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On R-regular and R-normal spaces

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Abstract

In this paper we introduce the R-regular and R-normal spaces through the generalized closed set, R-closed sets. we compare it with regular and normal spaces. Also we introduce R-Hausdorff spaces and study its properties in R-regular and R-normal spaces.

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1.Introduction

As a generalization of closed sets, R-closed sets were introduced and studied by the same author[5]. In this paper, we introduce R-regular spaces and R-normal spaces in topological spaces. We obtain several characterizations of R-regular and R-normal spaces. We prove some preservation theorems for R-regular and R-normal spaces.

Throughout this paper(X,τ),(Y,σ) and (Z,η) will always denote topological spaces on which no separation axioms are assumed, unless otherwise mentioned. When A is a subset of (X,τ),cl(A),Int(A) denote the closure, the interior of A.We recall some known definitions needed in this paper.

2.Preliminieries

Definition 2.1: Let (X,τ) be a topological space. A subset A of the space X is said to be

1) Pre open [6] if $A \subseteq \text{Int}(\text{cl}(A))$ and preclosed if $\text{cl}(\text{Int}(A)) \subseteq A$.

2) Semi open [5] if $A \subseteq \text{cl}(\text{Int}(A))$ and semiclosed if $\text{Int}(\text{cl}(A)) \subseteq A$.

3) α -open [10] if $A \subseteq \text{Int}(\text{cl}(\text{Int}(A)))$ and α-closed if $\text{cl}(\text{Int}(\text{cl}(A)) \subseteq A$.

Definition 2.2: Let (X, τ) be a topological space.A subset $A\subseteq X$ is said to be

(i) a generalized closed set [4] (briefly g-closed) if $\text{cl}(A) \subseteq U$ whenever $A\subseteq U$ and U is open in (X,τ);the complement of a g- closed set is called a g-open set.

(ii) an R-closed [5] in (X, τ) if $\alpha\text{cl}(A)\subseteq \text{Int}(U)$ whenever $A\subseteq U$ and U is ω-open in(X, τ).

Definition 2.3: A function $f: (X,\tau)\to(Y,\sigma)$ is called

1. g-continuous [2] if $f^{-1}(V)$ is g-closed in(X,τ) for every closed set V in(Y,σ)
2. ω-continuous [11] if $f^{-1}(V)$ is ω-closed in(X,τ) for everyclosed set V in(Y,σ)
3. R-continuous[1] if $f^{-1}(V)$ is R-closed in (X, τ) for every closed set V of (Y,σ).
4. α-irresolute [8] if $f^{-1}(V)$ is an α-open set in (X,τ) for each α-open set V of (Y,σ).
5. α-quotient map [8] if f is α-continuous and $f^{-1}(V)$ is open set in (X,τ) implies V is an α-open set in (Y,σ).
6. ω-irresolute [11] if $f^{-1}(V)$ is ω-closed in(X,τ) for everyω- closed set V in(Y,σ)
7. α-continuous [13] if $f^{-1}(V)$ is α-closed in(X,τ) for every closed set V in(Y,σ)
8. weakly continuous [7] if for each point $x \in X$ and each open set $V\subseteq Y$ containing f(x), there exists an open set $U\subseteq X$ containing x such that $f(U)\subseteq \text{cl}(V)$.

Definition 2.4: Let x be a point of (X,τ) and V be subset of X.Then V is called a R-neighbourhood of x in (X,τ) if there exist a R-open set U of (X,τ) such that $x\epsilon U\subseteq V$.

Definition 2.5: A space (X,τ) is called

1. a $T_{1/2}$ space [4] if every g-closed set is closed.
2. a T_ω space [12] if every ω-closed set is closed.
3. a T_R-space [1] if every R-closed set is closed.
4. a αT_R-space if R-closed set is α-closed.
5. semi-normal [3](resp. s-normal [5] if for each pair of disjoint semi-closed (resp. closed) sets A and B, their exists disjoint semi-open sets U, V such that $A\subseteq U$ and $B\subseteq V$.

Result 2.6: A space (X,τ) Symmetric if and only if $\{x\}$ is g-closed in (X,τ) for each point x of (X,τ).

Result 2.7: Suppose $B\subseteq A\subseteq X$, B is a R-closed set relative to A and A is open and R-closed in (X,τ). Then B is R-closed in (X,τ).

Recall that a topological space (X,τ) is

1. Compact if every open cover for X has a finite subcover.
2. Hausdorff space if for each pair of distinct points x and y in X, their exist an open neighborhood U of x and an open neighborhood V of y such that $U\cap V=\emptyset$.
3. Regular space if for each point $x\in X$ and for each closed set F in X not containing x, their exist an open neighborhood U of X and an open neighborhood V of F such that $U\cap V=\emptyset$.
4. Normal space if for each pair of disjoint closed sets U and V in X, their exist an open neighborhood U_1 of U and an open neighborhood V_1 of V such that $U_1\cap V_1=\emptyset$.
5. The map $f: (X,\tau)\to(Y,\sigma)$ is open if f(V) is open in (Y,σ) for every open set V in (X,τ).

3. R-regular spaces

Definition 3.1: A space (X,τ) is said to be R-regular if for every R-closed set F and each point $x\notin F$, there exists disjoint α-open sets U and V such that $F\subseteq U$ and $x\in V$.

Example 3.2: Let $X=\{a,b,c\}$ and $\tau=\{X, \emptyset,\{a\},\{b,c\}\}$. Let $F=\{a\}$ and $b\notin F$. Then there exists α-open sets $U=\{a\}$ and $V=\{b,c\}$ such that $F\subseteq U$ and $b\in V$. Thus (X,τ) is R-regular. If $X=\{a,b,c\}$ and $\tau=\{X, \emptyset,\{a\}$, then (X,τ) is not R-regular.

Theorem 3.3: Let (X,τ) be a topological space.Then the following statements are equivalent.

(i) (X,τ) is R-regular.

(ii) For each point and for each R-open neighborhood W of x , there exist an α-open set V of x such that $\alpha cl(V)\subseteq W$.

(iii) For each point $x\in X$ and for each R-closed set F not containing x, there exist an α-open set V of x such that $\alpha cl(V)\cap F=\emptyset$

Proof: (i)⇒(ii) Let W be any R-neighborhood of x.Then there exist an R-open set G such that $x\in G\subseteq W$.Since G^c is R-closed and $x\notin G^c$, by hypothesis there exist α-open sets U and V such that $G^c\subseteq U, x\in V$ and $U\cap V=\emptyset$ and so $V\subseteq U^c$. Now $\alpha cl(V)\subseteq \alpha cl(U^c)=U^c$ and $G^c\subseteq U$.Thus $U^c\subseteq G\subseteq W$.Thus $\alpha cl(V)\subseteq W$.

(ii)⇒(i) Let F be any R-closed set and $x\notin F$.Then $x\in F^c$ and F^c is R-open and so F^c is a R-open neighborhood of x. By hypothesis there exist an α-open set V of x such that $x\in V$ and $\alpha cl(V)\subseteq F^c$. Thus $F\subseteq (\alpha cl(V))^c$. Thus $(\alpha cl(V))^c$ is an α-open set containing F and $V\cap (\alpha cl(V))^c=\emptyset$.Thus X is R-regular.

(ii) ⇒(iii) Let $x\in X$ and F be an R-closed set not containing x.Then F^c is an R-open neighborhood of x and by hypothesis there exist α-open set V of x such that $\alpha cl(V)\subseteq F^c$. Hence $(\alpha cl(V))\cap F=\emptyset$.

(iii)⇒(ii)Let $x\in X$ and W be a R-neighborhood of x.Then there exist an R-open set G such that $x\in G\subseteq W$.Since G^c is R-closed and $x\notin G^c$, by hypothesis there exist an α-open set V of x such that $\alpha cl(V)\cap G^c=\emptyset$.Thus $\alpha cl(V)\subseteq G\subseteq W$.

Definition 3.4: R-Hausdorff Space: A topological space (X,τ) is a R-hausdorff space if for each pair of distinct points x and y in X,there exist an open R-neighborhood U of x and an open R-neighborhood V of Y such that $U\cap V=\emptyset$.

Lemma 3.5: If Y is a R-compact subspace of the αT_R, R-hausdorff space X and x_0 is not in Y, then there exist disjoint α-open sets U and V of x containing x_0 and Y respectively.

Proof: Since X is R-hausdorff space and $x_0 \notin Y$ for each $x \in Y$,there exist disjoint R-open sets U_x and V_x such that $x_0 \in U_x$ and $x \in V_x$ .The collection $\{ V_x / x \in Y \}$ is evidently anR-open cover of Y.Since Y is R-compact subspace of X,finitely many points $x_1, x_2, \ldots x_n$ of Y such that $Y \subseteq \cup\{V_{x_i}, i=1,2,3,\ldots n\}$.Let $U=\cap\{U_{x_i}, i=1,2,3,\ldots\}$ and $V=\cup\{V_{x_i}, i=1,2,3,\ldots n\}$.Then by assumption U and V are disjoint R-open sets of X such that $x_0 \in U$ and $Y \subseteq V$.Since X is αT_R space, the sets U and V are disjoint α-open sets of X containing x_0 and Y respectively.

Theorem 3.6: If (X,τ) is R-regular space and $f:(X,\tau) \rightarrow (Y,\sigma)$ is bijective, ω-open, α-irresolute and α-open then (Y,σ) is R-regular.

Proof: Let F be a R-closed subset (Y,σ) and $y \notin F$.Since f is R-irresolute,$f^{-1}(F)$ is R-closed in (X,τ).Since f is bijective, let $f(x)=y$ then $y \notin F$ implies $x \notin f^{-1}(F)$.By hypothesis there exist disjoint α-open sets U and V such that $x \in U$ and $f^{-1}(F) \subseteq V$.Since f is α-open and bijective, we have $y \in f(U)$,$F \subseteq f(V)$ and $f(U) \cap f(V)=\emptyset$.This shows that the space(Y,σ) is also R-regular.

Theorem 3.7: Every R-compact αT_R,R-hausdorff space is regular.

Proof: Let X be a R-compact αT_R,R-hausdorff space.Let x be a point of X and B be a R-closed set in X not containing x. By theorem 5.6[1],B is compact. Thus B is a R-compact subspace of the αT_R,R-hausdorff space X. By the above lemma, there exists disjoint α-open sets about x and B respectively.Hence X is regular.

Theorem 3.8: Let (X,τ) be a T_R-space. Then the following are equivalent.

(i) (X,τ) is R-regular.

(ii) For every closed set A and each $x \in X-A$, there exist disjoint R-open sets U and V such that $x \in U$ and $A \subseteq V$.

Proof : (i)⇒(ii) Let (X,τ) be a T_R-space. Let A be a closed set and x be a point of X-A. Then by hypothesis, there exist disjoint open sets U and V such that $x \in U$ and $A \subseteq V$.Hence there exists disjoint R-open sets U and V such that $x \in U$ and $A \subseteq V$.

(ii)⇒(i) Let (X,τ) be a T_R-space. Let A be a closed set and $x \notin A$.Then by hypothesis, there exists disjoint R-open sets U and V such that $x \in U$ and $A \subseteq V$.Since the space is T_R-space, U and V are open sets in (X,τ) such that $x \in U$ and $A \subseteq V$.

Theorem 3.9: The closure of a compact subset of a compact hausdorff T_R-space is R-regular.

Proof: Let X be a compact hausdorff T_R-space. Let A be a compact subset of X. Since compact subset of a hausdorff space is closed, A is closed. Hence A is R-closed in (X,τ).Now to prove cl(A)=A is R-regular. Suppose $x \in A$ and F is a R-closed set such that $F \subseteq A$ and not containing x. Then F is R-closed in X. Since X is T_R-space, F is closed in X. Hence F is compact in X because closed subset of a compact Hausdorff space is compact. Therefore there exists disjoint open sets U and V of F and x respectively. Since every open set is α-open, we get X is R-regular.

Theorem 3.10: If (X,τ) is a R-regular space and $f:(X,\tau) \rightarrow (Y,\sigma)$ is bijective, ω-open , α –irresolute and α-quotient then (Y,σ) is R-regular.

Proof: Let F be any R-closed subset of (Y,σ) and $y \notin F$.Then by the above theorem f is R-irresolute.Thus $f^{-1}(F)$ is R-closed in (X,τ).Since f is bijective, let $f(x)=y$ then $x \notin f^{-1}(F)$.By hypothesis, there exists α-open sets U and V such that $x \in U$ and $f^{-1}(F)$.Since f is α –irresolute, f(V) is α-open.Since f is bijective, $y \in f(U)$, $F \subseteq f(U)$ and $f(U) \cap f(V)=\emptyset$.Thus the subspace (Y,σ) is R-regular.

4. R-Normal spaces

Definition 4.1: A topological space (X,τ) is said to be R-normal if for any pair of disjoint R-closed sets A and B , there exist α-open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Theorem 4.2: If (X,τ) is a R-normal space and Y is an open and R-closed subset of (X,τ) then the space Y is R-normal.

Proof: Let A and B be any disjoint R-closed set of Y. By result 2.7, A and B are R-closed in (X,τ). Since (X,τ) is R-normal, there exists α-open sets U and V of (X,τ) such that $A \subseteq U$ and $B \subseteq V$.Thus $U \cap Y$ and $V \cap Y$ are disjoint α-open sets in Y and also $A \subseteq U \cap Y$ and $B \subseteq V \cap Y$. Hence the subspace Y is R-normal.

Theorem 4.3: Let (X,τ) be a topological space.Then the following statements are equivalent:

(i) (X,τ) is R-normal.

(ii) For each R-closed set F and for each R-open set U containing F, there exist an α-open set V containing F such that αcl(V) ⊆U.
(iii) For each pair of disjoint closed sets A and B in (X,τ), there exist an α-open set U containing A such that αcl(U) ∩B =∅.
Proof: (i)⇒(ii)Let F be a R-closed set and U be a R-open set such that F⊆U.Then F∩U^c=∅.By assumption there exist α-open sets V and W such that F⊆V, U^c⊆W and V∩W=∅.Thus αcl(V) ∩W=∅.Now αcl(V) ∩U^c⊆ αcl(V) ∩W =∅.Thus αcl(V) ⊆U.
(ii) ⇒(iii) Let A and B be disjoint R-closed sets of (X,τ).Since A∩B =∅, A⊆B^c and B^c is R-open.By assumption, there exist an α-open set U containing A such that αcl(U) ⊆B^c and so αcl(U) ∩B=∅.
(iii)⇒(i) Let A and B be disjoint R-closed sets of (X,τ).Then by assumption there exist an α-open set U containing A such that αcl(U) ∩ B= ∅ .Again by assumption, there exist an α-open set U containing B such that αcl(V) ∩A=∅.Also αcl(U) ∩ αcl(V)=∅.Thus U∩V =∅.
Theorem 4.4: Let (X,τ) be a T_R-space.Then the following are equivalent:
(i) (X,τ) is normal.
(ii) For every disjoint closed sets A and B, there exist disjoint R-open sets U and V such that A⊆U and B⊆V.
Proof: (i)⇒(ii)
Let (X,τ) be a T_R-space. Let A and B be disjoint closed subsets of (X,τ).Then by hypothesis, there exist disjoint open sets U and V such that A⊆U and B⊆V.Since every open set is an R-open set we get (ii).
(ii)⇒ (i) Let (X,τ) be a T_R-space. Let A and B be disjoint closed subsets of (X,τ).Then by assumption there exist disjoint R-open sets U and V such that A⊆U and B⊆V. Since (X,τ) is a T_R-space, Every R-open set is open and hence we get the conclusion.
Theorem 4.5: If (X,τ) is seminormal and F∩A=∅ where F is ω-closed and A is R- closed then there exist semi open sets U and V such that F⊆U and A⊆V.
Proof: Let F be ω -closed and A be R- closed such that F ∩ A= ∅ .Then A ⊆ F^c and F^c is ω -open.Thus αcl(A) ⊆int(F^c).Thus αcl(A) ∩F=∅, where F is ω-closed. Also αcl(A) is semi-closed.Hence there exist semiopen sets U and V such that F⊆U and A⊆V.
Definition 4.6: A space (X,τ) will be termed symmetric if and only if for x, y∈ (X, τ), x∈cl(y)⇒y∈cl(x).
Definition 4.7: We define gT_R-space as a space in (X, τ) in which every g-closed set is R-closed.
Proposition 4.8: A gT_R-space (X, τ) is symmetric if and only if {x} is R-closed in (X, τ) for each point x of (X, τ).
By result 2.6, a space (X, τ) is symmetric if and only if {x} is g-closed in (X, τ) for each point x of (X, τ), we get the proof.
Theorem 4.9: Every seminormal, symmetric, gT_R-space (X, τ) is semi-regular.
Proof: Let F be a closed subset of (X, τ) and x∈X such that x∉F. Since (X, τ) is symmetric and gT_R-space, {x} is R-closed. Since F is closed, it is ω-closed and since (X, τ) is seminormal, by theorem 4.5, there exist disjoint semi-open sets U and V such that F⊆U, {x}⊆V.Thus (X, τ) is semi-regular.
Theorem 4.10: The closure of a compact subset of a compact Hausdorff T_R-space is R-normal.
Proof: Let X be a compact Hausdorff T_R-space and A be a compact subset of X. Then A is closed in X, Since compact subset of a hausdorff space is closed. Hence A is closed in X and hence R-closed in X. Now to prove cl(A) is normal. Suppose for any pair of disjoint R-closed sets F and G are given. Since X is T_R-space, F and G are closed sets in X. Since closed subset of a compact Hausdorff space is compact, F and G are compact sets in X. Thus, there exist open sets U and V of F and G .Thus there exist disjoint α-open sets U and V of F and G respectively. Thus X is R-normal.
Theorem 4.11: If (X, τ) is R-normal space and f :(X, τ) → (Y, σ) is bijective, ω-open, α-irresolute and f^{-1} is α-irresolute then (Y, σ) is R-normal.
Proof: Let A and B be disjoint R-closed sets of (Y, σ).By theorem we have f^{-1}(A) and f^{-1}(B) are disjoint R-closed sets of (X,τ). Since (X,τ) is normal there exist α-open sets U and V such that f^{-1}(A) ⊆U and f^{-1}(B) ⊆V.Since f is bijection and f^{-1} is α-irresolute we get A ⊆f(U),B ⊆f(V) and f(U)∩f(V)=∅. This shows that (Y, σ) is R-normal.

Theorem 4.12: If f :$(X,\tau) \to (Y,\sigma)$ is α-irresolute, open and α-closed, f(A) and f(B) are disjoint R-closed sets of (Y,σ). Since (Y,σ) is R-normal, there exist α-open sets U and V in (Y,σ) such that f(A) ⊆U and f(B) ⊆V. By hypothesis $f^{-1}(U)$ and $f^{-1}(V)$ are α-open sets of (X,τ) such that $A \subseteq f^{-1}(U), B \subseteq f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V)=\emptyset$. Thus (X,τ) is R-normal.

Theorem 4.13: If (X,τ) is an R-regular space and Y is a pre-open and R-closed subset of (X,τ), then the subspace Y is regular.

Proof: Let F be any R-closed subset of Y and y∉F. By the following theorem, F is a R-closed subset of (X,τ) and (X,τ) is R-regular. Thus there exist α-open sets U and V of (X,τ) such that y∈U and F⊆V.Thus U∩Y and V∩Y are disjoint α-open sets of the subspace Y such that y∈ U ∩ Y and F⊆ V ∩ Y .Hence the subspace Y is R-regular.

Proposition 4.14: If f: $(X,\tau) \to (Y,\sigma)$ is injection, α-irresolute, ω-irresolute, open and α-closed then (X,τ) is R-regular if (Y,σ) is R-regular.

Proof: Let F be any R-closed set of (X,τ) and x∉F.Since f is ω-irresolute, open and α-closed, f(F) is R-closed in (Y,σ) and f(x)∉f(F).Since (Y,σ) is R-regular there exist disjoint α-open sets U and V in (Y,σ) such that f(x)∈U and f(F) ⊆V.Also since f is injection and α-irresolute, we get α-open sets $f^{-1}(U)$ and $f^{-1}(V)$ in (X,τ) such that $x \in f^{-1}(U)$ and $F \subseteq f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. Thus (X,τ) is R-regular.

Theorem 4.15: If (X,τ) is ω-normal, then the following statements are true.

(i) For each ω-closed set A and every R-open set B such that A⊆B, then there exist an ω-open set U such that A⊆U⊆ ωcl(U) ⊆B.

(ii) For every R-closed set A and every open set B containing A, there exist an ω-open set U such that A⊆U⊆ ωcl(U) ⊆B.

(iii) For every pair consisting of disjoint sets A and B one of which is ω-closed set and the other is R-closed set then there exist ω-open sets U and V such that A⊆U , B⊆V and ωcl(U)∩ ωcl(V)= ∅.

Theorem 4.16: Let (X,τ) be a topological space. (X,τ) is R-regular if and only if for each R-closed set F of (X,τ) and each $x \in F^c$, there exist α –open sets U and V of (X,τ) such that x∈U, F⊆ V and αcl(U) ∩ αcl(V)= ∅.

Proof: Let F be an R-closed set of (X,τ) and $x \in F^c$. Then there exist an α-open set U of (X,τ) such that αcl(U)∩F=∅.Then αcl(U) is α-closed and hence R-closed. For all y∈F, y∉ αcl(U). Thus there exist an α–open set V containing F such that αcl(U)∩ αcl(V)=∅. The converse part is trivial.

Corollary 4.17: If a space (X,τ) is R-regular, symmetric and ${}_gT_R$-space then it is Urysohn.

Proof: Let x and y be any two disjoint points of (X,τ). Since (X,τ) is symmetric and ${}_gT_R$-space, {x} is R-closed. Then by the above theorem there exist α–open sets U and V of (X,τ) such that x∈U, y∈V and α cl(U) ∩ α cl(V)= ∅.

Corollary 4.18: If a space (X,τ) is ${}_gT_R$-space, R-regular and symmetric, then it is Hausdorff.

Proof: Similar to the above corollary.

Theorem 4.19: Let (X,τ) be a topological space. Then the following statements are equivalent:

(i) (X,τ) is R-normal.

(ii) For each R-closed set F and for each R-open set U containing F there exist an α-open set V containing F such that αcl(V) ⊆U.

(iii) For each pair of disjoint R-closed sets A and B in (X,τ) there exist an α-open set U containing A such that αcl(U) ∩B= ∅.

(iv) For each pair of disjoint R-closed sets A and B in (X,τ) there exist α-open sets U containing A and V containing B such that αcl(U) ∩ αcl(V)= ∅.

Proof: (iii)⇒ (iv) Let A and B be any two disjoint R-closed sets of (X,τ). Then by assumption there exist an an α-open set U containing A such that αcl(U) ∩B= ∅. Since αcl(U) is R-closed, again by assumption exist an α-open set V containing B such that αcl(U) ∩ αcl(V)= ∅.

(iv)⇒(i)Let A and B be any two disjoint R-closed sets of (X,τ). By assumption there exist -open sets U containing A and V containing B such that αcl(U) ∩ αcl(V)= ∅.Hence U∩V=∅ and thus (X,τ) is R-normal.

Theorem 4.20: If f: $(X,\tau) \to (Y,\sigma)$ is bijective, continuous, α-quotient and ω-open, then (Y,σ) is R-normal if (X,τ) is R-normal.

Proof: Let A and B be any two disjoint R-closed subsets of (Y, σ). Since f is R-irresolute, by theorem 4.12 [1] $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint R-closed sets of (X,τ). Since (X,τ) is R-normal, there exist disjoint α-open sets U and V such that $f^{-1}(A)\subseteq U$ and $f^{-1}(B)\subseteq V$. Since f is a bijection, A⊆f(U) and B⊆f(V) where f(U) and f(V) are α-open sets. Hence (Y, σ) is R-normal.

Theorem 4.21: If f: (X,τ) → (Y, σ) is an injective function,ω-irresolute, open, α-closed and α-irresolute, then (X,τ) is R-normal if (Y, σ) is R-normal.

Proof: Let A and B be any two disjoint R-closed subsets of (X,τ). Since f is ω-irresolute, open, α-closed, by proposition 4.11[1] f(A) and f(B) are R-closed in (Y, σ). Since (Y, σ) is R-normal, there exist disjoint α-open sets U and V such that f(A)⊆U and f(B)⊆V. Hence $A\subseteq f^{-1}(U)$ and $B\subseteq f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V)=\emptyset$. Since f is α-irresolute, $f^{-1}(U)$ and $f^{-1}(V)$ are α-open sets in (X,τ). Thus (X,τ) is R-normal.

Proposition 4.22: If f: (X,τ) → (Y, σ) is weakly continuous, R-closed injection and (Y, σ) is a R-normal, α-space then (X,τ) is normal.

Proof: Let A and B be any two disjoint closed sets of (X,τ). Since f is injective and R-closed, f(A) and f(B) are disjoint R-closed sets of (Y, σ). Since (Y, σ) is R-normal, there exist α-open sets U and V such that f(A) ⊆U, f(B) ⊆V and αcl(U) ∩ αcl(V)= ∅. Since (Y, σ) is α-space, there exist open sets U and V such that f(A) ⊆U, f(B) ⊆V and cl(U) ∩cl(V)= ∅. Since f is weakly continuous it follows that, $A\subseteq f^{-1}(U) \subseteq int(f^{-1}(cl(U)))$, $B\subseteq f^{-1}(V) \subseteq int(f^{-1}(cl(V)))$ and
$int(f^{-1}(cl(U))) \cap int(f^{-1}(cl(V)))$. Thus (X,τ) is normal.

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