# Certain transformations of a class of integrals involving the multivariable I-function

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ABSTRACT

Jaimini et al [3] have given results concerning certain tranformations of a class of integrals involving the multivariable H-function. In this paper two unified integrals and their multiple integral transformations formulae involving general classes of polynomials of one and several variables with the multivariable I-function defined by Prasad [5] are established.

Keywords: General class of polynomial, multiple integral transformations, multivariable I-function, multivariable H-function, class of polynomials

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## 1.Introduction and preliminaries.

The multivariable I-function of r-variables is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1, z_2, \cdots, z_r) = I^{0,n_2;0,n_3;\cdots;0,n_r:m^{(1)},n^{(1)};\cdots;m^{(r)},n^{(r)}}_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p^{(1)},q^{(1)};\cdots;p^{(r)},q^{(r)}} \left( \begin{array}{c} z_1 \\ . \\ . \\ . \\ z_r \end{array} \right| \begin{array}{c} (a_{2j}; \alpha'_{2j}, \alpha''_{2j})_{1,p_2}; \cdots ; \\ \\ \\ \\ (b_{2j}; \beta'_{2j}, \beta''_{2j})_{1,q_2}; \cdots ; \end{array}$$

$$(a_{rj}; \alpha^{(1)}_{rj}, \cdots, \alpha^{(r)}_{rj})_{1,p_r} : (a^{(1)}_j, \alpha^{(1)}_j)_{1,p^{(1)}}; \cdots ; (a^{(r)}_j, \alpha^{(r)}_j)_{1,p^{(r)}}$$

$$(b_{rj}; \beta^{(1)}_{rj}, \cdots, \beta^{(r)}_{rj})_{1,q_r} : (b^{(1)}_j, \beta^{(1)}_j)_{1,q^{(1)}}; \cdots ; (b^{(r)}_j, \beta^{(r)}_j)_{1,q^{(r)}} \right) \qquad (1.1)$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \phi(s_1, \cdots, s_r) \prod_{i=1}^{r} \theta_i(s_i) z_i^{s_i} ds_1 \cdots ds_r \qquad (1.2)$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [5]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$$|arg z_i| < \frac{1}{2}\Omega_i\pi \ , \ \text{where}$$

$$\Omega_i = \sum_{k=1}^{n^{(i)}} \alpha^{(i)}_k - \sum_{k=n^{(i)}+1}^{p^{(i)}} \alpha^{(i)}_k + \sum_{k=1}^{m^{(i)}} \beta^{(i)}_k - \sum_{k=m^{(i)}+1}^{q^{(i)}} \beta^{(i)}_k + \left( \sum_{k=1}^{n_2} \alpha^{(i)}_{2k} - \sum_{k=n_2+1}^{p_2} \alpha^{(i)}_{2k} \right) + \cdots +$$

$$\left( \sum_{k=1}^{n_s} \alpha_{sk}^{(i)} - \sum_{k=n_s+1}^{p_s} \alpha_{sk}^{(i)} \right) - \left( \sum_{k=1}^{q_2} \beta_{2k}^{(i)} + \sum_{k=1}^{q_3} \beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_s} \beta_{sk}^{(i)} \right) \tag{1.3}$$

where $i = 1, \cdots, r$

The complex numbers $z_i$ are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1, \cdots, z_r) = 0( |z_1|^{\alpha_1}, \cdots, |z_r|^{\alpha_r}), max( |z_1|, \cdots, |z_r| ) \to 0$$

$$I(z_1, \cdots, z_r) = 0( |z_1|^{\beta_1}, \cdots, |z_r|^{\beta_r}), min( |z_1|, \cdots, |z_r| ) \to \infty$$

where $k = 1, \cdots, r : \alpha'_k = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta'_k = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, n_k$$

The multivariable I-function of r-variables is defined by Prasad [5] in term of multiple Mellin-Barnes type integral :

$$I(z'_1, \cdots, z'_s) = I_{p'_2,q'_2,p'_3,q'_3;\cdots;p'_s,q'_s:p'^{(1)},q'^{(1)};\cdots;p'^{(s)},q'^{(s)}}^{0,n'_2;0,n'_3;\cdots;0,n'_s:m'^{(1)},n'^{(1)};\cdots;m'^{(s)},n'^{(s)}} \left( \begin{array}{c|c} z'_1 & (\text{a'}_{2j}; \alpha_{2j}'^{(1)}, \alpha_{2j}'^{(2)})_{1,p_2}; \cdots; \\ . & \\ . & \\ . & \\ z'_s & (\text{b'}_{2j}; \beta_{2j}'^{(1)}, \beta_{2j}'^{(2)})_{1,q_2}; \cdots; \end{array} \right.$$

$$(\text{a'}_{sj}; \alpha_{sj}'^{(1)}, \cdots, \alpha_{sj}'^{(s)})_{1,p'_s} : (a_j'^{(1)}, \alpha_j'^{(1)})_{1,p'^{(1)}}; \cdots; (a_j'^{(s)}, \alpha_j'^{(s)})_{1,p'^{(s)}}$$

$$\left. (\text{b'}_{rj}; \beta_{sj}'^{(1)}, \cdots, \beta_{sj}'^{(s)})_{1,q_s} : (b_j'^{(1)}, \beta_j'^{(1)})_{1,q'^{(1)}}; \cdots; (b_j'^{(s)}, \beta_j'^{(s)})_{1,q'^{(s)}} \right) \tag{1.4}$$

$$= \frac{1}{(2\pi\omega)^s} \int_{L'_1} \cdots \int_{L'_s} \psi(t_1, \cdots, t_s) \prod_{i=1}^{s} \zeta_i(t_i) z_i'^{t_i} \mathrm{d}t_1 \cdots \mathrm{d}t_s \tag{1.5}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [5]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$$|arg z'_i| < \frac{1}{2} \Omega'_i \pi, \text{ where}$$

$$\Omega'_i = \sum_{k=1}^{n'^{(i)}} \alpha'^{(i)}_k - \sum_{k=n'^{(i)}+1}^{p'^{(i)}} \alpha'^{(i)}_k + \sum_{k=1}^{m'^{(i)}} \beta'^{(i)}_k - \sum_{k=m'^{(i)}+1}^{q'^{(i)}} \beta'^{(i)}_k + \left( \sum_{k=1}^{n'_2} \alpha'^{(i)}_{2k} - \sum_{k=n'_2+1}^{p'_2} \alpha'^{(i)}_{2k} \right) +$$

$$+ \cdots + \left( \sum_{k=1}^{n'_s} \alpha'^{(i)}_{sk} - \sum_{k=n'_s+1}^{p'_s} \alpha'^{(i)}_{sk} \right) - \left( \sum_{k=1}^{q'_2} \beta'^{(i)}_{2k} + \sum_{k=1}^{q'_3} \beta'^{(i)}_{3k} + \cdots + \sum_{k=1}^{q'_s} \beta'^{(i)}_{sk} \right) \tag{1.6}$$

where $i = 1, \cdots, s$

The complex numbers $z_i$ are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z'_1, \cdots, z'_s) = 0( |z'_1|^{\alpha_1}, \cdots, |z'_s|^{\alpha_r} ), max( |z'_1|, \cdots, |z'_s| ) \to 0$$

$$I(z'_1, \cdots, z'_s) = 0( |z'_1|^{\beta_1}, \cdots, |z'_s|^{\beta_r} ), min( |z'_1|, \cdots, |z'_s| ) \to \infty$$

where $k = 1, \cdots, r : \alpha''_k = min[Re(b'^{(k)}_j / \beta'^{(k)}_j)], j = 1, \cdots, m'_k$ and

$$\beta''_k = max[Re((a'^{(k)}_j - 1)/\alpha'^{(k)}_j)], j = 1, \cdots, n'_k$$

The finite series representation for $\phi_n(x)$ is also obtained there [4,page 55 31. eq(2.1)]

$$\phi_n(x) = \sum_{k=0}^{[n/m]} \frac{(-)^k c^{-\upsilon-n+(m-1)k} (\upsilon)_{n+(1-m)k} (ax)^{n-mk} \left[ b(2x-1)^d \right]^k}{k!(n-mk)!} \tag{1.7}$$

$$= \sum_{k=0}^{[n/m]} \sum_{l=0}^{[dk]} \frac{(-dk)_l 2^l c^{-\upsilon-n+(m-1)k} (\upsilon)_{n+(1-m)k} (a)^{n-mk} b^k (-)^{(d+1)k} x^{n-mk+l}}{l!k!(n-mk)!} \tag{1.8}$$

The generalized polynomials defined by Srivastava [7], is given in the following manner :

$$S^{M_1,\cdots,M_r}_{N_1,\cdots,N_r}[y_1, \cdots, y_r] = \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_r=0}^{[N_r/M_r]} \frac{(-N_1)_{M_1 K_1}}{K_1!} \cdots \frac{(-N_r)_{M_r K_r}}{K_r!}$$

$$A[N_1, K_1; \cdots ; N_r, K_r] y_1^{K_1} \cdots y_r^{K_r} \tag{1.9}$$

Where $M_1, \cdots, M_r$ are arbitrary positive integers and the coefficients $A[N_1, K_1; \cdots ; N_r, K_r]$ are arbitrary constants, real or complex.

The another general class of polynomials of K-variables defined by Srivastava et al [8,page 686,eq.(1.4)] and denoted as follows

$$T_n^{m_1,\cdots,m_k}[x_1,\cdots,x_k] = \sum_{h_1,\cdots,h_k=0}^{M\leqslant n}\left[(-n)_M B(n,h_1,\cdots,h_k)\prod_{i=1}^{k}\frac{x_i^{h_i}}{h_i!}\right] \tag{1.10}$$

## 2. Required result

The following Beta integral

$$\int_0^\infty y^{\upsilon-1}[b+cy+d(1-y)]^{-\rho}\mathrm{d}y = (c-d)^{-\upsilon}(b+d)^{\upsilon-\rho}B(\upsilon,\rho-\upsilon) \tag{2.1}$$

provided that $Re(\rho-\upsilon)>0, Re(\upsilon)>0, c\neq d\neq 0$

The following result [1, page172-173], see also [10, page 358 eq.(2.4)]

$$\int_0^\infty\cdots\int_0^\infty x_1^{\sigma_1-1}\cdots x_n^{\sigma_n-1}\ f[(k_1 x_1^{\rho_1}+\cdots+k_n x_n^{\rho_n})]\mathrm{d}x_1\cdots\mathrm{d}x_n$$

$$= \Psi(k_1,\cdots,k_n)\frac{\Gamma(\sigma_1/\rho_1)\cdots\Gamma(\sigma_n/\rho_n)}{\Gamma(\sigma_1/\rho_1+\cdots+\sigma_n/\rho_n)}\int_0^\infty z^{\sigma_1/\rho_1+\cdots+\sigma_n/\rho_n-1}f(z)\mathrm{d}z \tag{2.2}$$

where $\Psi(k_1,\cdots,k_n) = (\sigma_1\cdots\sigma_n)^{-1}k_1^{-\sigma_1/\rho_1}\cdots k_n^{-\sigma_n/\rho_n}$

## 3. Main integrals

Let $U = p_2,q_2;p_3,q_3;\cdots;p_{r-1},q_{r-1}$ \hfill (3.1)

$V = 0,n_2;0,n_3;\cdots;0,n_{r-1}$ \hfill (3.2)

$X = m^{(1)},n^{(1)};\cdots;m^{(r)},n^{(r)}$ \hfill (3.3)

$Y = p^{(1)},q^{(1)};\cdots;p^{(r)},q^{(r)}$ \hfill (3.4)

$$A = \left(a_{2k};\alpha_{2k}^{(1)},\alpha_{2k}^{(2)}\right)_{1,p_2};\cdots;\left(a_{(r-1)k};\alpha_{(r-1)k}^{(1)},\alpha_{(r-1)k}^{(2)},\cdots,\alpha_{(r-1)k}^{(r-1)}\right)_{1,p_{r-1}} \tag{3.5}$$

$$B = \left(b_{2k};\beta_{2k}^{(1)},\beta_{2k}^{(2)}\right)_{1,q_2};\cdots;\left(b_{(r-1)k};\beta_{(r-1)k}^{(1)},\beta_{(r-1)k}^{(2)},\cdots,\beta_{(r-1)k}^{(r-1)}\right)_{1,q_{r-1}} \tag{3.6}$$

$$\mathbb{A} = \left(a_{rk};\alpha_{rk}^{(1)},\alpha_{rk}^{(2)},\cdots,\alpha_{rk}^{(r)}\right)_{1,p_r},\left(1-\gamma-\sigma(n-mk+l)-\sum_{i=1}^{r}\sigma_i h_i,\omega_1,\cdots,\omega_r\right)$$

$$\left(1+\gamma-\sigma+(\sigma-\lambda)(n-mk+l)+\sum_{i=1}^{r}(\sigma_i-\lambda_i)h_i;\mu_1-\omega_1,\cdots,\mu_r-\omega_r\right) \tag{3.7}$$

$$\mathbb{B} = \left(b_{rk};\beta_{rk}^{(1)},\beta_{rk}^{(2)},\cdots,\beta_{rk}^{(r)}\right)_{1,q_r},\left(1-\rho-\lambda(n-mk+l)-\sum_{i=1}^{r}\lambda_i h_i;\mu_1,\cdots,\mu_r\right) \tag{3.8}$$

$$A' = (a_k^{(1)}, \alpha_k^{(1)})_{1,p^{(1)}} ; \cdots ; (a_k^{(r)}, \alpha_k^{(r)})_{1,p^{(r)}} \tag{3.9}$$

$$B' = (b_k^{(1)}, \beta_k^{(1)})_{1,q^{(1)}} ; \cdots ; (b_k^{(r)}, \beta_k^{(r)})_{1,q^{(r)}} \tag{3.10}$$

$$a_r = \frac{(-N_1)_{M_1 K_1}}{K_1!} \cdots \frac{(-N_r)_{M_r K_r}}{K_r!} A[N_1, K_1; \cdots ; N_r, K_r] \tag{3.11}$$

We have

**Theorem 1**

$$\int_0^\infty y^{\upsilon-1}[b+cy+d(1-y)]^{-\rho} \phi_n[yx^\sigma\{u+vx+w(1-x)\}^{-\lambda}] S_{N_1,\cdots,N_r}^{M_1,\cdots,M_r} \begin{pmatrix} \mathrm{y}_1 x^{\sigma_1}\{u+vx+w(1-x)\}^{-\lambda_1} \\ \cdots \\ \cdots \\ \mathrm{y}_r x^{\sigma_r}\{u+vx+w(1-x)\}^{-\lambda_r} \end{pmatrix}$$

$$I_{U;p_r,q_r:Y}^{V;0,n_r:X} \begin{pmatrix} \mathrm{z}_1 x^{\omega_1}\{u+vx+w(1-x)\}^{-\mu_1} \\ \cdots \\ \cdots \\ \mathrm{z}_r x^{\omega_r}\{u+vx+w(1-x)\}^{-\mu_r} \end{pmatrix} \mathrm{d}x = \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_r=0}^{[N_r/M_r]} \sum_{k=0}^{[n/m]} \sum_{l=0}^{[dk]} a_r y^{n-mk+l}$$

$$\frac{(-dk)_l 2^l c^{-\upsilon-n+(m-1)k}(\upsilon)_{n+(1-m)k}(a)^{n-mk} b^k(-)^{(d+1)k} x^{n-mk+l}}{l!k!(n-mk)!}$$

$$\prod_{i=1}^r \left[y_i(v-w)^{-\sigma_i}(u+w)^{\sigma_i-\lambda_i}\right]^{K_i} (v-w)^{-\gamma-\sigma(n-mk+l)}(u+w)^{\gamma-\rho+(\sigma-\lambda)(n-mk+l)}$$

$$I_{U;p_r+2,q_r+1:Y}^{V;0,n_r+2:X} \begin{pmatrix} \mathrm{z}_1(v-w)^{-\omega_1}(u+w)^{-\omega_1-\mu_1} & \Big| & \mathrm{A} \ ; \mathbb{A} : \ \mathrm{A'} \\ \cdots & & \\ \cdots & & \\ \mathrm{z}_r(v-w)^{-\omega_r}(u+w)^{-\omega_r-\mu_r} & \Big| & \mathrm{B} \ ; \ \mathbb{B} : \ \mathrm{B'} \end{pmatrix} \tag{3.12}$$

Provided that

(a) $u,v,w>0, v>\omega, \mu_i>\omega_i, \sigma_i, \lambda_i>0, i=1,\cdots,r, \lambda>\sigma, \lambda_i>\sigma_i$

(b) $Re\left[\gamma + \sum_{i=1}^r \omega_i \min_{1\leqslant j\leqslant m^{(i)}} \frac{b_j^{(i)}}{\beta_j^{(i)}}\right] > 0; Re\left[\rho + \sum_{i=1}^r \mu_i \min_{1\leqslant j\leqslant m^{(i)}} \frac{b_j^{(i)}}{\beta_j^{(i)}} + 1\right] > 0;$

$$Re\left[\rho - \gamma + \sum_{i=1}^r (\mu_i - \omega_i) \min_{1\leqslant j\leqslant m^{(i)}} \frac{b_j^{(i)}}{\beta_j^{(i)}}\right] > 0$$

Let $U = p_2, q_2; p_3, q_3; \cdots ; p_{r-1}, q_{r-1}$ \tag{3.13}

$$V = 0, n_2; 0, n_3; \cdots ; 0, n_{r-1} \tag{3.14}$$

$$X = m^{(1)}, n^{(1)}; \cdots ; m^{(r)}, n^{(r)} \tag{3.15}$$

$$Y = p^{(1)}, q^{(1)}; \cdots ; p^{(r)}, q^{(r)} \tag{3.16}$$

$$A = (a_{2k}; \alpha_{2k}^{(1)}, \alpha_{2k}^{(2)})_{1,p_2}; \cdots ; (a_{(r-1)k}; \alpha_{(r-1)k}^{(1)}, \alpha_{(r-1)k}^{(2)}, \cdots , \alpha_{(r-1)k}^{(r-1)})_{1,p_{r-1}} \tag{3.17}$$

$$B = (b_{2k}; \beta_{2k}^{(1)}, \beta_{2k}^{(2)})_{1,q_2}; \cdots ; (b_{(r-1)k}; \beta_{(r-1)k}^{(1)}, \beta_{(r-1)k}^{(2)}, \cdots , \beta_{(r-1)k}^{(r-1)})_{1,q_{r-1}} \tag{3.18}$$

$$\mathbb{A} = (a_{rk}; \alpha_{rk}^{(1)}, \alpha_{rk}^{(2)}, \cdots , \alpha_{rk}^{(r)})_{1,p_r}, (1 - \gamma - \sigma(n - mk + l) - \sum_{i=1}^{r} \sigma_i h_i, \omega_1, \cdots , \omega_r)$$

$$(1 + \gamma - \rho + (\sigma - \lambda)(n - mk + l) + \sum_{i=1}^{r}(\sigma_i - \lambda_i)h_i; \mu_1 - \omega_1, \cdots , \mu_r - \omega_r) \tag{3.19}$$

$$\mathbb{B} = (b_{rk}; \beta_{rk}^{(1)}, \beta_{rk}^{(2)}, \cdots , \beta_{rk}^{(r)})_{1,q_r}, (1 - \rho - \lambda(n - mk + l) - \sum_{i=1}^{r} \lambda_i h_i; \mu_1, \cdots , \mu_r) \tag{3.20}$$

$$A' = (a_k^{(1)}, \alpha_k^{(1)})_{1,p^{(1)}}; \cdots ; (a_k^{(r)}, \alpha_k^{(r)})_{1,p^{(r)}} \tag{3.21}$$

$$B' = (b_k^{(1)}, \beta_k^{(1)})_{1,q^{(1)}}; \cdots ; (b_k^{(r)}, \beta_k^{(r)})_{1,q^{(r)}} \tag{3.22}$$

$$b_r = (-N)_M B(N, h_1, \cdots , h_r) \text{ with } M = \sum_{i=1}^{r} m_i h_i \tag{3.24}$$

We have

**Theorem 2**

$$\int_0^{\infty} y^{\upsilon-1}[b + cy + d(1-y)]^{-\rho} \phi_n[yx^{\sigma}\{u + vx + w(1-x)\}^{-\lambda}] T_N^{M_1, \cdots, M_r} \left( \begin{array}{c} \mathrm{y}_1 x^{\sigma_1}\{u + vx + w(1-x)\}^{-\lambda_1} \\ \cdots \\ \cdots \\ \mathrm{y}_r x^{\sigma_r}\{u + vx + w(1-x)\}^{-\lambda_r} \end{array} \right)$$

$$I_{U;p_r,q_r:Y}^{V;0,n_r:X} \left( \begin{array}{c} \mathrm{z}_1 x^{\omega_1}\{u + vx + w(1-x)\}^{-\mu_1} \\ \cdots \\ \cdots \\ \mathrm{z}_r x^{\omega_r}\{u + vx + w(1-x)\}^{-\mu_r} \end{array} \right) \mathrm{d}x = \sum_{h_1, \cdots, h_k=0}^{M \leqslant N} \sum_{k=0}^{[n/m]} \sum_{l=0}^{[dk]} y^{n-mk+l} b_r$$

$$\frac{(-dk)_l 2^l c^{-\upsilon-n+(m-1)k}(\upsilon)_{n+(1-m)k}(a)^{n-mk} b^k (-)^{(d+1)k} x^{n-mk+l}}{l! k! (n - mk)!}$$

$$\prod_{i=1}^{r}\left[\frac{\{y_i(v-w)^{-\sigma_i}(u+w)^{\sigma_i-\lambda_i}\}^{h_i}}{h_i!}\right](v-w)^{-v-\sigma(n-mk+l)}(u+w)^{\gamma-\rho+(\sigma-\lambda)(n-mk+l)}$$

$$I_{U;p_r+2,q_r+1:Y}^{V;0,n_r+2:X}\left(\begin{array}{c}z_1(v-w)^{-\omega_1}(u+w)^{-\omega_1-\mu_1}\\ \cdots\\ \cdots\\ z_r(v-w)^{-\omega_r}(u+w)^{-\omega_r-\mu_r}\end{array}\middle|\begin{array}{l}\text{A ;}\mathbb{A}\text{: A'}\\ \\ \\ \text{B ; }\mathbb{B}\text{ : B'}\end{array}\right) \tag{3.24}$$

Provided that

(a) $u,v,w>0, v>\omega, \mu_i>\omega_i, \sigma_i, \lambda_i>0, i=1,\cdots,r, \lambda>\sigma, \lambda_i>\sigma_i$

(b) $Re\left[\gamma+\sum_{i=1}^{r}\omega_i\min_{1\leqslant j\leqslant m^{(i)}}\frac{b_j^{(i)}}{\beta_j^{(i)}}\right]>0; Re\left[\rho+\sum_{i=1}^{r}\mu_i\min_{1\leqslant j\leqslant m^{(i)}}\frac{b_j^{(i)}}{\beta_j^{(i)}}+1\right]>0;$

$$Re\left[\rho-\gamma+\sum_{i=1}^{r}(\mu_i-\omega_i)\min_{1\leqslant j\leqslant m^{(i)}}\frac{b_j^{(i)}}{\beta_j^{(i)}}\right]>0$$

## 4. Multiple integrals

In this paper ,thus $0,\cdots,0$ would me r zeros and so on.

Let

$$\theta=\sum_{i=1}^{r}\eta_i x_i^{l_i}, X=\theta^{\sigma}\{u+v\theta+w(1-\theta)\}^{-\lambda}, g_i(X)=\theta^{\omega_i'}\{u+v\theta+w(1-\theta)\}^{-\mu_i'}, i=1,\cdots,s$$

and for $j=1,\cdots,r$, let $X_j=x_1^{\xi_1^{(i)}}\cdots x_r^{\xi_r^{(i)}}\theta^{w_j}\{u+v\theta+w(1-\theta)\}^{-\mu_j}$

$$X_j'=\theta^{\sigma_j}\{u+v\theta+w(1-\theta)\}^{-\lambda_j}, \tau_j=w_j+\sum_{i=1}^{r}\frac{\xi_i^{(j)}}{l_i!}, Z_j=z_j\prod_{i=1}^{r}\eta_i\left(\frac{-\xi_i^{(i)}}{l_i}\right)$$

$$U=\sum_{i=1}^{r}\frac{\gamma_i}{l_i} \text{ and } \phi(\eta_1,\cdots,\eta_r)=(l_1\cdots l_r)^{-1}\eta_1^{-\left(\frac{\tau_1+\sum_{j=1}^{r}\zeta_1^{(j)}\xi_1}{l_1}\right)}\cdots\eta_r^{-\left(\frac{\tau_r+\sum_{j=1}^{r}\zeta_r^{(j)}\xi_r}{l_r}\right)}$$

Let

$$U=p_2,q_2;p_3,q_3;\cdots;p_{r-1},q_{r-1};p_2',q_2';p_3',q_3';\cdots;p_{s-1}',q_{s-1}' \tag{4.1}$$

$$V=0,n_2;0,n_3;\cdots;0,n_{r-1};0,n_2';0,n_3';\cdots;0,n_{s-1}' \tag{4.2}$$

$$X=m^{(1)},n^{(1)};\cdots;m^{(r)},n^{(r)};m'^{(1)},n'^{(1)};\cdots;m'^{(s)},n'^{(s)} \tag{4.3}$$

$$Y=p^{(1)},q^{(1)};\cdots;p^{(r)},q^{(r)};p'^{(1)},q'^{(1)};\cdots;p'^{(s)},q'^{(s)} \tag{4.4}$$

$$A_1=(a_{2k};\alpha_{2k}^{(1)},\alpha_{2k}^{(2)})_{1,p_2};\cdots;(a_{(r-1)k};\alpha_{(r-1)k}^{(1)},\alpha_{(r-1)k}^{(2)},\cdots,\alpha_{(r-1)k}^{(r-1)})_{1,p_{r-1}};$$

$$(a'_{2k}; \alpha'^{(1)}_{2k}, \alpha'^{(2)}_{2k})_{1,p'_2}; \cdots ; (a'_{(s-1)k}; \alpha'^{(1)}_{(s-1)k}, \alpha'^{(2)}_{(s-1)k}, \cdots, \alpha'^{(s-1)}_{(s-1)k})_{1,p'_{s-1}} \tag{4.5}$$

$$B_1 = (b_{2k}; \beta^{(1)}_{2k}, \beta^{(2)}_{2k})_{1,q_2}; \cdots ; (b_{(r-1)k}; \beta^{(1)}_{(r-1)k}, \beta^{(2)}_{(r-1)k}, \cdots, \beta^{(r-1)}_{(r-1)k})_{1,q_{r-1}};$$

$$(b'_{2k}; \beta'^{(1)}_{2k}, \beta'^{(2)}_{2k})_{1,q'_2}; \cdots ; (b'_{(s-1)k}; \beta'^{(1)}_{(s-1)k}, \beta'^{(2)}_{(s-1)k}, \cdots, \beta'^{(s-1)}_{(s-1)k})_{1,q'_{s-1}} \tag{4.6}$$

$$\mathbb{A}_1 = (a_{rk}; \alpha^{(1)}_{rk}, \alpha^{(2)}_{rk}, \cdots, \alpha^{(r)}_{rk}, \underset{s}{0, \cdots, 0})_{1,p_r}; (a'_{sk}; \underset{r}{0, \cdots, 0}, \alpha'^{(1)}_{sk}, \alpha'^{(2)}_{sk}, \cdots, \alpha'^{(s)}_{sk})_{1,p'_s},$$

$$\left( 1 - \frac{\gamma_j}{l_j}; \frac{\xi^{(j)}_1}{l_1}, \cdots, \frac{\xi^{(j)}_r}{l_r}, \underset{s}{0, \cdots, 0} \right)_{1,r}, (1 - u - \gamma - \sigma(n - mk + l) - \sum_{j=1}^{r} \sigma_j h_j; \underset{r}{0, \cdots, 0}; \omega'_1, \cdots, \omega'_s),$$

$$(1 + u + \gamma - \rho + (\sigma - \lambda)(n - mk + l) + \sum_{j=1}^{r} (\sigma_j - \lambda_j) h_j; \underset{r}{0, \cdots, 0}; \mu'_1 - \omega'_1, \cdots, \mu'_s - \omega'_s) \tag{4.7}$$

$$\mathbb{B}_1 = (b_{rk}; \beta^{(1)}_{rk}, \beta^{(2)}_{rk}, \cdots, \beta^{(r)}_{rk}, \underset{s}{0, \cdots, 0})_{1,q_r}; (b'_{sk}; \underset{r}{0, \cdots, 0}, \beta'^{(1)}_{sk}, \beta'^{(2)}_{sk}, \cdots, \beta'^{(s)}_{sk}, 0)_{1,q'_s},$$

$$(1 - U; \tau_1 - \omega_1, \cdots, \tau_r - \omega_r; \underset{s}{0, \cdots, 0}), (1 - \rho - \lambda(n - mk + l) - \sum_{j=1}^{r} \lambda_j h_j; \underset{r}{0, \cdots, 0}; \mu'_1, \cdots, \mu'_s) \tag{4.8}$$

$$A'_1 = (a^{(1)}_k, \alpha^{(1)}_k)_{1,p^{(1)}}; \cdots ; (a^{(r)}_k, \alpha^{(r)}_k)_{1,p^{(r)}}; (a'^{(1)}_k, \alpha'^{(1)}_k)_{1,p'^{(1)}}; \cdots ; (a'^{(s)}_k, \alpha'^{(s)}_k)_{1,p'^{(s)}} \tag{4.9}$$

$$B'_1 = (b^{(1)}_k, \beta^{(1)}_k)_{1,q^{(1)}}; \cdots ; (b^{(r)}_k, \beta^{(r)}_k)_{1,q^{(r)}}; (b'^{(1)}_k, \beta'^{(1)}_k)_{1,q'^{(1)}}; \cdots ; (b'^{(s)}_k, \beta'^{(s)}_k)_{1,q'^{(s)}} \tag{4.10}$$

We have

**Theorem 3**

$$\int_0^\infty \cdots \int_0^\infty \prod_{i=1}^{r} x_i^{\gamma_i - 1} \theta^\gamma [u + v\theta + w(1-\theta)]^{-\rho} \phi_n(yX) S^{M_1, \cdots, M_r}_{N_1, \cdots, N_r}[y_1 X'_1, \cdots, y_r X'_r] I(z_1 X_1, \cdots, z_r X_r)$$

$$I(Y_1 g_1(X), \cdots, Y_s g_s(X)) \mathrm{d}x_1 \cdots \mathrm{d}x_r = \phi(\eta_1, \cdots, \eta_r) \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_r=0}^{[N_r/M_r]} \sum_{k=0}^{[n/m]} \sum_{l=0}^{[dk]} a_r y^{n-mk+l}$$

$$\frac{(-dk)_l 2^l c^{-\upsilon - n + (m-1)k} (\upsilon)_{n+(1-m)k} (a)^{n-mk} b^k (-)^{(d+1)k} x^{n-mk+l}}{l!k!(n-mk)!}$$

$$\prod_{i=1}^{r} \left[ y_i (v-w)^{-\sigma_i} (u+w)^{\sigma_i - \lambda_i} \right]^{K_i} (v-w)^{-U - \gamma - \sigma(n-mk+l)} (u+w)^{U + \gamma + (\sigma - \lambda)(n-mk+l)}$$

$$I_{U:p_r+p'_s+r+2,q_r+q'_s+2;Y}^{V;0,n_r+n'_s+r+2;X}\left(\begin{array}{c}z_1(v-w)^{-\tau_1}(u+w)^{\tau_1-\mu_1}\\ \cdots\\ \cdots\\ z_r(v-w)^{-\tau_r}(u+w)^{\tau_r-\mu_r}\\ y_1(v-w)^{-\omega'_1}(u+w)^{\omega'_1-\mu'_1}\\ \cdots\\ \cdots\\ y_s(v-w)^{-\omega'_s}(u+w)^{\omega'_s-\mu'_s}\end{array}\middle|\begin{array}{c}A_1;\mathbb{A}_1:A'_1\\ \vdots\\ \vdots\\ \vdots\\ \vdots\\ \vdots\\ B_1;\mathbb{B}_1:B'_1\end{array}\right)$$

(4.11)

Provided that

(a) $\min_{1\leqslant i\leqslant r}(\eta_i,l_i,\xi_i^{(j)},Re(\gamma_i))>0,\mu_j>\tau_j>\omega_j>0,\lambda_j>\sigma_j>0;j=1,\cdots,r$

(b) $Re\left[\rho+\sum_{j=1}^r\mu_j\min_{1\leqslant k\leqslant m^{(j)}}\dfrac{b_k^{(j)}}{\beta_k^{(j)}}\right]>Re\left[\gamma+U+(\sigma-\lambda)(n-mk+l)+\sum_{j=1}^r\tau_j\min_{1\leqslant k\leqslant m^{(j)}}\dfrac{b_k^{(j)}}{\beta_k^{(j)}}\right]>0$

We have

**Theorem 4**

$$\int_0^\infty\cdots\int_0^\infty\prod_{i=1}^r x_i^{\gamma_i-1}\theta^\gamma[u+v\theta+w(1-\theta)]^{-\rho}\phi_n(yX)T_N^{M_1,\cdots,M_r}[y_1X'_1,\cdots,y_rX'_r]I(z_1X_1,\cdots,z_rX_r)$$

$$I(Y_1g_1(X),\cdots,Y_sg_s(X))\mathrm{d}x_1\cdots\mathrm{d}x_r=\phi(\eta_1,\cdots,\eta_r)\sum_{h_1,\cdots,h_k=0}^{M\leqslant N}\sum_{k=0}^{[n/m]}\sum_{l=0}^{[dk]}b_ry^{n-mk+l}$$

$$\frac{(-dk)_l2^lc^{-\upsilon-n+(m-1)k}(\upsilon)_{n+(1-m)k}(a)^{n-mk}b^k(-)^{(d+1)k}x^{n-mk+l}}{l!k!(n-mk)!}$$

$$\prod_{i=1}^r\left[\frac{\{y_i(v-w)^{-\sigma_i}(u+w)^{\sigma_i-\lambda_i}\}^{h_i}}{h_i!}\right](v-w)^{-U-\gamma-\sigma(n-mk+l)}(u+w)^{U+\gamma+(\sigma-\lambda)(n-mk+l)}$$

$$I_{U:p_r+p'_s+r+2,q_r+q'_s+2;Y}^{V;0,n_r+n'_s+r+2;X}\left(\begin{array}{c}z_1(v-w)^{-\tau_1}(u+w)^{\tau_1-\mu_1}\\ \cdots\\ \cdots\\ z_r(v-w)^{-\tau_r}(u+w)^{\tau_r-\mu_r}\\ y_1(v-w)^{-\omega'_1}(u+w)^{\omega'_1-\mu'_1}\\ \cdots\\ \cdots\\ y_s(v-w)^{-\omega'_s}(u+w)^{\omega'_s-\mu'_s}\end{array}\middle|\begin{array}{c}A_1;\mathbb{A}_1:A'_1\\ \vdots\\ \vdots\\ \vdots\\ \vdots\\ \vdots\\ B_1;\mathbb{B}_1:B'_1\end{array}\right)$$

(4.12)

Provided that

(a) $\min\limits_{1\leqslant i\leqslant r}(\eta_i, l_i, \zeta_i^{(j)}, Re(\tau_i)) > 0, \mu_j > \tau_j > \omega_j > 0, \lambda_j > \sigma_j > 0; j = 1, \cdots, r$

(b) $Re\Big[\rho + \sum\limits_{j=1}^{r}\mu_j \min\limits_{1\leqslant k\leqslant m^{(j)}}\dfrac{b_k^{(j)}}{\beta_k^{(j)}}\Big] > Re\left[\gamma + U + (\sigma - \lambda)(n - mk + l) + \sum\limits_{j=1}^{r}\tau_j \min\limits_{1\leqslant k\leqslant m^{(j)}}\dfrac{b_k^{(j)}}{\beta_k^{(j)}}\right] > 0$

**Proofs**

To prove (3.12), we denote the L.H.S of (3.12) by L. To evaluate L, we first replace the multivariable I-function defined by Prasad [5] by Mellin-Barnes contour integrals with the help of (1.2), then use (1.9) to express $S_{N_1,\cdots,N_r}^{M_1,\cdots,M_r}[.]$ and (1.8) to express $\phi_n[.]$ in series therein interchange the order of summation and integration (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process). Now evaluate the inner most- integral with the help of (2.1). Finally interpreting the resulting Mellin-Barnes contour integral as the multivariable I-function defined by (1.2), we get the desired formula (3.12). The proof of (3.24) is similar by using the definition of the general class of polynomials $T_N^{M_1,\cdots,M_r}[.]$.

To prove (4.11), we let

$$L_1 = \int_0^\infty \cdots \int_0^\infty \prod_{i=1}^r x_i^{\gamma_i-1}f(\theta)I(z_1X_1, \cdots, z_rX_r)\mathrm{d}x_1\cdots\mathrm{d}x_r\,, \theta = \sum_{i=1}^r \eta_i x_i^{L_i}$$

We nest replace the multivariable I-function in $L_1$ by its Mellin-Barnes contour integrals with the help of (1.2), change the order of resulting multiple integrals and using (2.2) , we get

$$L_1 = \frac{\phi(\eta_1,\cdots,\eta_r)}{(2\pi w)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,\cdots,s_r)\prod_{i=1}^r \theta_i(s_i)z_i^{s_i}\frac{\prod_{i=1}^r \Gamma\left(\frac{\gamma_i}{l_i} + \sum_{j=1}^r \frac{\xi_i^{(j)}}{l_i}s_i\right)}{\Gamma\left(U + \sum_{j=1}^r (\tau_j - \omega_j)s_j\right)}$$

$$\left[\int_0^\infty z^{U+\sum_{j=1}^r \tau_j s_j - 1}\{u + vz + w(1-z)\}^{-\sum_{j=1}^r \mu_j s_j}f(z)\mathrm{d}z\right]\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{4.13}$$

Now we set

$$f(z) = z^\gamma[u + vz + w(1-z)]^{-\rho}\phi\left[yz^\sigma\{u + vz + w(1-z)\}^{-\lambda}\right]S_{N_1,\cdots,N_r}^{M_1,\cdots,M_r}\left(\begin{array}{c} y_1 x^{\sigma_1}\{u + vx + w(1-x)\}^{-\lambda_1} \\ \cdots \\ \cdots \\ y_r x^{\sigma_r}\{u + vx + w(1-x)\}^{-\lambda_r} \end{array}\right)$$

$$I\left(\begin{array}{c} y_1 x^{\omega_1'}\{u + vx + w(1-x)\}^{-\mu_1'} \\ \cdots \\ \cdots \\ y_s x^{\omega_s'}\{u + vx + w(1-x)\}^{-\mu_s'} \end{array}\right) \tag{4.14}$$

In (4.13), evaluate the inner z-integral with the help of (3.12) for s-variables and then we interprete the resulting Mellin-Barnes contour integral into the I-function of (r+s)-variables with the help of (1.2)

The proof of (4.12) is similar with

$$f(z) = z^\gamma [u + vz + w(1-z)]^{-\rho} \phi \left[ yz^\sigma \{u + vz + w(1-z)\}^{-\lambda} \right] T_N^{M_1,\cdots,M_r} \begin{pmatrix} y_1 x^{\sigma_1} \{u + vx + w(1-x)\}^{-\lambda_1} \\ \cdots \\ \cdots \\ y_r x^{\sigma_r} \{u + vx + w(1-x)\}^{-\lambda_r} \end{pmatrix}$$

$$I \begin{pmatrix} y_1 x^{\omega'_1} \{u + vx + w(1-x)\}^{-\mu'_1} \\ \cdots \\ \cdots \\ y_s x^{\omega'_s} \{u + vx + w(1-x)\}^{-\mu'_s} \end{pmatrix}$$

in (4.13) and using (3.24)

# 5. Multivariable H-function

If $U = V = A = B = 0$, the multivariable I-function defined by Prasad [5] reduces to multivariable H-function defined by Srivastava et al [9]. We obtain the following result.

**Corollary 1**

$$\int_0^\infty y^{v-1} [b + cy + d(1-y)]^{-\rho} \phi_n [yx^\sigma \{u + vx + w(1-x)\}^{-\lambda}] S_{N_1,\cdots,N_r}^{M_1,\cdots,M_r} \begin{pmatrix} y_1 x^{\sigma_1} \{u + vx + w(1-x)\}^{-\lambda_1} \\ \cdots \\ \cdots \\ y_r x^{\sigma_r} \{u + vx + w(1-x)\}^{-\lambda_r} \end{pmatrix}$$

$$H_{p_r,q_r:Y}^{0,n_r:X} \begin{pmatrix} z_1 x^{\omega_1} \{u + vx + w(1-x)\}^{-\mu_1} \\ \cdots \\ \cdots \\ z_r x^{\omega_r} \{u + vx + w(1-x)\}^{-\mu_r} \end{pmatrix} dx = \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_r=0}^{[N_r/M_r]} \sum_{k=0}^{[n/m]} \sum_{l=0}^{[dk]} a_r y^{n-mk+l}$$

$$\frac{(-dk)_l 2^l c^{-v-n+(m-1)k} (v)_{n+(1-m)k} (a)^{n-mk} b^k (-)^{(d+1)k} x^{n-mk+l}}{l!k!(n-mk)!}$$

$$\prod_{i=1}^r \left[ y_i (v-w)^{-\sigma_i} (u+w)^{\sigma_i-\lambda_i} \right]^{K_i} (v-w)^{-\gamma-\sigma(n-mk+l)} (u+w)^{\gamma-\rho+(\sigma-\lambda)(n-mk+l)}$$

$$H_{p_r+2,q_r+1:Y}^{0,n_r+2:X} \begin{pmatrix} z_1 (v-w)^{-\omega_1} (u+w)^{-\omega_1-\mu_1} & \bigg| \ \mathbb{A}: \text{A'} \\ \cdots \\ \cdots \\ z_r (v-w)^{-\omega_r} (u+w)^{-\omega_r-\mu_r} & \bigg| \ \mathbb{B} : \text{B'} \end{pmatrix} \tag{5.1}$$

under the same notations and conditions that (3.12) with $U = V = A = B = 0$

**Corollary 2**

$$\int_0^\infty y^{\upsilon-1}[b+cy+d(1-y)]^{-\rho}\phi_n[yx^\sigma\{u+vx+w(1-x)\}^{-\lambda}]T_N^{M_1,\cdots,M_r}\begin{pmatrix} \mathrm{y}_1 x^{\sigma_1}\{u+vx+w(1-x)\}^{-\lambda_1} \\ \cdots \\ \cdots \\ \mathrm{y}_r x^{\sigma_r}\{u+vx+w(1-x)\}^{-\lambda_r} \end{pmatrix}$$

$$H_{p_r,q_r:Y}^{0,n_r:X}\begin{pmatrix} \mathrm{z}_1 x^{\omega_1}\{u+vx+w(1-x)\}^{-\mu_1} \\ \cdots \\ \cdots \\ \mathrm{z}_r x^{\omega_r}\{u+vx+w(1-x)\}^{-\mu_r} \end{pmatrix}\mathrm{d}x = \sum_{h_1,\cdots,h_k=0}^{M\leqslant N}\sum_{k=0}^{[n/m]}\sum_{l=0}^{[dk]} y^{n-mk+l}b_r$$

$$\frac{(-dk)_l 2^l c^{-\upsilon-n+(m-1)k}(\upsilon)_{n+(1-m)k}(a)^{n-mk}b^k(-)^{(d+1)k}x^{n-mk+l}}{l!k!(n-mk)!}$$

$$\prod_{i=1}^r\left[\frac{\{y_i(v-w)^{-\sigma_i}(u+w)^{\sigma_i-\lambda_i}\}^{h_i}}{h_i!}\right](v-w)^{-\upsilon-\sigma(n-mk+l)}(u+w)^{\gamma-\rho+(\sigma-\lambda)(n-mk+l)}$$

$$H_{p_r+2,q_r+1:Y}^{0,n_r+2:X}\begin{pmatrix} \mathrm{z}_1(v-w)^{-\omega_1}(u+w)^{-\omega_1-\mu_1} \\ \cdots \\ \cdots \\ \mathrm{z}_r(v-w)^{-\omega_r}(u+w)^{-\omega_r-\mu_r} \end{pmatrix}\begin{array}{|l} \mathbb{A}\colon \text{A'} \\ \\ \mathbb{B}\colon \text{B'} \end{array}\Bigg) \tag{5.2}$$

under the same notations and conditions that (3.24) with $U=V=A=B=0$

Remark : If $n_r=0$, we obtain the results of Jaimini et al [3] concerning the simple integrals.

**Corollary 3**

$$\int_0^\infty\cdots\int_0^\infty\prod_{i=1}^r x_i^{\gamma_i-1}\theta^\gamma[u+v\theta+w(1-\theta)]^{-\rho}\phi_n(yX)S_{N_1,\cdots,N_r}^{M_1,\cdots,M_r}[y_1 X_1',\cdots,y_r X_r']H(z_1 X_1,\cdots,z_r X_r)$$

$$H(Y_1 g_1(X),\cdots,Y_s g_s(X))\mathrm{d}x_1\cdots\mathrm{d}x_r = \phi(\eta_1,\cdots,\eta_r)\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_r=0}^{[N_r/M_r]}\sum_{k=0}^{[n/m]}\sum_{l=0}^{[dk]} a_r y^{n-mk+l}$$

$$\frac{(-dk)_l 2^l c^{-\upsilon-n+(m-1)k}(\upsilon)_{n+(1-m)k}(a)^{n-mk}b^k(-)^{(d+1)k}x^{n-mk+l}}{l!k!(n-mk)!}$$

$$\prod_{i=1}^r\left[y_i(v-w)^{-\sigma_i}(u+w)^{\sigma_i-\lambda_i}\right]^{K_i}(v-w)^{-U-\gamma-\sigma(n-mk+l)}(u+w)^{U+\gamma+(\sigma-\lambda)(n-mk+l)}$$

$$H^{0,n_r+n'_s+r+2;X}_{p_r+p'_s+r+2,q_r+q'_s+2;Y} \left( \begin{array}{c|c} \begin{array}{c} \mathrm{z}_1(v-w)^{-\tau_1}(u+w)^{\tau_1-\mu_1} \\ \cdots \\ \cdots \\ \mathrm{z}_r(v-w)^{-\tau_r}(u+w)^{\tau_r-\mu_r} \\ \mathrm{y}_1(v-w)^{-\omega'_1}(u+w)^{\omega'_1-\mu'_1} \\ \cdots \\ \cdots \\ \mathrm{y}_s(v-w)^{-\omega'_s}(u+w)^{\omega'_s-\mu'_s} \end{array} & \begin{array}{c} \mathbb{A}_1 : A'_1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \mathbb{B}_1 : B'_1 \end{array} \end{array} \right) \tag{5.3}$$

under the same notations and conditions that (4.11) with $U = V = A_1 = B_1 = 0$

**Corollary 4**

$$\int_0^\infty \cdots \int_0^\infty \prod_{i=1}^r x_i^{\gamma_i-1} \theta^\gamma [u+v\theta+w(1-\theta)]^{-\rho} \phi_n(yX) T_N^{M_1,\cdots,M_r}[y_1 X'_1,\cdots,y_r X'_r] H(z_1 X_1,\cdots,z_r X_r)$$

$$H(Y_1 g_1(X),\cdots,Y_s g_s(X)) \mathrm{d}x_1 \cdots \mathrm{d}x_r = \phi(\eta_1,\cdots,\eta_r) \sum_{h_1,\cdots,h_k=0}^{M\leqslant N} \sum_{k=0}^{[n/m]} \sum_{l=0}^{[dk]} b_r y^{n-mk+l}$$

$$\frac{(-dk)_l 2^l c^{-\upsilon-n+(m-1)k}(\upsilon)_{n+(1-m)k}(a)^{n-mk}b^k(-)^{(d+1)k}x^{n-mk+l}}{l!k!(n-mk)!}$$

$$\prod_{i=1}^r \left[ \frac{\{y_i(v-w)^{-\sigma_i}(u+w)^{\sigma_i-\lambda_i}\}^{h_i}}{h_i!} \right] (v-w)^{-U-\gamma-\sigma(n-mk+l)}(u+w)^{U+\gamma+(\sigma-\lambda)(n-mk+l)}$$

$$H^{0,n_r+n'_s+r+2;X}_{p_r+p'_s+r+2,q_r+q'_s+2;Y} \left( \begin{array}{c|c} \begin{array}{c} \mathrm{z}_1(v-w)^{-\tau_1}(u+w)^{\tau_1-\mu_1} \\ \cdots \\ \cdots \\ \mathrm{z}_r(v-w)^{-\tau_r}(u+w)^{\tau_r-\mu_r} \\ \mathrm{y}_1(v-w)^{-\omega'_1}(u+w)^{\omega'_1-\mu'_1} \\ \cdots \\ \cdots \\ \mathrm{y}_s(v-w)^{-\omega'_s}(u+w)^{\omega'_s-\mu'_s} \end{array} & \begin{array}{c} \mathrm{A}_1 ; \mathbb{A}_1 : A'_1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \mathrm{B}_1 ; \mathbb{B}_1 : B'_1 \end{array} \end{array} \right) \tag{5.4}$$

under the same notations and conditions that (4.12) with $U = V = A_1 = B_1 = 0$

Remarks : If $n_s = 0$, we obtain the results of Jaimini et al [3] concerning the multiple integrals.

The results (3.12), (3.24), (4.11) and (4.12) reduce to the results involving the Sinha polynomials [6], Humbert

polynomials [2], Pincherle polynomials [2],Gould polynomials [4] etc.

## 6. Conclusion

In this paper we have evaluated two unified and two multiple integrals involving the multivariable I-function defined by Prasad [5], a sequence of polynomials and a class of multivariable polynomials. These integrals established in this paper is of very general nature as it contains multivariable I-function, which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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