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Double integrals involving the multivariable Gimel-function

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Abstract
In this paper some double integrals involving the multivariable Gimel-function and hypergeometric function have been evaluated. Few interesting results have been deduced as particular cases.

Keywords :Multivariable Gimel-function, integrals contour, hypergeometric function, double integrals.

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1. Introduction and preliminaries.

The object of this document is to evaluate several finite double integrals involving the multivariable Gimel-function with general arguments.

Throughout this paper, let $\mathbb{C}, \mathbb{R}$ and $\mathbb{N}$ be set of complex numbers, real numbers and positive integers respectively. Also $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

We define a generalized transcendental function of several complex variables noted $\gimel$.

$$\gimel(z_1, \cdots, z_r) = \gimel^{0,n_2;0,n_3;\cdots;0,n_r:m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)}}_{p_{i_2},q_{i_2},\tau_{i_2};R_2;p_{i_3},q_{i_3},\tau_{i_3};R_3;\cdots;p_{i_r},q_{i_r},\tau_{i_r}:R_r:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}} \left(\begin{array}{c} \mathrm{z}_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r \end{array} \middle| \right.$$

$$[(\mathrm{a}_{2j}; \alpha^{(1)}_{2j}, \alpha^{(2)}_{2j}; A_{2j})]_{1,n_2}, [\tau_{i_2}(a_{2ji_2}; \alpha^{(1)}_{2ji_2}, \alpha^{(2)}_{2ji_2}; A_{2ji_2})]_{n_2+1,p_{i_2}}; [(a_{3j}; \alpha^{(1)}_{3j}, \alpha^{(2)}_{3j}, \alpha^{(3)}_{3j}; A_{3j})]_{1,n_3},$$
$$[\tau_{i_2}(b_{2ji_2}; \beta^{(1)}_{2ji_2}, \beta^{(2)}_{2ji_2}; B_{2ji_2})]_{1,q_{i_2}};$$

$$[\tau_{i_3}(a_{3ji_3}; \alpha^{(1)}_{3ji_3}, \alpha^{(2)}_{3ji_3}, \alpha^{(3)}_{3ji_3}; A_{3ji_3})]_{n_3+1,p_{i_3}}; \cdots ; [(\mathrm{a}_{rj}; \alpha^{(1)}_{rj}, \cdots, \alpha^{(r)}_{rj}; A_{rj})_{1,n_r}],$$
$$[\tau_{i_3}(b_{3ji_3}; \beta^{(1)}_{3ji_3}, \beta^{(2)}_{3ji_3}, \beta^{(3)}_{3ji_3}; B_{3ji_3})]_{1,q_{i_3}}; \cdots ; \quad \cdot \ \cdot \ \cdot$$

$$[\tau_{i_r}(a_{rji_r}; \alpha^{(1)}_{rji_r}, \cdots, \alpha^{(r)}_{rji_r}; A_{rji_r})_{n_r+1,p_r}] : \ [(\mathrm{c}^{(1)}_j, \gamma^{(1)}_j; C^{(1)}_j)_{1,n^{(1)}}], [\tau_{i^{(1)}}(c^{(1)}_{ji^{(1)}}, \gamma^{(1)}_{ji^{(1)}}; C^{(1)}_{ji^{(1)}})_{n^{(1)}+1,p^{(1)}_i}]$$
$$[\tau_{i_r}(b_{rji_r}; \beta^{(1)}_{rji_r}, \cdots, \beta^{(r)}_{rji_r}; B_{rji_r})_{1,q_r}] : \ [(\mathrm{d}^{(1)}_j), \delta^{(1)}_j; D^{(1)}_j)_{1,m^{(1)}}], [\tau_{i^{(1)}}(d^{(1)}_{ji^{(1)}}, \delta^{(1)}_{ji^{(1)}}; D^{(1)}_{ji^{(1)}})_{m^{(1)}+1,q^{(1)}_i}]$$

$$\left. \begin{array}{l} ;\cdots ; [(c^{(r)}_j, \gamma^{(r)}_j; C^{(r)}_j)_{1,m^{(r)}}], [\tau_{i^{(r)}}(c^{(r)}_{ji^{(r)}}, \gamma^{(r)}_{ji^{(r)}}; C^{(r)}_{ji^{(r)}})_{m^{(r)}+1,p^{(r)}_i}] \\ ;\cdots ; [(d^{(r)}_j, \delta^{(r)}_j; D^{(r)}_j)_{1,n^{(r)}}], [\tau_{i^{(r)}}(d^{(r)}_{ji^{(r)}}, \delta^{(r)}_{ji^{(r)}}; D^{(r)}_{ji^{(r)}})_{n^{(r)}+1,q^{(r)}_i}] \end{array} \right)$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \psi(s_1, \cdots, s_r) \prod_{k=1}^{r} \theta_k(s_k) z_k^{s_k} \, \mathrm{d}s_1 \cdots \mathrm{d}s_r \tag{1.1}$$

with $\omega = \sqrt{-1}$

$$\psi(s_1, \cdots, s_r) = \frac{\prod_{j=1}^{n_2} \Gamma^{A_{2j}}(1 - a_{2j} + \sum_{k=1}^{2} \alpha^{(k)}_{2j} s_k)}{\sum_{i_2=1}^{R_2} [\tau_{i_2} \prod_{j=n_2+1}^{p_{i_2}} \Gamma^{A_{2ji_2}}(a_{2ji_2} - \sum_{k=1}^{2} \alpha^{(k)}_{2ji_2} s_k) \prod_{j=1}^{q_{i_2}} \Gamma^{B_{2ji_2}}(1 - b_{2ji2} + \sum_{k=1}^{2} \beta^{(k)}_{2ji2} s_k)]}$$

$$\frac{\prod_{j=1}^{n_3}\Gamma^{A_{3j}}(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k)}{\sum_{i_3=1}^{R_3}[\tau_{i_3}\prod_{j=n_3+1}^{p_{i_3}}\Gamma^{A_{3ji_3}}(a_{3ji_3}-\sum_{k=1}^{3}\alpha_{3ji_3}^{(k)}s_k)\prod_{j=1}^{q_{i_3}}\Gamma^{B_{3ji_3}}(1-b_{3ji3}+\sum_{k=1}^{3}\beta_{3ji3}^{(k)}s_k)]}$$

$$\cdot \qquad \cdot \qquad \cdot \qquad \cdot$$

$$\cdot \qquad \cdot \qquad \cdot \qquad \cdot$$

$$\cdot \qquad \cdot \qquad \cdot \qquad \cdot$$

$$\frac{\prod_{j=1}^{n_r}\Gamma^{A_{rj}}(1-a_{rj}+\sum_{k=1}^{r}\alpha_{rj}^{(k)}s_k)}{\sum_{i_r=1}^{R_r}[\tau_{i_r}\prod_{j=n_r+1}^{p_{i_r}}\Gamma^{A_{rji_r}}(a_{rji_r}-\sum_{k=1}^{r}\alpha_{rji_r}^{(k)}s_k)\prod_{j=1}^{q_{i_r}}\Gamma^{B_{rji_r}}(1-b_{rjir}+\sum_{k=1}^{r}\beta_{rjir}^{(k)}s_k)]} \tag{1.2}$$

and

$$\theta_k(s_k)=\frac{\prod_{j=1}^{m^{(k)}}\Gamma^{D_j^{(k)}}(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n^{(k)}}\Gamma^{C_j^{(k)}}(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m^{(k)}+1}^{q_{i^{(k)}}}\Gamma^{D_{ji^{(k)}}^{(k)}}(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n^{(k)}+1}^{p_{i^{(k)}}}\Gamma^{C_{ji^{(k)}}^{(k)}}(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

1) $[(c_j^{(1)};\gamma_j^{(1)}]_{1,n_1}$ stands for $(c_1^{(1)};\gamma_1^{(1)}),\cdots,(c_{n_1}^{(1)};\gamma_{n_1}^{(1)})$.

2) $n_2,\cdots,n_r,m^{(1)},n^{(1)},\cdots,m^{(r)},n^{(r)},p_{i_2},q_{i_2},R_2,\tau_{i_2},\cdots,p_{i_r},q_{i_r},R_r,\tau_{i_r},p_{i^{(r)}},q_{i^{(r)}},\tau_{i^{(r)}},R^{(r)}\in\mathbb{N}$ and verify :

$0\leqslant m_2,\cdots,0\leqslant m_r,0\leqslant n_2\leqslant p_{i_2},\cdots,0\leqslant n_r\leqslant p_{i_r},0\leqslant m^{(1)}\leqslant q_{i^{(1)}},\cdots,0\leqslant m^{(r)}\leqslant q_{i^{(r)}}$

$0\leqslant n^{(1)}\leqslant p_{i^{(1)}},\cdots,0\leqslant n^{(r)}\leqslant p_{i^{(r)}}$.

3) $\tau_{i_2}(i_2=1,\cdots,R_2)\in\mathbb{R}^+;\tau_{i_r}\in\mathbb{R}^+(i_r=1,\cdots,R_r);\tau_{i^{(k)}}\in\mathbb{R}^+(i=1,\cdots,R^{(k)}),(k=1,\cdots,r)$.

4) $\gamma_j^{(k)},C_j^{(k)}\in\mathbb{R}^+;(j=1,\cdots,n^{(k)});(k=1,\cdots,r);\delta_j^{(k)},D_j^{(k)}\in\mathbb{R}^+;(j=1,\cdots,m^{(k)});(k=1,\cdots,r)$.

$\mathrm{C}_{ji^{(k)}}^{(k)}\in\mathbb{R}^+,(j=m^{(k)}+1,\cdots,p^{(k)});(k=1,\cdots,r);$

$\mathrm{D}_{ji^{(k)}}^{(k)}\in\mathbb{R}^+,(j=n^{(k)}+1,\cdots,q^{(k)});(k=1,\cdots,r)$.

$\alpha_{kj}^{(l)},A_{kj}\in\mathbb{R}^+;(j=1,\cdots,n_k);(k=2,\cdots,r);(l=1,\cdots,k)$.

$\alpha_{kji_k}^{(l)},A_{kji_k}\in\mathbb{R}^+;(j=n_k+1,\cdots,p_{i_k});(k=2,\cdots,r);(l=1,\cdots,k)$.

$\beta_{kji_k}^{(l)},B_{kji_k}\in\mathbb{R}^+;(j=m_k+1,\cdots,q_{i_k});(k=2,\cdots,r);(l=1,\cdots,k)$.

$\delta_{ji^{(k)}}^{(k)}\in\mathbb{R}^+;(i=1,\cdots,R^{(k)});(j=m^{(k)}+1,\cdots,q_{i^{(k)}});(k=1,\cdots,r)$.

$\gamma_{ji^{(k)}}^{(k)}\in\mathbb{R}^+;(i=1,\cdots,R^{(k)});(j=n^{(k)}+1,\cdots,p_{i^{(k)}});(k=1,\cdots,r)$.

5) $c_j^{(k)}\in\mathbb{C};(j=1,\cdots,n^{(k)});(k=1,\cdots,r);d_j^{(k)}\in\mathbb{C};(j=1,\cdots,m^{(k)});(k=1,\cdots,r)$.

$a_{kji_k}\in\mathbb{C};(j=n_k+1,\cdots,p_{i_k});(k=2,\cdots,r)$.

$b_{kji_k} \in \mathbb{C}; (j = 1, \cdots, q_{i_k}); (k = 2, \cdots, r).$

$d^{(k)}_{ji^{(k)}} \in \mathbb{C}; (i = 1, \cdots, R^{(k)}); (j = m^{(k)} + 1, \cdots, q_{i^{(k)}}); (k = 1, \cdots, r).$

$\gamma^{(k)}_{ji^{(k)}} \in \mathbb{C}; (i = 1, \cdots, R^{(k)}); (j = n^{(k)} + 1, \cdots, p_{i^{(k)}}); (k = 1, \cdots, r).$

The contour L_k is in the $s_k(k = 1, \cdots, r)$- plane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ if is a real number with loop, if necessary to ensure that the poles of $\Gamma^{A_{2j}}\left(1 - a_{2j} + \sum_{k=1}^{2} \alpha^{(k)}_{2j} s_k\right) (j = 1, \cdots, n_2), \Gamma^{A_{3j}}\left(1 - a_{3j} + \sum_{k=1}^{3} \alpha^{(k)}_{3j} s_k\right)$ $(j = 1, \cdots, n_3), \cdots, \Gamma^{A_{rj}}\left(1 - a_{rj} + \sum_{i=1}^{r} \alpha^{(i)}_{rj}\right) (j = 1, \cdots, n_r)$, $\Gamma^{C^{(k)}_j}\left(1 - c^{(k)}_j + \gamma^{(k)}_j s_k\right) (j = 1, \cdots, n^{(k)})(k = 1, \cdots, r)$to the right of the contour L_k and the poles of $\Gamma^{D^{(k)}_j}\left(d^{(k)}_j - \delta^{(k)}_j s_k\right) (j = 1, \cdots, m^{(k)})(k = 1, \cdots, r)$ lie to the left of the contour L_k. The condition for absolute convergence of multiple Mellin-Barnes type contour (1.1) can be obtained of the corresponding conditions for multivariable H-function given by as :

$|arg(z_k)| < \frac{1}{2} A^{(k)}_i \pi$ where

$$A^{(k)}_i = \sum_{j=1}^{m^{(k)}} D^{(k)}_j \delta^{(k)}_j + \sum_{j=1}^{n^{(k)}} C^{(k)}_j \gamma^{(k)}_j - \tau_{i^{(k)}} \left(\sum_{j=m^{(k)}+1}^{q^{(k)}_i} D^{(k)}_{ji^{(k)}} \delta^{(k)}_{ji^{(k)}} + \sum_{j=n^{(k)}+1}^{p^{(k)}_i} C^{(k)}_{ji^{(k)}} \gamma^{(k)}_{ji^{(k)}} \right) +$$

$$-\tau_{i_2} \left(\sum_{j=n_2+1}^{p_{i_2}} A_{2ji_2} \alpha^{(k)}_{2ji_2} + \sum_{j=1}^{q_{i_2}} B_{2ji_2} \beta^{(k)}_{2ji_2} \right) - \cdots - \tau_{i_r} \left(\sum_{j=n_r+1}^{p_{i_r}} A_{rji_r} \alpha^{(k)}_{rji_r} + \sum_{j=1}^{q_{i_r}} B_{rji_r} \beta^{(k)}_{rji_r} \right) \tag{1.4}$$

Following the lines of Braaksma ([2] p. 278), we may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1, \cdots, z_r) = 0(|z_1|^{\alpha_1}, \cdots, |z_r|^{\alpha_r}), max(|z_1|, \cdots, |z_r|) \rightarrow 0$

$\aleph(z_1, \cdots, z_r) = 0(|z_1|^{\beta_1}, \cdots, |z_r|^{\beta_r}), min(|z_1|, \cdots, |z_r|) \rightarrow \infty$ where $i = 1, \cdots, r :$

$\alpha_i = \min_{1 \leqslant j \leqslant m^{(i)}} Re\left[D^{(i)}_j \left(\frac{d^{(i)}_j}{\delta^{(i)}_j}\right)\right]$ and $\beta_i = \max_{1 \leqslant j \leqslant n^{(i)}} Re\left[C^{(i)}_j \left(\frac{c^{(i)}_j - 1}{\gamma^{(i)}_j}\right)\right]$

Remark 1.
If $n_2 = \cdots = n_{r-1} = p_{i_2} = q_{i_2} = \cdots = p_{i_{r-1}} = q_{i_{r-1}} = 0$ and $A_{2j} = A_{2ji_2} = B_{2ji_2} = \cdots = A_{rj} = A_{rji_r} = B_{rji_r} = 1$ $A_{rj} = A_{rji_r} = B_{rji_r} = 1$, then the multivariable Gimel-function reduces in the multivariable Aleph- function defined by Ayant [1].

Remark 2.
If $n_2 = \cdots = n_r = p_{i_2} = q_{i_2} = \cdots = p_{i_r} = q_{i_r} = 0$ and $\tau_{i_2} = \cdots = \tau_{i_r} = \tau_{i^{(1)}} = \cdots = \tau_{i^{(r)}} = R_2 = \cdots = R_r = R^{(1)} = \cdots = R^{(r)} = 1$, then the multivariable Gimel-function reduces in a multivariable I-function defined by Prathima et al. [6].

Remark 3.
If$A_{2j} = A_{2ji_2} = B_{2ji_2} = \cdots = A_{rj} = A_{rji_r} = B_{rji_r} = 1$and $\tau_{i_2} = \cdots = \tau_{i_r} = \tau_{i^{(1)}} = \cdots = \tau_{i^{(r)}} = R_2 = \cdots = R_r = R^{(1)} = \cdots = R^{(r)} = 1$, then the generalized multivariable Gimel-function reduces in multivariable I-function defined by Prasad [5].

Remark 4.
If the three above conditions are satisfied at the same time, then the generalized multivariable Gimel-function reduces in the multivariable H-function defined by Srivastava and panda [8,9].

In your investigation, we shall use the following notations.

$$\mathbb{A} =[(\mathrm{a}_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}},[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},$$

$$[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{(r-1)j};\alpha_{(r-1)j}^{(1)},\cdots,\alpha_{(r-1)j}^{(r-1)};A_{(r-1)j})_{1,n_{r-1}}],$$

$$[\tau_{i_{r-1}}(a_{(r-1)ji_{r-1}};\alpha_{(r-1)ji_{r-1}}^{(1)},\cdots,\alpha_{(r-1)ji_{r-1}}^{(r-1)};A_{(r-1)ji_{r-1}})_{n_{r-1}+1,p_{i_{r-1}}}] \tag{1.5}$$

$$\mathbf{A} = [(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)};A_{rj})_{1,n_r}],[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)};A_{rji_r})_{\mathfrak{n}+1,p_{i_r}}] \tag{1.6}$$

$$A=[(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,n^{(1)}}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})_{n^{(1)}+1,p_i^{(1)}}];\cdots;$$

$$[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_{ji^{(r)}}^{(r)})_{m^{(r)}+1,p_i^{(r)}}] \tag{1.7}$$

$$\mathbb{B}=[\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{1,q_{i_2}},[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{1,q_{i_3}};\cdots;$$

$$[\tau_{i_{r-1}}(b_{(r-1)ji_{r-1}};\beta_{(r-1)ji_{r-1}}^{(1)},\cdots,\beta_{(r-1)ji_{r-1}}^{(r-1)};B_{(r-1)ji_{r-1}})_{1,q_{i_{r-1}}}] \tag{1.8}$$

$$\mathbf{B}= [\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)};B_{rji_r})_{1,q_{i_r}}] \tag{1.9}$$

$$\mathrm{B}=[(\mathrm{d}_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,m^{(1)}}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)};D_{ji^{(1)}}^{(1)})_{m^{(1)}+1,q_i^{(1)}}];\cdots;$$

$$[(\mathrm{d}_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)};D_{ji^{(r)}}^{(r)})_{m^{(r)}+1,q_i^{(r)}}] \tag{1.10}$$

$$U=0,n_2;0,n_3;\cdots;0,n_{r-1};V=m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)} \tag{1.11}$$

$$X=p_{i_2},q_{i_2},\tau_{i_2};R_2;\cdots;p_{i_{r-1}},q_{i_{r-1}},\tau_{i_{r-1}}:R_{r-1};Y=p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)} \tag{1.12}$$

2. Required results.

In this section, we give three finite integrals. These results will utilized in the following section.

Lemma 1. ([3], Erdelyi, p. 450-452)

$$\int_0^{\frac{\pi}{2}} e^{\omega(a+b)\theta}(\sin\theta)^{a-1}(\cos\theta)^{b-1}\mathrm{d}\theta=e^{\frac{1}{2}\omega\pi a}\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} \tag{2.1}$$

provided $\min\{Re(a),Re(b)\}>0$

Lemma 2. ([3], Erdelyi, p. 253)

$$\int_0^{\frac{\pi}{2}}\frac{2^{\alpha+\beta+1}}{\pi}e^{\omega(\alpha-\beta)\theta}(\cos\theta)^{\alpha+\beta}\mathrm{d}\theta=\frac{\pi}{2^{\alpha+\beta+1}}\frac{\Gamma(\alpha+\beta+1)}{\Gamma(\alpha+1)\Gamma(\beta+1)} \tag{2.2}$$

$Re(\alpha+\beta)>-1$

Lemma 3 . ([4], MacRobert).

$$\int_0^1 \frac{x^{p-1}(1-x)^{q-1}}{[a'x+b'(1-x)]^{p+q}}\,{}_2F_1\left(e,f;p;\frac{a'x}{a'x+b'(1-x)}\right)\mathrm{d}x=\frac{\Gamma(p)\Gamma(q)\Gamma(p+q-e-f)}{a'^pb'^q\Gamma(p+q-e)\Gamma(p+q-f)} \tag{2.3}$$

provided $Re(p)>0, Re(q)>0, Re(p+q-e-f)>0, |a'x+b'(1-x)]\neq 0, 0\leqslant x\leqslant 1$.

Lemma. 4 ([3], Erdelyi)

$$\int_0^1 x^{\gamma-1}[(a+x)(b+x))]^{-\gamma}\mathrm{d}x=\sqrt{\pi}(\sqrt{a}+\sqrt{b})^{1-2\gamma}\frac{\Gamma\left(\gamma-\frac{1}{2}\right)}{\Gamma(\gamma)} \tag{2.4}$$

provided $Re(\gamma)>\frac{1}{2}$

3. Main integrals.

In this section,we evaluate several finite double integrals with general arguments.

Theorem 1.

$$\int_0^1\int_0^{\frac{\pi}{2}} e^{\omega(\alpha+\beta)\theta}(\sin\theta)^{\alpha-1}(\cos\theta)^{\beta-1}x^{\gamma-1}(1-x)^{\delta-1}[ax+b(1-x)]^{-\gamma-\delta}{}_2F_1(c,d;\gamma;ax+b(1-x))$$

$$\gimel\left(z_1\frac{e^{\omega(a_1+b_1)\theta}(\sin\theta)^{a_1}(\cos\theta)^{b_1}x^{c_1}(1-x)^{d_1}}{[ax+b(1-x)]^{c_1+d_1}},\cdots,z_r\frac{e^{\omega(a_r+b_r)\theta}(\sin\theta)^{a_r}(\cos\theta)^{b_r}x^{c_r}(1-x)^{d_r}}{[ax+b(1-x)]^{c_r+d_r}}\right)\mathrm{d}\theta\mathrm{d}x$$

$$=\frac{\sqrt{e}\omega^{\pi\alpha}}{a^\gamma b^\sigma}\gimel^{U;0,n_r+5:V}_{X;p_{i_r}+5,q_{i_r}+3,\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c} \mathrm{z}_1\frac{e^{\omega\pi\frac{a_1}{2}}}{a^{c_1}b^{d_1}} & \mathbb{A};(1-\alpha;a_1,\cdots,a_r;1),(1-\beta;b_1,\cdots,b_r;1),(1-\gamma;c_1,\cdots,c_r;1), \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r\frac{e^{\omega\pi\frac{a_r}{2}}}{a^{c_r}b^{d_r}} & \mathbb{B};\mathbf{B},(1-\alpha-\beta;a_1+b_1,\cdots,a_r+b_r;1), \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-}\delta;d_1,\cdots,d_r;1),(1+c+d-\gamma-\alpha;q+e+f;c_1+d_1,\cdots,c_r+d_r;1),\mathbf{A}:A \\ \cdot \\ \cdot \\ (1\text{+c+d-}\gamma-\delta;c_1+d_1,\cdots,c_r+d_r;1),(1-\gamma-\delta+d;c_1+d_1,\cdots,c_r+d_r;1):B \end{array}\right) \tag{3.1}$$

provided

$Re(\alpha), Re(\beta), a_i, b_i, c_i, d_i>0 (i=1,\cdots,r), [ax+b(1-x)]\neq 0$, $Re(\alpha)+\sum_{i=1}^r a_i \min\limits_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>0.$

$Re(\gamma)+\sum_{i=1}^r c_i \min\limits_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>0.$ $Re(\beta)+\sum_{i=1}^r b_i \min\limits_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>0.$ and

$\left|arg\left(z_i\frac{e^{\omega(a_i+b_i)\theta}\sin^{a_i}\theta\cos^{b_i}\theta x^{c_i}(1-x)^{d_i}}{[ax+b(1-x)]^{c_i+d_i}}\right)\right|<\frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4).

Proof
To establish the theorem 1, expressing the multivariable Gimel-function in the Mellin-Barnes multiple integrals contour with the help of (1.1) and interchanging the order of integrations, which is justified under the conditions mentioned above, we get (say I)

$$I=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}$$

$$\left[\int_0^{\frac{\pi}{2}}e^{\omega(\alpha+\beta+\sum_{i=1}^{r}(a_i+b_i)s_i)\theta}(\sin\theta)^{\alpha+\sum_{i=1}^{r}a_is_i-1}(\cos\theta)^{\beta+\sum_{i=1}^{r}b_is_i-1}\mathrm{d}x\right]$$

$$\left[\int_0^1 x^{\gamma+\sum_{i=1}^{r}c_is_i-1}(1-x)^{\delta+\sum_{i=1}^{r}d_is_i-1}[ax+b(1-x)]^{-\gamma-\delta-\sum_{i=1}^{r}(c_i+d_i)s_i}{}_2F_1\left[c,d;\gamma;\frac{ax}{ax+b(1-x)}\right]\mathrm{d}x\right]$$

$$\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{3.2}$$

Now evaluating the inner integrals with the help of lemmae 1and 3 and interpreting the resulting expression with the help of (1.1), we obtain the desired theorem 1.

Theorem 2.

$$\int_0^1\int_0^{\frac{\pi}{2}}e^{\omega(\alpha-\beta)\theta}(\cos\theta)^{\alpha+\beta}x^{\gamma-1}(1-x)^{\delta-1}[ax+b(1-x)]^{-\gamma-\delta}{}_2F_1\left(c,d;a;\frac{ax}{ax+b(1-x)}\right)$$

$$\gimel\left(z_1\frac{e^{\omega(a_1-b_1)\theta}cos\theta)^{a_1+b_1}x^{c_1}(1-x)^{d_1}}{[ax+b(1-x)]^{c_1+d_1}},\cdots,z_r\frac{e^{\omega(a_r-b_r)\theta}(\cos\theta)^{a_r+b_r}x^{c_r}(1-x)^{d_r}}{[ax+b(1-x)]^{c_r+d_r}}\right)\mathrm{d}\theta\mathrm{d}x$$

$$=\frac{\pi}{a^\gamma b^\sigma}\gimel^{U;0,n_r+4:V}_{X;p_{i_r}+4,q_{i_r}+4,\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c}\mathrm{z}_1\frac{2^{-a_1-b_1}}{a^{c_1}b^{d_1}} & \mathbb{A};(1-\alpha-\beta;a_1+b_1,\cdots,a_r+b_r;1),(1-\gamma;c_1,\cdots,c_r;1),\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_r\frac{2^{-a_r-b_r}}{a^{c_r}b^{d_r}} & \mathbb{B};\mathbf{B},(1-\alpha;a_1,\cdots,a_r;1),(1-\beta;b_1,\cdots,b_r;1),\end{array}\right.$$

$$\left.\begin{array}{c}(1\text{-}\delta;d_1,\cdots,d_r;1),(1+c+d-\gamma-\delta;c_1+d_1,\cdots,c_r+d_r;1),\mathbf{A}:A\\ \cdot\\ \cdot\\ (1\text{+c-}\gamma-\delta;c_1+d_1,\cdots,c_r+d_r;1),(1+d-\gamma-\delta;c_1+d_1,\cdots,c_r+d_r;1):B\end{array}\right) \tag{3.3}$$

provided

$Re(\alpha+\beta)>-1,a_i,b_i,c_i,d_i>0(i=1,\cdots,r),[ax+b(1-x)]\neq 0$,

$$Re(\gamma)+\sum_{i=1}^{r}c_i\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>0.$$

$$Re(\delta)+\sum_{i=1}^{r}d_i\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>0.\quad Re(\alpha+\beta)+\sum_{i=1}^{r}(a_i+b_i)\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>0.$$

$\left|arg\left(z_i\frac{e^{\omega(a_i-b_i)\theta}\cos^{a_i+b_i}\theta x^{c_i}(1-x)^{d_i}}{[ax+b(1-x)]^{c_i+d_i}}\right)\right|<\frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4).

Proof

To establish the theorem 1, expressing the multivariable Gimel-function in the Mellin-Barnes multiple integrals contour with the help of (1.1) and interchanging the order of integrations, which is justified under the conditions mentioned above, we get (say I)

$$I=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\left[\int_0^{\frac{\pi}{2}}e^{\omega(\alpha+\beta+\sum_{i=1}^{r}(a_i-b_i)s_i)\theta}(\cos\theta)^{\beta+\sum_{i=1}^{r}(a_i+b_i)s_i-1}\mathrm{d}x\right]$$

$$\left[\int_0^1 x^{\gamma+\sum_{i=1}^r c_i s_i-1}(1-x)^{\delta+\sum_{i=1}^r d_i s_i-1}[ax+b(1-x)]^{-\gamma-\delta-\sum_{i=1}^r(c_i+d_i)s_i}{}_2F_1\left[c,d;\gamma;\frac{ax}{ax+b(1-x)}\right]\mathrm{d}x\right]$$

$$\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{3.4}$$

Now evaluating the inner integrals with the help of lemmae21and 3 and interpreting the resulting expression with the help of (1.1), we obtain the desired theorem 2.

Theorem 3.

$$\int_0^1\int_0^{\frac{\pi}{2}} e^{\omega(\alpha-\beta)\theta}(\cos\theta)^{\alpha+\beta}x^{\gamma-\frac{1}{2}}[(a+x)(b+x)]^{-\gamma}$$

$$\gimel\left(z_1\frac{e^{\omega(a_1-b_1)\theta}cos\theta)^{a_1+b_1}x^{c_1}}{[(a+x)(b+x)]^{c_1}},\cdots,z_r\frac{e^{\omega(a_r-b_r)\theta}cos\theta)^{a_r+b_r}x^{c_r}}{[(a+x)(b+x)]^{c_r}}\right)\mathrm{d}\theta\mathrm{d}x=\pi^{\frac{3}{2}}(\sqrt{a}+\sqrt{b})^{1-2\gamma}$$

$$\gimel^{U;0,n_r+2:V}_{X;p_{i_r}+2,q_{i_r}+3,\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c}\frac{z_1}{2^{a_1+b_1}} & \mathbb{A};(1-\alpha-\beta;a_1+b_1,\cdots,a_r+b_r;1),\left(\frac{3}{2}-\gamma;c_1,\cdots,c_r;1\right),\mathbf{A}:A\\ \cdot & \cdot\\ \cdot & \cdot\\ \frac{z_r}{2^{a_r+b_r}} & \mathbb{B};\mathbf{B},(1-\gamma;c_1,\cdots,c_r;1),(1-\alpha;a_1,\cdots,a_r;1),(1-\beta;b_1,\cdots,b_r;1):B\end{array}\right) \tag{3.5}$$

provided

$$Re(\alpha+\beta)>-1,a_i,b_i,c_i>0(i=1,\cdots,r),\ Re(\gamma)+\sum_{i=1}^r c_i\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>0.$$

$$Re(\alpha+\beta)+\sum_{i=1}^r(a_i+b_i)\min_{1\leqslant j\leqslant m^{(i)}}Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>0.$$

$\left|arg\left(z_i\frac{e^{\omega(a_i-b_i)\theta}\cos^{a_i+b_i}\theta x^{c_i}}{[(a+x)(b+x)]^{c_i}}\right)\right|<\frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4).

Proof

To establish the theorem 1, expressing the multivariable Gimel-function in the Mellin-Barnes multiple integrals contour with the help of (1.1) and interchanging the order of integrations, which is justified under the conditions mentioned above, we get (say I)

$$I=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^r\theta_k(s_k)z_k^{s_k}\left[\int_0^{\frac{\pi}{2}}e^{\omega(\alpha+\beta+\sum_{i=1}^r(a_i-b_i)s_i)\theta}(\cos\theta)^{\alpha+\beta+\sum_{i=1}^r(a_i+b_i)s_i}\mathrm{d}x\right]$$

$$\left[\int_0^1 x^{\gamma-1}[(a+x)(b+x))]^{-\gamma-\sum_{i=1}^r c_i s_i}\mathrm{d}x\right]\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{3.6}$$

Now evaluating the inner integrals with the help of lemmae 2 and 4 and interpreting the resulting expression with the help of (1.1), we obtain the desired theorem 3.

Theorem 4.

$$\int_0^1 \int_0^{\frac{\pi}{2}} e^{\omega(\alpha+\beta)\theta} (\sin\theta)^{\alpha+1} (\cos\theta)^{\beta-1} x^{\gamma-\frac{1}{2}} [(a+x)(b+x)]^{-\gamma}$$

$$\gimel\left(z_1 \frac{e^{\omega(a_1+b_1)\theta}(\sin\theta)^{a_1}(cos\theta)^{b_1} x^{c_1}}{[(a+x)(b+x)]^{c_1}}, \cdots, z_r \frac{e^{\omega(a_r+b_r)\theta}(\sin\theta)^{a_r}(\cos\theta)^{b_r} x^{c_r}}{[(a+x)(b+x)]^{c_r}} \right) \mathrm{d}\theta \mathrm{d}x = \sqrt{\pi} e^{\frac{1}{2}\omega\pi\theta} (\sqrt{a}+\sqrt{b})^{1-2\gamma}$$

$$\gimel^{U;0,n_r+3:V}_{X;p_{i_r}+3,q_{i_r}+2,\tau_{i_r}:R_r:Y} \left(\begin{array}{c|c} z_1 e^{\frac{1}{2}\omega\pi a_1} & \mathbb{A}; \left(\frac{3}{2}-\gamma; c_1,\cdots,c_r;1\right), \left(1-\alpha; a_1,\cdots,a_r;1\right), \left(1-\beta; b_1,\cdots,b_r;1\right) \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r e^{\frac{1}{2}\omega\pi a_r} & \mathbb{B}; \mathbf{B}, (1-\gamma; c_1,\cdots,c_r;1), (1-\alpha-\beta; a_1+b_1,\cdots,a_r+b_r;1) : B \end{array} \right) \tag{3.7}$$

provided

$$Re(\alpha), Re(\beta), a_i, b_i, c_i > 0 (i=1,\cdots,r),\ Re(\gamma) + \sum_{i=1}^r c_i \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)} \left(\frac{d_j^{(i)}}{\delta_j^{(i)}} \right) \right] > 0.$$

$$Re(\alpha) + \sum_{i=1}^r a_i \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)} \left(\frac{d_j^{(i)}}{\delta_j^{(i)}} \right) \right] > 0.\ Re(\beta) + \sum_{i=1}^r b_i \min_{1\leqslant j\leqslant m^{(i)}} Re\left[D_j^{(i)} \left(\frac{d_j^{(i)}}{\delta_j^{(i)}} \right) \right] > 0.$$

$\left| arg\left(z_i \frac{e^{\omega(a_i+b_i)\theta} \sin^{a_i}\theta \cos^{b_i}\theta x^{c_i}}{[(a+x)(b+x)]^{c_i}} \right) \right| < \frac{1}{2} A_i^{(k)} \pi$ where $A_i^{(k)}$ is defined by (1.4).

Proof

To establish the theorem 1, expressing the multivariable Gimel-function in the Mellin-Barnes multiple integrals contour with the help of (1.1) and interchanging the order of integrations, which is justified under the conditions mentioned above, we get (say I)

$$I = \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \psi(s_1,\cdots,s_r) \prod_{k=1}^r \theta_k(s_k) z_k^{s_k}$$

$$\left[\int_0^{\frac{\pi}{2}} e^{\omega(\alpha+\beta+\sum_{i=1}^r (a_i-b_i)s_i)\theta} (\sin\theta)^{\alpha+1+\sum_{i=1}^r a_i s_i} (\cos\theta)^{\beta-1+\sum_{i=1}^r b_i s_i} \mathrm{d}x \right]$$

$$\left[\int_0^1 x^{\gamma-1} [(a+x)(b+x))]^{-\gamma-\sum_{i=1}^r c_i s_i} \mathrm{d}x \right] \mathrm{d}s_1 \cdots \mathrm{d}s_r \tag{3.8}$$

Now evaluating the inner integrals with the help of lemmae 1and 4 and interpreting the resulting expression with the help of (1.1), we obtain the desired theorem 4.

4. Special case.

Considering the theorem 1 and taking $c = d = 0$ and simplifying the result, we obtain.

Corollary.

$$\int_0^1 \int_0^{\frac{\pi}{2}} e^{\omega(\alpha+\beta)\theta} (\sin\theta)^{\alpha-1} (\cos\theta)^{\beta-1} x^{\gamma-1} [ax+b(1-x)]^{-\gamma-\delta} {}_0F_1\left(-,-;-a; \frac{ax}{b(1-x)} \right)$$

$$\gimel\left(z_1\frac{e^{\omega(a_1+b_1)\theta}(\sin\theta)^{a_1}(\cos\theta)^{b_1}x^{c_1}(1-x)^{d_1}}{[ax+b(1-x)]^{c_1+d_1}},\cdots,z_r\frac{e^{\omega(a_r+b_r)\theta}(\sin\theta)^{a_r}(\cos\theta)^{b_r}x^{c_r}(1-x)^{d_r}}{[ax+b(1-x)]^{c_r+d_r}}\right)\mathrm{d}\theta\mathrm{d}x$$

$$=\frac{e^{\frac{\pi}{2}}\alpha}{a^{\gamma}b^{\sigma}}\gimel^{U;0,n_r+4:V}_{X;p_{i_r}+4,q_{i_r}+2,\tau_{i_r}:R_r:Y}\left(\begin{array}{c}z_1e^{\omega\pi\frac{a_1}{2}}\\ \cdot\\ \cdot\\ z_re^{\omega\pi\frac{a_r}{2}}\end{array}\left|\begin{array}{c}\mathbb{A};(1-\alpha;a_1,\cdots,a_r;1),(1-\beta;b_1,\cdots,b_r;1),(1-\gamma;c_1,\cdots,c_r;1),\\ \cdot\\ \cdot\\ \mathbb{B};\mathbf{B},(1-\alpha-\beta;a_1+b_1,\cdots,a_r+b_r;1),\end{array}\right.\right.$$

$$\left.\begin{array}{c}(1\text{-}\delta;d_1,\cdots,d_r;1),\mathbf{A}:A\\ \cdot\\ \cdot\\ (1\text{-}\gamma-\alpha;c_1+d_1,\cdots,c_r+d_r;1):B\end{array}\right) \tag{4.1}$$

under the same conditions that theorem 1.

Remark 5.

The double finite integrals involved in this paper are general character, we can given a large corollaries by specializing the parameters.

Remark 6.
If $m_2=n_2=\cdots=m_{r-1}=n_{r-1}=p_{i_2}=q_{i_2}=\cdots=p_{i_{r-1}}=q_{i_{r-1}}=0$ and $A_{2j}=B_{2j}=A_{2ji_2}=B_{2ji_2}=\cdots=A_{rj}=B_{rj}=A_{rji_r}=B_{rji_r}=1$, then we can obtain the same double finite integrals in the generalized multivariable Aleph- function (extension of multivariable Aleph-function defined by Ayant [1]).

Remark 7.
If $m_2=n_2=\cdots=m_r=n_r=p_{i_2}=q_{i_2}=\cdots=p_{i_r}=q_{i_r}=0$ and $\tau_{i_2}=\cdots=\tau_{i_r}=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=R_2==\cdots=R_r=R^{(1)}=\cdots=R^{(r)}=1$, then we can obtain the same double finite integrals in a generalized multivariable I-function (extension of multivariable I-function defined by Prathima et al. [6]).

Remark 8.
If$A_{2j}=B_{2j}=A_{2ji_2}=B_{2ji_2}=\cdots=A_{rj}=B_{rj}=A_{rji_r}=B_{rji_r}=1$and $\tau_{i_2}=\cdots=\tau_{i_r}=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=R_2=\cdots=R_r=R^{(1)}=\cdots=R^{(r)}=1$, then we can obtain the same double finite integrals in generalized of multivariable I-function (extension of multivariable I-function defined by Prasad [5]).

Remark 9.
If the three above conditions are satisfied at the same time, then the generalized multivariable Gimel-function reduces in the generalized multivariable H-function (extension of multivariable H-function defined by Srivastava and Panda [8,9] and then we can obtain the same double finite integrals, see Ronghe for more details [7].

5. Conclusion.

The importance of our all the results lies in their manifold generality. Firstly, in view of general arguments utilized in these double integrals, we can obtain a large simpler double or single finite integrals, Secondly by specialising the various parameters as well as variables in the generalized multivariable Gimel-function, we get a several formulae involving remarkably wide variety of useful functions (or product of such functions) which are expressible in terms of E, F, G, H, I, Aleph-function of one and several variables and simpler special functions of one and several variables. Hence the formulae derived in this paper are most general in character and may prove to be useful in several interesting cases appearing in literature of Pure and Applied Mathematics and Mathematical Physics.

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