Int. J. of Mathematical Sciences and Applications,
Vol. 1, No. 2, May 2011

www.ijmsa.yolasite.com

ON FUZZY rw-CLOSED SETS AND FUZZY rw-OPEN SETS IN FUZZY TOPOLOGICAL SPACES

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Abtract: In this paper, a new class of sets called fuzzy rw-closed sets and fuzzy rw-open sets in fuzzy topological spaces are introduced and studied. A fuzzy set α of a fuzzy topological space X is said to be fuzzy regular w-closed (briefly, fuzzy rw-closed) if $cl(\alpha) \leq \sigma$ whenever $\alpha \leq \sigma$ and σ is fuzzy regular semiopen in fuzzy topological space X. The complement of fuzzy rw-closed set is called a fuzzy regular w-open (briefly, fuzzy rw-open) set in fuzzy topological spaces X. Some properties of new concept have been studied.

2000 Mathematics Subject Classification: 54A40.

Key words and phrases: Fuzzy regular closed sets, fuzzy rw-closed sets.

1. Introduction

The concept of a fuzzy subset was introduced and studied by L.A.Zadeh [12] in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. C.L.Chang [04] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces. Many researchers like R.H.Warren ([10], [11]), K.K.Azad ([01],[02],) G.Balasubramanian and P.Sundaram [03], S.R.Malghan and S.S.Benchalli ([06], [07]), M.N.Mukherjee and B.Ghosh [09], Anjan.Mukherjee [08], A.N.Zahren [13], J.A.Goguen [05] and many others have contributed to the development of fuzzy topological spaces.

The purpose of this paper is to introduce a new class of fuzzy sets called fuzzy rw-closed sets in fuzzy topological spaces and investigate certain basic properties of these fuzzy sets. Among many other results it is observed that every fuzzy closed set is fuzzy rw-closed but not conversely. Also we introduce fuzzy rw-open sets in fuzzy topological spaces and study some of their properties.

2. Preliminaries

2.1 Definition: [02] A fuzzy set A in a fts X is said to be fuzzy semiopen if and only if there exists a fuzzy open set V in X such that $V \le A \le cl(V)$.

2.2 Definition: [02] A fuzzy set A in a fts X is said to be fuzzy semi-closed if and only if there exists a fuzzy closed set V in X such that $int(V) \le A \le V$.

It is seen that a fuzzy set A is fuzzy semiopen if and only if $1-A$ is a fuzzy semi-closed.

2.3 Theorem: [02] The following are equivalent:

(a) λ is a fuzzy semiclosed set,

(b) λ^C is a fuzzy semiopen set,

(c) $int(cl(\lambda)) \le \lambda$

(d) $cl(int(\lambda^C)) \ge \lambda^C$

2.4 Theorem: [02] (a) Any union of fuzzy semiopen sets is a fuzzy semiopen set and (b) any intersection of fuzzy semi closed sets is a fuzzy semi closed.

2.5 Remark: [02] (i) Every fuzzy open set is a fuzzy semiopen but not conversely.

(ii) Every fuzzy closed set is a fuzzy semi-closed set but not conversely.

(iii) The closure of a fuzzy open set is fuzzy semiopen set

(iv) The interior of a fuzzy closed set is fuzzy semi-closed set

2.6 Definition: [02] A fuzzy set λ of a fts X is called a fuzzy regular open set of X if $int(cl(\lambda)=\lambda$

2.7 Definition: [02] A fuzzy set λ of fts X is called a fuzzy regular closed set of X if $cl(int(\lambda)) = \lambda$.

2.8 Theorem: [02] A fuzzy set λ of a fts X is a fuzzy regular open if and only if λ^C fuzzy regular closed set.

2.9 Remark: [02] (i) Every fuzzy regular open set is a fuzzy open set but not conversely.

(ii) Every fuzzy regular closed set is a fuzzy closed set but not conversely.

2.10 Theorem: [02] (i) The closure of a fuzzy open set is a fuzzy regular closed.

(ii) The interior of a fuzzy closed set is a fuzzy regular open set

2.11 Definition: [13] A fuzzy set α of a fuzzy topological space X is said to be a fuzzy regular semiopen set in fts X if there exists a fuzzy regular open set σ in X such that $\sigma \leq \alpha \leq cl(\sigma)$.We denote the class of fuzzy regular semiopen sets in fts X by FRSO(X).

2.12 Theorem: [13] (i) Every fuzzy regular semiopen set is a fuzzy semiopen set but not conversely.

(ii) Every fuzzy regular closed set is a fuzzy regular semiopen set but not conversely.

(iii) Every fuzzy regular open set is a fuzzy regular semiopen set but not conversely.

2.13 Theorem: [13] A fuzzy set α of fts X is fuzzy regular semiopen if and only if α is both fuzzy semiopen and fuzzy semi-closed

2.14 Theorem: [13] If α is fuzzy regular semiopen in fts X, then $1-\alpha$ is fuzzy regular semiopen in X.

2.15 Definition: [03] A fuzzy set α in fts X is called fuzzy generalized closed (gf-closed) if $cl(\alpha) \leq \mu$ whenever $\alpha \leq \mu$ and μ fuzzy open and α is fuzzy generalized open if $1-\alpha$ is fuzzy generalized closed.

3. Fuzzy rw-closed sets and fuzzy rw-open sets in fts.

3.1 Definition: Let (X, T) be a fuzzy topological space. A fuzzy set α of X is called fuzzy regular w-closed (briefly, fuzzy rw-closed) if $cl(\alpha) \leq \sigma$ whenever $\alpha \leq \sigma$ and σ is fuzzy regular semiopen in fts X.

We denote the class of all fuzzy regular w-closed sets in fts X by FRWC(X).

3.2 Theorem: Every fuzzy closed set is a fuzzy rw-closed set in a fts X.

Proof: Let α be a fuzzy closed set in a fts X. Let β be a fuzzy regular semiopen set in X such that $\alpha \leq \beta$. Since α is fuzzy closed, $cl(\alpha)=\alpha$. Therefore $cl(\alpha) \leq \beta$. Hence α is fuzzy rw-closed in fts X.

The converse of the above Theorem need not be true in general as seen from the following example.

3.3 Example: Let X= {a, b, c}. Define a fuzzy set α in X by

$$\alpha(x)=\begin{cases} 1 \text{ if } x = a \\ 0 \text{ otherwise} \end{cases}$$

Let T= {1, 0, α}. Then (X, T) is a fuzzy topological space. Define a fuzzy set β in X by

$$\beta(x) = \begin{cases} 1 \text{ if } x = b \\ 0 \text{ otherwise} \end{cases}$$

Then β is a fuzzy rw-closed set but it is not a fuzzy closed set in fts X.

3.4 Corollary: By Remark 2.8 (ii), it has been proved that every fuzzy regular closed set is a fuzzy closed set but not conversely. By Theorem 3.2 every fuzzy closed set is a fuzzy rw-closed set but not conversely and hence every fuzzy regular closed set is a fuzzy rw-closed set but not conversely.

3.5 Remark: Fuzzy generalized closed sets and fuzzy rw-closed sets are independent.

3.6 Example: Let X= {a, b, c, d} and the functions

$\alpha,\beta,\gamma : X\rightarrow [0, 1]$ be defined as

$$\alpha(x) = \begin{cases} 1 \text{ if } x = a \\ 0 \text{ otherwise} \end{cases} \qquad \beta(x) = \begin{cases} 1 \text{ if } x = b \\ 0 \text{ otherwise} \end{cases}$$

$$\gamma(x) = \begin{cases} 1 \text{ if } x = a, b \\ 0 \text{ otherwise} \end{cases}$$

Consider T= {1, 0,α, β, γ}. Then (X, T) is a fuzzy topological space. In this fts X, the fuzzy set $\lambda:X \rightarrow[0, 1]$ define by

$$\lambda(x) = \begin{cases} 1 \text{ if } x = c \\ 0 \text{ otherwise} \end{cases}$$

Then λ is a fuzzy generalized closed set in fts X. In this fts, the fuzzy set $\delta : X \to [0, 1]$ define by

$$\delta(x) = \begin{cases} 1 \text{ if } x = a, c, \\ 0 \text{ otherwise} \end{cases}$$

Then δ is a fuzzy regular semiopen set containingλ, but δ does not contain cl(λ) which is γ^C. Therefore δ is not a fuzzy rw-closed set in fts X.

3.7 Example: Let X = I = [0, 1]. Define a fuzzy set λ in X by

$$\lambda(x) = \begin{cases} ½ \text{ if } x = 2/3 \\ 0 \text{ otherwise} \end{cases}$$

Let T = {1, 0, λ}. Then (X, T) is a fuzzy topological space.

Let $\alpha(x) = \begin{cases} 1/3 \text{ if } x = 2/3 \\ 0 \text{ otherwise} \end{cases}$

Then α is a fuzzy rw-closed set in fts X. Now cl(α) = λ^C and λ is a fuzzy open set containing α but λ does not contain cl(α) which is λ^C. Therefore α is not a fuzzy generalized closed

3.8 Remark: Fuzzy rw-closed sets and fuzzy semi-closed sets are independent.

3.9 Example: Consider the fuzzy topological space (X, T) defined in Example 3.3. Then the fuzzy set α= {(a, 1), (b, 0), (c, 0)} is a fuzzy rw-closed but it is not a fuzzy semi-closed set in fts X.

3.10 Example: Consider the fuzzy topological space (X, T) defined in Example 3.6. In this fts X, the fuzzy set $\mu : X \rightarrow [0, 1]$ is define by

$$\mu(x) = \begin{cases} 1 \text{ if } x = a, c \\ 0 \text{ otherwise} \end{cases}$$

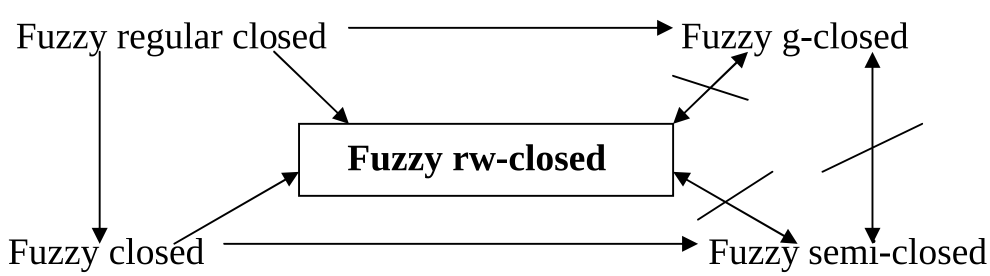
Then μ is a fuzzy semi-closed in fts X. μ is also fuzzy regular semiopen set containing μ which is does not contain $cl(\mu)=\beta^c=\{(a, 1), (b, 0), (c, 1), (d, 1)\}$. Therefore μ is not a fuzzy rw-closed set in fts X.

3.11 Remark: From the above discussions and known results we have the following implications

In the following diagram, by

A $\longrightarrow$ B we mean A implies B but not conversely and

A $\longleftrightarrow$ B means A and B are independent of each other.



3.12 Theorem: If α and β are fuzzy rw-closed sets in fts X, then $\alpha\vee\beta$ is fuzzy rw-closed set in fts X.

Proof: Let σ be a fuzzy regular semiopen set in fts X such that $\alpha\vee\beta \leq \sigma$. Now $\alpha \leq \sigma$ and $\beta \leq \sigma$. Since α and β are fuzzy rw-closed sets in fts X, $cl(\alpha) \leq \sigma$ and $cl(\beta) \leq \sigma$. Therefore $cl(\alpha)\vee cl(\beta) \leq \sigma$. But $cl(\alpha)\vee cl(\beta) = cl(\alpha\vee\beta)$. Thus $cl(\alpha\vee\beta) \leq \sigma$. Hence $\alpha\vee\beta$ is a fuzzy rw-closed set in fts X.

3.13 Remark: If α and β are fuzzy rw-closed sets in fts X, then $\alpha \wedge \beta$ need not be a fuzzy rw-closed set in general as seen from the following example.

3.14 Example: Consider the fuzzy topological space (X, T) defined in Example 3.6. In this fts X, the fuzzy sets δ_1, δ_2 : $X \rightarrow [0, 1]$ are defined by

$$\delta_1(x) = \begin{cases} 1 \text{ if } x = c, d \\ 0 \text{ otherwise} \end{cases} \qquad \delta_2(x) = \begin{cases} 1 \text{ if } x = a, b, c \\ 0 \text{ otherwise} \end{cases}$$

Then δ_1 and δ_2 are the fuzzy rw-closed sets in fts in X.

Let $\lambda = \delta_1 \wedge \delta_2$. Then $\lambda(x) = \begin{cases} 1 \text{ if } x = c \\ 0 \text{ otherwise} \end{cases}$

Then $\lambda = \delta_1 \wedge \delta_2$ is not a fuzzy rw-closed set in fts X.

3.15 Theorem: If a fuzzy set α of fts X is both fuzzy regular open and fuzzy rw-closed, then α is a fuzzy regular closed set in fts X.

Proof: Suppose a fuzzy set α of fts X is both fuzzy regular open and fuzzy rw-closed. As every fuzzy regular open set is a fuzzy regular semiopen set and $\alpha \leq \alpha$, we have $cl(\alpha) \leq \alpha$. Also $\alpha \leq cl(\alpha)$. Therefore $cl(\alpha) = \alpha$. That is α is fuzzy closed. Since α is fuzzy regular open, $int(\alpha)=\alpha$. Now $cl(int(\alpha))= cl(\alpha) = \alpha$.Therefore α is a fuzzy regular closed set in fts X.

3.16 Theorem: If a fuzzy set α of a fts X is both fuzzy regular semiopen and fuzzy rw-closed, then α is a fuzzy closed set in fts X.

Proof: Suppose a fuzzy set α of a fts X is both fuzzy regular semiopen and fuzzy rw-closed. Now $\alpha \leq \alpha$, we have $cl(\alpha) \leq \alpha$. Also $\alpha \leq cl(\alpha)$. Therefore $cl(\alpha) = \alpha$ and hence α is a fuzzy closed set in fts X.

3.17 Theorem: If a fuzzy set α of a fts X is both fuzzy open and fuzzy generalized closed, then α is a fuzzy rw-closed set in fts X.

Proof: Suppose a fuzzy set α of a fts X is both fuzzy open and fuzzy generalized closed. Now $\alpha \leq \alpha$, by hypothesis we have $cl(\alpha) \leq \alpha$. Also $\alpha \leq cl(\alpha)$. Therefore $cl(\alpha) = \alpha$. That is α is a fuzzy closed set and hence α is a fuzzy rw-closed set in fts X, as every fuzzy closed set is a fuzzy rw-closed set.

3.18 Remark: If a fuzzy set γ is both fuzzy regular open and fuzzy rw-closed set in a fts X, then γ need not be a fuzzy generalized closed set in general as seen from the following example.

3.19 Example: Let X= {a, b, c} and the functions $\alpha, \beta, \gamma : X \to [0, 1]$ be defined as

$$\alpha(x) = \begin{cases} 1 \text{ if } x = a \\ 0 \text{ otherwise} \end{cases} \qquad \beta(x) = \begin{cases} 1 \text{ if } x = b \\ 0 \text{ otherwise} \end{cases}$$

$$\gamma(x) = \begin{cases} 1 \text{ if } x = a, b \\ 0 \text{ otherwise} \end{cases}$$

Consider T={1, 0, α, β, γ}. Then (X, T) is a fuzzy topological space. In this fts X, γ is both fuzzy open and fuzzy rw-closed set in fts X but it is not fuzzy generalized closed.

3.20 Theorem: Let α be a fuzzy rw-closed set of a fts X and suppose $\alpha \leq \beta \leq cl(\alpha)$. Then β is also a fuzzy rw-closed set in fts X.

Proof: Let $\alpha \leq \beta \leq cl(\alpha)$ and α be a fuzzy rw-closed set of fts X. Let σ be any fuzzy regular semiopen set such that $\beta \leq \sigma$. Then $\alpha \leq \sigma$ and α is fuzzy rw-closed, we have $cl(\alpha) \leq \sigma$. But $cl(\beta) \leq cl(\alpha)$ and thus $cl(\beta) \leq \sigma$. Hence β is a fuzzy rw-closed set in fts X.

3.21 Theorem: In a fuzzy topological space X if FRSO(X) = {1, 0}, where FRSO(X) is the family of all fuzzy regular semiopen sets then every fuzzy subset of X is fuzzy rw-closed

Proof: Let X be a fuzzy topological space and FRSO(X) = {1, 0}. Let α be any fuzzy subset of X. Suppose $\alpha = 0$. Then 0 is a fuzzy rw-closed set in fts X. Suppose $\alpha \neq 0$. Then 1 is the only fuzzy regular semiopen set containing α and so $cl(\alpha) \leq 1$. Hence α is a fuzzy rw-closed set in fts X.

3.22 Remark: The converse of the above Theorem 3.21 need not be true in general as seen from the following example.

3.23 Example: Let X= {a, b, c} and the functions $\alpha, \beta : X \to [0, 1]$ be defined as

$$\alpha(x) = \begin{cases} 1 \text{ if } x = a \\ 0 \text{ otherwise} \end{cases} \qquad \beta(x) = \begin{cases} 1 \text{ if } x = b, c \\ 0 \text{ otherwise} \end{cases}$$

Consider T= {1,0,$\alpha\beta$}. Then (X, T) is a fuzzy topological space. In this fts X, every fuzzy subset of X is a fuzzy rw-closed set in fts X, but FRSO= {1, 0, α, β}.

3.24 Theorem: If α is a fuzzy rw-closed set of fts X and $cl(\alpha)\wedge(1-cl(\alpha))=0$, then $cl(\alpha)-\alpha$ does not contain any non-zero fuzzy regular semiopen set in fts X.

Proof: Suppose α is a fuzzy rw-closed set of fts X and $cl(\alpha)\wedge(1-cl(\alpha))=0$. We prove the result by contradiction. Let β be a fuzzy regular semiopen set such that $cl(\alpha)-\alpha \geq \beta$ and $\beta \neq 0$. Now $\beta \leq cl(\alpha) - \alpha$, i.e $\beta \leq 1-\alpha$ which implies $\alpha \leq 1-\beta$. Since β is a fuzzy regular semiopen set, by Theorem 6.1.30, $1-\beta$ is also fuzzy regular semiopen set in fts X. Since α is a fuzzy rw-closed set in fts X, by definition $cl(\alpha) \leq 1-\beta$. So $\beta \leq 1-cl(\alpha)$. Therefore $\beta \leq (cl(\alpha)\wedge(1-cl(\alpha))=0$, by hypothesis. This shows that $\beta=0$ which is a contradiction. Hence $cl(\alpha)-\alpha$ does not contain any non-zero fuzzy regular semiopen set in fts X..

3.25 Corollary: If α is a fuzzy rw-closed set of fts X and $cl(\alpha)\wedge(1-cl(\alpha))=0$, then $cl(\alpha)-\alpha$ does not contain any non-zero fuzzy regular open set in fts X.

Proof: Follows form the Theorem 3.24 and the fact that every fuzzy regular open set is a fuzzy regular semiopen set in fts X.

3.26 Corollary: If α is a fuzzy rw-closed set of a fts X and $cl(\alpha)\wedge(1-cl(\alpha))=0$, then $cl(\alpha)-\alpha$ does not contain any non-zero fuzzy regular closed set in fts X.

Proof: Follows form the Theorem 3.24 and the fact that every fuzzy regular closed set is a fuzzy regular semiopen set in fts X.

3.27 Theorem: Let α be a fuzzy rw-closed set of fts X and $cl(\alpha)\wedge(1-cl(\alpha))=0$ Then α is a fuzzy closed set if and only if $cl(\alpha)-\alpha$ is a fuzzy regular semiopen set in fts X.

Proof: Suppose α is a fuzzy closed set in fts X. Then $cl(\alpha) = \alpha$ and so $cl(\alpha)-\alpha = 0$, which is a fuzzy regular semiopen set in fts X.

Conversely suppose $cl(\alpha)-\alpha$ is a fuzzy regular semiopen set in fts X. Since α is fuzzy rw-closed, by Theorem 3.24 $cl(\alpha)-\alpha$ does not contain any-non zero fuzzy regular open set in fts X. Then $cl(\alpha)-\alpha=0$. That is $cl(\alpha) = \alpha$ and hence α is a fuzzy closed set in fts X.

We introduce a fuzzy rw-open set in fuzzy topological space X as follows.

3.28 Definition: A fuzzy set α of a fuzzy topological space X is called a fuzzy regular w-open (briefly, fuzzy rw-open) set if its complement α^C is a fuzzy rw-closed set in fts X.

We denote the family of all fuzzy rw-open sets in fts X by FRWO(X).

3.29 Theorem: If a fuzzy set α of a fuzzy topological space X is fuzzy open, then it is fuzzy rw-open but not conversely.

Proof: Let α be a fuzzy open set of fts X. Then α^C is fuzzy closed. Now by Theorem 3.2, α^C is fuzzy rw-closed. Therefore α is a fuzzy rw-open set in fts X.

The converse of the above Theorem need not be true in general as seen from the following example.

3.30 Example: Let X= {a, b, c}. Define a fuzzy set α in X by

$$\alpha(x)= \begin{cases} 1 \text{ if } x = a, b \\ 0 \text{ otherwise} \end{cases}$$

Let T= {1, 0, α}. Then (X, T) is a fuzzy topological space. Define a fuzzy set β in X by

$$\beta(x) = \begin{cases} 1 \text{ if } x = b \\ 0 \text{ otherwise} \end{cases}$$

Then β is a fuzzy rw-open set but it is not fuzzy open set in fts X.

3.31 Corollary: By Remark 2.8. (i), it has been proved that every fuzzy regular open set is a fuzzy open set but not conversely. By Theorem 3.29, every fuzzy open set is a fuzzy rw-open set but not conversely and hence every fuzzy regular open set is a fuzzy rw-open set but not conversely.

3.32 Theorem: A fuzzy set α of a fuzzy topological space X is fuzzy rw-open if and only if $\delta \leq \text{int}(\alpha)$ whenever $\delta \leq \alpha$ and δ is a fuzzy regular semiopen set in fts X.

Proof: Suppose that $\delta \leq \text{int}(\alpha)$ whenever $\delta \leq \alpha$ and δ is a fuzzy regular semiopen set in fts X. To prove that α is fuzzy rw-open in fts X. Let $\alpha^C \leq \beta$ and β is any fuzzy regular semiopen set in fts X. Then $\beta^C \leq \alpha$. By Theorem 2.13, β^C is also fuzzy regular semiopen set in fts X. By hypothesis, $\beta^C \leq \text{int}(\alpha)$ which implies $(\text{int}(\alpha))^C \leq \beta$. That is $\text{cl}(\alpha^C) \leq \beta$, since $\text{cl}(\alpha^C) = (\text{int}(\alpha))^C$. Thus α^C is a fuzzy rw-closed and hence α is fuzzy rw-open in fts X.

Conversely, suppose that α is fuzzy rw-open. Let $\beta \leq \alpha$ and β is any fuzzy regular semiopen in fts X. Then $\alpha^C \leq \beta^C$. By Theorem 6.1.30, β^C is also fuzzy regular semiopen. Since α^C is fuzzy rw-closed, we have $\text{cl}(\alpha^C) \leq \beta^C$ and so $\beta \leq \text{int}(\alpha)$, since $\text{cl}(\alpha^C) = (\text{int}(\alpha))^C$.

3.33 Theorem: If α and β are fuzzy rw-open sets in a fts X, then $\alpha\wedge\beta$ is also a fuzzy rw-open set in fts X.

Proof: Let α and β be two fuzzy rw-open sets in a fts X. Then α^C and β^C are fuzzy rw-closed sets in fts X. By Theorem 3.12, $\alpha^C\vee \beta^C$ is also a fuzzy rw-closed set in fts X. That is $(\alpha^C\vee \beta^C) = (\alpha\wedge\beta)^C$ is a fuzzy rw-closed set in X. Therefore $\alpha\wedge\beta$ is also a fuzzy rw-open set in fts X.

3.34 Remark: The union of two fuzzy rw-open sets in a fts X is generally not a fuzzy rw-open set in fts X.

3.35 Example: Consider the fuzzy topological space (X, T) defined as in Example 6.2.19. In this fts X, the fuzzy sets $\delta_1, \delta_2 : X \to [0\ 1]$ are defined by

$$\delta_1(x) = \begin{cases} 1 \text{ if } x = a \\ 0 \text{ 0therwise} \end{cases} \qquad \delta_2(x) = \begin{cases} 1 \text{ if } x = c \\ 0 \text{ otherwise} \end{cases}$$

Then δ_1 and δ_2 are the rw-open fuzzy sets in fts X. Let $\lambda = \delta_1 \vee \delta_2$ Then

$$\lambda(x) = \begin{cases} 1 \text{ if } x= a, c \\ 0 \text{ otherwise} \end{cases}$$

Then $\lambda = \delta_1 \vee \delta_2$ is not a fuzzy rw-open set in fts X.

3.36 Theorem: If $\text{int}(\alpha) \leq \beta \leq \alpha$ and α is a fuzzy rw-open set in a fts X, then β is also a fuzzy rw-open set in fts X.

Proof: Suppose $\text{int}(\alpha) \leq \beta \leq \alpha$ and α is a fuzzy rw-open set in a fts X. To prove that β is a fuzzy rw-open set in fts X. Let σ be any fuzzy regular semiopen set in fts X such that $\sigma \leq \beta$. Now $\sigma \leq \beta \leq \alpha$. That is $\sigma \leq \alpha$. Since α is fuzzy rw-open set of fts X, $\sigma \leq \text{int}(\alpha)$ by Theorem 6.2.32. By hypothesis $\text{int}(\alpha) \leq \beta$. Then $\text{int}(\text{int}(\alpha)) \leq \text{int}(\beta)$. That is $\text{int}(\alpha) \leq \text{int}(\beta)$. Then $\sigma \leq \text{int}(\beta)$. Again by Theorem 6.2.32 β is a fuzzy rw-open set in fts X.

3.37 Theorem: If a fuzzy subset α of a fts X is fuzzy rw-closed and $\text{cl}(\alpha) \wedge (1 - \text{cl}(\alpha)) = 0$, then $\text{cl}(\alpha) - \alpha$ is a fuzzy rw-open set in fts X.

Proof: Let α be a fuzzy rw-closed set in a fts X and $\text{cl}(\alpha) \wedge (1 - \text{cl}(\alpha)) = 0$. Let β be any fuzzy regular semiopen set of fts X such that $\beta \leq (\text{cl}(\alpha) - \alpha)$. Then by Theorem 3.24, $\text{cl}(\alpha) - \alpha$ does not contain any non-zero fuzzy regular semiopen set and so $\beta = 0$. Therefore $\beta \leq \text{int}(\text{cl}(\alpha) - \alpha)$. By Theorem 3.32, $\text{cl}(\alpha) - \alpha$ is fuzzy rw-open.

3.38 Theorem: Let α and β be two fuzzy subsets of a fts X. If β is a fuzzy rw-open set and $\alpha \geq int(\beta)$, then $\alpha \wedge \beta$ is a fuzzy rw-open set in fts X.

Proof: Let β be a fuzzy rw-open set of a fts X and $\alpha \geq int(\beta)$. That is $int(\beta) \leq \alpha \wedge \beta$. Also $int(\beta) \leq \alpha \wedge \beta \leq \beta$ and β is a fuzzy rw-open set. By Theorem 6.2.36, $\alpha \wedge \beta$ is also a fuzzy rw-open set in fts X.

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