+\mu')\eta_{G,g}-\sum_{i=1}^{v}(\lambda''_i+\mu''_i)K_i-\sum_{j=1}^{r}(\lambda'_j+\mu'_j)\eta_{G_j,g_j};\lambda_1+\mu_1,\cdots,\lambda_s+\mu_s\right),\mathbb{B} \tag{3.3}$$

Let $Y(t;\rho) = \dfrac{(t-a)^{\rho}}{[b-a+u(t-a)+w(b-t)]^{\rho}}$

We have the unified following result

Theorem 2

$$\int_a^b \frac{(t-a)^{\lambda-1}(b-t)^{\mu-1}}{[b-a+u(t-a)+w(b-t)]^{\lambda+\mu}}\ {}_2F_1\left[\begin{array}{c} \alpha,\beta \\ \mu \end{array};\frac{(b-t)(1+w)}{\{b-a+u(t-a)+w(b-t)\}}\right]\bar{H}\left(xY(t;\rho')\right)$$

$$S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\left(\begin{array}{c} \mathrm{y}_1Y(t;\rho''_1) \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{y}_vY(t;\rho''_v) \end{array}\right)\aleph\left(\begin{array}{c} \mathrm{z}_1Y(t;\rho'_1) \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}'_rY(t;\rho'_r) \end{array}\right)A\left(\begin{array}{c} \mathrm{z}'_1Y(t;\rho_1) \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_sX(t;\rho_s) \end{array}\right)\mathrm{d}t = (b-a)^{-1}(1+u)^{-\lambda}(1+w)^{-\mu}$$

$$\Gamma(u)\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{G_i=1}^{m_i}\sum_{g_i=0}^{\infty}\phi\frac{\prod_{i=1}^r\phi_i z_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^r g_i}}{\prod_{i=1}^r\delta_{G^{(i)}}^{(i)}\prod_{i=1}^r g_i!}\Omega_{P,Q}^{M,N}(\eta_{G,g})\frac{x^{\eta_{G,g}}(-)^g}{\delta_G g!}$$

$$a_v y_1^{K_1}\cdots y_v^{K_v}(1+u)^{-(\lambda+\rho'\eta_{G,g}+\sum_{i=1}^v\rho''_iK_i+\sum_{j=1}^s\rho'_j\eta_{G_j,g_j})}A_{\mathbf{p}+2,\mathbf{q}+2;Y}^{\mathbf{m},\mathbf{n}+2;X}\left(\begin{array}{c|c} \mathrm{z}'_1(1+u)^{-\rho_1} & \mathrm{A'} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \\ \mathrm{z}'_s(1+u)^{-\rho_s} & \mathrm{B'} \end{array}\right) \tag{3.4}$$

Provided

$min\{\rho',\rho''_i,\rho'_j,\rho_k\} > 0$; $i = 1,\cdots,v; j = 1,\cdots,r; k = 1,\cdots,s$

$Re\left(\lambda+\sum_{i=1}^{r}\rho'_i\eta_{G_i,g_i}+\rho'\eta_{G,g}\right)+\sum_{k=1}^{s}\rho_k\min_{1\leqslant K\leqslant m_k}Re\left(\dfrac{d_K^{(k)}}{D_K^{(k)}}\right) > 0$; $Re(\mu) > 0; Re(\lambda+\mu-\alpha-\beta) > 0$

$b \neq a$ and the constants u and w are such that none of the expression$1+u, 1+w, b-a+u(t-a)+w(b-t)$, where $a \leqslant t \leqslant b$ is zero.

$|arg(\Omega_i')z_k z| < \frac{1}{2}\eta_k'\pi, \xi'^* = 0, \eta_i' > 0$ and the multiple series in the left-hand side of (3.1) converges absolutely, where

$$A' = \left(1-\lambda-\rho'\eta_{G,g}-\sum_{i=1}^{v}\rho_i''K_i-\sum_{j=1}^{r}\rho_j'\eta_{G_j,g_j};\rho_1,\cdots,\rho_s\right)$$

$$,\left(1+\alpha+\beta-\lambda-\mu-\rho'\eta_{G,g}-\sum_{i=1}^{v}\rho_i''K_i-\sum_{j=1}^{r}\rho_j'\eta_{G_j,g_j};\rho_1,\cdots,\rho_s\right), \mathbb{A} \tag{3.5}$$

$$B' = \left(1+\alpha-\lambda-\mu-\rho'\eta_{G,g}-\sum_{i=1}^{v}\rho_i''K_i-\sum_{j=1}^{r}\rho_j'\eta_{G_j,g_j};\rho_1,\cdots,\rho_s\right),$$

$$\left(1+\beta-\lambda-\mu-\rho'\eta_{G,g}-\sum_{i=1}^{v}\rho_i''K_i-\sum_{j=1}^{r}\rho_j'\eta_{G_j,g_j};\rho_1,\cdots,\rho_s\right), \mathbb{B} \tag{3.6}$$

Proof of (3.1)

To evaluate the main integral (3.1), first we replace the class of multivariable polynomials $S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[.]$, the multivariable $\aleph$-function and the function $\bar{H}_{P,Q}^{M,N}(z)$ occurring in the left-hand side of the main integral in term of series with the help of equations (1.11), (1.5) and (1,3) respectively. Now we express the multivariable A-function in Mellin-Barnes integrals contour with the help of (1.17). Nest, we change the order of the $(t_1,\cdots,t_s)$-integrals and summations (which is justified under the conditions stated) and we obtain the following result (say L.H.S.) :

$$\text{L.H.S} = \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{G_i=1}^{m_i}\sum_{g_i=0}^{\infty}\phi\frac{\prod_{i=1}^{r}\phi_i z_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^{r}g_i}}{\prod_{i=1}^{r}\delta_{G^{(i)}}^{(i)}\prod_{i=1}^{r}g_i!}$$

$$\Omega_{P,Q}^{M,N}(\eta_{G,g})\frac{x^{\eta_{G,g}}(-)^g}{\delta_G g!}\ a_v y_1^{K_1}\cdots y_v^{K_v}\frac{1}{(2\pi\omega)^s}\int_{L_1'}\cdots\int_{L_s'}\phi'(t_1,\cdots,t_s)\prod_{i=1}^{s}\theta_i'(t_i)z_i'^{t_i}$$

$$\left[\int_a^b(b-t)^{\mu+\mu''\eta_{G,g}+\sum_{i=1}^{v}K_i\mu_i''+\sum_{j=1}^{r}\mu_j'\eta_{G_j,g_j}+\sum_{k=1}^{s}\mu_k t_i-1}(t-a)^{\lambda+\lambda'\eta_{G,g}+\sum_{i=1}^{v}K_i\lambda_i''+\sum_{j=1}^{r}\lambda_j'\eta_{G_j,g_j}+\sum_{k=1}^{s}\lambda_k t_i-1}\right.$$

$$\left.[b-a+u(t-a)+w(b-t)]^{-\left(\lambda+\mu+(\lambda'+\mu')\eta_{G,g}+\sum_{i=1}^{v}K_i(\lambda_i''+\mu_i'')+\sum_{j=1}^{r}(\lambda_j'+\mu_j')\eta_{G_j,g_j}+\sum_{k=1}^{s}(\lambda_k+\mu_k)t_k\right)}\mathrm{d}t\right]\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{3.7}$$

Now we evaluate the t-integral occurring in (3.7) with help of the lemma 1, finally reinterpreting the result thus obtained in terms of A-function of s-variables. We obtain the right side of (3.1) after algebric manipulations.

The integral (3.4) can be proved by proceeding in the similar manner with the help of the lemma 2.

4. Corollaries

If $S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[.] \rightarrow S_{\mathfrak{N}}^{\mathfrak{M}}[.]$ [12], the multivariable $\aleph$-function reduces to $\aleph$-function of one variable defined by Sudland [17,18], and the multivariable A-function reduces to A-function of one function defined by Gautam and Asgar [3], we get the following corollaries

Corollary 1

$$\int_a^b \frac{(t-a)^{\lambda-1}(b-t)^{\mu-1}}{[b-a+u(t-a)+w(b-t)]^{\lambda+\mu}}\bar{H}\left(xX(t;\lambda',\mu')\right)S_N^{\mathfrak{M}}\left(\ \mathrm{y}_1X(t;\lambda_1'',\mu_1'')\ \right)$$

$$\aleph\left(\ \mathrm{z}_1X(t;\lambda_1',\mu_1')\ \right)A\left(\ \mathrm{z}_1'X(t;\lambda_1,\mu_1)\ \right)\mathrm{d}t=(b-a)^{-1}(1+u)^{-\lambda}(1+w)^{-\mu}\sum_{K=0}^{[\mathfrak{N}/\mathfrak{M}]}\sum_{G=1}^{M}\sum_{g=0}^{\infty}$$

$$\sum_{G_1=1}^{m_1}\sum_{g_1=0}^{\infty}\frac{(-\mathfrak{N})_{\mathfrak{M}K}A_{\mathfrak{N},K}}{K!}\phi_1\frac{z_1^{\eta_{G_1,g_1}}(-)^{g_1}}{\delta_{G_1}g_1!}\,a_v y_1^K(1+u)^{-(\lambda+\lambda'\eta_{G,g}+\lambda_1''K+\lambda_1'\eta_{G_1,g_1})}\,\Omega_{P,Q}^{M,N}(\eta_{G,g})\frac{x^{\eta_{G,g}}(-)^g}{\delta_G g!}$$

$$(1+w)^{-(\mu+\mu'\eta_{G,g}+\mu_1''K+\mu_1'\eta_{G_1,g_1})}\,\Omega_{P,Q}^{M,N}(\eta_{G,g})\frac{x^{\eta_{G,g}}(-)^g}{\delta_G g!}A_{p_1+2,q_1+1}^{m_1,n_1+2}\left(\ \mathrm{z}_1'(1+u)^{-\lambda_1}(1+w)^{-\mu_1}\ \middle|\ \begin{matrix}\mathrm{A}_1\\ \cdot\\ \cdot\\ \mathrm{B}_1\end{matrix}\right) \tag{4.1}$$

Provided

$min\{\lambda',\mu',\lambda_1'',\mu_1'',\sigma_1'',\lambda_1',\mu_1',\lambda_1,\mu_1\}>0$

$$Re\,(\lambda+\lambda_1'\eta_{G_1,g_1}+\lambda'\eta_{G,g})+\lambda_1\min_{1\leqslant K\leqslant m_1}Re\left(\frac{d_K^{(1)}}{D_K^{(1)}}\right)>0$$

$$Re\,(\mu+\mu_1'\eta_{G_1,g_1}+\mu'\eta_{G,g})+\mu_1\min_{1\leqslant K\leqslant m_1}Re\left(\frac{d_K^{(1)}}{D_K^{(1)}}\right)>0$$

$b\neq a$ and the constants u and w are such that none of the expression$1+u,1+w,b-a+u(t-a)+w(b-t)$, where $a\leqslant t\leqslant b$ is zero.

$|arg(\Omega_i')z_1'|<\frac{1}{2}\eta_1'\pi,\xi'^*=0,\eta_i'>0$ and the multiple series in the left-hand side of (3.1) converges absolutely, where

$$A=(1-\lambda-\lambda'\eta_{G,g}-\lambda_1''K-\lambda_1'\eta_{G_1,g_1};\lambda_1),(1-\mu-\mu'\eta_{G,g}-\mu_1''K-\mu_1'\eta_{G_1,g_1};\mu_1),(c_j^{(1)},C_j^{(1)})_{1,p_1} \tag{4.2}$$

$$B=(1-(\lambda+\mu)-(\lambda'+\mu')\eta_{G,g}-K(\lambda_1''+\mu_1'')-(\lambda_1'+\mu_1')\eta_{G_1,g_1};\lambda_1+\mu_1)\ ,(d_j^{(1)},D_j^{(1)})_{1,q_1} \tag{4.3}$$

Corollary 2

$$\int_a^b \frac{(t-a)^{\lambda-1}(b-t)^{\mu-1}}{[b-a+u(t-a)+w(b-t)]^{\lambda+\mu}}\,{}_2F_1\left[\begin{matrix}\alpha,\beta\\ \mu\end{matrix}\ ;\frac{(b-t)(1+w)}{\{b-a+u(t-a)+w(b-t)\}}\right]H\left(xY(t;\rho')\right)$$

$$S_N^{\mathfrak{M}}\left(\ \mathrm{y}_1X(t;\lambda_1'',\mu_1'')\ \right)\aleph\left(\ \mathrm{z}_1X(t;\lambda_1',\mu_1')\ \right)A\left(\ \mathrm{z}_1'X(t;\lambda_1,\mu_1)\ \right)\mathrm{d}t=(b-a)^{-1}(1+u)^{-\lambda}(1+w)^{-\mu}$$

$$\sum_{K=0}^{[\mathfrak{N}/\mathfrak{M}]}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{G_1=1}^{m_1}\sum_{g_1=0}^{\infty}\frac{(-\mathfrak{N})_{\mathfrak{M}K}A_{\mathfrak{N},K}}{K!}\phi_1\frac{z_1^{\eta_{G_1,g_1}}(-)^{g_1}}{\delta_{G_1}g_1!}\,a_v y_1^K(1+u)^{-(\lambda+\lambda'\eta_{G,g}+\lambda_1''K+\lambda_1'\eta_{G_1,g_1})}$$

$$\Omega_{P,Q}^{M,N}(\eta_{G,g})\frac{x^{\eta_{G,g}}(-)^g}{\delta_G g!}A_{p_1+2,q_1+2}^{m_1,n_1+2}\left(\begin{array}{c} z_1'(1+u)^{-\lambda_1}(1+w)^{-\mu_1} \end{array}\middle|\begin{array}{c} \mathrm{A}_1' \\ \cdot \\ \cdot \\ \mathrm{B}_1' \end{array}\right) \tag{4.4}$$

Provided

$min\{\rho,\rho_1'',\rho_1',\rho_1\}>0$

$$Re\left(\lambda+\rho_1'\eta_{G_1,g_1}+\rho'\eta_{G,g}\right)+\rho_1\min_{1\leqslant K\leqslant m_1}Re\left(\frac{d_K^{(1)}}{D_K^{(1)}}\right)>0\ ;\ Re(\mu)>0; Re(\lambda+\mu-\alpha-\beta)>0$$

$b\neq a$ and the constants u and w are such that none of the expression$1+u, 1+w, b-a+u(t-a)+w(b-t)$, where $a\leqslant t\leqslant b$ is zero.

$|arg(\Omega_i')z_1'|<\frac{1}{2}\eta_1'\pi, \xi'^*=0, \eta_i'>0$ and the multiple series in the left-hand side of (3.1) converges absolutely, where

$$A'=(1-\lambda-\rho'\eta_{G,g}-\rho_1''K-\rho_1'\eta_{G_1,g_1};\rho_1),(1+\alpha+\beta-\lambda-\mu-\rho'\eta_{G,g}-\rho_1''K-\rho_1'\eta_{G_1,g_1};\rho_1),(c_j^{(1)},C_j^{(1)})_{1,p_1} \tag{4.5}$$

$$B'=(1+\alpha-\lambda-\mu-\rho'\eta_{G,g}-\rho_1''K-\rho_1'\eta_{G_1,g_1};\rho_1);\ (1+\beta-\lambda-\mu-\rho'\eta_{G,g}-\rho_1''K-\rho_1'\eta_{G_1,g_1};\rho_1)$$

$$(d_j^{(1)},D_j^{(1)})_{1,q_1} \tag{4.6}$$

By applying our result given in (4.1) and (4.4) to the case the Laguerre polynomials ([19], page 101, eq.(15.1.6)) and ([16] ,page 159) and by setting

$$S_N^1(x)\rightarrow L_N^{\alpha'}(x)$$

In which case $\mathfrak{M}=1, A_{N,K}=\binom{N+\alpha'}{N}\frac{1}{(\alpha'+1)_K}$ we have the following two corollaries.

Corollary 3

$$\int_a^b\frac{(t-a)^{\lambda-1}(b-t)^{\mu-1}}{[b-a+u(t-a)+w(b-t)]^{\lambda+\mu}}\,\bar{H}\left(xX(t;\lambda',\mu')\right)L_N^{\alpha'}\left(\ \mathrm{y}_1X(t;\lambda_1'',\mu_1'')\ \right)$$

$$\aleph\left(\ \mathrm{z}_1X(t;\lambda_1',\mu_1')\ \right)A\left(\ \mathrm{z}_1'X(t;\lambda_1,\mu_1)\ \right)\mathrm{d}t=(b-a)^{-1}(1+u)^{-\lambda}(1+w)^{-\mu}$$

$$\sum_{K=0}^{N}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{G_1=1}^{m_1}\sum_{g_1=0}^{\infty}\binom{N+\alpha'}{N}\frac{1}{(\alpha'+1)_K}\,a_v y_1^K(1+u)^{-(\lambda+\lambda'\eta_{G,g}+\lambda_1''K+\lambda_1'\eta_{G_1,g_1})}$$

$$(1+w)^{-(\mu+\mu'\eta_{G,g}+\mu_1''K+\mu_1'\eta_{G_1,g_1})}\,\Omega_{P,Q}^{M,N}(\eta_{G,g})\frac{x^{\eta_{G,g}}(-)^g}{\delta_G g!}A_{p_1+2,q_1+1}^{m_1,n_1+2}\left(\begin{array}{c|c} \mathrm{z}_1'(1+u)^{-\lambda_1}(1+w)^{-\mu_1} & \begin{array}{c}\mathrm{A}_1\\ \cdot\\ \cdot\\ \mathrm{B}_1\end{array}\end{array}\right) \tag{4.7}$$

under the same notations and conditions that (4.1).

Corollary 4

$$\int_a^b \frac{(t-a)^{\lambda-1}(b-t)^{\mu-1}}{[b-a+u(t-a)+w(b-t)]^{\lambda+\mu}}\ {}_2F_1\left[\begin{array}{c} \alpha,\beta \\ \mu \end{array};\frac{(b-t)(1+w)}{\{b-a+u(t-a)+w(b-t)\}}\right]\bar{H}\left(xY(t;\rho')\right)$$

$$L_N^{\alpha'}\left(\ \mathrm{y}_1X(t;\lambda_1'',\mu_1'')\ \right)\aleph\left(\ \mathrm{z}_1X(t;\lambda_1',\mu_1')\ \right)A\left(\ \mathrm{z}_1'X(t;\lambda_1,\mu_1)\ \right)\mathrm{d}t=(b-a)^{-1}(1+u)^{-\lambda}(1+w)^{-\mu}$$

$$\sum_{K=0}^{[\mathfrak{N}/\mathfrak{M}]}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{G_1=1}^{m_1}\sum_{g_1=0}^{\infty}\binom{N+\alpha'}{N}\frac{1}{(\alpha'+1)_K}\phi_1\frac{z_1^{\eta_{G_1,g_1}}(-)^{g_1}}{\delta_{G_1}g_1!}\,a_v y_1^K(1+u)^{-(\lambda+\lambda'\eta_{G,g}+\lambda_1''K+\lambda_1'\eta_{G_1,g_1})}$$

$$\Omega_{P,Q}^{M,N}(\eta_{G,g})\frac{x^{\eta_{G,g}}(-)^g}{\delta_G g!}A_{p_1+2,q_1+2}^{m_1,n_1+2}\left(\begin{array}{c|c} \mathrm{z}_1'(1+u)^{-\lambda_1}(1+w)^{-\mu_1} & \begin{array}{c}\mathrm{A}_1'\\ \cdot\\ \cdot\\ \mathrm{B}_1'\end{array}\end{array}\right) \tag{4.8}$$

under the same notations and conditions that (4.4).

6. Conclusion

In this paper we have evaluated two unified Eulerian integrals involving the product of a expansion of the multivariable Aleph-function, a multivariable A-function defined by Gautam and Asgar [4] and a class of multivariable polynomials defined by Srivastava [13]. The formulae established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

References

[1] R.G. Buschman and H.M. Srivastava, The H-Function associated with a certain class of Feynman integrals, J. Phys. A: Math. Gen., 23 (1990), 4707-4710.

[2] A. Erdelyi, A table of integral transforms, Vol II, Mcgraw-Hill, New York, 1954.

[3] B.P. Gautam, A.S. Asgar and A.N.Goyal. The A-function. Revista. Mathematica. Tucuman (1980).

[4] B.P. Gautam, A.S. Asgar and Goyal A.N, On the multivariable A-function. Vijnana Parishas Anusandhan Patrika 29(4) 1986, 67-81.

[5] K.C. Gupta, R. Jain and R. Agrawal, On existence conditions for generalized Mellin-Barnes type integral, Nat. Acad. Sci. Lett., 30(5 & 6) (2007), 169-172.

[6] V.G. Gupta and N.K. Jangid, Some integrals involving generalized polynomials and the generalized H-function, International Journal of Mathematical Archive, 3(12), (2012), 4972-4979.

[7] A. Inayat-Hussain, New properties of hypergeometric series derivable from Feynman integrals: II. A generalization of the H-Function, J. Phys. A: Math. Gen., 20 (1987), 4119-4128.

[8] A.M. Mathai and R.K. Saxena, Generalized hypergeometric functions with applications in statistics and physical

sciences, Springer-Verlag, Lecture Notes No.348, Heidelberg, 1973.

[9] A.K. Rathie, A new generalization of generalized hypergeometric functions, Le Matematiche Fasc. II, 52 (1997), 297-310.

[10] R.K. Saxena, S.L. Kalla and R. Saxena, Multivariate analogue of Generalized Mittag- Leffler function, Integral Transform and special functions I First (2011), 1-6.

[11] C.K. Sharma and S.S. Ahmad, On the multivariable I-function. Acta ciencia Indica Math , 1994 vol 20,no2, 113-116.

[12] H.M. Srivastava, A contour integral involving Fox's H-function. Indian. J. Math, (14) (1972), 1-6.

[13] H.M. Srivastava, A multilinear generating function for the Konhauser set of biorthogonal polynomials suggested by Laguerre polynomial, Pacific. J. Math. 177(1985), 183-191.

[14] H.M.Srivastava and R. Panda, Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24 (1975),119-137.

[15] H.M.Srivastava and R.Panda, Some expansion theorems and generating relations for the H-function of several complex variables II. Comment. Math. Univ. St. Paul. 25 (1976)., 167-197.

[16] H.M. Srivastava and N.P. Singh, The integration of certain products of the multivariable H-function with a general class of polynomials. Rend. Circ. Mat. Palermo. Vol 32 (No 2) (1983), 157-187.

[17] N. Südland, B. Baumann and T.F. Nonnenmacher T , Open problem : who knows about the Aleph-functions? Fract. Calc. Appl. Anal, 1(4) (1998): 401-402.

[18] N.Sudland, B. Baumann and T.F. Nannenmacher, Fractional drift-less Fokker-Planck equation with power law diffusion coefficients, in V.G. Gangha, E.W. Mayr, W.G. Vorozhtsov (Eds.), Computer Algebra in Scientific Computing (CASC Konstanz 2001), Springer, Berlin, 2001, 513–525.

[19] C. Szego, (1975), Orthogonal polynomials. Amer. Math. Soc. Colloq. Publ. 23 fourth edition. Amer. Math. Soc. Providence. Rhodes Island, 1975.