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Fractional Integration of the Product of Two Prasad's Multivariable I-Functions and a General Class of Polynomials I

$F.Y.\ AYANT^1$

1 Teacher in High School , France
E-mail : fredericayant@gmail.com

Abstract : A significantly large number of earlier works on the subject of fractional calculus give interesting account of the theory and applications of fractional calculus operators in many different areas of mathematical analysis (such as ordinary and partial differential equations, integral equations, special functions, summation of series, etc.). The main object of the present paper is to study and develop the Saigo operators. First, we establish two results that give the images of the product of two multivariable I-functions defined by Prasad and a general class of multivariable polynomials in Saigo operators. On account of the general nature of the Saigo operators, multivariable I-functions and a general class multivariable polynomials a large number of new and Known Images involving Riemann-Liouville and Erde'lyi-Kober fractional integral operators and several special functions notably generalized Wright hypergeometric function, Mittag-Leffler function, Whittaker function follow as special cases of our main findings. Results given by Kilbas, Kilbas and Sebastian, Saxena et al. and Gupta et al., follow as special cases of our findings.

Keywords: General class of multivariable polynomial, Saigo operator, multivariable I-function, multivariable H-function.

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1.Introduction and preliminaries.

The fractional integral operator involving various special functions has found significant importance and applications in various subfields of applicable mathematical analysis. Since last four decades, a number of workers like Love [14], McBride [16], Kalla [4,5], Kalla and Saxena [6,7], Saxena et al. [24], Saigo [20-21], Kilbas [8], Kilbas and Sebastian [10] and Kiryakova [12,13] have studied in depth the properties, applications and different extensions of various hypergeometric operators of fractional integration. A detailed account of such operators along with their properties and applications can be found in the research monographs by Samko, Kilbas and Marichev [23], Miller and Ross [17]; Kiryakova [12,13], Kilbas, Srivastava and Trujillo [11] and Debnath and Bhatta [2]. A useful generalization of the hypergeometric fractional integrals, including the Saigo operators [18-20], has been introduced by Marichev [15] (see details in Samko et al. [23] and also see Kilbas and Saigo [9])as follows:
Let α,β,η be complex numbers and $x>0$, then the generalized fractional integral operators (the Saigo operators [20]) involving Gaussian hypergeometric function are defined by the following equations:

$$I_{0^+}^{\alpha,\beta,\eta}f(x)=\frac{x^{-\alpha-\beta}}{\Gamma(\alpha)}\int_0^x(x-t)^{\alpha-1}F\left(\alpha+\beta,-\eta;\alpha;1-\frac{t}{x}\right)f(t)\mathrm{d}t(Re(\alpha))>0) \tag{1.1}$$

and

$$I_{-}^{\alpha,\beta,\eta}f(x)=\frac{1}{\Gamma(\alpha)}\int_x^\infty t^{-\alpha-\beta}(x-t)^{\alpha-1}F\left(\alpha+\beta,-\eta;\alpha;1-\frac{x}{t}\right)f(t)\mathrm{d}t(Re(\alpha))>0) \tag{1.2}$$

When $\beta=-\alpha$, equations (1.1) and (1.2) reduce to the following classical Riemann–Liouville fractional integral operator (see Samko et al. [23], p. 94, (5.1), (5.3)):

$$I_{0^+}^{\alpha,-\alpha,\eta}f(x)=I_{0+}^{\alpha}f(x)=\frac{1}{\Gamma(\alpha)}\int_0^x(x-t)^{\alpha-1}f(t)\mathrm{d}t(x>0) \tag{1.3}$$

$$I_{0^-}^{\alpha,-\alpha,\eta}f(x)=I_{0^-}^{\alpha}f(x)=\frac{1}{\Gamma(\alpha)}\int_x^\infty(x-t)^{\alpha-1}f(t)\mathrm{d}t(x>0) \tag{1.4}$$

Again, if $\beta=0$ Equations (1.1) and (1.2) reduce to the following Erdelyi–Kober fractional integral operator (see Samko et al. [23], p.322, Eqns. (18.5), (18.6)):

$$I_{0^+}^{\alpha,0,\eta}f(x)=I_{\alpha,\eta}^{+}f(x)=\frac{x^{-\alpha-\beta}}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1}t^{\eta}f(t)\mathrm{d}t(x>0) \tag{1.5}$$

and

$$I_{-}^{\alpha,0,\eta}f(x)=K_{\alpha,\eta}^{-}f(x)=\frac{x^{\eta}}{\Gamma(\alpha)}\int_x^{\infty} (x-t)^{\alpha-1}t^{-\alpha-\eta}f(t)\mathrm{d}t(x>0) \tag{1.6}$$

Recently, Gupta et al. [3] have obtained the images of the product of two H-functions in Saigo operator given by (1.1) and (1.2) and thereby generalized several important results obtained earlier by Kilbas, Kilbas and Sebastian and Saxena et al. as mentioned in this paper cited above. It has recently become a subject of interest for many researchers in the field of fractional calculus and its applications. Motivated by these avenues of applications, a number of workers have made use of the fractional calculus operators to obtain the image formulas. The aim of this paper is to obtain two results that give the Theorems of the product of two multivariable I-functions defined by Prasad [19] and a general class of multivariable polynomials [26] in Saigo operators.
The multivariable I-function of r-variables is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,z_2,\cdots,z_r)=I_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p^{(1)},q^{(1)};\cdots;p^{(r)},q^{(r)}}^{0,n_2;0,n_3;\cdots;0,n_r:m^{(1)},n^{(1)};\cdots;m^{(r)},n^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)})_{1,p_r}:(a_j^{(1)},\alpha_j^{(1)})_{1,p^{(1)}};\cdots;(a_j^{(r)},\alpha_j^{(r)})_{1,p^{(r)}} \\ \\ (\mathrm{b}_{rj};\beta_{rj}^{(1)},\cdots,\beta_{rj}^{(r)})_{1,q_r}:(b_j^{(1)},\beta_j^{(1)})_{1,q^{(1)}};\cdots;(b_j^{(r)},\beta_j^{(r)})_{1,q^{(r)}} \end{array}\right) \tag{1.7}$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.8}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [19]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.8) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_i|<\frac{1}{2}\Omega_i\pi$, where

$$\Omega_i=\sum_{k=1}^{n^{(i)}}\alpha_k^{(i)}-\sum_{k=n^{(i)}+1}^{p^{(i)}}\alpha_k^{(i)}+\sum_{k=1}^{m^{(i)}}\beta_k^{(i)}-\sum_{k=m^{(i)}+1}^{q^{(i)}}\beta_k^{(i)}+\left(\sum_{k=1}^{n_2}\alpha_{2k}^{(i)}-\sum_{k=n_2+1}^{p_2}\alpha_{2k}^{(i)}\right)+\cdots+$$

$$\left(\sum_{k=1}^{n_s}\alpha_{sk}^{(i)}-\sum_{k=n_s+1}^{p_s}\alpha_{sk}^{(i)}\right)-\left(\sum_{k=1}^{q_2}\beta_{2k}^{(i)}+\sum_{k=1}^{q_3}\beta_{3k}^{(i)}+\cdots+\sum_{k=1}^{q_s}\beta_{sk}^{(i)}\right) \tag{1.9}$$

where $i=1,\cdots,r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1,\cdots,z_r)=0(\,|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\,),max(\,|z_1|,\cdots,|z_r|\,)\rightarrow 0$$

$$I(z_1,\cdots,z_r)\,=0(\,|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\,),min(\,|z_1|,\cdots,|z_r|\,)\rightarrow\infty$$

where $k = 1, \cdots, r : \alpha'_k = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta'_k = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, n_k$$

We shall note $I(z_1, \cdots, z_r) = I_1(z_1, \cdots, z_r)$

The multivariable I-function of s-variables is defined by Prasad [19] in term of multiple Mellin-Barnes type integral :

$$I(z'_1, \cdots, z'_s) = I^{0,n'_2;0,n'_3;\cdots;0,n'_s:m'^{(1)},n'^{(1)};\cdots;m'^{(s)},n'^{(s)}}_{p'_2,q'_2,p'_3,q'_3;\cdots;p'_s,q'_s:p'^{(1)},q'^{(1)};\cdots;p'^{(s)},q'^{(s)}} \left(\begin{array}{c|c} \text{z'}_1 & (\text{a'}_{2j}; \alpha'^{(1)}_{2j}, \alpha'^{(2)}_{2j})_{1,p_2}; \cdots ; \\ \cdot & \\ \cdot & \\ \cdot & \\ \text{z'}_s & (\text{b'}_{2j}; \beta'^{(1)}_{2j}, \beta'^{(2)}_{2j})_{1,q_2}; \cdots ; \end{array} \right.$$

$$\left. \begin{array}{c} (\text{a'}_{sj}; \alpha'^{(1)}_{sj}, \cdots, \alpha'_{sj}{}^{(s)})_{1,p'_s} : (a'^{(1)}_j, \alpha'^{(1)}_j)_{1,p'^{(1)}}; \cdots ; (a'_j{}^{(s)}, \alpha'_j{}^{(s)})_{1,p'^{(s)}} \\ \\ (\text{b'}_{rj}; \beta'^{(1)}_{sj}, \cdots, \beta'_{sj}{}^{(s)})_{1,q_s} : (b'^{(1)}_j, \beta'^{(1)}_j)_{1,q'^{(1)}}; \cdots ; (b'_j{}^{(s)}, \beta'_j{}^{(s)})_{1,q'^{(s)}} \end{array} \right) \tag{1.10}$$

$$= \frac{1}{(2\pi\omega)^s} \int_{L'_1} \cdots \int_{L'_s} \psi(t_1, \cdots, t_s) \prod_{i=1}^{s} \zeta_i(t_i) z_i'^{t_i} \mathrm{d}t_1 \cdots \mathrm{d}t_s \tag{1.11}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [19]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz'_i| < \frac{1}{2}\Omega'_i\pi$, where

$$\Omega'_i = \sum_{k=1}^{n'^{(i)}} \alpha'_k{}^{(i)} - \sum_{k=n'^{(i)}+1}^{p'^{(i)}} \alpha'_k{}^{(i)} + \sum_{k=1}^{m'^{(i)}} \beta'_k{}^{(i)} - \sum_{k=m'^{(i)}+1}^{q'^{(i)}} \beta'_k{}^{(i)} + \left(\sum_{k=1}^{n'_2} \alpha'_{2k}{}^{(i)} - \sum_{k=n'_2+1}^{p'_2} \alpha'_{2k}{}^{(i)} \right) +$$

$$+ \cdots + \left(\sum_{k=1}^{n'_s} \alpha'_{sk}{}^{(i)} - \sum_{k=n'_s+1}^{p'_s} \alpha'_{sk}{}^{(i)} \right) - \left(\sum_{k=1}^{q'_2} \beta'_{2k}{}^{(i)} + \sum_{k=1}^{q'_3} \beta'_{3k}{}^{(i)} + \cdots + \sum_{k=1}^{q'_s} \beta'_{sk}{}^{(i)} \right) \tag{1.12}$$

where $i = 1, \cdots, s$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z'_1, \cdots, z'_s) = 0(\, |z'_1|^{\alpha_1}, \cdots, |z'_s|^{\alpha_r}\,), max(\, |z'_1|, \cdots, |z'_s|\,) \to 0$$

$$I(z'_1, \cdots, z'_s) \; = 0(\, |z'_1|^{\beta_1}, \cdots, |z'_s|^{\beta_r}\,), min(\, |z'_1|, \cdots, |z'_s|\,) \to \infty$$

where $k = 1, \cdots, r : \alpha''_k = min[Re(b'^{(k)}_j/\beta'^{(k)}_j)], j = 1, \cdots, m'_k$ and

$$\beta''_k = max[Re((a'^{(k)}_j - 1)/\alpha'^{(k)}_j)], j = 1, \cdots, n'_k$$

We shall note $I(z'_1, \cdots, z'_s) = I_2(z'_1, \cdots, z'_s)$

Remark : The multivariable I-function is an extension of the multivariable H-function defined by Srivastava and Panda [27,28].

The generalized polynomials of multivariables defined by Srivastava [26], is given in the following manner :

$$S^{\mathfrak{M}_1, \cdots, \mathfrak{M}_v}_{N_1, \cdots, N_v}[y_1, \cdots, y_v] = \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_v=0}^{[N_v/\mathfrak{M}_v]} \frac{(-N_1)_{\mathfrak{M}_1 K_1}}{K_1!} \cdots \frac{(-N_v)_{\mathfrak{M}_v K_v}}{K_v!}$$

$$A[N_1, K_1; \cdots; N_v, K_v] y_1^{K_1} \cdots y_v^{K_v} \tag{1.13}$$

where $\mathfrak{M}_1, \cdots, \mathfrak{M}_v$ are arbitrary positive integers and the coefficients $A[N_1, K_1; \cdots; N_v, K_v]$ are arbitrary constants, real or complex.

We shall note $a_v = \frac{(-N_1)_{\mathfrak{M}_1 K_1}}{K_1!} \cdots \frac{(-N_v)_{\mathfrak{M}_v K_v}}{K_v!} A[N_1, K_1; \cdots; N_v, K_v]$

2. Lemma

Lemma 1 (Kilbas and Sebastien ([10], page 871, (15)-(18))

$$\left(I^{\alpha,\beta,\eta}_{0^+} t^{\mu-1}\right)(x) = \frac{\Gamma(\mu)\Gamma(\mu+\eta-\beta)}{\Gamma(\mu+\alpha+\eta)\Gamma(\mu-\beta)} x^{\mu-\beta-1} \tag{2.1}$$

where $\alpha, \beta, \eta \in \mathbb{C}, Re(\alpha) > 0$ and $Re(\mu) > max\{0, Re(\beta - \eta)\}$

in particular, if $\beta = -\alpha$ and $\beta = 0$ in (2.1), we have respectively

$$\left(I^{\alpha}_{0^+} t^{\mu-1}\right)(x) = \frac{\Gamma(\mu)}{\Gamma(\mu+\alpha)} x^{\mu-\beta-1}, Re(\alpha) > 0, Re(\mu) > 0, \tag{2.2}$$

$$\left(I^{+}_{\eta,\alpha} t^{\mu-1}\right)(x) = \frac{\Gamma(\mu+\eta)}{\Gamma(\mu+\alpha+\eta)} x^{\mu-1}, Re(\alpha) > 0, Re(\mu) > -Re(\eta). \tag{2.3}$$

Lemma 2 (Kilbas and Sebastien ([10], page 872, (21)-(24))

$$\left(I^{\alpha,\beta,\eta}_{0^-} t^{\mu-1}\right)(x) = \frac{\Gamma(\beta-\mu+1)\Gamma(\eta-\mu+1)}{\Gamma(1-\mu)\Gamma(\alpha+\beta+\eta-\mu+1)} x^{\mu-\beta-1} \tag{2.4}$$

where $\alpha, \beta, \eta \in \mathbb{C}, Re(\alpha) > 0$ and $Re(\mu) < 1 + min\{Re(\beta), Re(\eta)\}$

in particular, if $\beta = -\alpha$ and $\beta = 0$ in (2.4), we have respectively

$$\left(I^{\alpha}_{-} t^{\mu-1}\right)(x) = \frac{\Gamma(1-\alpha-\mu)}{\Gamma(1-\mu)} x^{\mu-\beta-1}, 1 - Re(\mu) > Re(\alpha) > 0 \tag{2.5}$$

$$\left(K^{-}_{\eta,\alpha} t^{\mu-1}\right)(x) = \frac{\Gamma(\eta-\mu+1)}{\Gamma(1-\mu+\alpha+\eta)} x^{\mu-1}, Re(\mu) < Re(\eta) + 1. \tag{2.6}$$

3. Main results

We have the following result

Theorem 1

$$\left\{I_{0^+}^{\alpha,\beta,\eta}\left(t^{\mu-1}(b-at)^{-\upsilon}S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\begin{pmatrix}\mathrm{c}_1t^{\lambda_1}(b-at)^{-\delta_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{c}_vt^{\lambda_v}(b-at)^{-\delta_v}\end{pmatrix}I_1\begin{pmatrix}\mathrm{z}_1t^{\sigma_1}(b-at)^{-\omega_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rt^{\sigma_r}(b-at)^{-\omega_r}\end{pmatrix}I_2\begin{pmatrix}\mathrm{z}'_1t^{\sigma'_1}(b-at)^{-\omega'_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}'_st^{\sigma'_s}(b-at)^{-\omega'_s}\end{pmatrix}\right)\right\}(x)$$

$$=b^{-\upsilon}x^{\mu-\beta-1}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}a_v\,c_1^{K_1}\cdots c_v^{K_v}b^{-\sum_{j=1}^{v}\delta_jK_j}x^{\sum_{j=1}^{v}\lambda_jK_j}I_{U:p_r+p'_s+3,q_r+q'_s+3;Y}^{V;0,n_r+n'_s+3;X}\left(\begin{array}{c|c}\mathrm{z}_1\frac{x^{\sigma_1}}{b^{\omega_1}} & \mathrm{A}\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_r\frac{x^{\sigma_r}}{b^{\omega_r}} & \cdot\\ \mathrm{z}'_1\frac{x^{\sigma'_1}}{b^{\omega'_1}} & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}'_s\frac{x^{\sigma'_s}}{b^{\omega'_s}} & \mathrm{B}\\ -\frac{ax}{b} & \end{array}\right) \quad (3.1)$$

where

$$U=p_2,q_2;p_3,q_3;\cdots;p_{r-1},q_{r-1};p'_2,q'_2;p'_3,q'_3;\cdots;p'_{s-1},q'_{s-1} \quad (3.2)$$

$$V=0,n_2;0,n_3;\cdots;0,n_{r-1};0,n'_2;0,n'_3;\cdots;0,n'_{s-1} \quad (3.3)$$

$$X=m^{(1)},n^{(1)};\cdots;m^{(r)},n^{(r)};m'^{(1)},n'^{(1)};\cdots;m'^{(s)},n'^{(s)};1,0 \quad (3.4)$$

$$Y=p^{(1)},q^{(1)};\cdots;p^{(r)},q^{(r)};p'^{(1)},q'^{(1)};\cdots;p'^{(s)},q'^{(s)};0,1 \quad (3.5)$$

$$A=(a_{2k};\alpha_{2k}^{(1)},\alpha_{2k}^{(2)})_{1,p_2};\cdots;(a_{(r-1)k};\alpha_{(r-1)k}^{(1)},\alpha_{(r-1)k}^{(2)},\cdots,\alpha_{(r-1)k}^{(r-1)})_{1,p_{r-1}};$$

$$(a'_{2k};\alpha'^{(1)}_{2k},\alpha'^{(2)}_{2k})_{1,p'_2};\cdots;(a'_{(s-1)k};\alpha'^{(1)}_{(s-1)k},\alpha'^{(2)}_{(s-1)k},\cdots,\alpha'^{(s-1)}_{(s-1)k})_{1,p'_{s-1}}:$$

$$\left(1-\upsilon-\sum_{j=1}^{v}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,1\right),\left(1-\mu-\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1\right),$$

$$\left(1-\mu-\eta+\beta-\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1\right),(a_{rk};\alpha_{rk}^{(1)},\alpha_{rk}^{(2)},\cdots,\alpha_{rk}^{(r)},0,\cdots,0,0)_{1,p_r},$$

$$(a'_{sk};0,\cdots,0,\alpha'^{(1)}_{sk},\alpha'^{(2)}_{sk},\cdots,\alpha'^{(s)}_{sk},0)_{1,p'_s}:(a_k^{(1)},\alpha_k^{(1)})_{1,p^{(1)}};\cdots;(a_k^{(r)},\alpha_k^{(r)})_{1,p^{(r)}};$$

$$(a'^{(1)}_k,\alpha'^{(1)}_k)_{1,p'^{(1)}};\cdots;(a'^{(s)}_k,\alpha'^{(s)}_k)_{1,p'^{(s)}} \quad (3.6)$$

$$B=(b_{2k};\beta_{2k}^{(1)},\beta_{2k}^{(2)})_{1,q_2};\cdots;(b_{(r-1)k};\beta_{(r-1)k}^{(1)},\beta_{(r-1)k}^{(2)},\cdots,\beta_{(r-1)k}^{(r-1)})_{1,q_{r-1}};$$

$$(b'_{2k};\beta_{2k}^{\prime(1)},\beta_{2k}^{\prime(2)})_{1,q'_2};\cdots;(b'_{(s-1)k};\beta_{(s-1)k}^{\prime(1)},\beta_{(s-1)k}^{\prime(2)},\cdots,\beta_{(s-1)k}^{\prime(s-1)})_{1,q'_{s-1}}:$$

$$\left(1-\upsilon-\sum_{j=1}^{v}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,0\right),\left(1-\mu+\beta-\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1\right),$$

$$\left(1-\mu-\eta-\alpha-\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1\right),$$

$$(b_{rk};\beta_{rk}^{(1)},\beta_{rk}^{(2)},\cdots,\beta_{rk}^{(r)},0,\cdots,0,0)_{1,q_r};(b'_{sk};0,\cdots,0,\beta_{sk}^{\prime(1)},\beta_{sk}^{\prime(2)},\cdots,\beta_{sk}^{\prime(s)},0)_{1,q'_s},$$

$$(b_k^{(1)},\beta_k^{(1)})_{1,q^{(1)}};\cdots;(b_k^{(r)},\beta_k^{(r)})_{1,q^{(r)}};(b_k^{\prime(1)},\beta_k^{\prime(1)})_{1,q^{\prime(1)}};\cdots;(b_k^{\prime(s)},\beta_k^{\prime(s)})_{1,q^{\prime(s)}};(0,1) \tag{3.7}$$

Provided that

$a,b,c,\alpha,\beta,\eta,\mu,\upsilon,\delta_k,\omega_i,\omega'_j\in\mathbb{C},k=1,\cdots,v;i=1,\cdots,r;j=1,\cdots,s$

$\lambda_k,\sigma_i,\sigma'_j>0,k=1,\cdots,v;i=1,\cdots,r;j=1,\cdots,s$

$|argz_i|<\frac{1}{2}\Omega_i\pi$, where Ω_i is defined by (1.9).

$|argz'_i|<\frac{1}{2}\Omega'_i\pi$, where Ω'_i is defined by (1.12).

$$Re\left[\mu+\sum_{i=1}^{r}\sigma_i\min_{1\leqslant l\leqslant m^{(i)}}\frac{b_l^{(i)}}{\beta_l^{(i)}}+\sum_{j=1}^{s}\sigma'_j\min_{1\leqslant l'\leqslant m^{\prime(j)}}\frac{b_{l'}^{\prime(j)}}{\beta_{l'}^{\prime(j)}}\right]>max\{0,Re(\beta-\eta)\}$$

$$Re\left[\upsilon+\sum_{i=1}^{r}\omega_i\min_{1\leqslant l\leqslant m^{(i)}}\frac{b_l^{(i)}}{\beta_l^{(i)}}+\sum_{j=1}^{s}\omega'_j\min_{1\leqslant l'\leqslant m^{\prime(j)}}\frac{b_{l'}^{\prime(j)}}{\beta_{l'}^{\prime(j)}}\right]>max\{0,Re(\beta-\eta)\}\text{ and }\left|\frac{a}{b}x\right|<1.$$

Proof

To prove (3.1), we first express the class of multivariable polynomials $S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[.]$ in series with the help of (1.13), the multivariable I-functions of Prasad in terms of Mellin-Barnes type contour integrals with the help of (1.8) and (1.11) respectively. Now interchange the order of summations and two multiple Mellin-Barnes contour integrals, respectively and taking the fractional integral operator inside (which is permissible under the stated conditions) and make simplifications. Next, we express the terms $(b-ax)^{-\upsilon-\sum_{k=1}^{v}\delta_kK_k-\sum_{j=1}^{r}\omega_is_i-\sum_{j=1}^{s}\omega'_jt_j}$ in the terms of Mellin-Barnes contour integral (Srivastava et al [25], page 18, (2.6.3) ; page 10, (2.1.1)) and after algebric manipulations, we obtain

$$\text{L.H.S. of (3.1) }=b^{-\upsilon}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}a_v\,c_1^{K_1}\cdots c_v^{K_v}\,b^{-\sum_{j=1}^{v}\delta_jK_j}\frac{1}{(2i\pi)^{r+s+1}}\int_{L_1}\cdots\int_{L_r}\int_{L'_1}\cdots\int_{L'_s}\phi(s_1,\cdots,s_r)$$

$$\psi(t_1,\cdots,t_s)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}\prod_{i=1}^{s}\zeta_i(t_i)z_i'^{t_i}b^{-\sum_{j=1}^{r}\omega_i s_i-\sum_{j=1}^{s}\omega_j' t_j}$$

$$\int_L \frac{\Gamma(\upsilon+\sum_{k=1}^{v}\delta_k K_k+\sum_{i=1}^{r}\omega_i s_i+\sum_{j=1}^{s}\omega_j' t_j+u)}{\Gamma(\upsilon+\sum_{k=1}^{v}\delta_k K_k+\sum_{i=1}^{r}\omega_i s_i+\sum_{j=1}^{s}\omega_j' t_j)\Gamma(1+u)}\left(-\frac{a}{b}\right)^u$$

$$\left(I_{0+}^{\alpha,\beta,\eta}t^{\mu+\sum_{k=1}^{v}\lambda_k K_k+\sum_{i=1}^{r}\sigma_i s_i+\sum_{j=1}^{s}\sigma_j t_j+u-1}\right)(x)\mathrm{d}u\mathrm{d}s_1\cdots\mathrm{d}s_r\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{3.8}$$

Now using the Lemma 1 and interpreting the $(r+s+1)-$Mellin-barnes contour integral of (3.8) in multivariable I-function of $(r+s+1)-$variables, we obtain the desired result.

If we put $\beta=-\alpha$ in Theorem 1, we arrive at the following new and interesting corollary concerning Riemann–Liouville fractional integral operator defined by (1.3) and using (2.2):

Corollary 1

$$\left\{I_{0+}^{\alpha}\left(t^{\mu-1}(b-at)^{-\upsilon}S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\begin{pmatrix}\mathrm{c}_1t^{\lambda_1}(b-at)^{-\delta_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{c}_vt^{\lambda_v}(b-at)^{-\delta_v}\end{pmatrix}I_1\begin{pmatrix}\mathrm{z}_1t^{\sigma_1}(b-at)^{-\omega_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rt^{\sigma_r}(b-at)^{-\omega_r}\end{pmatrix}I_2\begin{pmatrix}\mathrm{z}_1't^{\sigma_1'}(b-at)^{-\omega_1'}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_s't^{\sigma_s'}(b-at)^{-\omega_s'}\end{pmatrix}\right)\right\}(x)$$

$$=b^{-\upsilon}x^{\mu-\beta-1}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}a_v\,c_1^{K_1}\cdots c_v^{K_v}b^{-\sum_{j=1}^{v}\delta_jK_j}x^{\sum_{j=1}^{v}\lambda_jK_j}I_{U:p_r+p_s'+2,q_r+q_s'+2;Y}^{V;0,n_r+n_s'+2;X}\left(\begin{array}{c|c}\mathrm{z}_1\frac{x^{\sigma_1}}{b^{\omega_1}} & \mathrm{A'}\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_r\frac{x^{\sigma_r}}{b^{\omega_r}} & \cdot\\ \mathrm{z}_1'\frac{x^{\sigma_1'}}{b^{\omega_1'}} & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_s'\frac{x^{\sigma_s'}}{b^{\omega_s'}} & \mathrm{B'}\\ -\frac{ax}{b} & \end{array}\right) \tag{3.9}$$

where

$$A'=(a_{2k};\alpha_{2k}^{(1)},\alpha_{2k}^{(2)})_{1,p_2};\cdots;(a_{(r-1)k};\alpha_{(r-1)k}^{(1)},\alpha_{(r-1)k}^{(2)},\cdots,\alpha_{(r-1)k}^{(r-1)})_{1,p_{r-1}};$$

$$(a_{2k}';\alpha_{2k}'^{(1)},\alpha_{2k}'^{(2)})_{1,p_2'};\cdots;(a_{(s-1)k}';\alpha_{(s-1)k}'^{(1)},\alpha_{(s-1)k}'^{(2)},\cdots,\alpha_{(s-1)k}'^{(s-1)})_{1,p_{s-1}'}:$$

$$\left(1-\upsilon-\sum_{j=1}^{v}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega_1',\cdots,\omega_s',1\right),\ \left(1-\mu-\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma_1',\cdots,\sigma_s',1\right),$$

$$(a_{rk};\alpha_{rk}^{(1)},\alpha_{rk}^{(2)},\cdots,\alpha_{rk}^{(r)},0,\cdots,0,0)_{1,p_r},(a_{sk}';0,\cdots,0,\alpha_{sk}'^{(1)},\alpha_{sk}'^{(2)},\cdots,\alpha_{sk}'^{(s)},0)_{1,p_s'}:$$

$$(a_k^{(1)},\alpha_k^{(1)})_{1,p^{(1)}};\cdots;(a_k^{(r)},\alpha_k^{(r)})_{1,p^{(r)}};(a_k'^{(1)},\alpha_k'^{(1)})_{1,p'^{(1)}};\cdots;(a_k'^{(s)},\alpha_k'^{(s)})_{1,p'^{(s)}} \tag{3.10}$$

$$B'=(b_{2k};\beta_{2k}^{(1)},\beta_{2k}^{(2)})_{1,q_2};\cdots;(b_{(r-1)k};\beta_{(r-1)k}^{(1)},\beta_{(r-1)k}^{(2)},\cdots,\beta_{(r-1)k}^{(r-1)})_{1,q_{r-1}};$$

$$(b'_{2k};\beta'^{(1)}_{2k},\beta'^{(2)}_{2k})_{1,q'_2};\cdots;(b'_{(s-1)k};\beta'^{(1)}_{(s-1)k},\beta'^{(2)}_{(s-1)k},\cdots,\beta'^{(s-1)}_{(s-1)k})_{1,q'_{s-1}}:$$

$$\left(1-\upsilon-\sum_{j=1}^{v}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,0\right),\left(1-\mu-\alpha-\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1\right),$$

$$(b_{rk};\beta^{(1)}_{rk},\beta^{(2)}_{rk},\cdots,\beta^{(r)}_{rk},0,\cdots,0,0)_{1,q_r};(b'_{sk};0,\cdots,0,\beta'^{(1)}_{sk},\beta'^{(2)}_{sk},\cdots,\beta'^{(s)}_{sk},0)_{1,q'_s},$$

$$(b^{(1)}_k,\beta^{(1)}_k)_{1,q^{(1)}};\cdots;(b^{(r)}_k,\beta^{(r)}_k)_{1,q^{(r)}};(b'^{(1)}_k,\beta'^{(1)}_k)_{1,q'^{(1)}};\cdots;(b'^{(s)}_k,\beta'^{(s)}_k)_{1,q'^{(s)}};(0,1) \tag{3.11}$$

with the same validity conditions that (3.1).

If $\beta=0$ in Theorem 1, we arrive at the following new and interesting corollary concerning Riemann–Liouville fractional integral operator defined by (1.5) and using (2.3) :

Corollary 2

$$\left\{I^{+}_{\eta,\alpha}\left(t^{\mu-1}(b-at)^{-\upsilon}\,S^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}_{N_1,\cdots,N_v}\begin{pmatrix}c_1t^{\lambda_1}(b-at)^{-\delta_1}\\ \cdot\\ \cdot\\ \cdot\\ c_vt^{\lambda_v}(b-at)^{-\delta_v}\end{pmatrix}I_1\begin{pmatrix}z_1t^{\sigma_1}(b-at)^{-\omega_1}\\ \cdot\\ \cdot\\ \cdot\\ z_rt^{\sigma_r}(b-at)^{-\omega_r}\end{pmatrix}I_2\begin{pmatrix}z'_1t^{\sigma'_1}(b-at)^{-\omega'_1}\\ \cdot\\ \cdot\\ \cdot\\ z'_st^{\sigma'_s}(b-at)^{-\omega'_s}\end{pmatrix}\right)\right\}(x)$$

$$=b^{-\upsilon}x^{\mu-\beta-1}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}a_vc_1^{K_1}\cdots c_v^{K_v}b^{-\sum_{j=1}^{v}\delta_jK_j}x^{\sum_{j=1}^{v}\lambda_jK_j}I^{V;0,n_r+n'_s+2;X}_{U:p_r+p'_s+2,q_r+q'_s+2;Y}\left(\begin{array}{c|c}z_1\frac{x^{\sigma_1}}{b^{\omega_1}} & \text{A"}\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ z_r\frac{x^{\sigma_r}}{b^{\omega_r}} & \cdot\\ z'_1\frac{x^{\sigma'_1}}{b^{\omega'_1}} & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ z'_s\frac{x^{\sigma'_s}}{b^{\omega'_s}} & \text{B"}\\ -\frac{ax}{b} & \end{array}\right) \tag{3.12}$$

where

$$A''=(a_{2k};\alpha^{(1)}_{2k},\alpha^{(2)}_{2k})_{1,p_2};\cdots;(a_{(r-1)k};\alpha^{(1)}_{(r-1)k},\alpha^{(2)}_{(r-1)k},\cdots,\alpha^{(r-1)}_{(r-1)k})_{1,p_{r-1}};$$

$$(a'_{2k};\alpha'^{(1)}_{2k},\alpha'^{(2)}_{2k})_{1,p'_2};\cdots;(a'_{(s-1)k};\alpha'^{(1)}_{(s-1)k},\alpha'^{(2)}_{(s-1)k},\cdots,\alpha'^{(s-1)}_{(s-1)k})_{1,p'_{s-1}}:$$

$$\left(1-\upsilon-\sum_{j=1}^{v}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,1\right),\quad\left(1-\mu-\eta-\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1\right),$$

$$(a_{rk};\alpha^{(1)}_{rk},\alpha^{(2)}_{rk},\cdots,\alpha^{(r)}_{rk},0,\cdots,0,0)_{1,p_r},\quad(a'_{sk};0,\cdots,0,\alpha'^{(1)}_{sk},\alpha'^{(2)}_{sk},\cdots,\alpha'^{(s)}_{sk},0)_{1,p'_s}:$$

$$(a^{(1)}_k,\alpha^{(1)}_k)_{1,p^{(1)}};\cdots;(a^{(r)}_k,\alpha^{(r)}_k)_{1,p^{(r)}};(a'^{(1)}_k,\alpha'^{(1)}_k)_{1,p'^{(1)}};\cdots;(a'^{(s)}_k,\alpha'^{(s)}_k)_{1,p'^{(s)}} \tag{3.13}$$

$$B''=(b_{2k};\beta^{(1)}_{2k},\beta^{(2)}_{2k})_{1,q_2};\cdots;(b_{(r-1)k};\beta^{(1)}_{(r-1)k},\beta^{(2)}_{(r-1)k},\cdots,\beta^{(r-1)}_{(r-1)k})_{1,q_{r-1}};$$

$$(b'_{2k};\beta'^{(1)}_{2k},\beta'^{(2)}_{2k})_{1,q'_2};\cdots;(b'_{(s-1)k};\beta'^{(1)}_{(s-1)k},\beta'^{(2)}_{(s-1)k},\cdots,\beta'^{(s-1)}_{(s-1)k})_{1,q'_{s-1}};(0,0):$$

$$\left(1-\upsilon-\sum_{j=1}^{v}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,0\right),\left(1-\mu-\alpha-\eta-\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1\right),$$

$$(b_{rk};\beta^{(1)}_{rk},\beta^{(2)}_{rk},\cdots,\beta^{(r)}_{rk},0,\cdots,0,0)_{1,q_r};(b'_{sk};0,\cdots,0,\beta'^{(1)}_{sk},\beta'^{(2)}_{sk},\cdots,\beta'^{(s)}_{sk},0)_{1,q'_s},$$

$$(b^{(1)}_k,\beta^{(1)}_k)_{1,q^{(1)}};\cdots;(b^{(r)}_k,\beta^{(r)}_k)_{1,q^{(r)}};(b'^{(1)}_k,\beta'^{(1)}_k)_{1,q'^{(1)}};\cdots;(b'^{(s)}_k,\beta'^{(s)}_k)_{1,q'^{(s)}};(0,1) \tag{3.14}$$

Provided that

$a,b,c,\alpha,\beta,\eta,\mu,\upsilon,\delta_k,\omega_i,\omega'_j\in\mathbb{C},k=1,\cdots,v;i=1,\cdots,r;j=1,\cdots,s$

$\lambda_k,\sigma_i,\sigma'_j>0,k=1,\cdots,v;i=1,\cdots,r;j=1,\cdots,s$

$|argz_i|<\frac{1}{2}\Omega_i\pi$, where Ω_i is defined by (1.9).

$|argz'_i|<\frac{1}{2}\Omega'_i\pi$, where Ω'_i is defined by (1.12).

$$Re\left[\mu+\sum_{i=1}^{r}\sigma_i\min_{1\leqslant l\leqslant m^{(i)}}\frac{b^{(i)}_l}{\beta^{(i)}_l}+\sum_{j=1}^{s}\sigma'_j\min_{1\leqslant l'\leqslant m'^{(j)}}\frac{b'^{(j)}_{l'}}{\beta'^{(j)}_{l'}}\right]>max\{0,Re(-\eta)\}$$

$$Re\left[\upsilon+\sum_{i=1}^{r}\omega_i\min_{1\leqslant l\leqslant m^{(i)}}\frac{b^{(i)}_l}{\beta^{(i)}_l}+\sum_{j=1}^{s}\omega'_j\min_{1\leqslant l'\leqslant m'^{(j)}}\frac{b'^{(j)}_{l'}}{\beta'^{(j)}_{l'}}\right]>max\{0,Re(-\eta)\}\text{ and }\left|\frac{a}{b}x\right|<1.$$

Theorem 2

$$\left\{I^{\alpha,\beta,\eta}_{0^-}\left(t^{\mu-1}(b-at)^{-\upsilon}S^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}_{N_1,\cdots,N_v}\begin{pmatrix}c_1t^{\lambda_1}(b-at)^{-\delta_1}\\ .\\ .\\ .\\ c_vt^{\lambda_v}(b-at)^{-\delta_v}\end{pmatrix}I_1\begin{pmatrix}z_1t^{\sigma_1}(b-at)^{-\omega_1}\\ .\\ .\\ .\\ z_rt^{\sigma_r}(b-at)^{-\omega_r}\end{pmatrix}I_2\begin{pmatrix}z'_1t^{\sigma'_1}(b-at)^{-\omega'_1}\\ .\\ .\\ .\\ z'_st^{\sigma'_s}(b-at)^{-\omega'_s}\end{pmatrix}\right)\right\}(x)$$

$$=b^{-\upsilon}x^{\mu-\beta-1}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}a_v\,c_1^{K_1}\cdots c_v^{K_v}b^{-\sum_{j=1}^{v}\delta_jK_j}x^{\sum_{j=1}^{v}\lambda_jK_j}I^{V;0,n_r+n'_s+3;X}_{U:p_r+p'_s+3,q_r+q'_s+3;Y}\left(\begin{array}{c|c}z_1\frac{x^{\sigma_1}}{b^{\omega_1}} & \mathbb{A}\\ . & .\\ . & .\\ . & .\\ z_r\frac{x^{\sigma_r}}{b^{\omega_r}} & .\\ z'_1\frac{x^{\sigma'_1}}{b^{\omega'_1}} & .\\ . & .\\ . & .\\ . & .\\ z'_s\frac{x^{\sigma'_s}}{b^{\omega'_s}} & \mathbb{B}\\ -\frac{ax}{b} & \end{array}\right) \tag{3.15}$$

where

$$\mathbb{A}=(a_{2k};\alpha_{2k}^{(1)},\alpha_{2k}^{(2)})_{1,p_2};\cdots;(a_{(r-1)k};\alpha_{(r-1)k}^{(1)},\alpha_{(r-1)k}^{(2)},\cdots,\alpha_{(r-1)k}^{(r-1)})_{1,p_{r-1}};$$

$$(a'_{2k};\alpha_{2k}^{\prime(1)},\alpha_{2k}^{\prime(2)})_{1,p'_2};\cdots;(a'_{(s-1)k};\alpha_{(s-1)k}^{\prime(1)},\alpha_{(s-1)k}^{\prime(2)},\cdots,\alpha_{(s-1)k}^{\prime(s-1)})_{1,p'_{s-1}}:$$

$$\left(1-\upsilon-\sum_{j=1}^{v}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,1\right),\left(\mu-\eta+\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1\right),$$

$$\left(-\beta+\mu+\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1\right),(a_{rk};\alpha_{rk}^{(1)},\alpha_{rk}^{(2)},\cdots,\alpha_{rk}^{(r)},0,\cdots,0,0)_{1,p_r},$$

$$(a'_{sk};0,\cdots,0,\alpha_{sk}^{\prime(1)},\alpha_{sk}^{\prime(2)},\cdots,\alpha_{sk}^{\prime(s)},0)_{1,p'_s}:(a_k^{(1)},\alpha_k^{(1)})_{1,p^{(1)}};\cdots;(a_k^{(r)},\alpha_k^{(r)})_{1,p^{(r)}};$$

$$(a_k^{\prime(1)},\alpha_k^{\prime(1)})_{1,p^{\prime(1)}};\cdots;(a_k^{\prime(s)},\alpha_k^{\prime(s)})_{1,p^{\prime(s)}} \tag{3.16}$$

$$\mathbb{B}=(b_{2k};\beta_{2k}^{(1)},\beta_{2k}^{(2)})_{1,q_2};\cdots;(b_{(r-1)k};\beta_{(r-1)k}^{(1)},\beta_{(r-1)k}^{(2)},\cdots,\beta_{(r-1)k}^{(r-1)})_{1,q_{r-1}};$$

$$(b'_{2k};\beta_{2k}^{\prime(1)},\beta_{2k}^{\prime(2)})_{1,q'_2};\cdots;(b'_{(s-1)k};\beta_{(s-1)k}^{\prime(1)},\beta_{(s-1)k}^{\prime(2)},\cdots,\beta_{(s-1)k}^{\prime(s-1)})_{1,q'_{s-1}}:$$

$$\left(1-\upsilon-\sum_{j=1}^{v}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,0\right),\left(\mu+\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1\right),$$

$$\left(-\alpha-\beta-\eta+\mu+\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1\right),(b_{rk};\beta_{rk}^{(1)},\beta_{rk}^{(2)},\cdots,\beta_{rk}^{(r)},0,\cdots,0,0)_{1,q_r};$$

$$(b'_{sk};0,\cdots,0,\beta_{sk}^{\prime(1)},\beta_{sk}^{\prime(2)},\cdots,\beta_{sk}^{\prime(s)},0)_{1,q'_s}:(b_k^{(1)},\beta_k^{(1)})_{1,q^{(1)}};\cdots;(b_k^{(r)},\beta_k^{(r)})_{1,q^{(r)}};$$

$$(b_k^{\prime(1)},\beta_k^{\prime(1)})_{1,q^{\prime(1)}};\cdots;(b_k^{\prime(s)},\beta_k^{\prime(s)})_{1,q^{\prime(s)}};(0,1) \tag{3.17}$$

Provided that

$a,b,c,\alpha,\beta,\eta,\mu,\upsilon,\delta_k,\omega_i,\omega'_j\in\mathbb{C},k=1,\cdots,v;i=1,\cdots,r;j=1,\cdots,s$

$\lambda_k,\sigma_i,\sigma'_j>0,k=1,\cdots,v;i=1,\cdots,r;j=1,\cdots,s$

$|argz_i|<\dfrac{1}{2}\Omega_i\pi$, where Ω_i is defined by (1.9).

$|argz'_i|<\dfrac{1}{2}\Omega'_i\pi$, where Ω'_i is defined by (1.12).

$$Re\left[\mu-\sum_{i=1}^{r}\sigma_i\min_{1\leqslant l\leqslant m^{(i)}}\frac{b_l^{(i)}}{\beta_l^{(i)}}-\sum_{j=1}^{s}\sigma'_j\min_{1\leqslant l'\leqslant m^{\prime(j)}}\frac{b_{l'}^{\prime(j)}}{\beta_{l'}^{\prime(j)}}\right]<1+min\{Re(\beta),Re(\eta)\}$$

$$Re\left[\upsilon-\sum_{i=1}^{r}\omega_i\min_{1\leqslant l\leqslant m^{(i)}}\frac{b_l^{(i)}}{\beta_l^{(i)}}-\sum_{j=1}^{s}\omega'_j\min_{1\leqslant l'\leqslant m'^{(j)}}\frac{b_{l'}^{\prime(j)}}{\beta_{l'}^{\prime(j)}}\right]<1+min\{Re(\beta),Re(\eta)\}\text{ and }\left|\frac{a}{b}x\right|<1.$$

The proof of the Theorem 2 is similar that Theorem 1 (use the Lemma 2).

If we put $\beta=-\alpha$ in Theorem 2, we arrive at the following new and interesting corollary concerning Riemann–Liouville fractional integral operator defined by (1.4) and using (2.5):

Corollary 3

$$\left\{I_{-}^{\alpha}\left(t^{\mu-1}(b-at)^{-\upsilon}\,S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_\mathfrak{v}}\begin{pmatrix}\mathrm{c}_1t^{\lambda_1}(b-at)^{-\delta_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{c}_vt^{\lambda_v}(b-at)^{-\delta_v}\end{pmatrix}I_1\begin{pmatrix}\mathrm{z}_1t^{\sigma_1}(b-at)^{-\omega_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rt^{\sigma_r}(b-at)^{-\omega_r}\end{pmatrix}I_2\begin{pmatrix}\mathrm{z}'_1t^{\sigma'_1}(b-at)^{-\omega'_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}'_st^{\sigma'_s}(b-at)^{-\omega'_s}\end{pmatrix}\right)\right\}(x)$$

$$=b^{-\upsilon}x^{\mu+\alpha-1}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_\mathfrak{v}]}a_v\,c_1^{K_1}\cdots c_v^{K_v}b^{-\sum_{j=1}^{v}\delta_jK_j}x^{\sum_{j=1}^{v}\lambda_jK_j}I_{U:p_r+p'_s+2,q_r+q'_s+2;Y}^{V;0,n_r+n'_s+2;X}\left(\begin{array}{c|c}\mathrm{z}_1\frac{x^{\sigma_1}}{b^{\omega_1}} & \mathbb{A}'\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_r\frac{x^{\sigma_r}}{b^{\omega_r}} & \cdot\\ \mathrm{z}'_1\frac{x^{\sigma'_1}}{b^{\omega'_1}} & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}'_s\frac{x^{\sigma'_s}}{b^{\omega'_s}} & \mathbb{B}'\\ -\frac{ax}{b} & \end{array}\right)\tag{3.18}$$

where

$$\mathbb{A}'=(a_{2k};\alpha_{2k}^{(1)},\alpha_{2k}^{(2)})_{1,p_2};\cdots;(a_{(r-1)k};\alpha_{(r-1)k}^{(1)},\alpha_{(r-1)k}^{(2)},\cdots,\alpha_{(r-1)k}^{(r-1)})_{1,p_{r-1}};$$

$$(a'_{2k};\alpha_{2k}^{\prime(1)},\alpha_{2k}^{\prime(2)})_{1,p'_2};\cdots;(a'_{(s-1)k};\alpha_{(s-1)k}^{\prime(1)},\alpha_{(s-1)k}^{\prime(2)},\cdots,\alpha_{(s-1)k}^{\prime(s-1)})_{1,p'_{s-1}}:$$

$$\left(1-\upsilon-\sum_{j=1}^{v}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,1\right),\left(\alpha+\mu+\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1\right),$$

$$(a_{rk};\alpha_{rk}^{(1)},\alpha_{rk}^{(2)},\cdots,\alpha_{rk}^{(r)},0,\cdots,0,0)_{1,p_r},(a'_{sk};0,\cdots,0,\alpha_{sk}^{\prime(1)},\alpha_{sk}^{\prime(2)},\cdots,\alpha_{sk}^{\prime(s)},0)_{1,p'_s}:$$

$$(a_k^{(1)},\alpha_k^{(1)})_{1,p^{(1)}};\cdots;(a_k^{(r)},\alpha_k^{(r)})_{1,p^{(r)}};(a_k^{\prime(1)},\alpha_k^{\prime(1)})_{1,p'^{(1)}};\cdots;(a_k^{\prime(s)},\alpha_k^{\prime(s)})_{1,p'^{(s)}}\tag{3.19}$$

$$\mathbb{B}'=(b_{2k};\beta_{2k}^{(1)},\beta_{2k}^{(2)})_{1,q_2};\cdots;(b_{(r-1)k};\beta_{(r-1)k}^{(1)},\beta_{(r-1)k}^{(2)},\cdots,\beta_{(r-1)k}^{(r-1)})_{1,q_{r-1}};$$

$$(b'_{2k};\beta_{2k}^{\prime(1)},\beta_{2k}^{\prime(2)})_{1,q'_2};\cdots;(b'_{(s-1)k};\beta_{(s-1)k}^{\prime(1)},\beta_{(s-1)k}^{\prime(2)},\cdots,\beta_{(s-1)k}^{\prime(s-1)})_{1,q'_{s-1}}:$$

$$\left(1-\upsilon-\sum_{j=1}^{v}\delta_j K_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,0\right),\left(\mu+\sum_{j=1}^{v}\lambda_j K_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1\right),$$

$$(b_{rk};\beta_{rk}^{(1)},\beta_{rk}^{(2)},\cdots,\beta_{rk}^{(r)},0,\cdots,0,0)_{1,q_r};(b'_{sk};0,\cdots,0,\beta'^{(1)}_{sk},\beta'^{(2)}_{sk},\cdots,\beta'^{(s)}_{sk},0)_{1,q'_s}:$$

$$(b_k^{(1)},\beta_k^{(1)})_{1,q^{(1)}};\cdots;(b_k^{(r)},\beta_k^{(r)})_{1,q^{(r)}};(b'^{(1)}_k,\beta'^{(1)}_k)_{1,q'^{(1)}};\cdots;(b'^{(s)}_k,\beta'^{(s)}_k)_{1,q'^{(s)}};(0,1) \tag{3.20}$$

with the same validity conditions that (3.15).

If we put $\beta=0$ in Theorem 2, we arrive at the following new and interesting corollary concerning Riemann–Liouville fractional integral operator defined by (1.4) and using (2.6):

Corollary 4

$$\left\{K^{-}_{\eta,\alpha}\left(t^{\mu-1}(b-at)^{-\upsilon}S^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}_{N_1,\cdots,N_v}\begin{pmatrix}c_1t^{\lambda_1}(b-at)^{-\delta_1}\\ .\\ .\\ .\\ c_vt^{\lambda_v}(b-at)^{-\delta_v}\end{pmatrix}I_1\begin{pmatrix}z_1t^{\sigma_1}(b-at)^{-\omega_1}\\ .\\ .\\ .\\ z_rt^{\sigma_r}(b-at)^{-\omega_r}\end{pmatrix}I_2\begin{pmatrix}z'_1t^{\sigma'_1}(b-at)^{-\omega'_1}\\ .\\ .\\ .\\ z'_st^{\sigma'_s}(b-at)^{-\omega'_s}\end{pmatrix}\right)\right\}(x)$$

$$=b^{-\upsilon}x^{\mu-1}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}a_v\,c_1^{K_1}\cdots c_v^{K_v}b^{-\sum_{j=1}^{v}\delta_jK_j}x^{\sum_{j=1}^{v}\lambda_jK_j}I^{V;0,n_r+n'_s+2;X}_{U:p_r+p'_s+2,q_r+q'_s+2;Y}\left(\begin{array}{c|c}z_1\frac{x^{\sigma_1}}{b^{\omega_1}} & \mathbb{A}''\\ . & .\\ . & .\\ . & .\\ z_r\frac{x^{\sigma_r}}{b^{\omega_r}} & .\\ z'_1\frac{x^{\sigma'_1}}{b^{\omega'_1}} & .\\ . & .\\ . & .\\ . & .\\ z'_s\frac{x^{\sigma'_s}}{b^{\omega'_s}} & \mathbb{B}''\\ -\frac{ax}{b} & \end{array}\right) \tag{3.21}$$

where

$$\mathbb{A}''=(a_{2k};\alpha_{2k}^{(1)},\alpha_{2k}^{(2)})_{1,p_2};\cdots;(a_{(r-1)k};\alpha_{(r-1)k}^{(1)},\alpha_{(r-1)k}^{(2)},\cdots,\alpha_{(r-1)k}^{(r-1)})_{1,p_{r-1}};$$

$$(a'_{2k};\alpha'^{(1)}_{2k},\alpha'^{(2)}_{2k})_{1,p'_2};\cdots;(a'_{(s-1)k};\alpha'^{(1)}_{(s-1)k},\alpha'^{(2)}_{(s-1)k},\cdots,\alpha'^{(s-1)}_{(s-1)k})_{1,p'_{s-1}}:$$

$$\left(-\alpha-\eta+\mu+\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1\right),\left(1-\upsilon-\sum_{j=1}^{v}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,1\right),$$

$$(a_{rk};\alpha_{rk}^{(1)},\alpha_{rk}^{(2)},\cdots,\alpha_{rk}^{(r)},0,\cdots,0,0)_{1,p_r},(a'_{sk};0,\cdots,0,\alpha'^{(1)}_{sk},\alpha'^{(2)}_{sk},\cdots,\alpha'^{(s)}_{sk},0)_{1,p'_s}:$$

$$(a_k^{(1)},\alpha_k^{(1)})_{1,p^{(1)}};\cdots;(a_k^{(r)},\alpha_k^{(r)})_{1,p^{(r)}};(a'^{(1)}_k,\alpha'^{(1)}_k)_{1,p'^{(1)}};\cdots;(a'^{(s)}_k,\alpha'^{(s)}_k)_{1,p'^{(s)}} \tag{3.22}$$

$$\mathbb{B}''=(b_{2k};\beta_{2k}^{(1)},\beta_{2k}^{(2)})_{1,q_2};\cdots;(b_{(r-1)k};\beta_{(r-1)k}^{(1)},\beta_{(r-1)k}^{(2)},\cdots,\beta_{(r-1)k}^{(r-1)})_{1,q_{r-1}};$$

$$(b'_{2k};\beta'^{(1)}_{2k},\beta'^{(2)}_{2k})_{1,q'_2};\cdots;(b'_{(s-1)k};\beta'^{(1)}_{(s-1)k},\beta'^{(2)}_{(s-1)k},\cdots,\beta'^{(s-1)}_{(s-1)k})_{1,q'_{s-1}}:$$

$$\left(1-\upsilon-\sum_{j=1}^{v}\delta_jK_j;\omega_1,\cdots,\omega_r,\omega'_1,\cdots,\omega'_s,0\right),\left(\mu-\alpha-\eta+\sum_{j=1}^{v}\lambda_jK_j;\sigma_1,\cdots,\sigma_r,\sigma'_1,\cdots,\sigma'_s,1\right),$$

$$(b_{rk};\beta^{(1)}_{rk},\beta^{(2)}_{rk},\cdots,\beta^{(r)}_{rk},0,\cdots,0,0)_{1,q_r};(b'_{sk};0,\cdots,0,\beta'^{(1)}_{sk},\beta'^{(2)}_{sk},\cdots,\beta'^{(s)}_{sk},0)_{1,q'_s}:$$

$$(b^{(1)}_k,\beta^{(1)}_k)_{1,q^{(1)}};\cdots;(b^{(r)}_k,\beta^{(r)}_k)_{1,q^{(r)}};(b'^{(1)}_k,\beta'^{(1)}_k)_{1,q'^{(1)}};\cdots;(b'^{(s)}_k,\beta'^{(s)}_k)_{1,q'^{(s)}};(0,1) \tag{3.23}$$

with the same validity conditions that (3.15).

Remark : If the multivariable I-functions reduce in to multivariable H-functions defined by Srivastava and Panda [27,28] and $S^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}_{N_1,\cdots,N_v}\rightarrow\prod_{k=1}^{v}S^{\mathfrak{M}_k}_{N_k}$, we obtain the result of Agarwal [1].

4. Particular cases

The generalized fractional integral operator Theorem 1 and Theorem 2 established here are unified in nature and act as key formulae. Thus the product of general class of polynomials involved in Images 1 and 2 reduces to a large spectrum of polynomials listed by Srivastava and Singh ([29], pp. 158–161), and so from Theorem1 and Theorem 2 we can further obtain various fractional integral results involving a number of simpler polynomials. Again, the multivariable I-function occurring in these Theorems can be suitably specialized to a remarkably wide variety of useful functions (or product of several such functions) which are expressible in terms of generalized Wright hypergeometric function, generalized Mittag–Leffler function and Bessel functions of one variable. For example

If we reduce the I_2-function in to the Fox H-functions in Theorem 1 and then reduce one I_1-function to the exponential function by taking $\sigma_1=1,\omega_1\rightarrow 0$, we get the following result after a little simplification which is believe to be new:

$$\left\{I^{\alpha,\beta,\eta}_{0^+}\left(t^{\mu-1}(b-at)^{\upsilon}\prod_{j=1}^{v}S^{M_j}_{N_j}\left[d_jt^{\lambda_j}(b-at)^{-\delta_j}\right]e^{-z_1t}H^{M_2,N_2}_{P_2,Q_2}\left[\begin{array}{c}(\mathrm{c}_i,\gamma_i)_{1,P_2}\\(\mathrm{d}_i,\delta_i)_{1,Q_2}\end{array}\middle|\ \mathrm{z}_2t^{\sigma_2}(b-at)^{-\omega_2}\right]\right\}(x)$$

$$=b^{-\upsilon}x^{\mu-\beta-1}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}a'_v\,d_1^{K_1}\cdots d_v^{K_v}b^{-\sum_{j=1}^{v}\delta_jK_j}x^{\sum_{j=1}^{v}\lambda_jK_j}$$

$$H^{0,3:1,0;M_2,N_2;1,0}_{3,3:0,1;P_2,Q_2;0,1}\left(\begin{array}{c}\mathrm{z}_1x\\ \mathrm{z}_2\frac{x^{\sigma_2}}{b^{\omega_2}}\\ -\frac{ax}{b}\end{array}\middle|\begin{array}{c}\mathrm{A}_2\\ \cdot\\ \mathrm{B}_2\end{array}\right) \tag{4.1}$$

where

$$A_2=(1-\upsilon-\sum_{j=1}^{v}\delta_iK_i;1,\omega_2,1),(1-\mu-\sum_{j=1}^{v}\lambda_iK_i;1,\sigma_2,1),(1-\mu-\eta+\beta-\sum_{j=1}^{v}\lambda_iK_i;1,\sigma_2,1),(c_i;\gamma_i)_{1,P_2} \tag{4.2}$$

$$B_2=(1-\upsilon-\sum_{j=1}^{v}\delta_iK_i;1,\omega_2,0),(1-\mu+\beta-\sum_{j=1}^{v}\lambda_iK_i;1,\sigma_2,1),(1-\mu-\eta-\alpha-\sum_{j=1}^{v}\lambda_iK_i;1,\sigma_2,1);(0,1);$$

$$(d_i,\delta_i)_{1,Q_2};(0,1) \tag{4.3}$$

where the corresponding validity conditions, see Agarwal [1].

If we reduce the H-function of one variable to generalized Wright hypergeometric function ([25], p.19, (2.6.11)) in the result given by (4.1), we get the following new and interesting result, see Agarwal [1]:

$$\left\{I_{0+}^{\alpha,\beta,\eta}\left(t^{\mu-1}(b-at)^{\upsilon}\prod_{j=1}^{v}S_{N_j}^{M_j}\left[d_j t^{\lambda_j}(b-at)^{-\delta_j}\right]e^{-z_1 t}{}_{P_2}\psi_{Q_2}\left[\begin{array}{c}(1\text{-c}_i),\gamma_i)_{1,P_2}\\(0,1),\ (1\text{-d}_i,\delta_i)_{1,Q_2}\end{array}\middle|\ z_2 t^{\sigma_2}(b-at)^{-\omega_2}\right]\right\}(x)$$

$$=b^{-\upsilon}x^{\mu-\beta-1}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}a_v'\,d_1^{K_1}\cdots d_v^{K_v}b^{-\sum_{j=1}^{v}\delta_j K_j}x^{\sum_{j=1}^{v}\lambda_j K_j}$$

$$H_{3,3:0,1;P_2,Q_2;0,1}^{0,3:1,0;1,N_2;1,0}\left(\begin{array}{c}z_1 x\\ z_2\frac{x^{\sigma_2}}{b^{\omega_2}}\\ -\frac{ax}{b}\end{array}\middle|\begin{array}{c}A_2\\ .\\ B_2\end{array}\right)\tag{4.4}$$

where the corresponding validity conditions.

If we take $z_2,\sigma_2=1$, and $\omega_2=0$ in (4.1) and reduce the H-function of one variable occurring therein to generalized Mittag–Laffler function (Prabhakar) ([18], p. 19, (2.6.11)), we easily get after little simplification the following new and interesting result:

$$\left\{I_{0+}^{\alpha,\beta,\eta}\left(t^{\mu-1}(b-at)^{\upsilon}\prod_{j=1}^{v}S_{N_j}^{M_j}\left[d_j t^{\lambda_j}(b-at)^{-\delta_j}\right]e^{-z_1 t}E_{M_2,N_2}^{\rho}[t]\left[\begin{array}{c}(1\text{-c}_i),\gamma_i)_{1,P_2}\\(0,1),\ (1\text{-d}_i,\delta_i)_{1,Q_2}\end{array}\middle|\ z_2 t^{\sigma_2}(b-at)^{-\omega_2}\right]\right.(x)$$

$$=b^{-\upsilon}x^{\mu-\beta-1}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}a_v'\,d_1^{K_1}\cdots d_v^{K_v}b^{-\sum_{j=1}^{v}\delta_j K_j}(-x)^{\sum_{j=1}^{v}\lambda_j K_j}$$

$$H_{3,3:0,1;1,3;0,1}^{0,3:1,0;1,1;1,0}\left(\begin{array}{c}z_1 x\\ z_2\frac{x^{\sigma_2}}{b^{\omega_2}}\\ -\frac{ax}{b}\end{array}\middle|\begin{array}{c}A_3\\ .\\ B_3\end{array}\right)\tag{4.5}$$

where

$$A_3=(1-\upsilon-\sum_{j=1}^{v}\delta_i K_i;1,0,1),(1-\mu-\sum_{j=1}^{v}\lambda_i K_i;1,1,1),(1-\mu-\eta+\beta-\sum_{j=1}^{v}\lambda_i K_i;1,1,1),(1-\rho,1)\tag{4.6}$$

$$B_3=(1-\upsilon-\sum_{j=1}^{v}\delta_i K_i;1,0,0),(1-\mu+\beta-\sum_{j=1}^{v}\lambda_i K_i;1,1,1),(1-\mu-\eta-\alpha-\sum_{j=1}^{v}\lambda_i K_i;1,1,1);(0,1);$$

$$(0,1),(1-\upsilon;0),(1-N_2;M_2);(0,1)\tag{4.7}$$

where the corresponding validity conditions, see Agarwal [1].

Remark :By similar procedure, the results of this document can be extented to product of any finite number of multivariable I-functions defined by Prasad [19] and a class of multivariable polynomials defined by Srivastava [26].

5. Conclusion

In this paper, we have obtained the Theorems of the generalized fractional integral operators given by Saigo. The images have been developed in terms of the product of the two multivariables I-function defined by Prasad and a general class of multivariable polynomials in a compact and elegant form with the help of Saigo operators. Most of the results obtained in this paper are useful in deriving certain composition formulas involving Riemann–Liouville, Erdelyi–Kober fractional calculus operators and multivariable I-functions. The findings of this paper provide an extension of the results given earlier by Kilbas, Kilbas and Saigo, Kilbas and Sebastain, Saxena et al. and Gupta et al. as mentioned earlier.

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