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# integrals of certain products involving general class of polynomials, Aleph-function and the multivariable Aleph-functions

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ABSTRACT
The integrals evaluating here involve the product of Jacobi polynomials, Aleph-function, general class of polynomials and the multivariable Aleph-function. The mains results of our document are quite general in nature and capable of yielding a very large number of integrals involving polynomials and various special functions occuring in the problem of mathematical analysis and mathematical physics and mechanics.

Keywords :Multivariable Aleph-function, Aleph-function, Jacobi polynomial, general class of polynomials,.

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## 1. Introduction and preliminaries.

The Aleph- function , introduced by Südland [9] et al , however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral :

$$\aleph(z) = \aleph^{M,N}_{P_i,Q_i,c_i;r}\left( z \;\middle|\; \begin{matrix} (\mathrm{a}_j, A_j)_{1,\mathfrak{n}}, [c_i(a_{ji}, A_{ji})]_{\mathfrak{n}+1,p_i;r} \\ (\mathrm{b}_j, B_j)_{1,m}, [c_i(b_{ji}, B_{ji})]_{m+1,q_i;r} \end{matrix} \right)$$

$$= \frac{1}{2\pi\omega}\int_L \Omega^{M,N}_{P_i,Q_i,c_i;r}(s) z^{-s}\mathrm{d}s \tag{1.1}$$

for all $z$ different to $0$ and

$$\Omega^{\ M,N}_{\ P_i,Q_i,c_i;r}(s) = \frac{\prod_{j=1}^{M}\Gamma(b_j + B_j s)\prod_{j=1}^{N}\Gamma(1-a_j-A_j s)}{\sum_{i=1}^{r} c_i \prod_{j=N+1}^{P_i}\Gamma(a_{ji}+A_{ji}s)\prod_{j=M+1}^{Q_i}\Gamma(1-b_{ji}-B_{ji}s)} \tag{1.2}$$

With : $|argz| < \frac{1}{2}\pi\Omega$ Where $\Omega = \sum_{j=1}^{M}\beta_j + \sum_{j=1}^{N}\alpha_j - c_i(\sum_{j=M+1}^{Q_i}\beta_{ji} + \sum_{j=N+1}^{P_i}\alpha_{ji}) > 0, i = 1,\cdots,r$

For convergence conditions and other details of Aleph-function , see Südland et al [9]. The serie representation of Aleph-function is given by Chaurasia et al [2].

$$\aleph^{M,N}_{P_i,Q_i,c_i;r}(z) = \sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r}(s)}{B_G g!}z^{-s} \tag{1.3}$$

With $s = \eta_{G,g} = \dfrac{b_G+g}{B_G}, P_i < Q_i, |z| < 1$ and $\Omega^{M,N}_{P_i,Q_i,c_i;r}(s)$ is given in (1.2) (1.4)

The generalized polynomials defined by Srivastava [8], is given in the following manner :

$$S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}[y_1,\cdots,y_s] = \sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}$$

$$A[N_1,K_1;\cdots;N_s,K_s]y_1^{K_1}\cdots y_s^{K_s} \tag{1.5}$$

Where $M_1,\cdots,M_s$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_s,K_s]$ are arbitrary constants, real or complex. In the present paper, we use the following notations

$$A_1 = \frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}A[N_1,K_1;\cdots;N_s,K_s] \quad , \quad U_{22}=p_i+2,q_i+2,\tau_i;R$$

$U_{21}=p_i+2,q_i+1,\tau_i;R$ **and** $A' = A_1\, 2^{\eta+\rho+\sum_{i=1}^{s}K_i(\alpha_i+\beta_i)+(h+k)\eta_{G,g}+1}$

Srivastava [7] introduced the general class of polynomials :

$$S_N^M(x)=\sum_{k=0}^{[N/M]}\frac{(-N)_{Mk}}{k!}A_{N,k}x^k, N=0,1,2,... \tag{1.6}$$

Where $M$ is an arbtrary positive integer and the coefficient $A_{N,k}$ are arbitrary constants, real or complex.By suitably specialized the coefficient $A_{N,k}$ the polynomials $S_N^M(x)$ can be reduced to the classical orthogonal polynomials such as Jacobi, Hermite, Legendre and Laguerre polynomials etc.

The Aleph-function of several variables generalize the multivariable I-function defined by H.M. Sharma and Ahmad [5] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r \end{array}\right|$

$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}]\ ,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:$
$\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:$

$[(\mathrm{c}_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;\ ;[(\mathrm{c}_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]$
$[(\mathrm{d}_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;;[(\mathrm{d}_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\Bigg)$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.8}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{\sum_{i=1}^{R}[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k)]} \tag{1.9}$$

and $$\theta_k(s_k)=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.10}$$

where $j=1$ to $r$ and $k=1$ to $r$

Suppose , as usual , that the parameters

$a_j, j=1,\cdots,p; b_j, j=1,\cdots,q;$
$c_j^{(k)}, j=1,\cdots,n_k; c_{ji^{(k)}}^{(k)}, j=n_k+1,\cdots,p_{i^{(k)}};$
$d_j^{(k)}, j=1,\cdots,m_k; d_{ji^{(k)}}^{(k)}, j=m_k+1,\cdots,q_{i^{(k)}}$; with $k=1\cdots,r, i=1,\cdots,R\ , i^{(k)}=1,\cdots,R^{(k)}$
are complex numbers , and the $\alpha's,\beta's,\gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization

purpose such that

$$U_i^{(k)}=\sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)}+\tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)}+\sum_{j=1}^{n_k}\gamma_j^{(k)}+\tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}-\tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)}-\sum_{j=1}^{m_k}\delta_j^{(k)}$$

$$-\tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)}\leqslant 0 \qquad (1.11)$$

The reals numbers $\tau_i$ are positives for $i=1$ to $R$ , $\tau_{i^{(k)}}$ are positives for $i^{(k)}=1$ to $R^{(k)}$

The contour $L_k$ is in the $s_k$-p lane and run from $\sigma-i\infty$ to $\sigma+i\infty$ where $\sigma$ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)$ with $j=1$ to $m_k$ are separated from those of $\Gamma(1-a_j+\sum_{i=1}^{r}\alpha_j^{(k)}s_k)$ with $j=1$ to $n$ and $\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)$ with $j=1$ to $n_k$ to the left of the contour $L_k$ . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k|<\frac{1}{2}A_i^{(k)}\pi$ , where

$$A_i^{(k)}=\sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)}-\tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)}-\tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)}+\sum_{j=1}^{n_k}\gamma_j^{(k)}-\tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{m_k}\delta_j^{(k)}-\tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)}>0,\text{ with } k=1\cdots,r,i=1,\cdots,R\ ,i^{(k)}=1,\cdots,R^{(k)} \qquad (1.12)$$

The complex numbers $z_i$ are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.
We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\alpha_1}\ldots|z_r|^{\alpha_r}\,)\,,max(\,|z_1|\ldots|z_r|\,)\rightarrow 0$$

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\beta_1}\ldots|z_r|^{\beta_r}\,)\,,min(\,|z_1|\ldots|z_r|\,)\rightarrow\infty$$

where, with $k=1,\cdots,r:\alpha_k=min[Re(d_j^{(k)}/\delta_j^{(k)})],j=1,\cdots,m_k$ and

$$\beta_k=max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})],j=1,\cdots,n_k$$

We will use these following notations in this paper

$$U=p_i,q_i,\tau_i;R\ ;\ V=m_1,n_1;\cdots;m_r,n_r \qquad (1.13)$$

$$\mathrm{W}=p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)},\cdots,p_{i^{(r)}},q_{i^{(r)}},\tau_{i^{(r)}};R^{(r)} \qquad (1.14)$$

$$A=\{(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,n}\},\{\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{n+1,p_i}\} \qquad (1.15)$$

$$B=\{\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}\} \qquad (1.16)$$

$$C=\{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\},\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\},\cdots,\{(c_j^{(r)};\gamma_j^{(r)})_{1,n_r}\},\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \qquad (1.17)$$

$$D=\{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\},\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\},\cdots,\{(d_j^{(r)};\delta_j^{(r)})_{1,m_r}\},\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \qquad (1.18)$$

The multivariable Aleph-function write :

$$\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:V}_{U:W}\left(\begin{array}{c|c} z_1 & \mathrm{A}:\mathrm{C} \\ \cdot & \\ \cdot & \cdots \\ \cdot & \\ z_r & \mathrm{B}:\mathrm{D} \end{array}\right) \tag{1.19}$$

## 2. Jacobi polynomials formulas

**a )** $$\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\lambda}P_n^{(\alpha,\beta)}(x)\mathrm{d}x=\frac{2^{\alpha+\lambda+1}\Gamma(\lambda+1)\Gamma(\alpha+1)\Gamma(\lambda-\beta+1)}{\Gamma(\lambda-\beta-n+1)\Gamma(\alpha+\lambda+n+2)}, Re(\lambda)>-1 \tag{2.1}$$

**b )** $$\int_{-1}^{1}(1-x)^{\eta}(1+x)^{\rho}P_n^{(\alpha,\beta)}(x)\mathrm{d}x=2^{\rho+\eta+1}\sum_{l=1}^{n}\frac{(-n)_l(\alpha+\beta+n+1)_l}{(\alpha+1)_l l!} \tag{2.2}$$

The folowing results will be used throughout this paper ([1],p.945 and 946, [3] p.172)

**c)** $$P_k^{(\alpha,\beta)}(t+\rho)P_k^{(\alpha,\beta)}(t-\rho)=\frac{(-)^k(1+\alpha)_k(1+\beta)_k}{(k!)^2}\sum_{n=0}^{k}\frac{(-k)_n(1+\alpha+\beta+k)_n}{(1+\alpha)_n(1+\beta)_n}P_n^{(\alpha,\beta)}(x)t^n \tag{2.3}$$

**d)** $$\rho^k P_k^{(\alpha,\alpha)}(\frac{1-xt}{\rho})=\frac{(1+\alpha)_k}{k!}\sum_{n=0}^{k}\frac{(-k)_n}{(1+\alpha)_n}\quad P_n^{(\alpha,\alpha)}(x)t^n \tag{2.4}$$

**e)** $$\rho^{-1}(1-t+\rho)^{-\alpha}(1+t+\rho)^{-\beta}=2^{-(\alpha+\beta)}\sum_{n=0}^{\infty}P_n^{(\alpha,\beta)}(x)t^n \tag{2.5}$$

where $\rho=(1-2xt+t^2)^{\frac{1}{2}}$

## 3. Integral formulas

**a )** $$\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\lambda}P_n^{(\alpha,\beta)}(x)S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[x_1(1+x)^{\rho_1},\cdots,x_s(1+x)^{\rho_s}]\aleph_{P_i,Q_i,c_i;r'}^{M,N}\big(z(1+x)^h\big)$$

$$\aleph\left(\begin{array}{c}(1+\mathrm{x})^{h_1}z_1\\ \cdots\\ (1+\mathrm{x})^{h_r}z_r\end{array}\right)\mathrm{d}x=2^{\alpha+\lambda+1}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}2^{\sum_{i=1}^{s}K_i\rho_i+h\eta_{G,g}}$$

$$x_1^{K_1}\cdots x_s^{K_s}A_1\Gamma(\alpha+n+1)\,\aleph_{U_{22}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c}2^{h_1}z_1 & (\beta-\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g};h_1,\cdots,h_r),\\ \cdots & \cdots\\ \cdots & \\ 2^{h_r}z_r & (\beta-\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g}+n;h_1,\cdots,h_r),\end{array}\right.$$

$$\left.\begin{array}{c}(-\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g};h_1,\cdots,h_r),A:C\\ \cdots\\ (-\alpha-\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g}-n-1;h_1,\cdots,h_r),B:D\end{array}\right) \tag{3.1}$$

Provided that

a ) $\rho_1,\cdots,\rho_s>0;h_i>0,i=1,\cdots,r,h>0;Re(\lambda)>-1,Re(\alpha)>-1$

b ) $Re[\lambda+h\dfrac{b_i}{B_i}+\sum_{i=1}^{r}h_i\min_{1\leqslant j\leqslant m_i}\dfrac{d_j^{(i)}}{\delta_j^{(i)}}]>-1$

c ) $|argz| < \frac{1}{2}\pi\Omega$ Where $\Omega = \sum_{j=1}^{M}\beta_j + \sum_{j=1}^{N}\alpha_j - c_i(\sum_{j=M+1}^{Q_i}\beta_{ji} + \sum_{j=N+1}^{P_i}\alpha_{ji}) > 0$ with $i = 1, \cdots, r$

d ) $|argz_k| < \frac{1}{2}A_i^{(k)}\pi$ , where $A_i^{(k)}$ is defined by (1.12)

**Proof of (3.1)**

To establish the finite integral (3.1), express the generalized class of polynomials $S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}$ occuring on the L.H.S in the series form given by (1.5), the Aleph-function in serie form given by (1.3) and the multivariable Aleph-function involving there in terms of Mellin-Barnes contour integral by (1.8). We interchange the order of summation and integration (which is permissible under the conditions stated). Now evaluating the x-integral by using the formula (2.1), after simplifications and on reinterpreting the Mellin-Barnes contour integral, we get the desired result.

**b )** $$\int_{-1}^{1}(1-x)^{\eta}(1+x)^{\rho}P_n^{(\alpha,\beta)}(x)S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix}\mathrm{x}_1(1-x)^{\alpha_1}(1+x)^{\beta_1}\\ \cdots\\ \mathrm{x}_s(1-x)^{\alpha_s}(1+x)^{\beta_s}\end{pmatrix}\aleph_{P_i,Q_i,c_i;r'}^{M,N}\big(z(1-x)^h(1+x)^k\big)$$

$$\aleph\begin{pmatrix}(1\text{-x})^{h_1}(1+x)^{k_1}z_1\\ \cdots\\ (1\text{-x})^{h_r}(1+x)^{k_r}z_r\end{pmatrix}\mathrm{d}x = \sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{l=1}^{n}A_1B_l2^{\eta+\rho+\sum_{i=1}^{s}K_i(\alpha_i+\beta_i)+(h+k)\eta_{G,g}+1}$$

$$x_1^{K_1}\cdots x_s^{K_s}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_Gg!}z^{\eta_{G,g}}\aleph_{U_{21}:W}^{0,\mathrm{n}+2:V}\left(\begin{matrix}2^{h_1+k_1}z_1\\ \cdots\\ \cdots\\ 2^{h_r+k_r}z_r\end{matrix}\ \middle|\ \begin{matrix}(-\rho-\sum_{i=1}^{s}K_i\beta_i-k\eta_{G,g};k_1,\cdots,k_r),\\ \cdots\\ \cdots\cdots\\ \end{matrix}\right.$$

$$\left.\begin{matrix}(\text{-l-}\eta-\sum_{i=1}^{s}K_i\alpha_i-h\eta_{G,g};h_1,\cdots,h_r),A:C\\ \cdots\\ (\text{-1-l-}\eta-\sum_{i=1}^{s}K_i(\alpha_i+\beta_i)-(h+k)\eta_{G,g};h_1+k_1,\cdots,h_r+k_r),B:D\end{matrix}\right) \tag{3.2}$$

where $B_l = \frac{(-n)_l(\alpha+\beta+n+1)_l}{(\alpha+1)_l l!}$ . Provided that

a ) $\rho_1,\cdots,\rho_s > 0; h_i,k_i > 0, i = 1,\cdots,r, h > 0; Re(\eta) > -1, Re(\rho) > -1$

b ) $Re[\rho + k\frac{b_i}{B_i} + \sum_{i=1}^{r}k_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}] > -1$ ; $Re[\eta + h\frac{b_i}{B_i} + \sum_{i=1}^{r}h_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}] > -1$

c ) $|argz| < \frac{1}{2}\pi\Omega$ Where $\Omega = \sum_{j=1}^{M}\beta_j + \sum_{j=1}^{N}\alpha_j - c_i(\sum_{j=M+1}^{Q_i}\beta_{ji} + \sum_{j=N+1}^{P_i}\alpha_{ji}) > 0$ with $i = 1, \cdots, r$

d ) $|argz_k| < \frac{1}{2}A_i^{(k)}\pi$ , where $A_i^{(k)}$ is defined by (1.12)

**Proof of (3.2 )**

To establish the finite integral (3.2), express the generalized class of polynomials $S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}$ occuring on the L.H.S in the series form given by (1.5), the Aleph-function in serie form given by (1.3) and the multivariable Aleph-function involving there in terms of Mellin-Barnes contour integral by (1.8). We interchange the order of summation and integration (which is permissible under the conditions stated). Now evaluating the x-integral by using the formula (2.2), after simplifications and on reinterpreting the Mellin-Barnes contour integral, we get the desired result.

## 4. Main integrals

**a )** $\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\lambda}P_k^{(\alpha,\beta)}(t+\rho)\ P_k^{(\alpha,\beta)}(t-\rho)\ S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[x_1(1+x)^{\rho_1},\cdots,x_s(1+x)^{\rho_s}]$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\big(z(1+x)^h\big)\aleph\begin{pmatrix}(\ 1+\mathrm{x}\ )^{h_1}z_1\\ \cdot\ \cdot\ \cdot\\ (\ 1+\mathrm{x}\ )^{h_r}z_r\end{pmatrix}\mathrm{d}x=(-)^k\frac{\Gamma(1+\alpha+k)\Gamma(1+\beta+k)}{(k!)^2}\times$$

$$\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{n=0}^{k}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}A_1\ \frac{(-k)_n(1+\alpha+\beta+k)_n}{(1+\alpha)_n(1+\beta)_n}t^n x_1^{K_1}\cdots x_s^{K_s}$$

$$2^{\alpha+\lambda+\sum_{i=1}^s K_i\rho_i+h\eta_{G,g}+1}\aleph_{U_{22}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c}2^{h_1}z_1 & (\text{-}\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g};h_1,\cdots,h_r),\\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot & (\beta-\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g}+n;h_1,\cdots,h_r),\\ 2^{h_r}z_r & \end{array}\right.$$

$$\left.\begin{array}{c}(\beta-\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g};h_1,\cdots,h_r),A:C\\ \cdot\ \cdot\ \cdot\\ (\text{-}\alpha-\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g}-n-1;h_1,\cdots,h_r),B:D\end{array}\right) \tag{4.1}$$

valid under the same conditions as needed for (3.1)

**b )** $\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\lambda}\ \rho^k P_k^{(\alpha,\alpha)}(\frac{1-xt}{\rho})\ S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[x_1(1+x)^{\rho_1},\cdots,x_s(1+x)^{\rho_s}]$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\big(z(1+x)^h\big)\aleph\begin{pmatrix}(\ 1+\mathrm{x}\ )^{h_1}z_1\\ \cdot\ \cdot\ \cdot\\ (\ 1+\mathrm{x}\ )^{h_r}z_r\end{pmatrix}\mathrm{d}x=2^{\alpha+\lambda+1}\frac{\Gamma(1+\alpha+k)}{k!}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}$$

$$\sum_{n=0}^{k}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}A_1\ \frac{(-k)_n}{(1+\alpha)_n}\ t^n x_1^{K_1}\cdots x_s^{K_s}2^{\sum_{i=1}^s K_i\rho_i+h\eta_{G,g}}$$

$$\aleph_{U_{22}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c}2^{h_1}z_1 & (\text{-}\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g};h_1,\cdots,h_r),\\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot & (\text{-}\beta-\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g}+n;h_1,\cdots,h_r),\\ 2^{h_r}z_r & \end{array}\right.$$

$$\left.\begin{array}{c}(\beta-\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g};h_1,\cdots,h_r),A:C\\ \cdot\ \cdot\ \cdot\\ (\text{-}\alpha-\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g}-n-1;h_1,\cdots,h_r),B:D\end{array}\right) \tag{4.2}$$

which holds true under the same conditions as needed in (3.1)

**c )** $\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\lambda}\ \rho^{-1}(1-t+\rho)^{-\alpha}(1+t+\rho)^{-\beta}S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[x_1(1+x)^{\rho_1},\cdots,x_s(1+x)^{\rho_s}]$

$$\aleph^{M,N}_{P_i,Q_i,c_i;r'}\big(z(1+x)^h\big)\aleph\begin{pmatrix}(1+\text{x})^{h_1}z_1\\ \cdots\\ (1+\text{x})^{h_r}z_r\end{pmatrix}\mathrm{d}x=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{n=0}^{\infty}$$

$$\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(s)}{B_G g!}z^{\eta_{G,g}}A_1\ \Gamma(1+\alpha+n)t^n\,x_1^{K_1}\cdots x_s^{K_s}2^{-\beta-\alpha+\lambda+\sum_{i=1}^s K_i\rho_i+(h+k)\eta_{G,g}+1}$$

$$\aleph^{0,\mathfrak{n}+2:V}_{U_{22}:W}\left(\begin{matrix}2^{h_1}z_1\\ \cdots\\ \cdots\\ 2^{h_r}z_r\end{matrix}\ \left|\ \begin{matrix}(\text{-}\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g};h_1,\cdots,h_r),\\ \cdots\\ (\text{-}\beta-\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g}+n;h_1,\cdots,h_r),\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(\beta-\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g};h_1,\cdots,h_r),A:C\\ \cdots\\ (\text{-}\alpha-\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g}-n-1;h_1,\cdots,h_r),B:D\end{matrix}\right)\tag{4.3}$$

which holds true under the same conditions as needed in (3.1)

**d )** $\displaystyle\int_{-1}^{1}(1-x)^\eta(1+x)^\rho P_k^{(\alpha,\beta)}(t+\rho)P_k^{(\alpha,\beta)}(t-\rho)S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\begin{pmatrix}\text{x}_1(1-x)^{\alpha_1}(1+x)^{\beta_1}\\ \cdots\\ \text{x}_s(1-x)^{\alpha_s}(1+x)^{\beta_s}\end{pmatrix}$

$$\aleph^{M,N}_{P_i,Q_i,c_i;r'}\big(z(1-x)^h(1+x)^k\big)\quad\aleph\begin{pmatrix}(1\text{-x})^{h_1}(1+x)^{k_1}z_1\\ \cdots\\ (1\text{-x})^{h_r}(1+x)^{k_r}z_r\end{pmatrix}\mathrm{d}x=\frac{\Gamma(1+\alpha+k)\Gamma(1+\beta+k)}{(k!)^2}(-)^k$$

$$\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{n=0}^{k}\sum_{q=0}^{n}A'\ \frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(s)}{B_G g!}z^{\eta_{G,g}}t^n x_1^{K_1}\cdots x_s^{K_s}B_nB_q$$

$$\frac{(-k)_l(\alpha+\beta+k+1)_l}{(\alpha+1)_l l!}\aleph^{0,\mathfrak{n}+2:V}_{U_{21}:W}\left(\begin{matrix}2^{h_1+k_1}z_1\\ \cdots\\ \cdots\\ 2^{h_r+k_r}z_r\end{matrix}\ \left|\ \begin{matrix}(\text{-}\rho-\sum_{i=1}^s K_i\beta_i-k\eta_{G,g};k_1,\cdots,k_r),\\ \cdots\\ \cdots\cdots\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(\text{-}\eta-\sum_{i=1}^s K_i\alpha_i-h\eta_{G,g}-q;h_1,\cdots,h_r),A:C\\ \cdots\\ (\text{-}\eta-\rho-1-\sum_{i=1}^s K_i(\alpha_i+\beta_i)-(h+k)\eta_{G,g}-q;h_1+k_1,\cdots,h_r+k_r),B:D\end{matrix}\right)\tag{4.4}$$

**Where** $B_n=\dfrac{(-k)_n(1+\alpha+\beta+k)_n}{\Gamma(1+\alpha+n)\Gamma(1+\beta+n)}$ **and** $B_q=\dfrac{(-n)_q(\alpha+\beta+n+1)_q}{(\alpha+1)_q q!}$

valid under the same conditions as needed for (3.2)

**e )** $\int_{-1}^{1}\ (1-x)^{\eta}(1+x)^{\rho}\ \rho^k P_k^{(\alpha,\alpha)}(\frac{1-xt}{\rho})\ S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix}\mathrm{x}_1(1-x)^{\alpha_1}(1+x)^{\beta_1}\\ \cdot\ \cdot\ \cdot\\ \mathrm{x}_s(1-x)^{\alpha_s}(1+x)^{\beta_s}\end{pmatrix}$

$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\left(z(1-x)^h(1+x)^k\right)\ \ \aleph\begin{pmatrix}(1\text{-x})^{h_1}(1+x)^{k_1}z_1\\ \cdot\ \cdot\ \cdot\\ (1\text{-x})^{h_r}(1+x)^{k_r}z_r\end{pmatrix}\mathrm{d}x =$

$$\frac{\Gamma(1+\alpha+k)}{k!}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{n=0}^{k}\sum_{q=0}^{n}\ A'\,t^n x_1^{K_1}\cdots x_s^{K_s}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}B_nB_q$$

$$\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{matrix}2^{h_1+k_1}z_1\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ 2^{h_r+k_r}z_r\end{matrix}\ \middle|\ \begin{matrix}(\text{-}\rho-\sum_{i=1}^{s}K_i\beta_i-k\eta_{G,g};k_1,\cdots,k_r),\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\end{matrix}\right.$$

$$\left.\begin{matrix}(\text{-}\eta-\sum_{i=1}^{s}K_i\alpha_i-h\eta_{G,g};h_1,\cdots,h_r),A:C\\ \cdot\ \cdot\ \cdot\\ (\text{-}\eta-\rho-1-\sum_{i=1}^{s}K_i(\alpha_i+\beta_i)-(h+k)\eta_{G,g}-q;h_1+k_1,\cdots,h_r+k_r),B:D\end{matrix}\right) \tag{4.5}$$

where $B_n=\dfrac{(-k)_n}{\Gamma(1+\alpha+n)}$ and $B_q=\dfrac{(2\alpha+n+1)_q}{(\alpha+1)_q q!}$

which holds true under the same conditions as needed in (3.2)

**f)** $\int_{-1}^{1}\ (1-x)^{\alpha}(1+x)^{\eta}\ \rho^{-1}(1-t+\rho)^{-\alpha}(1+t+\rho)^{-\beta}\ S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix}\mathrm{x}_1(1-x)^{\alpha_1}(1+x)^{\beta_1}\\ \cdot\ \cdot\ \cdot\\ \mathrm{x}_s(1-x)^{\alpha_s}(1+x)^{\beta_s}\end{pmatrix}$

$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\left(z(1-x)^h(1+x)^k\right)\ \ \aleph\begin{pmatrix}(1\text{-x})^{h_1}(1+x)^{k_1}z_1\\ \cdot\ \cdot\ \cdot\\ (1\text{-x})^{h_r}(1+x)^{k_r}z_r\end{pmatrix}\mathrm{d}x =$

$$2^{-\alpha-\beta-\rho}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{n=0}^{\infty}\sum_{q=0}^{n}B_qA't^n\,x_1^{K_1}\cdots x_s^{K_s}$$

$$\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{matrix}2^{h_1+k_1}z_1\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ 2^{h_r+k_r}z_r\end{matrix}\ \middle|\ \begin{matrix}(\text{-}\rho-\sum_{i=1}^{s}K_i\beta_i-k\eta_{G,g};k_1,\cdots,k_r),\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\end{matrix}\right.$$

$$\left.\begin{matrix}(\text{-}\eta-\sum_{i=1}^{s}K_i\alpha_i-h\eta_{G,g}-q;h_1,\cdots,h_r),A:C\\ \cdot\ \cdot\ \cdot\\ (\text{-}\eta-\rho-1-\sum_{i=1}^{s}K_i(\alpha_i+\beta_i)-(h+k)\eta_{G,g}-q;h_1+k_1,\cdots,h_r+k_r),B:D\end{matrix}\right) \tag{4.6}$$

where $B_q = \dfrac{(-n)_q(\alpha+\beta+n+1)_q}{(\alpha+1)_q q!}$ , which holds true under the same conditions as needed in (3.2)

**Proofs :**

In order to derive (4.1), we multiply both the sides of (2.3) by $(1-x)^{\alpha}(1+x)^{\lambda}\aleph^{M,N}_{P_i,Q_i,c_i;r'}\big(z(1+x)x^h\big)$

$S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}[x_1(1+x)^{\rho_1},\cdots,x_s(1+x)^{\rho_s}]\aleph\begin{pmatrix}(1+x)^{h_1}z_1\\ \cdots\\ (1+x)^{h_r}z_r\end{pmatrix}$ and integrating both sides with respect to x between the limits $-1$ to $1$, we obtain

$$\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\lambda}P_k^{(\alpha,\beta)}(t+\rho)\ P_k^{(\alpha,\beta)}(t-\rho)\ S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}[x_1(1+x)^{\rho_1},\cdots,x_s(1+x)^{\rho_s}]$$

$$\aleph^{M,N}_{P_i,Q_i,c_i;r'}\big(z(1+x)^h\big)\aleph\begin{pmatrix}(1+x)^{h_1}z_1\\ \cdots\\ (1+x)^{h_r}z_r\end{pmatrix}\mathrm{d}x=\int_{-1}^{1}\frac{\Gamma(1+\alpha+k)\Gamma(1+\beta+k)}{(k!)^2}\sum_{n=0}^{k}\frac{(-)^k(-k)_n(1+\alpha+\beta+k)_n}{(1+\alpha)_n(1+\beta)_n}$$

$$P_n^{(\alpha,\beta)}(x)t^nS^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}[x_1(1+x)^{\rho_1},\cdots,x_s(1+x)^{\rho_s}]\aleph^{M,N}_{P_i,Q_i,c_i;r'}\big(z(1+x)x^h\big)\aleph\begin{pmatrix}(1+x)^{h_1}z_1\\ \cdots\\ (1+x)^{h_r}z_r\end{pmatrix}\mathrm{d}x \quad (4.7)$$

Now, we interchange the order of integration and summation on the right of (4.7),which is justified due to the absolute convergent of the integral involving in the process, then we evaluate the inner x-integral with the help of (3.1) and on reinterpreting the Mellin-Barnes contour integral, we get the desired result.

In order to derive (4.4), we multiply both the sides of (2.3) by $(1-x)^{\eta}(1+x)^{\rho}S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\begin{pmatrix}\mathrm{x}_1(1-x)^{\alpha_1}(1+x)^{\beta_1}\\ \cdots\\ \mathrm{x}_s(1-x)^{\alpha_s}(1+x)^{\beta_s}\end{pmatrix}$

$\aleph^{M,N}_{P_i,Q_i,c_i;r'}\big(z(1-x)^h(1+x)^k\big)\ \ \aleph\begin{pmatrix}(1\text{-x})^{h_1}(1+x)^{k_1}z_1\\ \cdots\\ (1\text{-x})^{h_r}(1+x)^{k_r}z_r\end{pmatrix}$ and integrating both sides with respect to x between the limits $-1$ to $1$, we obtain

$$\int_{-1}^{1}(1-x)^{\eta}(1+x)^{\rho}P_k^{(\alpha,\beta)}(t+\rho)P_k^{(\alpha,\beta)}(t-\rho)S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\begin{pmatrix}\mathrm{x}_1(1-x)^{\alpha_1}(1+x)^{\beta_1}\\ \cdots\\ \mathrm{x}_s(1-x)^{\alpha_s}(1+x)^{\beta_s}\end{pmatrix}$$

$$\aleph^{M,N}_{P_i,Q_i,c_i;r'}\big(z(1-x)^h(1+x)^k\big)\ \ \aleph\begin{pmatrix}(1\text{-x})^{h_1}(1+x)^{k_1}z_1\\ \cdots\\ (1\text{-x})^{h_r}(1+x)^{k_r}z_r\end{pmatrix}=\int_{-1}^{1}\frac{(-)^k\Gamma(1+\alpha+k)\Gamma(1+\beta+k)}{(k!)^2}$$

$$\sum_{n=0}^{k}\ \frac{(-k)_n(1+\alpha+\beta+k)_n}{(1+\alpha)_n(1+\beta)_n}P_n^{(\alpha,\beta)}(x)t^nS^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\begin{pmatrix}\mathrm{x}_1(1-x)^{\alpha_1}(1+x)^{\beta_1}\\ \cdots\\ \mathrm{x}_s(1-x)^{\alpha_s}(1+x)^{\beta_s}\end{pmatrix}\aleph^{M,N}_{P_i,Q_i,c_i;r'}\big(z(1-x)^h(1+x)^k\big)$$

$$\aleph\begin{pmatrix}(1\text{-x})^{h_1}(1+x)^{k_1}z_1\\ \cdots\\ (1\text{-x})^{h_r}(1+x)^{k_r}z_r\end{pmatrix}\mathrm{d}x \quad (4.8)$$

Now, we interchange the order of integration and summation on the right of (4.8),which is justified due to the absolute convergent of the integral involving in the process, then we evaluate the inner x-integral with the help of (3.2) and on reinterpreting the Mellin-Barnes contour integral, we get the desired result.

To prove (4.2) and (4.5), we use the equation (2.4) and the similar method that (4.1) and (4.4) respectively and to prove (4.3) and (4.6), we use the equation (2.5) and the similar method that (4.1) and (4.4) respectively.

## 5. Particular cases

If $x_2 = \cdots = x_s = 0$ , then the class of polynomials $S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}(\tau_1,\cdots,\tau_s)$ defined of (1.14) degenere to the class of polynomial $S_N^M(x_1)$ defined by Srivastava [7].Replace $x_1$ by $X$ in these section and $A = \frac{(-N)_{Mk}}{k!}A_{N,k}$

**a )** $$\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\lambda}P_k^{(\alpha,\beta)}(t+\rho)\ P_k^{(\alpha,\beta)}(t-\rho)\ S_N^M[X(1+x)^{\rho_1}]$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\left(z(1+x)^h\right)\aleph\begin{pmatrix}(\ 1+\mathrm{x}\ )^{h_1}z_1\\ \cdots\\ (\ 1+\mathrm{x}\ )^{h_r}z_r\end{pmatrix}\mathrm{d}x = \frac{\Gamma(1+\alpha+k)\Gamma(1+\beta+k)}{(k!)^2}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K=0}^{[N/M]}\sum_{n=0}^{k}$$

$$\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}A_1\frac{(-k)_n(1+\alpha+\beta+k)_n}{(1+\alpha)_n(1+\beta)_n}t^n X_1^K 2^{\alpha+\lambda+\sum_{i=1}^{s}K_i\rho_i+h\eta_{G,g}+1}$$

$$\aleph_{U_{22}:W}^{0,\mathfrak{n}+2:V}\left(\begin{matrix}2^{h_1}z_1\\ \cdots\\ \cdots\\ 2^{h_r}z_r\end{matrix}\ \middle|\ \begin{matrix}(\text{-}\lambda-K\rho_1-h\eta_{G,g};h_1,\cdots,h_r),\\ \cdots\\ (\text{-}\beta-\lambda-K\rho_1-h\eta_{G,g}+n;h_1,\cdots,h_r),\end{matrix}\right.$$

$$\left.\begin{matrix}(\beta-\lambda-K\rho_1-h\eta_{G,g};h_1,\cdots,h_r),A:C\\ \cdots\\ (\text{-}\alpha-\lambda-K\rho_1-h\eta_{G,g}-n-1;h_1,\cdots,h_r),B:D\end{matrix}\right) \tag{5.1}$$

valid under the same conditions as needed for (3.1)

**b )** $$\int_{-1}^{1}(1-x)^{\eta}(1+x)^{\rho}P_k^{(\alpha,\beta)}(t+\rho)P_k^{(\alpha,\beta)}(t-\rho)S_N^M[X(1-x)^{\alpha_1}(1+x)^{\beta_1}]$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\left(z(1-x)^h(1+x)^k\right)\ \aleph\begin{pmatrix}(\text{1-x})^{h_1}(1+x)^{k_1}z_1\\ \cdots\\ (\text{1-x})^{h_r}(1+x)^{k_r}z_r\end{pmatrix}\mathrm{d}x = \frac{\Gamma(1+\alpha+k)\Gamma(1+\beta+k)}{(k!)^2}(-)^k$$

$$\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K=0}^{[N/M]}\sum_{n=0}^{k}\sum_{q=0}^{n}A'\ \frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}t^n X_1^K B_n B_q$$

$$\frac{(-k)_l(\alpha+\beta+k+1)_l}{(\alpha+1)_l l!}\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{matrix}2^{h_1+k_1}z_1\\ \cdots\\ \cdots\\ 2^{h_r+k_r}z_r\end{matrix}\ \middle|\ \begin{matrix}(\text{-}\rho-K\beta_1-k\eta_{G,g};k_1,\cdots,k_r),\\ \cdots\\ \cdots\cdots\end{matrix}\right.$$

$$\left.\begin{matrix}(\text{-}\eta-K\alpha_1-h\eta_{G,g}-q;h_1,\cdots,h_r),A:C\\ \cdots\\ (\text{-}\eta-\rho-1-K(\alpha_1+\beta_1)-(h+k)\eta_{G,g}-q;h_1+k_1,\cdots,h_r+k_r),B:D\end{matrix}\right) \tag{5.2}$$

where $B_n=\dfrac{(-k)_n(+\alpha+\beta+k)_n}{\Gamma(+\alpha+n)\Gamma(+\beta+n)}$ **and** $B_q=\dfrac{(-n)_q(\alpha+\beta+n+1)_q}{(\alpha+1)_q q!}$

## 6. Multivariable I-function

In these section, we get six formulas concerning the multivariable I-function then defined by Sharma et al [4].

Let $\tau_i=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=1$

$$\textbf{a)}\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\lambda}P_k^{(\alpha,\beta)}(t+\rho)\ P_k^{(\alpha,\beta)}(t-\rho)\ S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[x_1(1+x)^{\rho_1},\cdots,x_s(1+x)^{\rho_s}]$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\big(z(1+x)^h\big)I\begin{pmatrix}(1+\mathrm{x})^{h_1}z_1\\ \cdots\\ (1+\mathrm{x})^{h_r}z_r\end{pmatrix}\mathrm{d}x=\quad(-)^k\frac{\Gamma(1+\alpha+k)\Gamma(1+\beta+k)}{(k!)^2}\sum_{G=1}^{M}\sum_{g=0}^{\infty}$$

$$\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{n=0}^{k}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}A_1\ \frac{(-k)_n(1+\alpha+\beta+k)_n}{(1+\alpha)_n(1+\beta)_n}t^n x_1^{K_1}\cdots x_s^{K_s}$$

$$2^{\alpha+\lambda+\sum_{i=1}^{s}K_i\rho_i+h\eta_{G,g}+1}I_{U_{22}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c}2^{h_1}z_1 & (\text{-}\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g};h_1,\cdots,h_r),\\ \cdots & \cdots\\ \cdots & \\ 2^{h_r}z_r & (\text{-}\beta-\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g}+n;h_1,\cdots,h_r),\end{array}\right.$$

$$\left.\begin{array}{c}(\beta-\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g};h_1,\cdots,h_r),A'':C''\\ \cdots\\ (\text{-}\alpha-\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g}-n-1;h_1,\cdots,h_r),B'':D''\end{array}\right)\tag{6.1}$$

valid under the same notations and conditions as needed for (4.1)

$$\textbf{b)}\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\lambda}\ \rho^k P_k^{(\alpha,\alpha)}(\frac{1-xt}{\rho})\ S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[x_1(1+x)^{\rho_1},\cdots,x_s(1+x)^{\rho_s}]$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\big(z(1+x)^h\big)I\begin{pmatrix}(1+\mathrm{x})^{h_1}z_1\\ \cdots\\ (1+\mathrm{x})^{h_r}z_r\end{pmatrix}\mathrm{d}x=\frac{\Gamma(1+\alpha+k)}{k!}\ \sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{n=0}^{k}$$

$$\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}A_1\ \frac{(-k)_n}{(1+\alpha)_n}t^n x_1^{K_1}\cdots x_s^{K_s}2^{\alpha+\lambda+\sum_{i=1}^{s}K_i\rho_i+h\eta_{G,g}+1}I_{U_{22}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|}2^{h_1}z_1\\ \cdots\\ \cdots\\ 2^{h_r}z_r\end{array}\right.$$

$$\begin{array}{c}(\text{-}\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g};h_1,\cdots,h_r),\\ \cdots\\ (\text{-}\beta-\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g}+n;h_1,\cdots,h_r),\end{array}$$

$$\left.\begin{array}{c}(\beta-\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g};h_1,\cdots,h_r),A'':C''\\ \cdots\\ (\text{-}\alpha-\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g}-n-1;h_1,\cdots,h_r),B'':D''\end{array}\right)\tag{6.2}$$

which holds true under the same conditions and notations as needed in (4.2)

$$\textbf{c)}\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\lambda}\ \rho^{-1}(1-t+\rho)^{-\alpha}(1+t+\rho)^{-\beta}S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[x_1(1+x)^{\rho_1},\cdots,x_s(1+x)^{\rho_s}]$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\big(z(1+x)^h\big)I\begin{pmatrix}(\ 1+\mathrm{x}\ )^{h_1}z_1\\ \cdot\ \cdot\ \cdot\\ (\ 1+\mathrm{x}\ )^{h_r}z_r\end{pmatrix}\mathrm{d}x = \sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{n=0}^{\infty}$$

$$\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}A_1\ \Gamma(1+\beta+n)t^n\,x_1^{K_1}\cdots x_s^{K_s}2^{-\beta-\alpha+\lambda+\sum_{i=1}^s K_i\rho_i+(h+k)\eta_{G,g}+1}$$

$$I_{U_{22}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c}2^{h_1}z_1 & (\text{-}\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g};h_1,\cdots,h_r),\\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot & \\ 2^{h_r}z_r & (\text{-}\beta-\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g}+n;h_1,\cdots,h_r),\end{array}\right.$$

$$\left.\begin{array}{c}(\beta-\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g};h_1,\cdots,h_r),A'':C''\\ \cdot\ \cdot\ \cdot\\ (\text{-}\alpha-\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g}-n-1;h_1,\cdots,h_r),B'':D''\end{array}\right) \tag{6.3}$$

which holds true under the same notations and conditions as needed in (4.3)

$$\textbf{d)}\int_{-1}^{1}(1-x)^{\eta}(1+x)^{\rho}P_k^{(\alpha,\beta)}(t+\rho)P_k^{(\alpha,\beta)}(t-\rho)S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix}\mathrm{x}_1(1-x)^{\alpha_1}(1+x)^{\beta_1}\\ \cdot\ \cdot\ \cdot\\ \mathrm{x}_s(1-x)^{\alpha_s}(1+x)^{\beta_s}\end{pmatrix}$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\big(z(1-x)^h(1+x)^k\big)\ \ I\begin{pmatrix}(1\text{-x})^{h_1}(1+x)^{k_1}z_1\\ \cdot\ \cdot\ \cdot\\ (1\text{-x})^{h_r}(1+x)^{k_r}z_r\end{pmatrix}\mathrm{d}x = \frac{\Gamma(1+\alpha+k)\Gamma(1+\beta+k)}{(k!)^2}$$

$$\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{n=0}^{k}\sum_{q=0}^{n}A_1\ \ \frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}t^n 2^{\eta+\rho+\sum_{i=1}^s K_i(\alpha_i+\beta_i)+(h+k)\eta_{G,g}+1}$$

$$x_1^{K_1}\cdots x_s^{K_s}B_nB_q\frac{(-k)_l(\alpha+\beta+k+1)_l}{(\alpha+1)_l l!}I_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c}2^{h_1+k_1}z_1 & (\text{-}\rho-\sum_{i=1}^s K_i\beta_i-k\eta_{G,g};k_1,\cdots,k_r),\\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\\ 2^{h_r+k_r}z_r & \end{array}\right.$$

$$\left.\begin{array}{c}(\text{-}\eta-\sum_{i=1}^s K_i\alpha_i-h\eta_{G,g}-q;h_1,\cdots,h_r),A'':C''\\ \cdot\ \cdot\ \cdot\\ (\text{-}\eta-\rho-1-\sum_{i=1}^s K_i(\alpha_i+\beta_i)-(h+k)\eta_{G,g}-q;h_1+k_1,\cdots,h_r+k_r),B'':D''\end{array}\right) \tag{6.4}$$

valid under the same notations and conditions as needed for (4.4)

**e )** $\int_{-1}^{1} \ (1-x)^{\eta}(1+x)^{\rho} \ \rho^k P_k^{(\alpha,\alpha)}(\frac{1-xt}{\rho}) \ S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix}\mathrm{x}_1(1-x)^{\alpha_1}(1+x)^{\beta_1}\\ \cdot\ \cdot\ \cdot\\ \mathrm{x}_s(1-x)^{\alpha_s}(1+x)^{\beta_s}\end{pmatrix}$

$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\big(z(1-x)^h(1+x)^k\big) \ \ I\begin{pmatrix}(1\text{-x})^{h_1}(1+x)^{k_1}z_1\\ \cdot\ \cdot\ \cdot\\ (1\text{-x})^{h_r}(1+x)^{k_r}z_r\end{pmatrix}\mathrm{d}x$ =

$$\frac{\Gamma(1+\alpha+k)}{k!}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{n=0}^{k}\sum_{q=0}^{n} \ A'\,t^n x_1^{K_1}\cdots x_s^{K_s}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}B_nB_q$$

$$I_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{matrix}2^{h_1+k_1}z_1\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ 2^{h_r+k_r}z_r\end{matrix}\ \middle|\ \begin{matrix}(\text{-}\rho-\sum_{i=1}^{s}K_i\beta_i-k\eta_{G,g};k_1,\cdots,k_r),\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\end{matrix}\right.$$

$$\left.\begin{matrix}(\text{-}\eta-\sum_{i=1}^{s}K_i\alpha_i-h\eta_{G,g}-q;h_1,\cdots,h_r),A'':C''\\ \cdot\ \cdot\ \cdot\\ (\text{-}\eta-\rho-1-\sum_{i=1}^{s}K_i(\alpha_i+\beta_i)-(h+k)\eta_{G,g}-q;h_1+k_1,\cdots,h_r+k_r),B'':D''\end{matrix}\right) \tag{6.5}$$

where $B_n=\dfrac{(-k)_n}{\Gamma(1+\alpha+n)}$ and $B_q=\dfrac{(2\alpha+n+1)_q}{(\alpha+1)_q q!}$

which holds true under the same notations and conditions as needed in (4.5)

**f)** $\int_{-1}^{1} \ (1-x)^{\alpha}(1+x)^{\rho} \ \rho^{-1}(1-t+\rho)^{-\alpha}(1+t+\rho)^{-\beta} \ S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix}\mathrm{x}_1(1-x)^{\alpha_1}(1+x)^{\beta_1}\\ \cdot\ \cdot\ \cdot\\ \mathrm{x}_s(1-x)^{\alpha_s}(1+x)^{\beta_s}\end{pmatrix}$

$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\big(z(1-x)^h(1+x)^k\big) \ \ I\begin{pmatrix}(1\text{-x})^{h_1}(1+x)^{k_1}z_1\\ \cdot\ \cdot\ \cdot\\ (1\text{-x})^{h_r}(1+x)^{k_r}z_r\end{pmatrix}\mathrm{d}x$ =

$$2^{-\alpha-\beta-\rho}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{n=0}^{\infty}\sum_{q=0}^{n}B_qA't^n\,x_1^{K_1}\cdots x_s^{K_s}$$

$$\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}I_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{matrix}2^{h_1+k_1}z_1\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ 2^{h_r+k_r}z_r\end{matrix}\ \middle|\ \begin{matrix}(\text{-}\rho-\sum_{i=1}^{s}K_i\beta_i-k\eta_{G,g};k_1,\cdots,k_r),\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\end{matrix}\right.$$

$$\left.\begin{matrix}(\text{-}\eta-\sum_{i=1}^{s}K_i\alpha_i-h\eta_{G,g}-q;h_1,\cdots,h_r),A'':C''\\ \cdot\ \cdot\ \cdot\\ (\text{-}\eta-\rho-1-\sum_{i=1}^{s}K_i(\alpha_i+\beta_i)-(h+k)\eta_{G,g}-q;h_1+k_1,\cdots,h_r+k_r),B'':D''\end{matrix}\right) \tag{6.6}$$

which holds true under the same notations and conditions as needed in (4.6)

## 7.Aleph-function of two variables

In these section, we get the six integrals concerning the Aleph-function of two variables defined by K. Sharma [6].

$$\textbf{a )}\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\lambda}P_k^{(\alpha,\beta)}(t+\rho)\ P_k^{(\alpha,\beta)}(t-\rho)\ S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[x_1(1+x)^{\rho_1},\cdots,x_s(1+x)^{\rho_s}]$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\big(z(1+x)^h\big)\aleph\begin{pmatrix}(1+\mathrm{x})^{h_1}z_1\\ \cdots\\ (1+\mathrm{x})^{h_2}z_2\end{pmatrix}\mathrm{d}x=\frac{\Gamma(1+\alpha+k)\Gamma(1+\beta+k)}{(k!)^2}\sum_{G=1}^{M}\sum_{g=0}^{\infty}$$

$$\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{n=0}^{k}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_Gg!}z^{\eta_{G,g}}A_1\frac{(-)^k(-k)_n(1+\alpha+\beta+k)_n}{(1+\alpha)_n(1+\beta)_n}t^n$$

$$2^{\alpha+\lambda+\sum_{i=1}^{s}K_i\rho_i+h\eta_{G,g}+1}x_1^{K_1}\cdots x_s^{K_s}\aleph_{U_{22}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c}2^{h_1}z_1 & (-\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g};h_1,h_2),\\ \cdots & \cdots\\ \cdots & (-\beta-\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g}+n;h_1,h_2),\\ 2^{h_2}z_2 & \end{array}\right.$$

$$\left.\begin{array}{c}(\beta-\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g};h_1,h_2),A_2:C_2\\ \cdots\\ (-\alpha-\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g}-n-1;h_1,h_2),B_2:D_2\end{array}\right)\tag{7.1}$$

valid under the same notations and conditions as needed for (4.1) with $r=2$

$$\textbf{b )}\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\lambda}\ \rho^kP_k^{(\alpha,\alpha)}(\frac{1-xt}{\rho})\ S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[x_1(1+x)^{\rho_1},\cdots,x_s(1+x)^{\rho_s}]$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\big(z(1+x)^h\big)\aleph\begin{pmatrix}(1+\mathrm{x})^{h_1}z_1\\ \cdots\\ (1+\mathrm{x})^{h_2}z_2\end{pmatrix}\mathrm{d}x=\frac{\Gamma(1+\alpha+k)}{k!}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}$$

$$\sum_{n=0}^{k}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_Gg!}z^{\eta_{G,g}}A_1\ \frac{(-k)_n}{(1+\alpha)_n}t^nx_1^{K_1}\cdots x_s^{K_s}2^{\alpha+\lambda+\sum_{i=1}^{s}K_i\rho_i+h\eta_{G,g}+1}$$

$$\aleph_{U_{22}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c}2^{h_1}z_1 & (-\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g};h_1,\cdots,h_r),\\ \cdots & \cdots\\ \cdots & (-\beta-\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g}+n;h_1,\cdots,h_r),\\ 2^{h_r}z_r & \end{array}\right.$$

$$\left.\begin{array}{c}(\beta-\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g};h_1,\cdots,h_r),A:C\\ \cdots\\ (-\alpha-\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g}-n-1;h_1,\cdots,h_r),B:D\end{array}\right)\tag{7.2}$$

which holds true under the same notations and conditions as needed in (4.2) with $r=2$

**c)** $$\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\lambda}\ \rho^{-1}(1-t+\rho)^{-\alpha}(1+t+\rho)^{-\beta}S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[x_1(1+x)^{\rho_1},\cdots,x_s(1+x)^{\rho_s}]$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\big(z(1+x)^h\big)\aleph\begin{pmatrix}(\ 1+\mathrm{x}\ )^{h_1}z_1\\ \cdot\ \cdot\ \cdot\\ (\ 1+\mathrm{x}\ )^{h_2}z_2\end{pmatrix}\mathrm{d}x = \sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{n=0}^{\infty}$$

$$\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}A\ \Gamma(1+\beta+n)t^n\,x_1^{K_1}\cdots x_s^{K_s}$$

$$2^{-\beta-\alpha+\lambda+\sum_{i=1}^s K_i\rho_i+(h+k)\eta_{G,g}+1}\aleph_{U_{22}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c}2^{h_1}z_1\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ 2^{h_2}z_2\end{array}\middle|\begin{array}{c}(\text{-}\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g};h_1,h_2),\\ \cdot\ \cdot\ \cdot\\ (\text{-}\beta-\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g}+n;h_1,h_2),\end{array}\right.$$

$$\left.\begin{array}{c}(\beta-\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g};h_1,h_2),A_2:C_2\\ \cdot\ \cdot\ \cdot\\ (\text{-}\alpha-\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g}-n-1;h_1,h_2),B_2:D_2\end{array}\right) \tag{7.3}$$

which holds true under the same notations and conditions as needed in (4.3) with $r=2$

**d)** $$\int_{-1}^{1}(1-x)^{\eta}(1+x)^{\rho}P_k^{(\alpha,\beta)}(t+\rho)P_k^{(\alpha,\beta)}(t-\rho)S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix}\mathrm{x}_1(1-x)^{\alpha_1}(1+x)^{\beta_1}\\ \cdot\ \cdot\ \cdot\\ \mathrm{x}_s(1-x)^{\alpha_s}(1+x)^{\beta_s}\end{pmatrix}$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\big(z(1-x)^h(1+x)^k\big)\quad\aleph\begin{pmatrix}(1\text{-x})^{h_1}(1+x)^{k_1}z_1\\ \cdot\ \cdot\ \cdot\\ (1\text{-x})^{h_2}(1+x)^{k_2}z_2\end{pmatrix}\mathrm{d}x = \frac{\Gamma(1+\alpha+k)\Gamma(1+\beta+k)}{(k!)^2}(-)^k$$

$$\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{n=0}^{k}\sum_{q=0}^{n}A'\ \frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}t^n x_1^{K_1}\cdots x_s^{K_s}B_nB_q$$

$$\frac{(-k)_l(\alpha+\beta+k+1)_l}{(\alpha+1)_l l!}\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c}2^{h_1+k_1}z_1\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ 2^{h_2+k_2}z_2\end{array}\middle|\begin{array}{c}(\text{-}\rho-\sum_{i=1}^s K_i\beta_i-k\eta_{G,g};k_1,k_2),\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\end{array}\right.$$

$$\left.\begin{array}{c}(\text{-}\eta-\sum_{i=1}^s K_i\alpha_i-h\eta_{G,g}-q;h_1,h_2),A_2:C_2\\ \cdot\ \cdot\ \cdot\\ (\text{-}\eta-\rho-1-\sum_{i=1}^s K_i(\alpha_i+\beta_i)-(h+k)\eta_{G,g}-q;h_1+k_1,h_2+k_2),B_2:D_2\end{array}\right) \tag{7.4}$$

valid under the same notations and conditions as needed for (4.4) with $r=2$

**e )** $$\int_{-1}^{1} (1-x)^{\eta}(1+x)^{\rho}\ \rho^k P_k^{(\alpha,\alpha)}(\frac{1-xt}{\rho})\ S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix} \mathrm{x}_1(1-x)^{\alpha_1}(1+x)^{\beta_1} \\ \cdot\ \cdot\ \cdot \\ \mathrm{x}_s(1-x)^{\alpha_s}(1+x)^{\beta_s} \end{pmatrix}$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\left(z(1-x)^h(1+x)^k\right)\ \ \aleph\begin{pmatrix} (1\text{-x})^{h_1}(1+x)^{k_1}z_1 \\ \cdot\ \cdot\ \cdot \\ (1\text{-x})^{h_2}(1+x)^{k_2}z_2 \end{pmatrix}\mathrm{d}x = \frac{\Gamma(1+\alpha+k)}{k!}\sum_{G=1}^{M}\sum_{g=0}^{\infty}$$

$$\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{n=0}^{k}\sum_{q=0}^{n} A'\,t^n x_1^{K_1}\cdots x_s^{K_s}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}B_nB_q\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c} 2^{h_1+k_1}z_1 \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ 2^{h_2+k_2}z_2 \end{array}\right|$$

$$\begin{array}{c}(\text{-}\eta-\sum_{i=1}^{s}K_i\alpha_i-h\eta_{G,g}-q;h_1,h_2), \\ \cdot\ \cdot\ \cdot \\ (\text{-}\eta-\rho-1-\sum_{i=1}^{s}K_i(\alpha_i+\beta_i)-(h+k)\eta_{G,g}-q;h_1+k_1,h_2+k_2),\end{array}$$

$$\left.\begin{array}{c}(\text{-}\rho-\sum_{i=1}^{s}K_i\beta_i-k\eta_{G,g};k_1,k_2),A_2:C_2 \\ \cdot\ \cdot\ \cdot \\ \mathrm{B}_2:D_2\end{array}\right) \tag{7.5}$$

which holds true under the same notations and conditions as needed in (4.5) with $r=2$

**f)** $$\int_{-1}^{1} (1-x)^{\alpha}(1+x)^{\rho}\ \rho^{-1}(1-t+\rho)^{-\alpha}(1+t+\rho)^{-\beta}\ S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix} \mathrm{x}_1(1-x)^{\alpha_1}(1+x)^{\beta_1} \\ \cdot\ \cdot\ \cdot \\ \mathrm{x}_s(1-x)^{\alpha_s}(1+x)^{\beta_s} \end{pmatrix}$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\left(z(1-x)^h(1+x)^k\right)\ \ \aleph\begin{pmatrix} (1\text{-x})^{h_1}(1+x)^{k_1}z_1 \\ \cdot\ \cdot\ \cdot \\ (1\text{-x})^{h_2}(1+x)^{k_2}z_2 \end{pmatrix}\mathrm{d}x =$$

$$2^{-\alpha-\beta-\rho}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{n=0}^{\infty}\sum_{q=0}^{n}B_qA't^n\,x_1^{K_1}\cdots x_s^{K_s}$$

$$\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}\ \aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c} 2^{h_1+k_1}z_1 \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ 2^{h_2+k_2}z_2 \end{array}\right|\begin{array}{c}(\text{-}\rho-\sum_{i=1}^{s}K_i\beta_i-k\eta_{G,g};k_1,k_2), \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\end{array}$$

$$\left.\begin{array}{c}(\text{-}\eta-\sum_{i=1}^{s}K_i\alpha_i-h\eta_{G,g}-q;h_1,h_2),A_2:C_2 \\ \cdot\ \cdot\ \cdot \\ (\text{-}\eta-\rho-1-\sum_{i=1}^{s}K_i(\alpha_i+\beta_i)-(h+k)\eta_{G,g}-q;h_1+k_1,h_2+k_2),B_2:D_2\end{array}\right) \tag{7.6}$$

which holds true under the same notations and conditions as needed in (4.6)

## 8. I-function of two variables

In these section, we get two results of double series concerning the I-function of two variables defined by Sharma and Mishra [5]. Let $\tau=\tau'=\tau''=1$. In these section, we get the six integrals concerning the Aleph-function of two variables defined by K. Sharma [6].

**a )** $$\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\lambda}P_k^{(\alpha,\beta)}(t+\rho)\ P_k^{(\alpha,\beta)}(t-\rho)\ S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[x_1(1+x)^{\rho_1},\cdots,x_s(1+x)^{\rho_s}]$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\big(z(1+x)^h\big)I\begin{pmatrix}(\ 1+\mathrm{x}\ )^{h_1}z_1\\ \cdots\\ (\ 1+\mathrm{x}\ )^{h_2}z_2\end{pmatrix}\mathrm{d}x=\frac{\Gamma(1+\alpha+k)\Gamma(1+\beta+k)}{(k!)^2}\sum_{G=1}^{M}\sum_{g=0}^{\infty}$$

$$\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{n=0}^{k}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}A_1\frac{(-)^k(-k)_n(1+\alpha+\beta+k)_n}{(1+\alpha)_n(1+\beta)_n}t^n$$

$$2^{\alpha+\lambda+\sum_{i=1}^s K_i\rho_i+h\eta_{G,g}+1}x_1^{K_1}\cdots x_s^{K_s}\,I_{U_{22}:W}^{0,\mathfrak{n}+2:V}\left(\begin{matrix}2^{h_1}z_1\\ \cdots\\ \cdots\\ 2^{h_2}z_2\end{matrix}\ \middle|\ \begin{matrix}(\text{-}\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g};h_1,h_2),\\ \cdots\\ (\text{-}\beta-\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g}+n;h_1,h_2),\end{matrix}\right.$$

$$\left.\begin{matrix}(\beta-\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g};h_1,h_2),A_2:C_2\\ \cdots\\ (\text{-}\alpha-\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g}-n-1;h_1,h_2),B_2:D_2\end{matrix}\right)\tag{8.1}$$

valid under the same notations and conditions as needed for (4.1) with $r=2$

**b )** $$\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\lambda}\ \rho^k P_k^{(\alpha,\alpha)}(\frac{1-xt}{\rho})\ S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[x_1(1+x)^{\rho_1},\cdots,x_s(1+x)^{\rho_s}]$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\big(z(1+x)^h\big)I\begin{pmatrix}(\ 1+\mathrm{x}\ )^{h_1}z_1\\ \cdots\\ (\ 1+\mathrm{x}\ )^{h_2}z_2\end{pmatrix}\mathrm{d}x=\frac{\Gamma(1+\alpha+k)}{k!}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}$$

$$\sum_{n=0}^{k}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}A_1\ \frac{(-k)_n}{(1+\alpha)_n}t^n x_1^{K_1}\cdots x_s^{K_s}2^{\alpha+\lambda+\sum_{i=1}^s K_i\rho_i+h\eta_{G,g}+1}$$

$$I_{U_{22}:W}^{0,\mathfrak{n}+2:V}\left(\begin{matrix}2^{h_1}z_1\\ \cdots\\ \cdots\\ 2^{h_r}z_r\end{matrix}\ \middle|\ \begin{matrix}(\text{-}\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g};h_1,\cdots,h_r),\\ \cdots\\ (\text{-}\beta-\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g}+n;h_1,\cdots,h_r),\end{matrix}\right.$$

$$\left.\begin{matrix}(\beta-\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g};h_1,\cdots,h_r),A:C\\ \cdots\\ (\text{-}\alpha-\lambda-\sum_{i=1}^s K_i\rho_i-h\eta_{G,g}-n-1;h_1,\cdots,h_r),B:D\end{matrix}\right)\tag{8.2}$$

which holds true under the same notations and conditions as needed in (4.2) with $r=2$

**c )** $$\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\lambda}\ \rho^{-1}(1-t+\rho)^{-\alpha}(1+t+\rho)^{-\beta}S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[x_1(1+x)^{\rho_1},\cdots,x_s(1+x)^{\rho_s}]$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\big(z(1+x)^h\big)I\begin{pmatrix}(\ 1+\mathrm{x}\ )^{h_1}z_1\\ \cdots\\ (\ 1+\mathrm{x}\ )^{h_2}z_2\end{pmatrix}\mathrm{d}x=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{n=0}^{\infty}$$

$$\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}A\ \Gamma(1+\beta+n)t^n x_1^{K_1}\cdots x_s^{K_s}2^{-\beta-\alpha+\lambda+\sum_{i=1}^{s}K_i\rho_i+(h+k)\eta_{G,g}+1}$$

$$I_{U_{22}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c}2^{h_1}z_1 & (\text{-}\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g};h_1,h_2),\\ \cdots & \cdots\\ \cdots & \\ 2^{h_2}z_2 & (\text{-}\beta-\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g}+n;h_1,h_2),\end{array}\right.$$

$$\left.\begin{array}{c}(\beta-\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g};h_1,h_2),A_2:C_2\\ \cdots\\ (\text{-}\alpha-\lambda-\sum_{i=1}^{s}K_i\rho_i-h\eta_{G,g}-n-1;h_1,h_2),B_2:D_2\end{array}\right) \tag{8.3}$$

which holds true under the same notations and conditions as needed in (4.3) with $r=2$

**d )** $$\int_{-1}^{1}(1-x)^{\eta}(1+x)^{\rho}P_k^{(\alpha,\beta)}(t+\rho)P_k^{(\alpha,\beta)}(t-\rho)S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix}\mathrm{x}_1(1-x)^{\alpha_1}(1+x)^{\beta_1}\\ \cdots\\ \mathrm{x}_s(1-x)^{\alpha_s}(1+x)^{\beta_s}\end{pmatrix}$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\big(z(1-x)^h(1+x)^k\big)\quad I\begin{pmatrix}(1\text{-x})^{h_1}(1+x)^{k_1}z_1\\ \cdots\\ (1\text{-x})^{h_2}(1+x)^{k_2}z_2\end{pmatrix}\mathrm{d}x=\frac{\Gamma(1+\alpha+k)\Gamma(1+\beta+k)}{(k!)^2}(-)^k$$

$$\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{n=0}^{k}\sum_{q=0}^{n}A'\ \frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}t^n x_1^{K_1}\cdots x_s^{K_s}B_nB_q$$

$$\frac{(-k)_l(\alpha+\beta+k+1)_l}{(\alpha+1)_l l!}I_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c}2^{h_1+k_1}z_1 & (\text{-}\rho-\sum_{i=1}^{s}K_i\beta_i-k\eta_{G,g};k_1,k_2),\\ \cdots & \cdots\\ \cdots & \cdots\cdots\\ 2^{h_2+k_2}z_2 & \end{array}\right.$$

$$\left.\begin{array}{c}(\text{-}\eta-\sum_{i=1}^{s}K_i\alpha_i-h\eta_{G,g}-q;h_1,h_2),A_2:C_2\\ \cdots\\ (\text{-}\eta-\rho-1-\sum_{i=1}^{s}K_i(\alpha_i+\beta_i)-(h+k)\eta_{G,g}-q;h_1+k_1,h_2+k_2),B_2:D_2\end{array}\right) \tag{8.4}$$

valid under the same notations and conditions as needed for (4.4) with $r=2$

$$\textbf{e)}\int_{-1}^{1}\ (1-x)^{\eta}(1+x)^{\rho}\ \rho^{k}P_{k}^{(\alpha,\alpha)}(\frac{1-xt}{\rho})\ S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix}\mathrm{x}_1(1-x)^{\alpha_1}(1+x)^{\beta_1}\\ \cdots\\ \mathrm{x}_s(1-x)^{\alpha_s}(1+x)^{\beta_s}\end{pmatrix}$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\left(z(1-x)^{h}(1+x)^{k}\right)\ \ I\begin{pmatrix}(1\text{-x})^{h_1}(1+x)^{k_1}z_1\\ \cdots\\ (1\text{-x})^{h_2}(1+x)^{k_2}z_2\end{pmatrix}\mathrm{d}x = \frac{\Gamma(1+\alpha+k)}{k!}\sum_{G=1}^{M}\sum_{g=0}^{\infty}$$

$$\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{n=0}^{k}\sum_{q=0}^{n}\ A'\,t^{n}x_1^{K_1}\cdots x_s^{K_s}\frac{(-)^{g}\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}B_nB_qI_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{matrix}2^{h_1+k_1}z_1\\ \cdots\\ \cdots\\ 2^{h_2+k_2}z_2\end{matrix}\ \middle|\ \begin{matrix}(\text{-}\eta-\sum_{i=1}^{s}K_i\alpha_i-h\eta_{G,g}-q;h_1,h_2),\\ \cdots\\ (\text{-}\eta-\rho-1-\sum_{i=1}^{s}K_i(\alpha_i+\beta_i)-(h+k)\eta_{G,g}-q;h_1+k_1,h_2+k_2),\end{matrix}\right.$$

$$\left.\begin{matrix}(\text{-}\rho-\sum_{i=1}^{s}K_i\beta_i-k\eta_{G,g};k_1,k_2),A_2:C_2\\ \cdots\\ \mathrm{B}_2:D_2\end{matrix}\right) \tag{8.5}$$

which holds true under the same notations and conditions as needed in (4.5) with $r=2$

$$\textbf{f)}\int_{-1}^{1}\ (1-x)^{\alpha}(1+x)^{\rho}\ \rho^{-1}(1-t+\rho)^{-\alpha}(1+t+\rho)^{-\beta}\ S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix}\mathrm{x}_1(1-x)^{\alpha_1}(1+x)^{\beta_1}\\ \cdots\\ \mathrm{x}_s(1-x)^{\alpha_s}(1+x)^{\beta_s}\end{pmatrix}$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\left(z(1-x)^{h}(1+x)^{k}\right)\ \ I\begin{pmatrix}(1\text{-x})^{h_1}(1+x)^{k_1}z_1\\ \cdots\\ (1\text{-x})^{h_2}(1+x)^{k_2}z_2\end{pmatrix}\mathrm{d}x = 2^{-\alpha-\beta-\rho}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}$$

$$\sum_{n=0}^{\infty}\sum_{q=0}^{n}B_qA't^{n}\,x_1^{K_1}\cdots x_s^{K_s}\frac{(-)^{g}\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{\eta_{G,g}}$$

$$I_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{matrix}2^{h_1+k_1}z_1\\ \cdots\\ \cdots\\ 2^{h_2+k_2}z_2\end{matrix}\ \middle|\ \begin{matrix}(\text{-}\eta-\sum_{i=1}^{s}K_i\alpha_i-h\eta_{G,g}-q;h_1,h_2),\\ \cdots\\ (\text{-}\eta-\rho-1-\sum_{i=1}^{s}K_i(\alpha_i+\beta_i)-(h+k)\eta_{G,g}-q;h_1+k_1,h_2+k_2),\end{matrix}\right.$$

$$\left.\begin{matrix}(\text{-}\rho-\sum_{i=1}^{s}K_i\beta_i-k\eta_{G,g};k_1,k_2),A_2:C_2\\ \cdots\\ \mathrm{B}_2,D_2\end{matrix}\right) \tag{8.6}$$

which holds true under the same notations and conditions as needed in (4.6)

## 9. Conclusion

Due to the nature of the multivariable Aleph-function and the general class of polynomials $S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}$ , we can get general product of Laguerre, Legendre, Jacobi and other polynomials, the special functions of one and several variables.

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