Int Jr. of Mathematical Sciences & Applications
Vol. 5, No. 2, (July-December, 2015)

ISSN No: 2230-9888
www.journalshub.com

On Intuitionistic $\tilde{g}^*$-extremally disconnectedness via lattice

G. Vasuki.
Associate Professor of Mathematics,
Sri Sarada College for Women (Autonomous), Salem, TamilNadu, India.
vasukigj@yahoo.in

T.Yogalakshmi.
Assistant Professor of Mathematics,
School of Advanced Sciences,
VIT University, Vellore, TamilNadu, India.
mathyoga14@gmail.com

Abstract: In this paper, the concept of intutionistic $\tilde{g}^*$-open set is introduced. Using lattice theory, characterization of an intutionistic $\tilde{g}^*$-extremally disconnected is established. Several properties and Tietz extension theorem on it are discussed.
2000 Mathematics Subject Classification. 54A10, 54G05
Keywords: intutionistic $\tilde{g}^*$-open set, intutionistic $\tilde{g}^*$-extremally disconnectedness.

1 Introduction:

Coker [1] introduced the fundamental concepts of intuitionistic sets. He also discussed the idea of intuitionistic topological space and investigated basic properties of continuous functions and compactness. He and his colleagues also examined separation axioms in intuitionistic topological spaces. Thomas Kubiak [4] introduced and studied about the properties of L-sets. The concepts of $\tilde{g}$-open set was introduced and discussed by Rajesh and Erdal Ekici [5]. L.Gillman, M.Jerison [2] studied the notion of extremally disconnected space.

In this paper, the notion of intutionistic $\tilde{g}^*$-open set is introduced. The concept of an intutionistic $\tilde{g}^*$-extremally disconnected L-space is introduced and several properties on it are discussed. Later, Tietz extension theorem is established, which is one of the most applicable one in digital space.

2 Preliminaries :

Definition: 2.1. [1] Let X be a non-empty set. An intuitionistic set (IS for short) A is an object having the form $A = \langle X, A^1, A^2\rangle$ where A^1 and A^2 are the subsets of X satisfying $A^1 \cap A^2 = \phi$. The set A^1 is called the set of members of A, while A^2 is called the set of non-members of A. Every crisp set A on a non-empty set X is obviously an IS having the form $\langle X, A, A^c\rangle$.

Definition: 2.2. [1] Let X be a non-empty set. Let $A = \langle X, A^1, A^2\rangle$ and $B = \langle X, B^1, B^2\rangle$ be an intuitionistic sets on X. Let $\{A_i = \langle X, A_i^1, A_i^2\rangle : i \in J\}$ be an arbitrary family of intuitionistic sets in X. Then,

(i) $A \subseteq B \Leftrightarrow A^1 \subseteq B^1$ and $A^2 \supseteq B^2$.

(ii) $A = B \Leftrightarrow A \subseteq B$ and $B \supseteq A$.

(iii) $A^c = \bar{A} = \langle X, A^2, A^1 \rangle$ if $\langle X, A^1, A^2 \rangle$.

(iv) $\cap A_i = \langle X, \cap A_i^1, \cup A_i^2 \rangle$; $\cup A_i = \langle X, \cup A_i^1, \cap A_i^2 \rangle$

(v) $\phi_\sim = \langle X, \phi, X \rangle$; $X_\sim = \langle X, X, \phi \rangle$.

Definition: 2.3. [1] Let X and Y be a non-empty set and $f : X \to Y$ be a function. Let $A = \langle X, A^1, A^2 \rangle$ and $B = \langle Y, B^1, B^2 \rangle$ be the ISs of X and Y respectively. Then, the pre-image $f^{-1}(B)$ is an IS in X is defined by $f^{-1}(B) = \langle X, f^{-1}(B^1), f^{-1}(B^2) \rangle$ and the image $f(A)$ is an IS in Y is defined by $f(A) = \langle Y, f(A^1), f(A^2) \rangle$ where $f(A^2) = (f((A^2)^c))^c$.

Definition: 2.4. [1] An intuitionistic topology (IT for short) on a non-empty set X is a family τ of ISs in X satisfying the following axioms:

(i) $\phi_\sim, X_\sim \in \tau$.

(ii) $A_1 \cap A_2 \in \tau$, for any $A_1, A_2 \in \tau$.

(iii) $\cup_i A_i \in \tau$, for any $\{A_i : i \in J\} \subseteq \tau$. Now the ordered pair (X, τ) is called an intuitionistic topological space (ITS for short) and any intuitionistic set in τ is called an intuitionistic open set (IOS for short) in X. The complement $\bar{A}$ of an IOS, A is called as an intuitionistic closed set (ICS, for short) in X.

Definition: 2.5. [1] Let (X, τ) be an ITS and $A = \langle X, A^1, A^2 \rangle$ be an IS in X. Then the interior and closure of A are defined by $cl(A) = \cap\{K : K \text{ is an } ICS \text{ in } X \text{ and } A \subseteq K\}$ and $int(A) = \cup\{G : G \text{ is an } IOS \text{ in } X \text{ and } G \subseteq A\}$.

Proposition: 2.1. *For any intuitionistic set A in (X, τ), the following properties hold:*

(i) $cl(\bar{A}) = \overline{int(A)}$.

(ii) $int(\bar{A}) = \overline{cl(A)}$.

Definition: 2.6. [2] A topological space (X, τ) is said to be an extremally disconnected space if the closure of any open set is open.

Definition: 2.7. [5] Let (X, τ) be any topological space. Let A and B be any two sets in X. Then, A is said to be

(i) $\hat{g}$-closed set if $Cl(A) \subseteq B$ whenever $A \subseteq B$ and B is a semi-open. The complement of $\hat{g}$-closed set is said to be $\hat{g}$-open set.

(ii) $g*$-closed if $cl(A) \subseteq B$ whenever $A \subseteq B$ and B is a $\hat{g}$-open. The complement of $g*$-closed is said to be $g*$-open.

(iii) $g^\sharp$ semi-closed if $scl(A) \subseteq B$ whenever $A \subseteq B$ and B is an $g*$-open. The complement of $g^\sharp$ semi-closed set is said to be $g^\sharp$ semi-open.

(iv) $\tilde{g}$-closed if $cl(A) \subseteq B$ whenever $A \subseteq B$ and B is an $g^\sharp$ semi-open. The complement of $\tilde{g}$-closed set is said to be $\tilde{g}$-open.

Definition: 2.8. Let X be any non-empty crisp set. Let A be any subset of X and χ_A: $X \to \{1, 0\}$. Then, the characteristic function of A, χ_A is defined as

$$\chi_A(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \notin A \end{cases} \quad , \quad \text{for all } x \in X$$

3 On intuitionistic $\tilde{g}^*$-open set :

Throughout this paper $\langle L, \cup, \cap, ' \rangle$ is an infinitely distributive lattice with an order-reversing involution. Such a lattice being complete has a least element 0 and greatest element 1.

Definition: 3.1. Let (X, τ) be an ITS. Let $A = \langle X, A^1, A^2 \rangle$ and $B = \langle X, B^1, B^2 \rangle$ be the intuitionistic sets in X. Then, A is said to be

(i) $I\hat{g}$-closed set if $Cl(A) \subseteq B$ whenever $A \subseteq B$ and B is an intuitionistic semi-open. The complement of $I\hat{g}$-closed set is said to be $I\hat{g}$-open set.

(ii) $Ig*$-closed if $cl(A) \subseteq B$ whenever $A \subseteq B$ and B is an $I\hat{g}$-open. The complement of $Ig*$-closed is said to be $Ig*$-open.

(iii) $Ig^\sharp$ semi-closed if $scl(A) \subseteq B$ whenever $A \subseteq B$ and B is an $Ig*$-open. The complement of $Ig^\sharp$ semi-closed set is said to be $Ig^\sharp$ semi-open.

(iv) $I\tilde{g}$-closed if $cl(A) \subseteq B$ whenever $A \subseteq B$ and B is an $Ig^\sharp$ semi-open. The complement of $I\tilde{g}$-closed set is said to be $I\tilde{g}$-open.

Definition: 3.2. Let (X, τ) be an ITS. Let $A = \langle X, A^1, A^2 \rangle$, $B = \langle X, B^1, B^2 \rangle$ and $C = \langle X, C^1, C^2 \rangle$ be the intuitionistic sets in X. Then, A is said to be an $I\tilde{g}^*$-open set (briefly $I\tilde{g}^*OS$) if $A = B \cap C$ where B is an $I\tilde{g}$-open set and C is an IOS.

Definition: 3.3. Let (X, τ) be an ITS. Let $A = \langle X, A^1, A^2 \rangle$ be an intuitionistic set in X. Then, the intuitionistic $\tilde{g}^*$-interior and intuitionistic $\tilde{g}^*$-closure of A are defined as $I\tilde{g}^*int(A)$=$\cup\{G : G$ is an $I\tilde{g}^*OS$ in X and $G \subseteq A\}$ and $I\tilde{g}^*cl(A)$=$\cap\{K : K$ is an $I\tilde{g}^*CS$ in X and $A \subseteq K\}$ respectively.

Remark: 3.1. *Let (X, τ) be an ITS. Let $A = \langle X, A^1, A^2 \rangle$ be an intuitionistic set in X. Then, the following statements are hold.*

(i) $I\tilde{g}^*int(A) \subseteq A \subseteq I\tilde{g}^*cl(A)$.

(ii) $I\tilde{g}^*int(\bar{A}) = \overline{I\tilde{g}^*cl(A)}$.

(iii) $I\tilde{g}^*cl(\bar{A}) = \overline{I\tilde{g}^*int(A)}$.

Definition: 3.4. Let $L^{\mathbb{R}}$ be the lattice of all L-sets on $\mathbb{R}$. Then, the lattice of L-real functions is defined as the lattice of all monotone decreasing element $\lambda \in \tilde{L}^{\mathbb{R}}$ such that $\cap\{\lambda(t) : t \in \mathbb{R}\} = 0$ and $\cup\{\lambda(t) : t \in \mathbb{R}\} = 1$ and is denoted as $\mathbb{R}_L$. For each $\lambda \in \mathbb{R}_L$ and $t \in \mathbb{R}$, we define

$$\lambda^+(t) = \lambda(t+) = \cup_{s>t} \lambda(s).$$
$$\lambda^-(t) = \lambda(t-) = \cap_{s>t} \lambda(s).$$

Clearly both + and - are order preserving self maps.

Definition: 3.5. For each $\lambda, \mu \in \mathbb{R}_L$, let $\lambda \sim \mu \Leftrightarrow \lambda^- = \mu^-$. Then, $\mathbb{R}_L | \sim$ is called the L-real line and it is denoted by $\mathbb{R}(L)$.

Definition: 3.6. For each $t \in \mathbb{R}$, let $L_t, R_t : \mathbb{R}(L) \to L$ be given by $L_t[\lambda] = \lambda(t-)$ and $R_t[\lambda] = \lambda(t+)$. Define $\mathcal{L} = \{L_t : t \in \mathbb{R}\} \cup \{0, 1\}$ and $\mathcal{R} = \{R_t : t \in \mathbb{R}\} \cup \{0, 1\}$. Then $\mathcal{L}$ and $\mathcal{R}$ are the topologies on $\mathbb{R}(L)$.

Definition: 3.7. A partial order on $\mathbb{R}_L$ is defined by $[\lambda] \leq [\mu] \Leftrightarrow \lambda(t-) \leq \mu(t-)$ and $\lambda(t+) \leq \mu(t+)$, for all $t \in \mathbb{R}$.

4 On intuitionistic $\tilde{g}^*$-extremally disconnectedness:

Definition: 4.1. Let (X, τ) be an intuitionistic topological space. If the intuitionistic $\tilde{g}^*$-closure of an intuitionistic open set is an intuitionistic $\tilde{g}^*$-open set, then (X, τ) is said to be an intuitionistic $\tilde{g}^*$-extremally disconnected space.

Proposition: 4.1. *Let (X, τ) be an intuitionistic topological space. Then, the following are equivalent:*

(a) (X, τ) is an intuitionistic $\tilde{g}^*$-extremally disconnected space.

(b) For each intuitionistic closed set A, $I\tilde{g}^*int(A)$ is an $I\tilde{g}^*CS$.

(c) For each intuitionistic open set A, we have $I\tilde{g}^*int(I\tilde{g}^*cl(A)) = I\tilde{g}^*cl(A)$.

(d) For each intuitionistic open set A and $I\tilde{g}^*$-open set B with $I\tilde{g}^*int(\bar{A}) = B$, we have $\overline{I\tilde{g}^*cl(A)} = I\tilde{g}^*cl(B)$.

Proof. $(a) \Rightarrow (b)$: Let A be an ICS. Now, $\bar{A}$ is an IOS. By (a), $I\tilde{g}^*cl(\bar{A})$ is an $I\tilde{g}^*OS$. Clearly, $I\tilde{g}^*cl(\bar{(A)}) = \overline{I\tilde{g}^*}$. This implies that, $I\tilde{g}^*int(A)$ is an $I\tilde{g}^*CS$.
$(b) \Rightarrow (c)$: Let A be an intuitionistic open set. Then, $\bar{A}$ is an intuitionistic closed set. By (b), $I\tilde{g}^*int(\bar{A})$ is an $I\tilde{g}^*CS$. Now, $I\tilde{g}^*int(I\tilde{g}^*cl(A)) = I\tilde{g}^*cl(I\tilde{g}^*int(\bar{A})) = I\tilde{g}^*int(\bar{A}) = \overline{I\tilde{g}^*cl(A)}$
$(c) \Rightarrow (d)$: Let A be an IOS and B be an $I\tilde{g}^*OS$ with $I\tilde{g}^*int(\bar{A}) = B$. By (c), we have $I\tilde{g}^*int(I\tilde{g}^*cl(A)) = I\tilde{g}^*cl(A)$. Now, $\overline{I\tilde{g}^*int(I\tilde{g}^*cl(A))} = \overline{I\tilde{g}^*cl(A)} = I\tilde{g}^*int(\bar{A}) = B$ This implies that, $I\tilde{g}cl(I\tilde{g}int(\bar{A})) = B$. It follows that, $I\tilde{g}^*cl(B) = B$. Therefore, $\overline{I\tilde{g}^*cl(A)} = I\tilde{g}^*int(\bar{A}) = B = I\tilde{g}^*cl(B)$.
$(d) \Rightarrow (a)$: Let A be any IOS and B be any $I\tilde{g}^*OS$ with $I\tilde{g}^*int(\bar{A}) = B$. By (d), $\overline{I\tilde{g}^*cl(A)} = I\tilde{g}^*cl(B)$. This implies that, $I\tilde{g}cl(A) = \overline{I\tilde{g}^*cl(B)}$. It follows that, $I\tilde{g}^*cl(A)$ is an $I\tilde{g}^*OS$. Hence, (X, τ) is an $I\tilde{g}^*$-extremally disconnected space. ☐

Proposition: 4.2. *Let (X, τ) be an ITS. Then, (X, τ) is an $I\tilde{g}^*$-extremally disconnected space iff for each $I\tilde{g}^*$-open set, A and $I\tilde{g}^*$-closed set, B such that $A \subseteq B$, we have $I\tilde{g}^*cl(A) \subseteq I\tilde{g}^*int(B)$.*

Definition: 4.2. An intuitionistic set which is both $I\tilde{g}^*$-open and $I\tilde{g}^*$-closed set is called as an $I\tilde{g}^*$-clopen set, denoted as $\mathbf{I\tilde{g}^*COS}$.

Remark: 4.1. *Let (X, τ) be an $I\tilde{g}^*$-extremally disconnected space. Let $\{A_i, B_j : i, j \in \mathbb{N}\}$ be a collection such that each A_i's are $I\tilde{g}^*OS$ and each B_j's are $S\tilde{L}Fc$-limCS. Let A, B be the $I\tilde{g}^*OS$ and intuitionistic closed set respectively. If $A_i \subseteq A \subseteq B_j$, for all $i, j \in \mathbb{N}$, then there exists an $I\tilde{g}^*COS$, $\mathfrak{M}$ such that $I\tilde{g}^*cl(A_i) \subseteq \mathfrak{M} \subseteq I\tilde{g}^*int(B_j)$, for all $i, j \in \mathbb{N}$.*

Proof. By the Proposition: 4.2, the proof is obvious. ☐

Proposition: 4.3. *Let (X, τ) be an $I\tilde{g}^*$-extremally disconnected space. Let $\{A_q\}_{q\in\mathbb{Q}}$ and $\{B_q\}_{q\in\mathbb{Q}}$ be the monotone increasing collections of $I\tilde{g}^*$-open sets and $I\tilde{g}^*$-closed sets of (X, τ) respectively. ($\mathbb{Q}$ is the set of all rational numbers). If $A_{q_1} \subseteq B_{q_2}$, whenever $q_1 < q_2$ ($q_1, q_2 \in \mathbb{Q}$), then there exists a monotone increasing collection $\{C_q\}_{q\in\mathbb{Q}}$ of $I\tilde{g}^*$ clopen sets of (X, τ) such that $I\tilde{g}^*cl(A_{q_1}) \subseteq C_{q_2}$ and $C_{q_1} \subseteq I\tilde{g}^*int(B_{q_2})$ whenever $q_1 < q_2$.*

Proof. Let us arrange into a sequence $\{q_n\}$ of rational numbers without repetitions. For every $n \geq 2$, define inductively a collection $\{C_{q_i} : 1 \leq i < n\}$ such that

$$\left.\begin{array}{ll} I\tilde{g}^*cl(A_q) \subseteq C_{q_i}, & \text{if } q < q_i \\ C_{q_i} \subseteq I\tilde{g}^*int(B_q), & \text{if } q_i < q \end{array}\right\} \dashrightarrow (S_n) \quad , \quad \text{for all } i < n.$$

By Proposition: 4.2, the countable collections $\{I\tilde{g}^*cl(A_q)\}_{q\in\mathbb{Q}}$ and $\{I\tilde{g}^*int(B_q)\}_{q\in\mathbb{Q}}$ satisfying $I\tilde{g}^*cl(A_{q_1}) \subseteq I\tilde{g}^*int(B_{q_2})$, if $q_1 < q_2$ ($q_1, q_2 \in \mathbb{Q}$). By the Remark: 4.1, there exists $I\tilde{g}^*$-clopen set, M such that $I\tilde{g}^*cl(A_{q_1}) \subseteq M \subseteq I\tilde{g}^*int(B_{q_2})$. By setting $H_{q_1} = M$, we get (S_2).

Assume that the intuitionistic sets H_{q_i} (already defined), for $i < n$ and satisfy (S_n). Define

$$\Phi = \cup\{H_{q_i} : i < n, q_i < q_n\} \cup A_{q_n}$$
$$\Omega = \cap\{H_{q_j} : j < n, q_j > q_n\} \cap B_{q_n}$$

Then, we have, $I\tilde{g}^*cl(H_{q_i}) \subseteq I\tilde{g}^*cl(\Phi) \subseteq I\tilde{g}^*int(H_{q_j})$ and $I\tilde{g}^*cl(H_{q_i}) \subseteq I\tilde{g}^*int(\Omega) \subseteq I\tilde{g}^*int(H_{q_j})$, whenever $q_i < q_n < q_j$ $(i, j < n)$, as well as $A_q \subseteq I\tilde{g}^*cl(\Phi) \subseteq B_{q'}$ and $A_q \subseteq I\tilde{g}^*int(\Omega) \subseteq B_{q'}$, whenever $q < q_n < q'$. This shows that the countable collections $\{H_{q_i} : i < n, q_i < q_n\} \bigcup \{A_q : q < q_n\}$ and $\{H_{q_j} : j < n, q_j > q_n\} \bigcup \{B_{q'} : q' > q_n\}$ together with Φ and Ω, fulfil all the conditions of the Remark: 4.1. Hence, there exists an $I\tilde{g}^*$-clopen set, M_n such that $I\tilde{g}^*cl(M_n) \subseteq B_q$ if $q_n < q$, and $A_q \subseteq I\tilde{g}^*int(M_n)$ if $q < q_n$. Also, $I\tilde{g}^*cl(H_{q_i}) \subseteq I\tilde{g}^*int(M_n)$ if $q_i < q_n$ and $I\tilde{g}^*cl(M_n) \subseteq I\tilde{g}^*int(H_{q_j})$ if $q_n < q_j$, where $1 \leq i, j \leq n-1$. Now setting $H_{q_n} = M_n$, we obtain the intuitionistic sets H_{q_1},H_{q_2},....H_{q_n} that satisfy (S_{n+1}). Therefore, the collection $\{H_{q_i} : i = 1, 2, 3, ...\}$ has the required property. This completes the proof. □

5 Tietz Extension Theorem :

Definition: 5.1. Let (X, τ) and (Y, σ) be any two intuitionistic topological spaces. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be an **intuitionistic $\tilde{g}^*$-continuous function** if the inverse image of an intuitionistic $\tilde{g}^*$-open set of (Y, σ) is an intuitionistic $\tilde{g}^*$-open set in (X, τ)

Definition: 5.2. Let (X, τ) be an intuitionistic topological space. A function $f : X \to \mathbb{R}(I)$ is called as the lower (upper) intuitionistic $\tilde{g}^*$-continuous function, if $f^{-1}R_t$ (respy. $f^{-1}L_t$) is an intuitionistic $\tilde{g}^*$-open set (intuitionistic $\tilde{g}^*$-closed set), for each $t \in \mathbb{R}$.

Proposition: 5.1. *Let (X, τ) be an intuitionistic topological space. Then the following statements are equivalent:*

(a) (X, τ) is an $I\tilde{g}^*$-extremally disconnected space.

(b) Let $g, h : X \to \mathbb{R}(L)$. If g is a lower $I\tilde{g}^*$-continuous function and h is a $I\tilde{g}^*$-continuous function with $g \leq h$, then there exists an intuitionistic $\tilde{g}^*$-continuous function, $f : X \to \mathbb{R}(L)$ such that $g \leq f \leq h$.

(c) Let $K', U \in \tau$ and $U \subseteq K$, then there exists an intuitionistic $\tilde{g}^*$-continuous function, $f : X \to \mathbb{R}(L)$ such that $U \subseteq f^{-1}(\bar{L}_t) \subseteq f^{-1}(R_0) \subseteq f^{-1}(\bar{L}_t) \subseteq K$.

Proof. $(a) \Rightarrow (b)$: Define $H_r = h^{-1}(L_r)$ and $G_r = g^{-1}(\bar{R}_r)$, $r \in \mathbb{Q}$. Then, H_r and G_r are the two monotone increasing families of $I\tilde{g}^*$-open and $I\tilde{g}^*$-closed sets of (X, τ). Moreover, $H_r \subseteq G_s$, if $r < s$. By Proposition: 4.3, there exists a monotone increasing family $\{F_r\}_{r\in\mathbb{Q}}$ of $I\tilde{g}^*$-clopen sets of (X, τ) such that $I\tilde{g}^*cl(H_r) \subseteq F_s$ and $F_r \subseteq I\tilde{g}^*int(G_s)$, whenever $r < s$. Let us define a monotone decreasing family $\{U_t : t \in \mathbb{R}\}$ of $I\tilde{g}^*$-clopen sets of (X, τ) such that $U_t = \cap_{r<t}\bar{F}_r$, for all $t \in \mathbb{R}$. Moreover, we have $I\tilde{g}^*cl(U_t) \subseteq I\tilde{g}^*int(U_s)$, whenever $s < t$. Now, $\cup_{t\in\mathbb{R}}U_t = \cup_{t\in\mathbb{R}} \cap_{r<t} \bar{F}_r \supseteq \cup_{t\in\mathbb{R}} \cap_{r<t} \bar{G}_r = \cup_{t\in\mathbb{R}} \cap_{r<t} g^{-1}(R_r) = \cup_{t\in\mathbb{R}}g^{-1}(R_t)$ $= g^{-1}(\cup_{t\in\mathbb{R}}R_t) = X$ Similarly, $\cap_{t\in\mathbb{R}}U_t = X$ Define a function $f : X \to \mathbb{R}(L)$ possessing the required properties. Let $f(x)(t) = \chi_{U_t}(x)$, for all $x \in X, t \in \mathbb{R}$. By the above discussion, it follows that f is well defined . Observe that, $\cup_{s>t}U_s = \cup_{s>t}I\tilde{g}^*int(U_s)$ and $\cap_{s<t}U_s = \cap_{s<t}I\tilde{g}^*cl(U_s)$ Then, $f^{-1}(R_t) = \cup_{s>t}U_s$ $= \cup_{s>t}I\tilde{g}^*int(U_s)$ is an $I\tilde{g}^*OS$ and $f^{-1}(\bar{L}_t) = \cap_{s<t}U_s = \cap_{s<t}I\tilde{g}^*cl(U_s)$ is an $I\tilde{g}^*CS$. Therefore, f is an intuitionistic $\tilde{g}^*$-continuous function. Now, for each $t \in \mathbb{R}$, we have

$$\begin{aligned} g^{-1}(\bar{L}_t) &= \cap_{s<t}g^{-1}(\bar{L}_s) \\ &= \cap_{s<t} \cap_{r<s} g^{-1}(R_r) \\ &= \cap_{s<t} \cap_{r<s} \bar{G}_r \\ &\subseteq \cap_{s<t} \cap_{r<s} \bar{F}_r \\ &= \cap_{s<t}U_s \\ &= f^{-1}(\bar{L}_t) \end{aligned}$$

Now,

$$\begin{aligned} f^{-1}(\bar{L_t}) &= \cap_{s<t} U_s \\ &= \cap_{s<t} \cap_{r<s} (\bar{F_r}) \\ &\subseteq \cap_{s<t} \cap_{r<s} (\bar{H_r}) \\ &= \cap_{s<t} \cap_{r<s} h^{-1}(\bar{L_r}) \\ &= \cap_{s<t} h^{-1}(\bar{L_s}) \\ &= h^{-1}(\bar{L_t}) \end{aligned}$$

Similarly, we obtain $g^{-1}(R_t) \subseteq f^{-1}(R_t) \subseteq h^{-1}(R_t)$. Thus, (b) is proved.
$(b) \Rightarrow (c)$: Suppose that F is an $I\tilde{g}^*$-closed set and G is an $I\tilde{g}^*$-open set such that $G \subseteq F$. Then χ_G and χ_F are the lower and upper $I\tilde{g}^*$-continuous functions with $\chi_G \leq \chi_F$ respectively. Hence, by (b), there exists an $I\tilde{g}^*$-continuous function, $f : X \to \mathbb{I}(L)$ such that $\chi_G \subseteq f \subseteq \chi_F$. Clearly, $f(x) \in \tilde{L}^X$, for all $x \in X$ and $G = \chi_G^{-1}(\bar{L_1}) \subseteq f^{-1}(\bar{L_1}) = f^{-1}(R_0) \subseteq \chi_F^{-1}(\bar{R_0}) \subseteq F$. Therefore, $G \subseteq f^{-1}(\bar{L_1}) \subseteq f^{-1}(R_0) \subseteq F$.
$(c) \Rightarrow (a)$: Let F be an $I\tilde{g}^*CS$ and G be an $I\tilde{g}^*OS$ such that $G \subseteq F$. Then, by hypothesis there exists an intuitionistic $\tilde{g}^*$-continuous function, $f : X \to I(L)$ such that $f^{-1}(\bar{L_1}) \subseteq f^{-1}(R_0)$. In fact, $\bar{L_1}$ is an $I\tilde{g}^*CS$ and R_0 is an $I\tilde{g}^*OS$. Since $G \subseteq f^{-1}(\bar{L_1}) \subseteq f^{-1}(R_0) \subseteq F$, it follows that, $I\tilde{g}^*cl(G) \subseteq I\tilde{g}^*cl(f^{-1}(\bar{L_1})) = f^{-1}(\bar{L_1})$. Similarly, $f^{-1}(R_0) = I\tilde{g}^*int(f^{-1}(R_0)) \subseteq I\tilde{g}^*int(F)$. This implies that, $I\tilde{g}^*cl(G) \subseteq I\tilde{g}^*int(F)$. By the Proposition: 4.2, (X, τ) is an $I\tilde{g}^*$-extremally disconnected space. □

Proposition: 5.2. *Let (X, τ) be an $I\tilde{g}^*$-extremally disconnected space. Let $A \subseteq X$ be an $I\tilde{g}^*COS$ and let $f : (A, \tau_A) \to [0, 1](L)$ be an $I\tilde{g}^*$-continuous function. Then, f has a continuous extension over X.*

Proof. Let $g, h : X \to [0, 1](L)$ be such that $g = f = h$ on A and $g(x) = 0$, if $x \notin A$ and $h(x) = 1$, if $x \notin A$. We have,

$$g^{-1}(R_t) = \begin{cases} G_t \cap A, & \text{if } t \geq 0 \\ X, & \text{if } t < 0 \end{cases} \quad , \quad \text{for all } t \in [0, 1]$$

where, H_t is an $I\tilde{g}^*OS$.
and,

$$h^{-1}(L_t) = \begin{cases} H_t \cap A, & \text{if } t \leq 1 \\ \phi, & \text{if } t > 1 \end{cases} \quad , \quad \text{for all } t \in [0, 1]$$

where, H_t is an $I\tilde{g}^*CS$. Thus, g is a lower intuitionistic $\tilde{g}^*$-continuous function and h is a upper intuitionistic $\tilde{g}^*$-continuous function with $g \leq h$. Now, by the Proposition: 5.1, there exists an $I\tilde{g}^*$-continuous function, $\mathcal{F} : X \to [0, 1](L)$ such that $g \leq \mathcal{F} \leq h$. Hence, $\mathcal{F} \equiv f$ on A. □

References

[1] Dogan Coker, A note on intuitionistic sets and intuitionistic points, *Turkish J. Math.*, **20** (1996), 343-351.

[2] L.Gillman, M.Jerison, Rings of continuous functions, *Univ.series in Higher Math., Van Nostrand, Princeton, NewYork,* 1960.

[3] S.Jafari, T.Noiri, N.Rajesh and M.L. Thivagar, Another generalization of closed sets, *Kochi Journal of Mathematics,* **III** (2008).

[4] Thomas Kubiak, Extending continuous L-Real functions, *Math. Japonica*, **31,6** (1986), 875-887.

[5] N.Rajesh, Erdal Ekici, $\tilde{g}$-locally closed sets in topological spaces, *Kochi. J. Math.*, **2** (2007), 1-9.