Int. Jr. of Mathematical Sciences & Applications
Vol. 8, No. 2, (July-December, 2018)

ISSN No: 2230-9888
www.ijmsa.yolasite.com

A new class of three-dimensional Fourier series of generalized multivariable Gimel-function

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ABSTRACT
In this paper, we present a new class of six three-dimensional Fourier series for the generalized multivariable Gimel-function.

KEYWORDS : Multivariable Gimel-function, multiple integral contours, three-dimensional Fourier series.

2010 Mathematics Subject Classification. 33C99, 33C60, 44A20

1. Introduction and preliminaries.

The object of this paper is to introduce a new class of three-dimensional Fourier series for generalized multivariable Gimel-function and present six Fourier series of the class.

Throughout this paper, let $\mathbb{C},\mathbb{R}$ and $\mathbb{N}$ be set of complex numbers, real numbers and positive integers respectively. Also $\mathbb{N}_0=\mathbb{N}\cup\{0\}$.

The subject of expansion formulae and Fourier series of special functions occupies a large place in the literature of special functions. Certain triple expansion formulae and triple Fourier series of generalized hypergeometric functions play an important rôle in the development of the theories of special functions. In this paper, we establish height triple expansion formulae for generalized multivariable Gimel-function.

We define a generalized transcendental function of several complex variables.

$$\gimel(z_1,\cdots,z_r)=\gimel^{m_2,n_2;m_3,n_3;\cdots;m_r,n_r:m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)}}_{p_{i_2},q_{i_2},\tau_{i_2};R_2;p_{i_3},q_{i_3},\tau_{i_3};R_3;\cdots;p_{i_r},q_{i_r},\tau_{i_r}:R_r:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c|}\mathrm{z}_1\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\end{array}\right.$$

$$[(\mathrm{a}_{2j};\alpha^{(1)}_{2j},\alpha^{(2)}_{2j};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha^{(1)}_{2ji_2},\alpha^{(2)}_{2ji_2};A_{2ji_2})]_{n_2+1,p_{i_2}},[(a_{3j};\alpha^{(1)}_{3j},\alpha^{(2)}_{3j},\alpha^{(3)}_{3j};A_{3j})]_{1,n_3},$$
$$[(\mathrm{b}_{2j};\beta^{(1)}_{2j},\beta^{(2)}_{2j};B_{2j})]_{1,m_2},[\tau_{i_2}(b_{2ji_2};\beta^{(1)}_{2ji_2},\beta^{(2)}_{2ji_2};B_{2ji_2})]_{m_2+1,q_{i_2}},[(b_{3j};\beta^{(1)}_{3j},\beta^{(2)}_{3j},\beta^{(3)}_{3j};B_{3j})]_{1,m_3},$$

$$[\tau_{i_3}(a_{3ji_3};\alpha^{(1)}_{3ji_3},\alpha^{(2)}_{3ji_3},\alpha^{(3)}_{3ji_3};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{rj};\alpha^{(1)}_{rj},\cdots,\alpha^{(r)}_{rj};A_{rj})_{1,n_r}],$$
$$[\tau_{i_3}(b_{3ji_3};\beta^{(1)}_{3ji_3},\beta^{(2)}_{3ji_3},\beta^{(3)}_{3ji_3};B_{3ji_3})]_{m_3+1,q_{i_3}};\cdots;[(\mathrm{b}_{rj};\beta^{(1)}_{rj},\cdots,\beta^{(r)}_{rj};B_{rj})_{1,m_r}],$$

$$[\tau_{i_r}(a_{rji_r};\alpha^{(1)}_{rji_r},\cdots,\alpha^{(r)}_{rji_r};A_{rji_r})_{n_r+1,p_r}]:\ [(\mathrm{c}^{(1)}_j,\gamma^{(1)}_j;C^{(1)}_j)_{1,n^{(1)}}],[\tau_{i^{(1)}}(c^{(1)}_{ji^{(1)}},\gamma^{(1)}_{ji^{(1)}};C^{(1)}_{ji^{(1)}})_{n^{(1)}+1,p^{(1)}_i}]$$
$$[\tau_{i_r}(b_{rji_r};\beta^{(1)}_{rji_r},\cdots,\beta^{(r)}_{rji_r};B_{rji_r})_{m_r+1,q_r}]:[(\mathrm{d}^{(1)}_j),\delta^{(1)}_j;D^{(1)}_j)_{1,m^{(1)}}],[\tau_{i^{(1)}}(d^{(1)}_{ji^{(1)}},\delta^{(1)}_{ji^{(1)}};D^{(1)}_{ji^{(1)}})_{m^{(1)}+1,q^{(1)}_i}]$$

$$\left.\begin{array}{l};\cdots;[(c^{(r)}_j,\gamma^{(r)}_j;C^{(r)}_j)_{1,n^{(r)}}],[\tau_{i^{(r)}}(c^{(r)}_{ji^{(r)}},\gamma^{(r)}_{ji^{(r)}};C^{(r)}_j)_{n^{(r)}+1,p^{(r)}_i}]\\ ;\cdots;[(d^{(r)}_j,\delta^{(r)}_j;D^{(r)}_j)_{1,m^{(r)}}],[\tau_{i^{(r)}}(d^{(r)}_{ji^{(r)}},\delta^{(r)}_{ji^{(r)}};D^{(r)}_j)_{m^{(r)}+1,q^{(r)}_i}]\end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega = \sqrt{-1}$

$$\psi(s_1,\cdots,s_r) = \frac{\prod_{j=1}^{m_2}\Gamma^{B_{2j}}(b_{2j}-\sum_{k=1}^{2}\beta_{2j}^{(k)}s_k)\prod_{j=1}^{n_2}\Gamma^{A_{2j}}(1-a_{2j}+\sum_{k=1}^{2}\alpha_{2j}^{(k)}s_k)}{\sum_{i_2=1}^{R_2}[\tau_{i_2}\prod_{j=n_2+1}^{p_{i_2}}\Gamma^{A_{2ji_2}}(a_{2ji_2}-\sum_{k=1}^{2}\alpha_{2ji_2}^{(k)}s_k)\prod_{j=m_2+1}^{q_{i_2}}\Gamma^{B_{2ji_2}}(1-b_{2ji2}+\sum_{k=1}^{2}\beta_{2ji2}^{(k)}s_k)]}$$

$$\frac{\prod_{j=1}^{m_3}\Gamma^{B_{3j}}(b_{3j}-\sum_{k=1}^{3}\beta_{3j}^{(k)}s_k)\prod_{j=1}^{n_3}\Gamma^{A_{3j}}(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k)}{\sum_{i_3=1}^{R_3}[\tau_{i_3}\prod_{j=n_3+1}^{p_{i_3}}\Gamma^{A_{3ji_3}}(a_{3ji_3}-\sum_{k=1}^{3}\alpha_{3ji_3}^{(k)}s_k)\prod_{j=m_3+1}^{q_{i_3}}\Gamma^{B_{3ji_3}}(1-b_{3ji3}+\sum_{k=1}^{3}\beta_{3ji3}^{(k)}s_k)]}$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\frac{\prod_{j=1}^{m_r}\Gamma^{B_{rj}}(b_{rj}-\sum_{k=1}^{r}\beta_{rj}^{(k)}s_k)\prod_{j=1}^{n_r}\Gamma^{A_{rj}}(1-a_{rj}+\sum_{k=1}^{r}\alpha_{rj}^{(k)}s_k)}{\sum_{i_r=1}^{R_r}[\tau_{i_r}\prod_{j=n_r+1}^{p_{i_r}}\Gamma^{A_{rji_r}}(a_{rji_r}-\sum_{k=1}^{r}\alpha_{rji_r}^{(k)}s_k)\prod_{j=m_r+1}^{q_{i_r}}\Gamma^{B_{rji_r}}(1-b_{rjir}+\sum_{k=1}^{r}\beta_{rjir}^{(k)}s_k)]} \tag{1.2}$$

and

$$\theta_k(s_k) = \frac{\prod_{j=1}^{m^{(k)}}\Gamma^{D_j^{(k)}}(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n^{(k)}}\Gamma^{C_j^{(k)}}(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m^{(k)}+1}^{q_{i^{(k)}}}\Gamma^{D_{ji^{(k)}}^{(k)}}(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n^{(k)}+1}^{p_{i^{(k)}}}\Gamma^{C_{ji^{(k)}}^{(k)}}(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

1) $[(c_j^{(1)};\gamma_j^{(1)})]_{1,n_1}$ stands for $(c_1^{(1)};\gamma_1^{(1)}),\cdots,(c_{n_1}^{(1)};\gamma_{n_1}^{(1)})$.

2) $m_2,n_2,\cdots,m_r,n_r,m^{(1)},n^{(1)},\cdots,m^{(r)},n^{(r)},p_{i_2},q_{i_2},R_2,\tau_{i_2},\cdots,p_{i_r},q_{i_r},R_r,\tau_{i_r},p_{i^{(r)}},q_{i^{(r)}},\tau_{i^{(r)}},R^{(r)}\in\mathbb{N}$ and verify :

$0\leqslant m_2\leqslant q_{i_2}, 0\leqslant n_2\leqslant p_{i_2},\cdots,0\leqslant m_r\leqslant q_{i_r}, 0\leqslant n_r\leqslant p_{i_r}, 0\leqslant m^{(1)}\leqslant q_{i^{(1)}},\cdots,0\leqslant m^{(r)}\leqslant q_{i^{(r)}}$

$0\leqslant n^{(1)}\leqslant p_{i^{(1)}},\cdots,0\leqslant n^{(r)}\leqslant p_{i^{(r)}}$.

3) $\tau_{i_2}(i_2=1,\cdots,R_2)\in\mathbb{R}^+;\tau_{i_r}\in\mathbb{R}^+(i_r=1,\cdots,R_r);\tau_{i^{(k)}}\in\mathbb{R}^+(i=1,\cdots,R^{(k)}),(k=1,\cdots,r)$.

4) $\gamma_j^{(k)},C_j^{(k)}\in\mathbb{R}^+;(j=1,\cdots,n^{(k)});(k=1,\cdots,r);\delta_j^{(k)},D_j^{(k)}\in\mathbb{R}^+;(j=1,\cdots,m^{(k)});(k=1,\cdots,r)$.

$\alpha_{kj}^{(l)},A_{kj}\in\mathbb{R}^+;(j=1,\cdots,n_k);(k=2,\cdots,r);(l=1,\cdots,k)$.

$\beta_{kj}^{(l)},B_{kj}\in\mathbb{R}^+;(j=1,\cdots,m_k);(k=2,\cdots,r);(l=1,\cdots,k)$.

$\alpha_{kji_k}^{(l)},A_{kji_k}\in\mathbb{R}^+;(j=n_k+1,\cdots,p_{i_k});(k=2,\cdots,r);(l=1,\cdots,k)$.

$\beta_{kji_k}^{(l)},B_{kji_k}\in\mathbb{R}^+;(j=m_k+1,\cdots,q_{i_k});(k=2,\cdots,r);(l=1,\cdots,k)$.

$\delta_{ji^{(k)}}^{(k)}\in\mathbb{R}^+;(i=1,\cdots,R^{(k)});(j=m^{(k)}+1,\cdots,q_{i^{(k)}});(k=1,\cdots,r)$.

$\gamma_{ji^{(k)}}^{(k)}\in\mathbb{R}^+;(i=1,\cdots,R^{(k)});(j=n^{(k)}+1,\cdots,p_{i^{(k)}});(k=1,\cdots,r)$.

5) $c_j^{(k)} \in \mathbb{C}; (j=1,\cdots,n_k); (k=1,\cdots,r); d_j^{(k)} \in \mathbb{C}; (j=1,\cdots,m_k); (k=1,\cdots,r)$.

$a_{kji_k} \in \mathbb{C}; (j=n_k+1,\cdots,p_{i_k}); (k=2,\cdots,r)$.

$b_{kji_k} \in \mathbb{C}; (j=m_k+1,\cdots,q_{i_k}); (k=2,\cdots,r)$.

$d_{ji^{(k)}}^{(k)} \in \mathbb{C}; (i=1,\cdots,R^{(k)}); (j=m^{(k)}+1,\cdots,q_{i^{(k)}}); (k=1,\cdots,r)$.

$\gamma_{ji^{(k)}}^{(k)} \in \mathbb{C}; (i=1,\cdots,R^{(k)}); (j=n^{(k)}+1,\cdots,p_{i^{(k)}}); (k=1,\cdots,r)$.

The contour L_k is in the $s_k(k=1,\cdots,r)$- plane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ if is a real number with loop, if necessary to ensure that the poles of $\Gamma^{A_{2j}}\left(1-a_{2j}+\sum_{k=1}^{2}\alpha_{2j}^{(k)}s_k\right)(j=1,\cdots,n_2), \Gamma^{A_{3j}}\left(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k\right)$ $(j=1,\cdots,n_3),\cdots,\Gamma^{A_{rj}}\left(1-a_{rj}+\sum_{i=1}^{r}\alpha_{rj}^{(i)}\right)(j=1,\cdots,n_r)$, $\Gamma^{C_j^{(k)}}\left(1-c_j^{(k)}+\gamma_j^{(k)}s_k\right)(j=1,\cdots,n^{(k)})(k=1,\cdots,r)$to the right of the contour L_k and the poles of $\Gamma^{B_{2j}}\left(b_{2j}-\sum_{k=1}^{2}\beta_{2j}^{(k)}s_k\right)(j=1,\cdots,m_2), \Gamma^{B_{3j}}\left(b_{3j}-\sum_{k=1}^{3}\beta_{3j}^{(k)}s_k\right)(j=1,\cdots,m_3)$ $,\cdots,\Gamma^{B_{rj}}\left(b_{rj}-\sum_{i=1}^{r}\beta_{rj}^{(i)}\right)(j=1,\cdots,m_r)$, $\Gamma^{D_j^{(k)}}\left(d_j^{(k)}-\delta_j^{(k)}s_k\right)(j=1,\cdots,m^{(k)})(k=1,\cdots,r)$ lie to the left of the contour L_k. The condition for absolute convergence of multiple Mellin-Barnes type contour (1.1) can be obtained of the corresponding conditions for multivariable H-function given by as :

$|arg(z_k)| < \frac{1}{2}A_i^{(k)}\pi$ where

$$A_i^{(k)} = \sum_{j=1}^{m^{(k)}} D_j^{(k)}\delta_j^{(k)} + \sum_{j=1}^{n^{(k)}} C_j^{(k)}\gamma_j^{(k)} - \tau_{i^{(k)}}\left(\sum_{j=m^{(k)}+1}^{q_i^{(k)}} D_{ji^{(k)}}^{(k)}\delta_{ji^{(k)}}^{(k)} + \sum_{j=n^{(k)}+1}^{p_i^{(k)}} C_{ji^{(k)}}^{(k)}\gamma_{ji^{(k)}}^{(k)}\right)+$$

$$\sum_{j=1}^{n_2} A_{2j}\alpha_{2j}^{(k)} + \sum_{j=1}^{m_2} B_{2j}\beta_{2j}^{(k)} - \tau_{i_2}\left(\sum_{j=n_2+1}^{p_{i_2}} A_{2ji_2}\alpha_{2ji_2}^{(k)} + \sum_{j=m_2+1}^{q_{i_2}} B_{2ji_2}\beta_{2ji_2}^{(k)}\right) + \cdots +$$

$$\sum_{j=1}^{n_r} A_{rj}\alpha_{rj}^{(k)} + \sum_{j=1}^{m_r} B_{rj}\beta_{rj}^{(k)} - \tau_{i_r}\left(\sum_{j=n_r+1}^{p_{i_r}} A_{rji_r}\alpha_{rji_r}^{(k)} + \sum_{j=m_r+1}^{q_{i_r}} B_{rji_r}\beta_{rji_r}^{(k)}\right) \tag{1.4}$$

Following the lines of Braaksma ([3] p. 278), we may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1,\cdots,z_r) = 0(\,|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\,), max(\,|z_1|,\cdots,|z_r|\,) \rightarrow 0$

$\aleph(z_1,\cdots,z_r) = 0(\,|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\,), min(\,|z_1|,\cdots,|z_r|\,) \rightarrow \infty$ where $i=1,\cdots,r$:

$\alpha_i = \min_{\substack{1\leqslant j\leqslant m_i \\ 1\leqslant j\leqslant m^{(i)}}} Re\left(\sum_{h=2}^{r}\sum_{h'=1}^{h} B_{hj}\frac{b_{hj}}{\beta_{hj}^{h'}} + D_j^{(i)}\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)$ and $\beta_i = \max_{\substack{1\leqslant j\leqslant n_i \\ 1\leqslant j\leqslant n^{(i)}}} Re\left(\sum_{h=2}^{r}\sum_{h'=1}^{h} A_{hj}\frac{a_{hj}-1}{\alpha_{hj}^{h'}} + C_j^{(i)}\frac{c_j^{(i)}-1}{\gamma_j^{(i)}}\right)$

Remark 1.
If $m_2=n_2=\cdots=m_{r-1}=n_{r-1}=p_{i_2}=q_{i_2}=\cdots=p_{i_{r-1}}=q_{i_{r-1}}=0$ and $A_{2j}=B_{2j}=A_{2ji_2}=B_{2ji_2}=\cdots=A_{rj}=B_{rj}=A_{rji_r}=B_{rji_r}=1$, then the generalized multivariable Gimel-function reduces in the generalized multivariable Aleph- function (extension of multivariable Aleph-function defined by Ayant [1]).

Remark 2.
If $m_2=n_2=\cdots=m_r=n_r=p_{i_2}=q_{i_2}=\cdots=p_{i_r}=q_{i_r}=0$ and $\tau_{i_2}=\cdots=\tau_{i_r}=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=R_2=$ $=\cdots=R_r=R^{(1)}=\cdots=R^{(r)}=1$, then the generalized multivariable Gimel-function reduces in a generalized multivariable I-function (extension of multivariable I-function defined by Prathima et al. [7]).

Remark 3.

If $A_{2j}=B_{2j}=A_{2ji_2}=B_{2ji_2}=\cdots=A_{rj}=B_{rj}=A_{rji_r}=B_{rji_r}=1$ and $\tau_{i_2}=\cdots=\tau_{i_r}=\tau_{i^{(1)}}=\cdots=\tau_{i^{(r)}}=R_2=\cdots=R_r=R^{(1)}=\cdots=R^{(r)}=1$, then the generalized multivariable Gimel-function reduces in generalized of multivariable I-function (extension of multivariable I-function defined by Prasad [6]).

Remark 4.

If the three above conditions are satisfied at the same time, then the generalized multivariable Gimel-function reduces in the generalized multivariable H-function (extension of multivariable H-function defined by Srivastava and panda [8,9]).

In your investigation, we shall use the following notations.

$$\mathbb{A}=[(\mathrm{a}_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}},[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},$$

$$[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{(r-1)j};\alpha_{(r-1)j}^{(1)},\cdots,\alpha_{(r-1)j}^{(r-1)};A_{(r-1)j})_{1,n_{r-1}}],$$

$$[\tau_{i_{r-1}}(a_{(r-1)ji_{r-1}};\alpha_{(r-1)ji_{r-1}}^{(1)},\cdots,\alpha_{(r-1)ji_{r-1}}^{(r-1)};A_{(r-1)ji_{r-1}})_{n_{r-1}+1,p_{i_{r-1}}}] \tag{1.5}$$

$$\mathbf{A}=[(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)};A_{rj})_{1,n_r}],[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)};A_{rji_r})_{\mathfrak{n}+1,p_{i_r}}] \tag{1.6}$$

$$A=[(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,n^{(1)}}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})_{n^{(1)}+1,p_i^{(1)}}];\cdots;$$

$$[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,n^{(r)}}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_j^{(r)})_{n^{(r)}+1,p_i^{(r)}}] \tag{1.7}$$

$$\mathbb{B}=[(b_{2j};\beta_{2j}^{(1)},\beta_{2j}^{(2)};B_{2j})]_{1,m_2},[\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{m_2+1,q_{i_2}},[(b_{3j};\beta_{3j}^{(1)},\beta_{3j}^{(2)},\beta_{3j}^{(3)};B_{3j})]_{1,m_3},$$

$$[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{m_3+1,q_{i_3}};\cdots;[(\mathrm{b}_{(r-1)j};\beta_{(r-1)j}^{(1)},\cdots,\beta_{(r-1)j}^{(r-1)};B_{(r-1)j})_{1,m_{r-1}}],$$

$$[\tau_{i_{r-1}}(b_{(r-1)ji_{r-1}};\beta_{(r-1)ji_{r-1}}^{(1)},\cdots,\beta_{(r-1)ji_{r-1}}^{(r-1)};B_{(r-1)ji_{r-1}})_{m_{r-1}+1,q_{i_{r-1}}}] \tag{1.8}$$

$$\mathbf{B}=[(\mathrm{b}_{rj};\beta_{rj}^{(1)},\cdots,\beta_{rj}^{(r)};B_{rj})_{1,m_r}],[\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)};B_{rji_r})_{m_r+1,q_{i_r}}] \tag{1.9}$$

$$\mathrm{B}=[(\mathrm{d}_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,m^{(1)}}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)};D_{ji^{(1)}}^{(1)})_{m^{(1)}+1,q_i^{(1)}}];\cdots;$$

$$[(\mathrm{d}_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)};D_j^{(r)})_{m^{(r)}+1,q_i^{(r)}}] \tag{1.10}$$

$$U=m_2,n_2;m_3,n_3;\cdots;m_{r-1},n_{r-1};V=m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)} \tag{1.11}$$

$$X=p_{i_2},q_{i_2},\tau_{i_2};R_2;\cdots;p_{i_{r-1}},q_{i_{r-1}},\tau_{i_{r-1}}:R_{r-1};Y=p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)} \tag{1.12}$$

2. Required results.

In this section, we four formulae. These results will be used in the following sections.

Lemma 1. ([4], p. 143).

$$\int_0^{\pi}\cos n\theta\left(\sin\frac{\theta}{2}\right)^{-2\zeta}\mathrm{d}\theta=\frac{\sqrt{\pi}\Gamma(\zeta+n)\Gamma\left(\frac{1}{2}-\zeta\right)}{\Gamma(\zeta)\Gamma(2-\zeta+n)} \tag{2.1}$$

provided $Re\,(1-2\zeta)>0, n\in\mathbb{N}_0$

Lemma 2. ([5], p. 80).

$$\int_0^{\pi}\sin(2n+1)\theta\,(\sin\theta)^{1-2\zeta}\,\mathrm{d}\theta=\frac{\sqrt{\pi}\Gamma(\zeta+n)\Gamma\left(\frac{3}{2}-\zeta\right)}{\Gamma(\zeta)\Gamma(2-\zeta+n)} \tag{2.2}$$

provided $Re\,(3-\zeta)>0, n\in\mathbb{N}_0$

The following orthogonality property ([2]. p. 32, Eq. (1.1))

Lemma 3.

$$\int_0^{\pi}e^{(2m+1)\omega x}\sin(2n+1)x\mathrm{d}x=\left[\begin{array}{c}0\text{ if m}\neq n\\ .\\ \frac{\pi\omega}{2}\text{ if m = n}\end{array}\right. \tag{2.3}$$

The following orthogonality property

Lemma 4.

$$\int_0^{\pi}e^{2m\omega x}\cos 2nx\mathrm{d}x=\left[\begin{array}{c}0\text{ if m}\neq n\\ \pi\text{if}m=n=0\\ \frac{\pi}{2}\text{ if m = n}\neq 0\end{array}\right. \tag{2.4}$$

3. Main integrals .

In this section, we gives two general integrals about generalized multivariable Gimel-function.

Theorem 1.

$$\int_0^{\pi}\cos(ux)\left(\sin\frac{x}{2}\right)^{-2\zeta}\gimel\left(z_1\left(\sin\frac{x}{2}\right)^{-2h_1},\cdots,z_r\left(\sin\frac{x}{2}\right)^{-2h_r}\right)\mathrm{d}x=\sqrt{\pi}$$

$$\gimel^{U;m_r+1,n_r+1:V}_{X;p_{i_r}+2,q_{i_r}+2,\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c}\mathrm{z}_1 & \mathbb{A};\ (1\text{-}\zeta-u;h_1,\cdots,h_r;1),\mathbf{A},(1-\zeta+u;h_1,\cdots,h_r;1):A\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_r & \mathbb{B};\ \left(\frac{1}{2}-\zeta;h_1,\cdots,h_r;1\right),\mathbf{B},(1-\zeta;h_1,\cdots,h_r;1):B\end{array}\right) \tag{3.1}$$

provided

$h_i>0(i=1,\cdots,r)$, $Re(1-2\zeta)-2\sum_{i=1}^{r}h_i\max_{\substack{1\leqslant j\leqslant n_i\\1\leqslant j\leqslant n^{(i)}}}Re\left(\sum_{h=2}^{r}\sum_{h'=1}^{h}A_{hj}\frac{a_{hj}-1}{\alpha_{hj}^{h'}}+C_j^{(i)}\frac{c_j^{(i)}-1}{\gamma_j^{(i)}}\right)>0.$

$|arg\left(z_i\left(\sin\frac{x}{2}\right)^{h_i}\right)|<\frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4).

Proof

To prove the theorem 1, we replace the generalized multivariable Gimel-function by this multiple integrals contour with the help of (1.1), change the order of integrations which is justified under the conditions mentioned above. We get

$$\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\int_0^{\pi}\cos(ux)\left(\sin\frac{x}{2}\right)^{-2w_1-2\sum_{i=1}^{r}h_is_i}\mathrm{d}x\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{3.2}$$

Evaluate the inner integral with the help of lemma 1 and interpreting the Mellin-Barnes multiple integrals contour in terms of the generalized multivariable Gimel-function, we get the desired result (3.1).

Theorem 2.

$$\int_0^{\pi}\sin(2v+1)x\,(\sin x)^{1-2\zeta}\,\gimel\left(z_1\left(\sin x\right)^{-2h_1},\cdots,z_r\left(\sin x\right)^{-2h_r}\right)\mathrm{d}x=\sqrt{\pi}$$

$$\gimel^{U;m_r+1,n_r+1:V}_{X;p_{i_r}+2,q_{i_r}+2,\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathbb{A};\,(1\text{-}\zeta-u;h_1,\cdots,h_r;1),\mathbf{A},(2-\zeta+u;h_1,\cdots,h_r;1):A\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_r & \mathbb{B};\,\left(\frac{3}{2}-\zeta;h_1,\cdots,h_r;1\right),\mathbf{B},(1-\zeta;h_1,\cdots,h_r;1):B \end{array}\right) \tag{3.3}$$

provided

$h_i>0(i=1,\cdots,r)$, $Re(3-2\zeta)-2\sum_{i=1}^{r}h_i\max_{\substack{1\leqslant j\leqslant n_i\\1\leqslant j\leqslant n^{(i)}}}Re\left(\sum_{h=2}^{r}\sum_{h'=1}^{h}A_{hj}\frac{a_{hj}-1}{\alpha_{hj}^{h'}}+C_j^{(i)}\frac{c_j^{(i)}-1}{\gamma_j^{(i)}}\right)>0.$

$|arg\left(z_i\left(\sin x\right)^{h_i}\right)|<\frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4).

Proof

To prove the theorem 1, we replace the generalized multivariable Gimel-function by this multiple integrals contour with the help of (1.1), change the order of integrations which is justified under the conditions mentioned above. We get

$$\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\int_0^{\pi}\sin(2v+1)x\,(\sin x)^{1-2\zeta-2\sum_{i=1}^{r}h_is_i}\,\mathrm{d}x\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{3.4}$$

Evaluate the inner integral with the help of lemma 2 and interpreting the Mellin-Barnes multiple integrals contour in terms of the generalized multivariable Gimel-function, we get the desired result (3.3).

4. Three dimensional Fourier series.

In this section, we shall note $\psi'=\dfrac{\psi}{2};\theta'=\dfrac{\theta}{2};\phi'=\dfrac{\phi}{2}$

Theorem 3.

$$(\sin\psi)^{1-2\zeta}(\sin\theta)^{1-2\rho}(\sin\phi')^{-2\sigma}\,\gimel^{U;m_r,n_r:V}_{X;p_{i_r},q_{i_r},\tau_{i_r}:R_r:Y}\left(\begin{array}{c} \mathrm{z}_1(\sin\psi)^{-2d_1}(\sin\theta)^{-2h_1}(\sin\phi')^{-2k_1}\\ \cdot\\ \cdot\\ \mathrm{z}_r(\sin\psi)^{-2d_r}(\sin\theta)^{-2h_r}(\sin\phi')^{-2k_r} \end{array}\right)=$$

$$\frac{8}{\pi\sqrt{\pi}\omega}\sum_{s=-\infty}^{\infty}\sum_{t,u=0}^{\infty}e^{(2s+1)\omega\psi}\sin(2t+1)\theta\cos u\phi$$

$$\gimel^{U;m_r+3,n_r+3:V}_{X;p_{i_r}+6,q_{i_r}+6,\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathbb{A};\,\mathrm{A}_1(\zeta,s),A_2(\rho,t),A_3(\sigma,u)\mathbf{A},A_4(\zeta,s),A_5(\rho,t),A_6(\sigma,u):A\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_r & \mathbb{B};\,\mathrm{B}_1(\zeta),B_2(\rho),B_3(\sigma),\mathbf{B},B_4(\zeta),B_5(\rho),B_6(\sigma):B \end{array}\right) \tag{4.1}$$

where

$$A_1(\zeta,s)=(1-\zeta-s;d_1,\cdots,d_r);A_2(\rho,t)=(1-\rho-t;h_1,\cdots,h_r);A_3(\sigma,u)=(1-\sigma-u;k_1,\cdots,k_r) \tag{4.2}$$

$$A_4(\zeta,s)=(2-\zeta+s;d_1,\cdots,d_r);A_5(\rho,t)=(2-\rho+t;h_1,\cdots,h_r);A_6(\sigma,u)=(1-\sigma+u;k_1,\cdots,k_r) \tag{4.3}$$

$$B_1(\zeta)=\left(\frac{3}{2}-\zeta;d_1,\cdots,d_r\right);B_2(\rho)=\left(\frac{3}{2}-\rho;h_1,\cdots,h_r\right);B_3(\sigma)=\left(\frac{3}{2}-\sigma;k_1,\cdots,k_r\right) \tag{4.4}$$

$$B_4(\zeta)=(1-\zeta;d_1,\cdots,d_r)\,;B_5(\rho)=(1-\rho;h_1,\cdots,h_r)\,;B_6(\sigma)=(1-\sigma;k_1,\cdots,k_r) \tag{4.5}$$

We shall adapt the above notations in your investigation.

provided

$d_i,h_i,k_i>0(i=1,\cdots,r),0<\phi<\pi,0<\theta<\pi,0<\psi<\pi$, the existence conditions (3.3) are satisfied about ψ and θ, the existence conditions (3.1) are satisfied about ϕ and

$|arg\left(z_i(\sin\psi)^{-2d_i}(\sin\theta)^{-2h_i}(\sin\phi')^{-2k_i}\right)|<\frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4).

Proof
To establish (4.1), let

$$(\sin\psi)^{1-2\zeta}(\sin\theta)^{1-2\rho}(\sin\phi')^{1-2\sigma}\,\beth_{X;p_{i_r},q_{i_r},\tau_{i_r}:R_r:Y}^{U;m_r,n_r:V}\left(\begin{array}{c}\mathrm{z}_1(\sin\psi)^{-2d_1}(\sin\theta)^{-2h_1}(\sin\phi')^{-2k_1}\\ \cdot\\ \cdot\\ \mathrm{z}_r(\sin\psi)^{-2d_r}(\sin\theta)^{-2h_r}(\sin\phi')^{-2k_r}\end{array}\right)=$$

$$\sum_{s=-\infty}^{\infty}\sum_{t,u=0}^{\infty}A_{s,t,u}e^{(2s+1)\psi}\sin(2t+1)\theta\cos u\phi \tag{4.6}$$

Multiplying both sides of (4.6) by $\cos v\phi$ and integrating with respect to ϕ from 0 to π, and using the theorem 1 and the orthogonality property of cosine functions. Then multiplying both sides of the resulting equation by $\sin(2w+1)\theta$ and integrating with respect to θ from 0 to π, and using the theorem 2 and the orthogonality property of sine functions. Now, multiplying both sides of the resulting expression by $\sin(2x+1)\psi$ and integrating with respect to ψ from 0 to π, and using the theorem 2 and lemma 3, we obtain the value of $A_{s,t,u}$. Substituting this value of $A_{s,t,u}$ in (4.6), we obtain the desired result (4.1).

Theorem 4.

$$(\sin\psi')^{-2\zeta}(\sin\theta)^{1-2\rho}(\sin\phi')^{-2\sigma}\,\beth_{X;p_{i_r},q_{i_r},\tau_{i_r}:R_r:Y}^{U;m_r,n_r:V}\left(\begin{array}{c}\mathrm{z}_1(\sin\psi')^{-2d_1}(\sin\theta)^{-2h_1}(\sin\phi')^{-2k_1}\\ \cdot\\ \cdot\\ \mathrm{z}_r(\sin\psi')^{-2d_r}(\sin\theta)^{-2h_r}(\sin\phi')^{-2k_r}\end{array}\right)=$$

$$\frac{8}{\pi\sqrt{\pi}}\sum_{s=-\infty}^{\infty}\sum_{t,u=0}^{\infty}e^{2s\omega\psi}\sin(2t+1)\theta\cos u\phi$$

$$\beth_{X;p_{i_r}+6,q_{i_r}+6,\tau_{i_r}:R_r:Y}^{U;m_r+3,n_r+3:V}\left(\begin{array}{c|c}\mathrm{z}_1 & \mathbb{A};\ \mathrm{A}_1(\zeta,2s),A_2(\rho,t),A_3(\sigma,u)\mathbf{A},A_4(\zeta,2s),A_5(\rho,t),A_6(\sigma,u):A\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_r & \mathbb{B};\ \mathrm{B}_1(\zeta+1),B_2(\rho),B_3(\sigma),\mathbf{B},B_4(\zeta),B_5(\rho),B_6(\sigma):B\end{array}\right) \tag{4.7}$$

provided

$d_i,h_i,k_i>0(i=1,\cdots,r),0<\phi<\pi,0<\theta<\pi,0<\psi<\pi$, the existence conditions (3.1) are satisfied about ψ and ϕ, the existence conditions (3.3) are satisfied about θ and

$|arg\left(z_i(\sin\psi')^{-2d_i}(\sin\theta)^{1-2h_i}(\sin\phi')^{-2k_i}\right)| < \frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4).

Theorem 5.

$$(\sin\psi)^{1-2\zeta}\,(\sin\phi')^{-2\sigma}\,(\sin\theta)^{1-2\rho}\,\daleth_{X;p_{i_r},q_{i_r},\tau_{i_r}:R_r:Y}^{U;m_r,n_r:V}\left(\begin{array}{c} \mathrm{z}_1(\sin\psi)^{-2d_1}(\sin\phi')^{-2h_1}(\sin\theta)^{-2k_1} \\ \cdot \\ \cdot \\ \mathrm{z}_r(\sin\psi)^{-2d_r}(\sin\phi')^{-2h_r}(\sin\theta)^{-2k_r} \end{array}\right) =$$

$$\frac{8}{\pi\sqrt{\pi}\omega}\sum_{s,t=-\infty}^{\infty}\sum_{u=0}^{\infty}e^{(2s+1)\omega\psi+2t\omega\phi}\sin(2u+1)\theta$$

$$\daleth_{X;p_{i_r}+6,q_{i_r}+6,\tau_{i_r}:R_r:Y}^{U;m_r+3,n_r+3:V}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathbb{A};\ \mathrm{A}_1(\zeta,s),A_3(\sigma,2t),A_2(\rho,u)\mathbf{A},A_4(\zeta,s),A_6(\sigma,2t),A_5(\rho,u):A \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r & \mathbb{B};\ \mathrm{B}_1(\zeta),B_3(\sigma+1),B_2(\rho),\mathbf{B},B_4(\zeta),B_6(\sigma),B_5(\rho):B \end{array}\right) \tag{4.8}$$

provided

$d_i,h_i,k_i > 0 (i = 1,\cdots,r), 0 < \phi < \pi, 0 < \theta < \pi, 0 < \psi < \pi$, the existence conditions (3.3) are satisfied about ψ and ϕ, the existence conditions (3.1) are satisfied about θ and

$|arg\left(z_i(\sin\psi)^{1-2d_i}(\sin\theta')^{-2h_i}(\sin\phi)^{1-2k_i}\right)| < \frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4).

Theorem 6.

$$(\sin\psi)^{1-2\zeta}\,(\sin\phi')^{-2\sigma}\,(\sin\theta')^{-2\rho}\,\daleth_{X;p_{i_r},q_{i_r},\tau_{i_r}:R_r:Y}^{U;m_r,n_r:V}\left(\begin{array}{c} \mathrm{z}_1(\sin\psi)^{-2d_1}(\sin\phi')^{-2h_1}(\sin\theta')^{-2k_1} \\ \cdot \\ \cdot \\ \mathrm{z}_r(\sin\psi)^{-2d_r}(\sin\phi')^{-2h_r}(\sin\theta')^{-2k_r} \end{array}\right) =$$

$$\frac{8}{\pi\sqrt{\pi}\omega}\sum_{s,t=-\infty}^{\infty}\sum_{u=0}^{\infty}e^{(2s+1)\omega\psi+2t\omega\phi}\cos u\theta$$

$$\daleth_{X;p_{i_r}+6,q_{i_r}+6,\tau_{i_r}:R_r:Y}^{U;m_r+3,n_r+3:V}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathbb{A};\ \mathrm{A}_1(\zeta,s),A_3(\sigma,2t),A_2(\rho,u)\mathbf{A},A_4(\zeta,s),A_6(\sigma,2t),A_5(\rho+1,u):A \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r & \mathbb{B};\ \mathrm{B}_1(\zeta),B_3(\sigma+1),B_2(\rho+1),\mathbf{B},B_4(\zeta),B_6(\sigma),B_5(\rho):B \end{array}\right) \tag{4.9}$$

provided

$d_i,h_i,k_i > 0 (i = 1,\cdots,r), 0 < \phi < \pi, 0 < \theta < \pi, 0 < \psi < \pi$, the existence conditions (3.3) are satisfied about ψ and the existence conditions (3.1) are satisfied about θ and ϕ

$|arg\left(z_i(\sin\psi)^{1-2d_i}(\sin\theta')^{1-2h_i}(\sin\phi')^{1-2k_i}\right)| < \frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4).

Theorem 7.

$$(\sin\psi)^{1-2\zeta}\,(\sin\theta)^{1-2\rho}\,(\sin\phi)^{1-2\sigma}\,\daleth_{X;p_{i_r},q_{i_r},\tau_{i_r}:R_r:Y}^{U;m_r,n_r:V}\left(\begin{array}{c} \mathrm{z}_1(\sin\psi)^{-2d_1}(\sin\theta)^{-2h_1}(\sin\phi)^{-2k_1} \\ \cdot \\ \cdot \\ \mathrm{z}_r(\sin\psi)^{-2d_r}(\sin\theta)^{-2h_r}(\sin\phi)^{-2k_r} \end{array}\right) =$$

$$-\frac{8}{\pi\sqrt{\pi}\omega}\sum_{s,t,u=-\infty}^{\infty} e^{(2s+1)\omega\psi+(2t+1)\omega\phi+(2u+1)\omega\psi}$$

$$\gimel^{U;m_r+3,n_r+3:V}_{X;p_{i_r}+6,q_{i_r}+6,\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathbb{A};\ \mathrm{A}_1(\zeta,u),A_2(\rho,s),A_3(\sigma,t)\mathbf{A},A_4(\zeta,u),A_5(\rho,s),A_6(\sigma-1,t):A \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r & \mathbb{B};\ \mathrm{B}_1(\zeta),B_2(\rho),B_3(\sigma),\mathbf{B},B_4(\zeta),B_5(\rho),B_6(\sigma):B \end{array}\right) \tag{4.10}$$

provided

$d_i,h_i,k_i>0(i=1,\cdots,r),0<\phi<\pi,0<\theta<\pi,0<\psi<\pi,$ the existence conditions (3.3) are satisfied about ψ, ϕ and θ and

$|arg\left(z_i(\sin\psi)^{1-2d_i}(\sin\theta)^{1-2h_i}(\sin\phi)^{1-2k_i}\right)|<\frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4).

Theorem 8.

$$(\sin\psi')^{-2\zeta}(\sin\theta')^{-2\rho}(\sin\phi')^{-2\sigma}\,\gimel^{U;m_r,n_r:V}_{X;p_{i_r},q_{i_r},\tau_{i_r}:R_r:Y}\left(\begin{array}{c} \mathrm{z}_1(\sin\psi')^{-2d_1}(\sin\theta')^{-2h_1}(\sin\phi')^{-2k_1} \\ \cdot \\ \cdot \\ \mathrm{z}_r(\sin\psi')^{-2d_r}(\sin\theta')^{-2h_r}(\sin\phi')^{-2k_r} \end{array}\right)=$$

$$\frac{8}{\pi\sqrt{\pi}}\sum_{s,t,u=-\infty}^{\infty} e^{2s\omega\psi+2t\omega\phi+2u\omega\psi}\gimel^{U;m_r+3,n_r+3:V}_{X;p_{i_r}+6,q_{i_r}+6,\tau_{i_r}:R_r:Y}$$

$$\gimel^{U;m_r+3,n_r+3:V}_{X;p_{i_r}+6,q_{i_r}+6,\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathbb{A};\ \mathrm{A}_1(\zeta,2u),A_2(\rho,2s),A_3(\sigma,2t)\mathbf{A},A_4(\zeta+1,2u),A_5(\rho+1,2s),A_6(\sigma,2t):A \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r & \mathbb{B};\ \mathrm{B}_1(\zeta+1),B_2(\rho+1),B_3(\sigma+1),\mathbf{B},B_4(\zeta),B_5(\rho),B_6(\sigma):B \end{array}\right) \tag{4.11}$$

provided

$d_i,h_i,k_i>0(i=1,\cdots,r),0<\phi<\pi,0<\theta<\pi,0<\psi<\pi,$ the existence conditions (3.1) are satisfied about ψ, ϕ and θ and

$|arg\left(z_i(\sin\psi')^{-2d_i}(\sin\theta')^{-2h_i}(\sin\phi')^{-2k_i}\right)|<\frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4).

The theorems 4 to 8 have been established, on applying the same procedure as theorem 3 with the help of theorems 1 and 2, and different combinations of the orthogonality properties.

Remark :
On applying the same procedure as above, we can establish a large numbers other three-dimensional Fourier series by combinations of this class.

5. Conclusion.

The generalized multivariable Gimel-function presented in this paper is a general character, therefore on specializing the parameters of this function, we can obtain a large number of three-dimensional Fourier series concerning a wide variety of special functions of one and several variables.

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