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Estimation of error variance under the LINEX loss function

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Abstract

The aim of this paper is to examine the behavior of Stein rule and usual estimator for the regression error variance using small sigma asymptotic under asymmetric loss function, which is called linear exponential (LINEX) loss function, in a linear regression model using the criterion of average loss. The expressions of the risk of estimators under the LINEX loss function are derived under small sigma asymptotics. Furthermore, a simulation study and numerical illustration are carried out for evaluating the risk performances of estimators of disturbance variance under the LINEX loss function.

Keywords Stein rule estimator, ordinary least squares estimator, error variance, LINEX loss function, small sigma asymptotics.
Mathematics Subject Classification Primary 62J05; Secondary 62J07.

1 Introduction

Zellner and Geisel[1], Aitchison and Dunsmore[2] and Berger[3] studied statistical properties and prediction analysis of some estimators, and suggested that the use of symmetric loss function may not be appropriate in some estimation and prediction problems. It means that a given positive error may be more serious than a given negative error of the same direction or vice versa. Keeping this in mind, Varian[4] introduced a very useful asymmetric linear loss function called the LINEX loss function that is approximately exponential on one side of zero and approximately linear on the other side. Much of literature is available where the LINEX loss function has been considered as a comparison criterion for comparing competing estimators in linear regression model. To cite a few, Zellner[5] examined the performance of least squares estimator of regression coefficient in a regression model using Bayesian approach under asymmetric loss function and Srivastava and Rao[6] studied the performance properties of some conventional estimators of error variance under asymmetric loss function. Ohtani[7] compared the risk performance of the feasible generalized ridge estimator with that of the ordinary least squares (OLS) estimator under asymmetric loss function. Giles and Giles[8] derived the risk of a pre-test estimator of the error variance under LINEX loss function. Further, Akdeniz[9] examined a new class of generalized Liu estimator which is biased and dominates the OLS estimator for large values of shape parameter under the LINEX loss function. Zou *et al*.[10] derived the necessary and sufficient condition for critical value of pretesting for exact restriction on the coefficients in linear regression model to minimize the risk of a pre-test estimator under asymmetric loss and squared error loss function. Recently, Shalabh *et al*.[11] evaluated the risk performances of the family of feasible generalized double k-class estimators under the LINEX loss function in linear regression model with non-spherical errors.

An estimation of a linear regression model consists of estimation of unknown parameters i.e., estimation of regression coefficients and error variance which are generally unknown. With regards to the estimation of disturbance variance, there are number of papers dealing with it. For instance, Ohtani[12], Dube and Chandra[13] examined the effect of Stein rule estimator of the disturbance variance when disturbances are normally distributed under the mean squared error criterion. It was observed that the Stein rule estimator is not only biased but also dominated by its counterparts stemming from the OLS estimator. Ünal[14] has shown that in a regression model with proxy variables, the mean squared error of the Stein rule estimator of the disturbance variance is greater than that of the disturbance variance related to the OLS estimator under certain conditions. Here, we will investigate whether the dominance of OLS based estimator of disturbance estimator over Stein rule based estimator still holds when compared under the LINEX loss function using small sigma asymptotics. The plan of the paper is as follows: Section 2 describes the model and the estimators. Section 3 provides the approximate form of the risk function of the estimators of disturbance variance under the LINEX loss function. The risk comparison of the estimators of disturbance variance is discussed with the help of a simulation study and numerical illustration in Sections 4 and 5, respectively. Lastly, summary of results is presented in Section 6. In appendix, a brief derivation of the risk function of estimators under the LINEX loss function is given.

2 The Model and the estimators

Consider a linear regression model

$$y = X\beta + u \tag{2.1}$$

where y is an $n \times 1$ vector of n observations on the dependent variable, X is a full column rank $n \times p$ matrix of n observations on p explanatory variables, β is a $p \times 1$ vector of regression coefficients and u is an $n \times 1$ vector of disturbances that are assumed to be independently and identically normally distributed with mean zero and unknown variance $\sigma^2 I$.

Application of least squares to (2.1) gives the OLS estimator of β, given by

$$b = (X'X)^{-1} X'y \tag{2.2}$$

An estimator of σ^2 based on the OLS estimator of β is

$$s^2 = \frac{1}{m}(y - Xb)'(y - Xb) \tag{2.3}$$

where m is any arbitrary scalar. It is well known that the OLS estimator is optimal in the class of linear and unbiased estimators. However, if we drop the linearity and unbiasedness, there exist estimators which perform better than the OLS estimator under the risk criterion. One such interesting and popularly known estimator of β is Stein- rule estimator which was proposed by James and Stein [15] and is given by

$$b_s = \left[1 - k\left(\frac{(y - Xb)'(y - Xb)}{b'X'Xb}\right)\right]b \qquad (2.4)$$

where k is a non stochastic scalar such that $0 \le k \le 2(p-2)/n - p + 2$ which is also called a shrinkage parameter.

The Stein rule estimator of σ^2 based on the Stein rule estimator of β which is also called an iterative estimator of error variance, is constructed as

$$\hat{\sigma}_s^2 = \frac{1}{m}(y - Xb_s)'(y - Xb_s). \qquad (2.5)$$

Following Ohtani[12] and Dube and Chandra[13], it can be concluded that the OLS based estimator of error variance is an unbiased estimator of σ^2 if $m = n - p$ and has minimum mean squared error for $m = n - p + 2$. However, the Stein rule based estimator of error variance is not only asymptotically biased if we choose $m = n - p$ but also dominated by the usual estimator of disturbance variance under small sigma asymptotics for $k > 0$.

3 Risk of the estimators of error variance under the LINEX loss function

In case where under-estimation is more serious than over-estimation or in other words, when positive and negative estimation errors have different results, in such situations, a strong fine should be put on under-estimation. This fine can be obtained by using an asymmetric loss function called LINEX loss function, which was developed by Varian[4] and is defined as

$$L(\varepsilon) = c[exp(a\varepsilon) - (a\varepsilon) - 1] \qquad (3.1)$$

where $a(\neq 0)$ is called the shape parameter and $c(\geq 0)$ determines the scale of the loss function and ε is the estimation error. From (3.1), it may be noted that larger the value of a, faster the loss increases. When $a > 0$, the larger loss is incurred for over-estimation. When $a < 0$, a larger loss is incurred for under-estimation. For small values of a, the LINEX loss function is approximated as a squared loss function. Thus, selecting appropriate values of a one can assign desired unequal weights to the positive and negative estimation errors. c is the proportionality factor and without any loss of generality it is assumed to be one.

Now, define an estimation error as

$$\varepsilon = (\hat{\sigma}^2 - \sigma^2) \qquad (3.2)$$

where $\hat{\sigma}^2$ is an estimator of σ^2. The risk function associated with $\hat{\sigma}^2$ under the LINEX loss function is obtained as

$$R(\varepsilon) = E(L(\varepsilon)). \qquad (3.3)$$

In order to derive the risk function of an estimator of disturbance variance based on the OLS estimator and Stein rule estimator, small sigma asymptotic is used. Assuming σ to be small is justifiable from the fact that if it is large, the model in (2.1) will not be well explained by the explanatory variables. This technique was first introduced by Kadane[16] and later used by many researchers, for instance, see Shalabh and Toutenburg[17], Dube and Chandra[13], and Qain and Giles[18]. The performance of the estimators is analyzed with respect to the criterion of risk under the LINEX loss function. The risk associated with the estimators of the disturbance variance based on the OLS estimator and Stein rule estimator under the LINEX loss function to the order o(σ^6) (see [13], [17]), is given in the form of the following theorems, respectively.

Theorem 3.1 *The risk function associated with the estimator* s^2 *under the LINEX loss function is given by*

$$R(\varepsilon_1) = \left(exp(-a\sigma^2)\right)\left[1 + \frac{a\sigma^2}{m}(n-p) + \frac{a^2\sigma^4}{2m^2}(n-p)(n-p+2) + \frac{a^3\sigma^6}{6m^3}(n-p)(n-p+2)(n-p+4)\right] - \frac{a\sigma^2}{m}(n-p-m) - 1. \quad (3.4)$$

where $\varepsilon_1 = (s^2 - \sigma^2)$.

Proof: *(See Appendix).*

Theorem 3.2 *The risk function associated with the estimator* $\hat{\sigma}_s^2$ *under the LINEX loss function is given by*

$$R(\varepsilon_2) = \left(exp(-a\sigma^2)\right)\left[1 + \frac{a\sigma^2}{m}(n-p) + \frac{ak^2\sigma^4}{m\beta'X'X\beta}(n-p)(n-p+2) + \frac{a^2\sigma^4}{2m^2}(n-p)(n-p+2) + \frac{a^2k^2\sigma^6}{m^2\beta'X'X\beta}(n-p)(n-p+2)(n-p+4) + \frac{a^3\sigma^6}{6m^3}(n-p)(n-p+2)(n-p+4)\right] - \frac{a\sigma^2}{m}(n-p-m) - \frac{ak^2\sigma^4}{m\beta'X'X\beta}(n-p)(n-p+2) - 1. \quad (3.5)$$

where $\varepsilon_2 = (\hat{\sigma}_s^2 - \sigma^2)$.

Proof: *(See Appendix).*

If a is small, using the infinite series expansion of the exponential function in (3.4) and (3.5) reduces the risk expressions to their quadratic loss counterparts neglecting the terms of third and higher order of a in the expansion, which are similar to those obtained by Dube and Chandra[13].

It is now a question of interest to obtain the optimum value of k for which the risk in (3.5) takes the minimum value. However, it is obvious from the risk expression in (3.5) that the value of k cannot be obtained by the derivative method, for which the risk of $\hat{\sigma}_s^2$ is minimized. The role of the shrinkage parameter in explaining the variations in the risk function in (3.5) is discussed through empirical results.

To check whether the dominance of s^2 over $\hat{\sigma}_s^2$ also holds under LINEX loss function, we compare (3.4) and (3.5), and find that

$$R(\varepsilon_2) - R(\varepsilon_1) \geq 0$$

if

$$exp(-a\sigma^2)\left[1 + \frac{a\sigma^2(n-p+4)}{m}\right] \geq 1 \quad (3.6)$$

provided $a \geq 0$ for $\sigma^2 \leq 1$, otherwise

$$exp(-a\sigma^2)\left[1 + \frac{a\sigma^2(n-p+4)}{m}\right] < 1 \quad (3.7)$$

for $\sigma^2 \geq 0.$

The risk functions in (3.4) and (3.5) are approximated to the order o(σ^6), therefore, the dominance condition obtained are not general dominance condition under the LINEX loss function. The dominance conditions obtained in (3.6) and (3.7) are verified numerically.

4 Simulation Study

In this section, the risk of $\hat{\sigma}_s^2$ is compared with the risk of s^2 under the LINEX loss function through a simulation study. In this study, the independent variables and error terms are generated from a standard normal distribution. The parametric values considered here are as follows, p = 4, 8; n=20, 100, and a = -0.6,-0.5,-0.2,0.2,0.3,0.5,0.6. Since for small values of a, the risk expressions in (3.4) and (3.5) reduces to the risk under squared error loss function, therefore, the range of k is chosen to be in the domain $0 \leq k \leq 2(p-2)/(n-p+2)$. The value of true error variance is taken to be 0.01, 0.05 and 0.1 in this simulation. The expression for the calculation of risk under the LINEX loss function is given by

$$R(\varepsilon) = \frac{1}{M}\sum_{i=1}^{M}[exp(a\varepsilon_i) - a\varepsilon_i - 1] \qquad (4.1)$$

where $\varepsilon_i = (\hat{\sigma}_i^2 - \sigma^2)$ and $\hat{\sigma}_i^2$ is an estimator of the error variance. The number of replications, M, is taken as 5000 and the results obtained are reported in the Tables 1-4.

It can be seen from the Tables 1-4, the risks of both the estimators do not show much difference for various parametric variations considered here. However, after a careful examination of the risk values, difference could be noticed at the second or third decimal places of the risk values. It may also be noted that the risk values increases as sample size increases and increase with the increase in the value of p. As expected, increase in the values of σ^2 increases the risk values of both the estimators of error variance. As regards variation in values of *a*, it can be clearly seen that as the loss function departs from symmetry, the risk under the LINEX loss function increases and it may be noted that over-estimation results in slightly higher risk than under-estimation.

5 Numerical Illustration

In this section, a numerical illustration has been provided to compare the risk performance of $\hat{\sigma}_s^2$ with that of s^2 under the LINEX loss function using average loss criterion. For illustration, data is taken from the report of National Sample Survey 50th round conducted in 1993-1994 on demand of potatoes in India in 1992-93 (see Gujarati *et al* .[19]), which comprises the response variable y to be per capita consumption of potatoes in kg, and four explanatory variables which are X_1: income per capita in thousand rupees at 1993-94 prices, X_2: prices of potatoes in rupees per kg, X_3: price of cabbage in rupees per kg and X_4: price of cauliflower in rupees per kg. The least squares estimation process is used to estimate the regression of y on explanatory variables. Considering the fact that the risk expressions obtained in (3.4) and (3.5) are approximated the order o(σ^6) , the numerical results are obtained for $\sigma^2 \leq 0.1$. All the numerical results are reported in Table 5.

The results based on the data set suggest that the difference in the risks of $\hat{\sigma}_s^2$ and s^2 looks substantial if observed upto the third and fourth decimal places. Also, increase in the chosen values of k and σ^2, increases the risk of both the estimators.

From Table 5, it can be observed that the risk values of s^2 is smaller than that of $\hat{\sigma}_s^2$. However, the dominance of $\hat{\sigma}_s^2$ over s^2 can be seen for $\sigma^2 = 0.05$ and 0.1; $k = 0.05$ (shown in the bold) for this data set.

6 Conclusion

In this paper, we have obtained the approximate expression for the risks of Stein rule and usual estimator for regression error variance under the LINEX loss function using small sigma asymptotics in a linear regression model. The risk performance of both the estimators is examined with the help of a simulation study and numerical illustration. It has been found that for moderate asymmetry in the loss function, the difference in the risks of the estimators considered here for estimating the error variance is not very substantial. In simple words it can be said that, in estimation of disturbance variance in linear regression model under the LINEX loss function working with Stein rule estimator is not a fruitful strategy than working with the usual estimator.

Table 1: Simulation results for $p = 4$, $n = 20$ and $0 \leq k \leq 0.22$

		$\sigma^2 = 0.01$		$\sigma^2 = 0.05$		$\sigma^2 = 0.1$	
k	*a*	*R_OLS*	*R_Stein*	*R_OLS*	*R_Stein*	*R_OLS*	*R_Stein*

0.05	-0.6	0.0229125	0.0229142	0.5716119	0.5718292	2.2805950	2.2823100
	-0.5	0.0159128	0.0159141	0.3971248	0.3972760	1.5850890	1.5862850
	-0.2	0.0025467	0.0025469	0.0636233	0.0636477	0.2542705	0.2544644
	0.2	0.0025476	0.0025478	0.0637363	0.0637608	0.2551739	0.2553712
	0.3	0.0057327	0.0057331	0.1434708	0.1435262	0.5746610	0.5751067
	0.5	0.0159270	0.0159282	0.3988893	0.3990438	1.5992060	1.6004550
	0.6	0.0229369	0.0229386	0.5746610	0.5748840	2.3049920	2.3067980
0.15	-0.6	0.0229125	0.0229283	0.5716119	0.5735718	2.2805950	2.2960980
	-0.5	0.0159128	0.0159238	0.3971248	0.3984888	1.5850890	1.5959020
	-0.2	0.0025467	0.0025485	0.0636233	0.0638430	0.2542705	0.2560231
	0.2	0.0025476	0.0025494	0.0637363	0.0639578	0.2551739	0.2569572
	0.3	0.0057327	0.0057367	0.1434708	0.1439704	0.5746610	0.5786908
	0.5	0.0159270	0.0159380	0.3988893	0.4002832	1.5992060	1.6104980
	0.6	0.0229369	0.0229528	0.5746610	0.5766725	2.3049920	2.3213230
0.22	-0.6	0.0229125	0.0229465	0.5716119	0.5758401	2.2805950	2.3141380
	-0.5	0.0159128	0.0159365	0.3971248	0.4000674	1.5850890	1.6084840
	-0.2	0.0025467	0.0025505	0.0636233	0.0640972	0.2542705	0.2580626
	0.2	0.0025476	0.0025514	0.0637363	0.0642143	0.2551739	0.2590323
	0.3	0.0057327	0.0057412	0.1434708	0.1445487	0.5746610	0.5833803
	0.5	0.0159270	0.0159507	0.3988893	0.4018964	1.5992060	1.6236380
	0.6	0.0229369	0.0229711	0.5746610	0.5790006	2.3049920	2.3403280

Table 2: Simulation results for $p = 4$, $n = 100$ and $0 \leq k \leq 0.04$

		$\sigma^2 = 0.01$		$\sigma^2 = 0.05$		$\sigma^2 = 0.1$	
k	*a*	*R_OLS*	*R_Stein*	*R_OLS*	*R_Stein*	*R_OLS*	*R_Stein*
0.02	-0.6	0.0037547	0.0037548	0.0938434	0.0938496	0.3752542	0.3753035
	-0.5	0.0026075	0.0026075	0.0651725	0.0651768	0.2606205	0.2606548
	-0.2	0.0004172	0.0004172	0.0104293	0.0104300	0.0417127	0.0417182
	0.2	0.0004172	0.0004172	0.0104316	0.0104323	0.0417311	0.0417367
	0.3	0.0009388	0.0009388	0.0234724	0.0234740	0.0939057	0.0939183
	0.5	0.0026078	0.0026078	0.0652086	0.0652130	0.2609092	0.2609445
	0.6	0.0037552	0.0037553	0.0939057	0.0939120	0.3757530	0.3758040
0.03	-0.6	0.0037547	0.0037548	0.0938434	0.0938573	0.3752542	0.3753651
	-0.5	0.0026075	0.0026075	0.0651725	0.0651822	0.2606205	0.2606978
	-0.2	0.0004172	0.0004172	0.0104293	0.0104309	0.0417127	0.0417251
	0.2	0.0004172	0.0004172	0.0104316	0.0104332	0.0417311	0.0417437
	0.3	0.0009388	0.0009388	0.0234724	0.0234760	0.0939057	0.0939341
	0.5	0.0026078	0.0026078	0.0652086	0.0652185	0.2609092	0.2609886
	0.6	0.0037552	0.0037553	0.0939057	0.0939199	0.3757530	0.3758677
0.04	-0.6	0.0037547	0.0037549	0.0938434	0.0938682	0.3752542	0.3754515
	-0.5	0.0026075	0.0026076	0.0651725	0.0651898	0.2606205	0.2607579
	-0.2	0.0004172	0.0004172	0.0104293	0.0104321	0.0417127	0.0417348
	0.2	0.0004172	0.0004172	0.0104316	0.0104344	0.0417311	0.0417536
	0.3	0.0009388	0.0009388	0.0234724	0.0234787	0.0939057	0.0939563
	0.5	0.0026078	0.0026079	0.0652086	0.0652261	0.2609092	0.2610505
	0.6	0.0037552	0.0037554	0.0939057	0.0939310	0.3757530	0.3759571

Table 3: Simulation results for $p = 8$, $n = 20$ and $0 \le k \le 0.86$

		$\sigma^2 = 0.01$		$\sigma^2 = 0.05$		$\sigma^2 = 0.1$	
k	*a*	*R_OLS*	*R_Stein*	*R_OLS*	*R_Stein*	*R_OLS*	*R_Stein*

0.05	-0.6	0.0305086	0.0305096	0.7605571	0.7606866	3.0317180	3.0327330
	-0.5	0.0211890	0.0211898	0.5284739	0.5285641	2.1077690	2.1084780
	-0.2	0.0033915	0.0033916	0.0847057	0.0847202	0.3384222	0.3385372
	0.2	0.0033931	0.0033932	0.0849089	0.0849236	0.3400477	0.3401650
	0.3	0.0076354	0.0076356	0.1911606	0.1911938	0.7660435	0.7663088
	0.5	0.0212144	0.0212152	0.5316489	0.5317413	2.1331730	2.1339180
	0.6	0.0305525	0.0305535	0.7660435	0.7661770	3.0756210	3.0766980
0.55	-0.6	0.0305086	0.0306366	0.7605571	0.7764301	3.0317180	3.1576030
	-0.5	0.0211890	0.0212780	0.5284739	0.5395242	2.1077690	2.1956230
	-0.2	0.0033915	0.0034057	0.0847057	0.0864870	0.3384222	0.3526899
	0.2	0.0033931	0.0034074	0.0849089	0.0867080	0.3400477	0.3546036
	0.3	0.0076354	0.0076675	0.1911606	0.1952188	0.7660435	0.7989592
	0.5	0.0212144	0.0213038	0.5316489	0.5429780	2.1331730	2.2255310
	0.6	0.0305525	0.0306812	0.7660435	0.7823983	3.0756210	3.2092890
0.86	-0.6	0.0305086	0.0308227	0.7605571	0.8000726	3.0317180	3.3504920
	-0.5	0.0211890	0.0214073	0.5284739	0.5559838	2.1077690	2.3302540
	-0.2	0.0033915	0.0034264	0.0847057	0.0891404	0.3384222	0.3745615
	0.2	0.0033931	0.0034281	0.0849089	0.0893883	0.3400477	0.3769271
	0.3	0.0076354	0.0077142	0.1911606	0.2012648	0.7660435	0.8494458
	0.5	0.0212144	0.0214337	0.5316489	0.5598575	2.1331730	2.3672240
	0.6	0.0305525	0.0308684	0.7660435	0.8067666	3.0756210	3.4143850

Table 4: Simulation results for $p = 8$, $n = 100$ and $0 \le k \le 0.13$

		$\sigma^2 = 0.01$		$\sigma^2 = 0.05$		$\sigma^2 = 0.1$	
k	*a*	*R_OLS*	*R_Stein*	*R_OLS*	*R_Stein*	*R_OLS*	*R_Stein*

0.02	-0.6	0.0038673	0.0038674	0.0966573	0.0966618	0.3865032	0.3865387
	-0.5	0.0026857	0.0026857	0.0671268	0.0671299	0.2684338	0.2684585
	-0.2	0.0004297	0.0004297	0.0107421	0.0107426	0.0429636	0.0429675
	0.2	0.0004297	0.0004297	0.0107445	0.0107450	0.0429831	0.0429871
	0.3	0.0009669	0.0009669	0.0241766	0.0241777	0.0967232	0.0967323
	0.5	0.0026860	0.0026860	0.0671650	0.0671681	0.2687389	0.2687643
	0.6	0.0038679	0.0038679	0.0967232	0.0967277	0.3870303	0.3870670
0.06	-0.6	0.0038673	0.0038677	0.0966573	0.0966977	0.3865032	0.3868232
	-0.5	0.0026857	0.0026859	0.0671268	0.0671549	0.2684338	0.2686567
	-0.2	0.0004297	0.0004298	0.0107421	0.0107466	0.0429636	0.0429995
	0.2	0.0004297	0.0004298	0.0107445	0.0107491	0.0429831	0.0430194
	0.3	0.0009669	0.0009670	0.0241766	0.0241868	0.0967232	0.0968052
	0.5	0.0026860	0.0026862	0.0671650	0.0671934	0.2687389	0.2689680
	0.6	0.0038679	0.0038682	0.0967232	0.0967642	0.3870303	0.3873612
0.13	-0.6	0.0038673	0.0038689	0.0966573	0.0968478	0.3865032	0.3880206
	-0.5	0.0026857	0.0026867	0.0671268	0.0672593	0.2684338	0.2694905
	-0.2	0.0004297	0.0004299	0.0107421	0.0107634	0.0429636	0.0431340
	0.2	0.0004297	0.0004299	0.0107445	0.0107659	0.0429831	0.0431555
	0.3	0.0009669	0.0009673	0.0241766	0.0242248	0.0967232	0.0971121
	0.5	0.0026860	0.0026870	0.0671650	0.0672993	0.2687389	0.2698252
	0.6	0.0038679	0.0038694	0.0967232	0.0969169	0.3870303	0.3885990

Table 5: Numerical results for $0 \leq k \leq 0.22$

		$\sigma^2 = 0.01$		$\sigma^2 = 0.05$		$\sigma^2 = 0.1$	
k	*a*	*R_OLS*	*R_Stein*	*R_OLS*	*R_Stein*	*R_OLS*	*R_Stein*

0.05	-0.6	0.0224877	0.0224993	0.5689214	**0.5619958**	2.3018732	**2.2416519**
	-0.5	0.0156180	0.0156260	0.3953018	**0.3905606**	1.6007231	**1.5600311**
	-0.2	0.0024996	0.0025008	0.0633286	**0.0626024**	0.2567759	**0.2507939**
	0.2	0.0025004	0.0025017	0.0633811	**0.0626971**	0.2567848	**0.2514782**
	0.3	0.0056263	0.0056292	0.1426158	**0.1411001**	0.5774408	**0.5658693**
	0.5	0.0156310	0.0156388	0.3961318	**0.3920501**	1.6011536	**1.5710203**
	0.6	0.0225102	0.0225214	0.5703639	**0.5645782**	2.3028815	**2.2609100**
0.15	-0.6	0.0224877	0.0227015	0.5689214	0.5891002	2.3018732	2.4773629
	-0.5	0.0156180	0.0157660	0.3953018	0.4091158	1.6007231	1.7192968
	-0.2	0.0024996	0.0025230	0.0633286	0.0654444	0.2567759	0.2742044
	0.2	0.0025004	0.0025236	0.0633811	0.0653737	0.2567848	0.2722414
	0.3	0.0056263	0.0056784	0.1426158	0.1470309	0.5774408	0.6111426
	0.5	0.0156310	0.0157747	0.3961318	0.4080200	1.6011536	1.6889029
	0.6	0.0225102	0.0227165	0.5703639	0.5872149	2.3028815	2.4250938
0.22	-0.6	0.0224877	0.0225510	0.5689214	0.5689214	2.3018732	2.3018732
	-0.5	0.0156180	0.0156618	0.3953018	0.3953018	1.6007231	1.6007231
	-0.2	0.0024996	0.0025065	0.0633286	0.0633286	0.2567759	0.2567759
	0.2	0.0025004	0.0025073	0.0633811	0.0633811	0.2567848	0.2567848
	0.3	0.0056263	0.0056417	0.1426158	0.1426158	0.5774408	0.5774408
	0.5	0.0156310	0.0156735	0.3961318	0.3961318	1.6011536	1.6011536
	0.6	0.0225102	0.0225712	0.5703639	0.5703639	2.3028815	2.3028815

Appendix:

Proof of the Theorem 3.1:

For the application of small sigma asymptotic approximations, consider the model in (2.1) as

$$y = X\beta + u \qquad (6.1)$$

where,

$$u = \sigma w$$

so that w follows a multivariate normal distribution having mean vector zero and variance covariance matrix I_n. We know that

$$E(w'Aw) = (trA)$$

$$E(w'Aw)^2 = (trA)(trA + 2)$$

$$E(w'Aw)^3 = (trA)((trA)+2)((trA)+4)$$

where A is any $n \times n$ symmetric matrix with non-stochastic elements.

Using (2.2) and $u = \sigma w$, we get

$$b'X'Xb = \beta'X'X\beta + 2\sigma\beta'X' + \sigma^2 w'P_x w \tag{6.2}$$

and

$$(b'X'Xb)^{-1} = \frac{1}{\beta'X'X\beta}\left[1 + 2\sigma\frac{\beta'X'w}{\beta'X'X\beta} + \sigma^2\frac{w'P_x w}{\beta'X'X\beta}\right]^{-1}. \tag{6.3}$$

Expanding and retaining terms to $o(\sigma)$ gives,

$$(b'X'Xb)^{-1} = \frac{1}{\beta'X'X\beta}\left[1 - 2\sigma\frac{\beta'X'w}{\beta'X'X\beta}\right]. \tag{6.4}$$

We notice from (2.3) that,

$$s^2 = \frac{1}{m} y'\overline{P}_x y \tag{6.5}$$

where $\overline{P}_x = 1 - X(X'X)^{-1}X'$.

From (2.3) and $u = \sigma w$, we can also write

$$s^2 = \frac{\sigma^2}{m} w'\overline{P}_x w, \tag{6.6}$$

so that using (6.4) and (6.6) and neglecting terms heigher than o(σ^4), we can write

$$\frac{s^4}{b'X'Xb} = \frac{\sigma^4}{m^2}\frac{(w'\overline{P}_x w)^2}{\beta'X'X\beta} \tag{6.7}$$

and considering terms up to o(σ^6), we get

$$\frac{s^6}{b'X'Xb} = \frac{\sigma^6}{m^3}\frac{(w'\overline{P}_x w)^3}{\beta'X'X\beta}. \tag{6.8}$$

Now, for the risk associated with the OLS estimator of σ^2 under the LINEX loss function, we have

$$a\varepsilon_1 = a(s^2 - \sigma^2). \tag{6.9}$$

Then, $$E(a\varepsilon_1) = aE(s^2) - a\sigma^2 = \frac{a\sigma^2(n-p)}{m} - a\sigma^2$$

$$= \frac{a\sigma^2}{m}(n-p-m) \tag{6.10}$$

and

$$E(exp(a\varepsilon_1)) = E[exp(-a\sigma^2)exp(as^2)]$$

$$= (exp(-a\sigma^2))E[1 + \frac{as^2}{1!} + \frac{a^2s^4}{2!} + \frac{a^3s^6}{3!} + ...]$$

$$= (exp(-a\sigma^2))[E(1) + \frac{aE(s^2)}{1!} + \frac{a^2E(s^4)}{2!} + \frac{a^3E(s^6)}{3!} + ...]. \tag{6.11}$$

$$E(exp(a\varepsilon_1)) = (exp(-a\sigma^2))\left[1 + \frac{a\sigma^2}{m}(n-p) + \frac{a^2\sigma^4}{2m^2}(n-p)(n-p+2)\right.$$

$$\left. + \frac{a^3\sigma^6}{6m^3}(n-p)(n-p+2)(n-p+4) + ...\right]. \tag{6.12}$$

Considering terms up to o(σ^6), (6.12) can also be written as

$$E(exp(a\varepsilon_1)) = (exp(-a\sigma^2))\left[1+\frac{a\sigma^2}{m}(n-p)+\frac{a^2\sigma^4}{2m^2}(n-p)(n-p+2)\right.$$
$$\left.+\frac{a^3\sigma^6}{6m^3}(n-p)(n-p+2)(n-p+4)\right]. \qquad (6.13)$$

Substituting (6.10) and (6.13) in (3.3), we obtain the expression for the risk of s^2 in (3.4).

Proof of the Theorem 3.2:

Similarly for the risk associated with $\hat{\sigma}_s^2$ under the LINEX loss criterion, we have

$$a\varepsilon_2 = a(\hat{\sigma}_s^2 - \sigma^2). \qquad (6.14)$$

Then,

$$E(a\varepsilon_2) = a\left[E\left(s^2 - \sigma^2 + \frac{k^2ms^4}{b'X'Xb}\right)\right] \qquad (6.15)$$

and

$$E(exp(a\varepsilon_2)) = (exp(-a\sigma^2))E\left[exp\left(as^2 + \frac{ak^2ms^4}{b'X'Xb}\right)\right] \qquad (6.16)$$

$$E(exp(a\varepsilon_2)) = (exp(-a\sigma^2))E\left[1+\frac{1}{1!}\left(as^2+\frac{ak^2ms^4}{b'X'Xb}\right)+\frac{1}{2!}\left(as^2+\frac{ak^2ms^4}{b'X'Xb}\right)^2\right.$$
$$\left.+\frac{1}{3!}\left(as^2+\frac{ak^2ms^4}{b'X'Xb}\right)^3+....\right]. \qquad (6.17)$$

$$E(exp(a\varepsilon_2)) = (exp(-a\sigma^2))E\left[1+\frac{1}{1!}as^2+\frac{1}{1!}\left(\frac{ak^2ms^4}{b'X'Xb}\right)+\frac{1}{2!}a^2s^4+\frac{1}{2!}\left(\frac{ak^2ms^4}{b'X'Xb}\right)^2\right.$$
$$\left.+\frac{1}{2!}\left(\frac{2a^2k^2ms^6}{b'X'Xb}\right)+\frac{1}{3!}a^3s^6+....\right]. \qquad (6.18)$$

$$E(exp(a\varepsilon_2)) = exp(-a\sigma^2)\left[1+\frac{a\sigma^2}{m}(n-p)+\frac{ak^2\sigma^4}{m\beta'X'X\beta}(n-p)(n-p+2)\right.$$
$$+\frac{a^2\sigma^4}{2m^2}(n-p)(n-p+2)+\frac{a^2k^2\sigma^6}{m^2\beta'X'X\beta}(n-p)(n-p+2)(n-p+4)$$
$$\left.+\frac{a^3\sigma^6}{6m^3}(n-p)(n-p+2)(n-p+4)+.......\right]. \qquad (6.19)$$

Considering terms up to o(σ^6), we can rewrite (6.19) as

$$E(exp(a\varepsilon_2)) = exp(-a\sigma^2)\left[1+\frac{a\sigma^2}{m}(n-p)+\frac{ak^2\sigma^4}{m\beta'X'X\beta}(n-p)(n-p+2)\right.$$

$$+\frac{a^2\sigma^4}{2m^2}(n-p)(n-p+2)+\frac{a^2k^2\sigma^6}{m^2\beta'X'X\beta}(n-p)(n-p+2)(n-p+4)$$
$$\left.+\frac{a^3\sigma^6}{6m^3}(n-p)(n-p+2)(n-p+4)\right]. \qquad (6.20)$$

By using (6.15) and (6.20) in (3.3), we obtain the expression for risk of $\hat{\sigma}_s^2$ in (3.5).

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