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TWO EXTERIOR GRIFFITH CRACKS IN AN INEIINITE LONG ELASTIC STRIP UNDER SHEAR

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ABSTRACT

The present paper deals with the distribution of stress and displacement in an infinitely long elastic strip with two exterior Griffith cracks under Shear. it is assumed that the cracks are by can: and the edges of strip are rigidly fixed. The problem reduced into pair of dual integral equations with sine and cosine kernels and a weightfunction. By using Fourier transform technique a closed form solution has been derived. Finally the analytical expression has been derived for the» stress intensity factor.

INTRODUCTION

In recent theory of fracture mechanic:, a lot of work has been done by various authors in solving two dimensional problems of single or two or more line cracks with complicated crack configuration. in most of the cases, the medium is isotropic in which the cracks lie. The theory of cracks in two dimensional elastic medium was first developed by Griffith [3]. Sneddon and Elliot [4] studied the distribution of stresses in the neighbourhood of a Griffith crack ic- are intenia tag-Assure. Also the workers lrwin [1], Barenblat [2], England and Green i6} have shown the importance of determining the stress in the neighbourhood of cracks in the theory of fracture mechanics. A detail survey of crack(s) have been done by Sneddon and Lowengrub [13] in their monograph. The case in which the medium is in the form of an infinite strip with finite thickness has not been so widely considered. Two types of problems of a Griffith crack in a thin elastic strip of infinite length have been considered by different workers such that the edges of the strip are assumed fft’i‘t’i‘ fren'i stresses. in first case the crack is assumed to be parallel to the edges of the strip and in the second case the crack is perpendicular to the edges of the strip. Lowengrub [12] discussed some crack problems in which a single crack lies in an elastic strip of infinite length but of finite thickness. Subsequently Lowengrub and Shrivastava [7] determined the stress distribution in an infinitely long elastic strip containing two coplaner Griffith Cracks parallel to the edge of the strip. Also the expressions for the stress intensity factors,crack shape and crack energy have also been derived. Also Sneddon and Lowengrub [13] considered the case of single crack perpendicular to the edge of the strip. The same problem with two Griffith cracks lying symmetrically and perpendicularly to the edge of the strip have been discussed by Fodder [19] reducmg the problem into a set of triple integral equations. By using finite Hilbert transform technique, these equations were reduced into a single Fredholm integral equation. Dhaliwal and Singh [14] determined the stress intensity factors and the crack energy in an infiinitely long elastic, homogeneous, isotropic strip containing two coplaner Griffith cracks. By using Fourier transform technique, the problem was reduced into a set of triple integral equations and a closed form solution had’been derived. Subsequently Lal and Jain [11] calculated the strees intesity factor atthe tip of an external line crack perpendicular to the surface of an elastric half-plane; They formulated‘the problem in terms of a pair of dual integral equations, which are further reduced to a single Fredholm integral equation with a non-singular kernel. in this paper, we consider the problem of determining the stress. intensity factors in an infinitely long elastic, homogeneous strip under shear containing two coplanar exterior Griffith cracks. It is assumed that the cracks are along the length of the strip and strip is rigidly clamped. The problem reduced into two pairs of dual integral equations. By using Fourier transform technique a closed form solution has been derived for all values of h, where 2h is the breadth of the strip. Finally an exact solution of stress intensity factors has alarm been derived for the case when shear stress be an odd function.

2. FORMULATION OF THE PROBLEM AND DERIVATION OF DUAL INTEGRAL EQUATIONS.

Let us consider an infinite strip of breadth 2h ($-\infty < x < \infty$, $-h \le y \le h$) and cracks located in the exterior of the material on the line $y = 0$, $a < x < \infty$, $-a < x < -\infty$. We assume that the cracks are along the length of the strip and opened by the internal pressure. It is assumed that the edges of the strip are rigidly fixed. Due to symmetry about the x - axis, the problem can be converted into a mixed boundary value problem of $O < y < h$ and $-\infty < x < \infty$ as following:-

$$u_z(x, o, z) = 0, \quad 0 \le |x| \le a, \qquad \text{..... (2.1)}$$

$$\sigma_{yz}(x, o, z) = -p(x) - q(x), \; |x| > a, \qquad \text{..... (2.2)}$$

$$u_z(x, h, z) = 0, \quad -\infty < x < \infty, \qquad \text{..... (2.3)}$$

where p(x) and q (x) are even and oldd functions of x respectively. The non-zero displacement and stress components are defined as :

$$u_z = w(x, y); \qquad \sigma_{zx} = \mu \frac{\partial w,}{\partial x}; \quad \sigma_{yz} = \frac{\partial w}{\partial y} \qquad \text{..... (2.4)}$$

where μ is the Lame's constant and w satisfie the Laplace equation

$$\nabla^2 w = 0 \qquad \text{..... (2.5)}$$

We see that the suitable expression for the displacement satisfying the boundary condition (2.5) we have

$$w(x,y) = F_c\left[\frac{A(\xi)\xi^{-1}\,\text{Sinh}\,(h-y)\,\xi}{\mu\,\text{Sinh}\,(\xi h)};\xi \to x\right] + F_s\left[\frac{B(\xi)\,\xi^{-1}\,\text{Sinh}\,(h-y)\,\xi}{\text{Sinh}\,(\xi h)};\xi \to x\right], \qquad \text{..... (2.6)}$$

where A (ξ), B (ξ) are unknown functions to be determined form the boundary condition (2.1) - (2.3) and the notations represent the Fourier cosine and sine transforms defined as following :

$$F_c[f(\xi,y); \xi \to x] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(\xi,y) \text{ Cos }(\xi,x)\, d\xi,$$

$$F_s[f(\xi,y); \xi \to x] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(\xi,y) \text{ Sin }(\xi,x)\, d\xi,$$

Now using equation (2.6) and (2.4) the shear stress can be written as

$$\sigma_{yz} = - F_c\left[\frac{A(\xi) \text{ Cosh}(h - y)\, \xi}{\text{Sin }(\xi h)} ; \xi \to x\right]$$

$$- F_s\left[\frac{B(\xi) \text{ Cosh }(h - y)\, \xi}{\text{Sin }(\xi h)} ; \xi \to x\right], \qquad \text{..... (2.7)}$$

We find the boundary condition (2.3) is identically satisfied with the help of equation (2.6). The boundary conditions (2.1) and (2.2) are reduced into a pair of dual integral equations :

$$F_c\, [\xi^{-1} A(\xi) ; \xi \to x] = 0,\ 0 \leq x < a, \qquad \text{..... (2.8)}$$

$$F_c\, [\text{Cot }(\xi h)\, A(\xi) ; \xi \to x] = p(x),\ x > a \qquad \text{..... (2.9)}$$

and

$$F_s\, [\xi^{-1} B(\xi) ; \xi \to x] = 0,\ 0 \leq x < a, \qquad \text{..... (2.10)}$$

$$F_s\, [\text{Cot h }(\xi h)\, B(\xi) : \xi \to x] = q(x),\ x > a \qquad \text{..... (2.11)}$$

3. SOLUTION OF DUAL INTEGRAL EQUATIONS.

Let us take

$$A(\xi) = \sqrt{\frac{\pi}{2}} \int_a^\infty \phi(t) \operatorname{Sin}(\xi t)\, dt \qquad \text{...... (3.1)}$$

with

$$\int_a^\infty \phi(t)\, dt = 0 \qquad \text{...... (3.2)}$$

as a trial solution of the dual integral equations (2.8) and (2.9), we find that the equation (2.8) is identically satisfied and equation (2.9) reduces to

$$k \int_a^\infty \frac{\tanh(kt)\, \phi(t)\, dt}{\tanh^2(kt) - \tanh^2(kx)} = \operatorname{Cosh}^2(kx)\, p(x);\ x > a, \qquad \text{..... (3.3)}$$

where $k = \pi/2h$. Now we change the following variables

$\alpha = \tanh^2(kx)$, $\beta = \tanh^2(ak)$, $\upsilon = \tan^2(kt)$,

we find that the equation (3.3) reduces to

$$\frac{1}{\pi} \int_\beta^1 \frac{\psi(\nu)\, d\upsilon}{(\nu - \alpha)} = \chi(\alpha) \qquad \text{..... (3.4)}$$

where $\phi(t) = \operatorname{Sec}^2(kt)\ \psi(\tanh^2 kt)$,

$$p(x) = -\frac{\pi}{2} \operatorname{Sec}^2(kx)\, \chi(\tan^2 kx),$$

then the equation (3.4) inverts to

$$\psi(\nu) = \frac{M}{[(1-\nu)(\nu-\beta)]^{1/2}} - \frac{1}{\pi}\int_{\beta}^{1} \frac{N(\alpha)\,\chi(\alpha)\,d\alpha}{N(\nu)(\alpha-\nu)}, \quad \beta < \nu < 1$$

where M is an arbitrary constant and N is defined as following :

$$N(\nu) = \sqrt{[\nu-\beta)/(1-\nu)]} \qquad \text{..... (3.5)}$$

Now reverting the original variables, the integral equation (3.3) may be written as :

$$\phi(t) = -\frac{4k\sqrt{(1-\tanh^2 kt)}\,\mathrm{Sech}^2(kt)}{\pi^2\sqrt{(\tanh^2 k) - \tanh^2 ka)}}$$

$$\times \int_a^{\infty}\left[p(x)\sqrt{\left(\frac{\tanh^2(kx) - \tanh^2(ka)}{1-\tanh^2 kx}\right)}\cdot\frac{\tanh(kx)}{\tanh^2 kx - \tanh^2 kt}\right]dx$$

$$+ \frac{2kM_1\,\mathrm{Sec}^2(kt)}{\sqrt{[\tanh^2 kt - \tanh^2 ka]}}, \quad a < t < \infty, \qquad \text{...... (3.6)}$$

where M_1 is an arbitrary constant. Now using the equations (3.2) and (3.6), we have

$$M_1 = \frac{k}{\pi^2 F[\pi/2, \mathrm{Sech}\,ka]}\int_a^{\infty}\frac{\mathrm{Sech}^2(kt)\sqrt{[1-\tanh^2 kt]}}{\sqrt{(\tanh^2 kt - \tanh^2 ka)}}\,dt$$

$$\times \int_a^{\infty}\frac{2\tanh(kx)\,p(x)}{[\tanh^2 kx - \tanh^2 kt]}\sqrt{\left[\frac{\tanh^2 kx - \tanh^2 ka}{1-\tanh^2 kx}\right]}\,dx, \qquad \text{..... (3.7)}$$

where F (π /2, Sech ka) denotes the elliptic integral of the second kind. Hence the complete solution of dual integral equations (2.8) and (2.9) have been derived. By assigning a value of p(x), we can finally solve the above integral.

Now we shall find the solution of dual integral equations (2.10) & (2.11)

Let

$$B(\xi) = B_1(\xi) + B_2(\xi),$$

where

$$B_1(\xi) = \sqrt{(\pi/2)}\tanh(\xi h)\int_a^\infty q(u)\,\mathrm{Sin}(\xi u)\,du,$$

$$F_s[\xi^{-1} B_2(\xi); \xi \to x] = q_2(x),\ 0 \le x < a \qquad \text{..... (3.8)}$$

$$F_s[\coth(\xi h) B_2(\xi); \xi \to x] = 0,\ x > a, \qquad \text{..... (3.9)}$$

where

$$q_2(x) = -F_s[\xi^{-1} B_1(\xi); \xi \to x]$$

$$= -\int_a^\infty q(u)\,du\int_0^\infty \xi^{-1}\tanh(\xi h)\,\mathrm{Sin}(\xi x)\,\mathrm{Sin}(\xi u)\,d\xi$$

$$= -\frac{1}{2}\int_0^\infty q(u)\log\left[\frac{\mathrm{Sinh}(ku) + \mathrm{Sinh}(kx)}{\mathrm{Sinh}(ku) - \mathrm{Sinh}(kx)}\right]du \qquad \text{..... (3.10)}$$

Let

$$B_2(\xi) = \sqrt{(\pi/2)}\tanh(\xi h)\int_0^a g(t).\,\mathrm{Sin}(\xi t)\,dt. \qquad \text{..... (3.11)}$$

By substituting the value of $B_2(\xi)$ from equation (3.11) into (3.9), we see that equation (3.9) is identically satisfied. Again equation (3.8) can be reduced with the help of equation (3.11) as following :

$$\frac{1}{\pi}\int_0^a b(t)\log\left[\frac{\text{Sinh } kt + \text{Sinh } kx}{\text{Sinh } kt - \text{Sinh } kx}\right]dt = q_2(x),\ 0 < x < a \qquad \text{..... (3.12)}$$

Again the above integral equation can be written with the help of the result Cooke ([15] pp 13) :

$$b(t) = -\frac{2\,k\text{Sinh }(kt)\text{ Cosh }(kt)}{\pi\sqrt{(\text{Sinh}^2ka - \text{Sinh}^2kt)}}\int_0^a \frac{\sqrt{(\text{Sinh}^2ka - \text{Sinh}^2kx)}\; q_2'(x)}{(\text{Sinh}^2kx - \text{Sinh}^2kt)}\,dx \qquad \text{..... (3.13)}$$

where the prime in $q_2(x)$ denotes the differentiation with respect to x. Then from equation (3.10), we find that

$$\frac{dq_2(x)}{dx} = \frac{2\,k\text{ Cosh }(kx)}{\pi}\int_a^\infty \frac{q(u)\text{ Sinh }(ku)\,du}{(\text{Sinh}^2kx - \text{Sinh}^2\,ku)} \qquad \text{..... (3.14)}$$

Hence the following integral can be written in the form

$$\int_0^a \frac{\sqrt{[\text{Sin}^2ka - \text{Sinh}^2hx]}\; q_2'(x)}{[\text{Sinh}^2\,kx - \text{Sinh}^2kt]}\,dx\,.$$

$$= \frac{1}{\pi}\int_a^\infty q(u)\text{ Sinh }(ku)\,du\int_0^\infty \frac{2\,k\text{ Cosh}(ku)\,[\text{Sinh}^2ka - \text{Sin}^2kx)]^{1/2}\,dx}{(\text{Sinh}^2kx - \text{Sinh}^2\,kt)\,(\text{Sinh}^2kx - \text{Sinh}^2\,ku)}\,.$$

.... (3.15)

Now using, the result of Gradshteyn and Ryzhik [16], the above integral changes to

$$\int_0^a \frac{2\,k\text{ Cosh}(kx)\,(\text{Sinh}^2ka - \text{Sinh}^2kx)\,dx}{(\text{Sinh}^2kx - \text{Sin}^2kt)\,(\text{Sinh}^2kx - \text{Sinh}^2ku)}$$

$$= \frac{\pi\sqrt{(\text{Sinh}^2 ku - \text{Sinh}^2 ka)}}{\text{Sinh}(ku)\,(\text{Sinh}^2 ku - \text{Sin}^2 kt)} \qquad \text{..... (3.16)}$$

Using the equations (3.15), (3.16), then equation (3.13) reduces to :

$$b(t) = -\frac{k\text{Sinh}(2kt)}{\pi\sqrt{[\text{Sinh}^2 ka - \text{Sinh}^2 kt]}} \cdot \int_0^\infty \frac{q(u)\,[\text{Sinh}^2 ku - \text{Sinh}^2 ka]^{1/2}}{[\text{Sinh}^2 ku - \text{Sin}^2 kt]}\,du$$

$$0 < t < a \qquad \text{..... (3.17)}$$

Hence the problem is completly solved.

4. EXPRESSON FOR STRESS INTENSITY FACTOR.

If we choose

$$q(u) = R\ \delta\ (b - u),\ b > a \qquad \text{..... (4.1)}$$

where R is constant and δ is Dirac delta function, then the shear stress can be written as

$$\sigma_{yz}(x, 0, z) = \frac{kR\ \text{Sinh}(2kx)\sqrt{[\text{Sinh}^2 kb - \text{Sinh}^2 ka]}}{2\sqrt{[\text{Sinh}^2 ka - \text{Sin}^2 kx]\,[\text{Sinh}^2 kb - \text{Sin}^2 kx]}},$$

$$0 < x < a \qquad \text{..... (4.2)}$$

Now we shall derive stress intensity factor when $p(x) = 0$ and $q(x) = p\ \delta\ (b - x)$. Then the stress intensity factor can be calculated by using the formula

$$K = \lim_{x \to a} [\{2(a - x)\}^{1/2}\ \sigma_{yz}(x, 0, z)]$$

$$K = R\sqrt{(k / \text{Sinh}\ ka)}\ \frac{\sinh(ka)}{\sqrt{[\text{Sinh}^2 kb - \text{Sinh}^2 ka]}} \qquad \text{..... (4.3)}$$

Thus the exact value of stress intensity factor is known when the shearing stress is an odd function and even function is supposed to be zero.

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