International Journal of Mathematical Sciences and Applications
Vol. 1 No. 2 (May, 2011)

www.ijmsa.yolasite.com

A NEW TYPE OF HOMEOMORPHISM IN A BITOPOLOGICAL SPACE

***O. Ravi, **S. Pious Missier, ***T. Salai Parkunan**

*Department of Mathematics, P.M. Thevar College, Usilampatti,
Madurai District, Tamil Nadu, India, e.mail: siingam@yahoo.com

**Department of Mathematics, V. O. Chidambaram College, Tuticorin
Tamil Nadu, India, e.mail: spmissier@yahoo.com

***Department of Mathematics, Arul Anandar College, Karumathur,
Madurai District, Tamil Nadu, India, e.mail: parkunan@yahoo.com

Abstract

In this paper, we introduce a new class of sets namely (1,2)*-g**-open which is stronger than (1,2)*-semi-open sets and investigate some of their properties. The notion of (1,2)*-g**-continuity, (1,2)*-g**-irresolute maps are introduced and some of their properties are investigated. Finally we introduce the concept of (1,2)*-g**-homeomorphism in bitopological space and study some of their properties.

2000 Mathematics Subject Classification: 54E55

Keywords and Phrases: *(1,2)*-g**-open set, (1,2)*-g**-continuity and (1,2)*-g**-homeomorphism*

1. Introduction

The study of generalized closed (g-closed) sets in a topological space was initiated by Levine (1963) and the concept of $T_{½}$-spaces was introduced. Dunham (1977, 1982) further investigated the properties of $T_{½}$-spaces and defined a new closure operator cl* using generalized closed sets. Again Levine (1970) introduced the concept of semi-open sets and semi-continuity in a topological space. Further, Bhattacharrya and Lahiri introduced a new class of semi-generalized open sets by means of semi-open sets introduced by Levine (1963). In continuation of the previous studies, Balachandran *et al* (1991) introduced the concept of generalized continuous maps and generalized homeomorphism in a topological space.

In this paper, let us introduce a new class of open sets namely (1,2)*-g**-open sets and investigate some of their properties. Further, we introduce the concept of (1,2)*-g**-continuous maps which includes the class of continuous maps in a bitopological space. Also we introduce (1,2)*-g**-irresolute maps analogy to irresolute maps in a topological space and investigate some of their properties. Finally we introduce a new class of maps namely (1,2)*-g**-homeomorphism in a bitopological space and study some of their properties.

Throughout this paper (X, τ_1, τ_2), (Y, σ_1, σ_2) and (Z, ρ_1, ρ_2) (or simply X, Y and Z) denote bitopological spaces.

2. Preliminaries

We recall the following.

Definition 2.1 [8]. Let S be a subset of X. Then S is said to be $\tau_{1,2}$-open if $S = A \cup B$ where $A \in \tau_1$ and $B \in \tau_2$.

The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed.

Definition 2.2 [8]. Let S be a subset of X. Then

(i) The $\tau_{1,2}$-closure of S, denoted by $\tau_{1,2}$-cl(S), is defined as

$\cap$ {F: $S \subseteq F$ and F is $\tau_{1,2}$-closed};

(ii) The $\tau_{1,2}$-interior of S, denoted by $\tau_{1,2}$-int(S), is defined as

$\cup$ {F / $F \subseteq S$ and F is $\tau_{1,2}$-open}.

Note 2.3 [8]. Notice that $\tau_{1,2}$-open sets need not necessarily form a topology.

Definition 2.4. Let S be a subset of X. Then S is said to be

(i) (1,2)*-g-closed [10] if and only if $\tau_{1,2}$-cl(S) $\subseteq$ F whenever $S \subseteq F$ and F is $\tau_{1,2}$-open in X;

(ii) (1,2)*-g-open [10] if and only if X\S is (1,2)*-g-closed;

(iii) (1,2)*-semi-open [1] if and only if there exists an $\tau_{1,2}$-open set F such that $F \subseteq S \subseteq \tau_{1,2}$-cl(F).

Definition 2.5 [9]. A map $f: X \to Y$ is called

(i) (1,2)*-g-continuous if $f^{-1}(V)$ is (1,2)*-g-open whenever V is $\sigma_{1,2}$-open in Y;

(ii) (1,2)*-gc-irresolute if the inverse image of every (1,2)*-g-closed set in Y is (1,2)*-g-closed in X.

Theorem 2.6 [10]. If A is (1,2)*-g-closed set in X and if f: X $\rightarrow$ Y is (1,2)*-continuous and (1,2)*-closed, then f(A) is (1,2)*-g-closed.

Theorem 2.7 [10]. If f: X $\rightarrow$ Y is (1,2)*-continuous and (1,2)*-closed and if B is a (1,2)*-g-closed ((or) (1,2)*-g-open) subset of Y, then $f^{-1}(B)$ is (1,2)*-g-closed ((or) (1,2)*-g-open) in X.

3. Characterizations of (1,2)*-g**-open sets.

Definition 3.1. Let S be a subset of X. Then

$$(1,2)^*\text{-cl}^{**}(S) = \cap \{F \,/\, S \subseteq F \text{ and } F \text{ is } (1,2)^*\text{-g-closed}\}.$$

Remark 3.2. (1,2)*-cl**(S) is a Kuratowki closure operator on X.

Definition 3.3. Let S be a subset of X. Then S is said to be (1,2)*-g**-open if and only if there exists an $\tau_{1,2}$-open set U of X such that U $\subseteq$ S $\subseteq$ (1,2)*-cl**(U).

Example 3.4. If X = {a, b, c}, τ_1 = {φ, X, {a}} and τ_2 = {φ, X, {a, b}}, then (1,2)*-g**-open sets of (X, τ_1, τ_2) are {φ, X, {a}, {a,b}, {a,c}}.

Remark 3.5. If A and B are subsets of X such that A $\subseteq$ B, then (1,2)*-cl**(A) $\subseteq$ (1,2)*-cl**(B).

Theorem 3.6. Let S be a subset of X. Then S is (1,2)*-g**-open in X if and only if S $\subseteq$ (1,2)*-cl** ($\tau_{1,2}$-int(S)).

Proof. If S is a (1,2)*-g**-open set of X, then there exists an $\tau_{1,2}$-open set U such that U $\subseteq$ S $\subseteq$ (1,2)*-cl** (U). U $\subseteq$ S implies U $\subseteq$ $\tau_{1,2}$-int(S). Hence by Remark 3.5, (1,2)*-cl**(U) $\subseteq$ (1,2)*-cl**($\tau_{1,2}$-int(S)). Therefore S $\subseteq$ (1,2)*-cl**($\tau_{1,2}$-int(S)). Conversely, let S $\subseteq$ (1,2)*-cl**($\tau_{1,2}$-int(S)). Let U = $\tau_{1,2}$-int(S). Then U $\subseteq$ S $\subseteq$ (1,2)*-cl**(U). Hence S is a (1,2)*-g**-open set in X.

Proposition 3.7. If S is an $\tau_{1,2}$-open set in X, then S is a (1,2)*-g**-open.

Proof. Let S be a $\tau_{1,2}$-open set in X. It implies S = $\tau_{1,2}$-int(S) $\subseteq$ (1,2)*-cl** ($\tau_{1,2}$-int(S)). Hence S is a (1,2)*-g**-open in X.

Example 3.8. The converse of Proposition 3.7 need not be true.

Refer Example 3.4, clearly S = {a, c} is a (1,2)*-g**-open set in X but it is not an $\tau_{1,2}$-open in X.

Definition 3.9. A bitopological space X is said to be (1,2)*-$T_{½}$ space if every (1,2)*-g-open set of X is $\tau_{1,2}$-open in X.

Definition 3.10. A bitopological space X is said to be (1,2)*-g**-$T_{½}$-space if every (1,2)*-g**-open set of X is $\tau_{1,2}$-open in X.

Remark 3.11. In (1,2)*-$T_{½}$ space, every (1,2)*-semi-open set is a (1,2)*-g**-open.

Remark 3.12. (1,2)*-cl**(A) $\subseteq$ $\tau_{1,2}$-cl(A) if A is a subset of X.

Theorem 3.13. If S is a (1,2)*-g**-open set in X, then A is (1,2)*-semi-open in X.

Proof. Given S is (1,2)*-g**-open set in X. Therefore there exists $\tau_{1,2}$-open set U such that U $\subseteq$ S $\subseteq$ (1,2)*-cl**(U). By Remark 3.12 (1,2)*-cl**(U) $\subseteq$ $\tau_{1,2}$-cl(U). Hence U $\subseteq$ S $\subseteq$ $\tau_{1,2}$-cl(U) implies S is (1,2)*-semi-open.

Example 3.14. The converse of Theorem 3.13 need not be true.

Let X = {a, b, c}, τ_1 = {φ, X, {a}} and τ_2 = {φ, X}. Then S = {a, b} is a (1,2)*-semi-open set but it is not a (1,2)*-g**-open.

Remark 3.15. If A and B are subsets of X, then (1,2)*-cl**(A $\cup$ B) = (1,2)*-cl**(A) $\cup$ (1,2)*-cl**(B).

Theorem 3.16. If A and B are (1,2)*-g**-open sets of X, then A $\cup$ B is also a (1,2)*-g**-open set in X.

Proof. Given A and B are (1,2)*-g**-open sets in X. Then there exists $\tau_{1,2}$-open sets U and V respectively such that U $\subseteq$ A $\subseteq$ (1,2)*-cl**(U) and V $\subseteq$ B $\subseteq$ (1,2)*-cl**(V). By Remark 3.15, (1,2)*-cl**(U) $\cup$ (1,2)*-cl**(V) = (1,2)*-cl**(U $\cup$ V). Hence U $\cup$ V $\subseteq$ A $\cup$ B $\subseteq$ (1,2)*-cl**(U $\cup$ V). Hence A $\cup$ B is also (1,2)*-g**-open set in X.

Example 3.17. If A and B are (1,2)*-g**-open in X, then A $\cap$ B need not be (1,2)*-g**-open in X.

Let $X = \{a, b, c\}$, $\tau_1 = \{\varphi, X, \{a\}\}$ and $\tau_2 = \{\varphi, X, \{b\}\}$. Then $A = \{a, c\}$ and $B = \{b, c\}$ are (1,2)*-g**-open sets in X and $A \cap B = \{c\}$ is not a (1,2)*-g**-open set in X.

Theorem 3.18. Let A be a (1,2)*-g**-open set in X and B be any set such that $A \subseteq B \subseteq$ (1,2)*-cl**($\tau_{1,2}$-int(A)). Then B is a (1,2)*-g**-open set in X.

Proof. Given A is a (1,2)*-g**-open set in X. Therefore by Theorem 3.6, $A \subseteq$ (1,2)*-cl**($\tau_{1,2}$-int(A)). $A \subseteq B$ implies $\tau_{1,2}$-int(A) $\subseteq$ $\tau_{1,2}$-int(B), hence, (1,2)*-cl** ($\tau_{1,2}$-int(A)) $\subseteq$ (1,2)*-cl**($\tau_{1,2}$-int(B)). Therefore $B \subseteq$ (1,2)*-cl**($\tau_{1,2}$-int(A)) $\subseteq$ (1,2)*-cl**($\tau_{1,2}$-int(B)). Thus, B is a (1,2)*-g**-open set in X.

Remark 3.19. If f: $X \rightarrow Y$ is (1,2)*-g-continuous map, then f((1,2)*-cl**(A)) $\subseteq$ $\sigma_{1,2}$-cl(f(A)).

Theorem 3.20. Let f: $X \rightarrow Y$ be (1,2)*-g-continuous (1,2)*-open map. If A is a (1,2)*-g**-open set in X, then f(A) is a (1,2)*-semi-open set in Y.

Proof. Given A is (1,2)*-g*-open set in X. Therefore there exist an $\tau_{1,2}$-open set U such that $U \subseteq A \subseteq$ (1,2)*-cl**(U). By Remark 3.19, we have f((1,2)*-cl**(A)) $\subseteq$ $\sigma_{1,2}$ -cl(f(A)). Hence f(U) $\subseteq$ f(A) $\subseteq$ f((1,2)*-cl**(U)) $\subseteq$ $\sigma_{1,2}$-cl(f(U)). Since f is an (1,2)*-open map, f(U) is $\sigma_{1,2}$-open in Y. This implies f(A) is a (1,2)*-semi-open set in Y.

Theorem 3.21. Let f: $X \rightarrow Y$ be a (1,2)*-homeomorphism. If A is a (1,2)*-g**-open set in X, then f(A) is (1,2)*-g**-open set in Y.

Proof. Given A is a (1,2)*-g**-open set in X. Therefore there exists an $\tau_{1,2}$-open set U such that $U \subseteq A \subseteq$ (1,2)*-cl**(U), implies f(U) $\subseteq$ f(A) $\subseteq$ f((1,2)*-cl**(U)). Since f is (1,2)*-homeomorphism and by Theorem 2.6, we have f((1,2)*-cl**(U)) $\subseteq$ (1,2)*-cl**(f(U)). Therefore f(U) $\subseteq$ f(A) $\subseteq$ (1,2)*-cl**(f(U)) and hence f(A) is a (1,2)*-g**-open set in Y.

Theorem 3.22. Let f: $X \rightarrow Y$ be a (1,2)*-homeomorphism. If A is a (1,2)*-g**-open set in Y, then f^{-1}(A) is (1,2)*-g**-open set in X.

Proof. Given A is a (1,2)*-g**-open set in Y. Therefore there exists an $\sigma_{1,2}$-open set U such that $U \subseteq A \subseteq$ (1,2)*-cl**(U), implies f^{-1}(U) $\subseteq$ f^{-1}(A) $\subseteq$ f^{-1}((1,2)*-cl**(U)). Since f is (1,2)*-homeomorphism and by Theorem 2.7, we f^{-1}((1,2)*-cl**(U)) $\subseteq$ (1,2)*-cl**(f^{-1}(U)). Therefore f^{-1}(U) $\subseteq$ f^{-1}(A) $\subseteq$ (1,2)*-cl**(f^{-1}(U)) and hence f^{-1}(A) is (1,2)*-g**-open in X.

Definition 3.23. A subset S of X is said to be (1,2)*-g**-closed if and only if X\S is (1,2)*-g**-open.

Definition 3.24. Let S be a subset of X. Then the $\tau_{1,2}$-g**-closure of S is defined as $\tau_{1,2}$-g**-cl(S) = $\cap$\{F: S $\subseteq$ F and F is (1,2)*-g**-closed\}.

Remark 3.25. From the above definition, $\tau_{1,2}$-g**-cl(S) is the smallest (1,2)*-g**-closed set containing S.

Definition 3.26. Let S be a subset of X. Let x $\in$ X. Then x is said to be a (1,2)*-g**-limit point of A if and only if every (1,2)*-g**-open set containing x contains at least one point other than x.

Definition 3.27. Let S be a subset of X. Then the set of all (1,2)*-g**-limit points of S is said to be (1,2)*-g**-derived set of S and it is denoted by (1,2)*-Dg**(S).

Theorem 3.28. Let A be a subset of X. Then x $\in$ $\tau_{1,2}$-g**-cl(A) if and only if every (1,2)*-g**-open set U contains x intersect with A.

Proof. We prove this theorem in contra positive way. If x $\notin$ $\tau_{1,2}$-g**-cl(A), then x $\in$ X\$\tau_{1,2}$-g**-cl(A). Let U = X\$\tau_{1,2}$-g**-cl(A). Then by Remark 3.25 U is (1,2)*-g**-open set which does not intersect with A. Conversely, if U is a (1,2)*-g**-open set of x which does not intersect with A, then X\U is a (1,2)*-g**-closed set containing A. This implies x $\notin$ $\tau_{1,2}$-g**-cl(A).

Theorem 3.29. Le A be a subset of X. Let (1,2)*-Dg**(A) be the set of all (1,2)*-g**-limit points of A. Then $\tau_{1,2}$-g**-cl(A) = A $\cup$ (1,2)*-Dg**(A).

Proof. Let x $\in$ A $\cup$ (1,2)*-Dg**(A). This implies either x $\in$ A or x $\in$ (1,2)*-Dg**(A). If x $\in$ A, then x $\in$ $\tau_{1,2}$-g**-cl(A). If x $\in$ (1,2)*-Dg**(A), then every (1,2)*-g**-open set contains x will intersect with A. Therefore x $\in$ $\tau_{1,2}$-g**-cl(A). This implies A $\cup$ (1,2)*-Dg**(A) $\subseteq$ $\tau_{1,2}$-g**-cl(A).

If x $\in$ $\tau_{1,2}$-g**-cl(A), then to prove x $\in$ A $\cup$ (1,2)*-Dg**(A). If x $\in$ A, then x $\in$ A $\cup$ (1,2)*-Dg**(A). If x $\notin$ A, since x $\in$ $\tau_{1,2}$-g**-cl(A) implies every (1,2)*-g**-open set of x intersects with A. Hence x $\in$ (1,2)*-Dg**(A). Therefore $\tau_{1,2}$-g**-cl(A) = A $\cup$ (1,2)*-Dg**(A).

4. Properties of (1,2)*-g**-continuous maps.

Definition 4.1. A map f: X $\to$ Y is said to be (1,2)*-g**-continuous if the inverse image of every $\sigma_{1,2}$-open set in Y is (1,2)*-g**-open in X.

Theorem 4.2. If f: $X \to Y$ is a (1,2)*-continuous map, then it is (1,2)*-g**-continuous.

Proof. Let U be an $\sigma_{1,2}$-open set in Y. Since f is (1,2)*-continuous, $f^{-1}(U)$ is $\tau_{1,2}$-open in X. By Proposition 3.7, $f^{-1}(U)$ is (1,2)*-g**-open in X. Hence f is (1,2)*-g**-continuous.

Example 4.3. The converse of Theorem 4.2 need not be true.

Let X = {a, b, c}, τ_1 = {φ, X, {a}} and τ_2 = {φ, X, {a, b}}.

Let Y = {p, q}, σ_1 = {φ. Y, {p}} and σ_2 = {φ, Y}.

Let f: $X \to Y$ be a map defined by f(a) = f(c) = p, f(b) = q. Then f is (1,2)*-g**-continuous but it is not a (1,2)*-continuous map.

Remark 4.4. In (1,2)*-g**-$T_{½}$ space, the concept of (1,2)*-continuous and (1,2)*-g**-continuous maps coincide.

Theorem 4.5. Let f: $X \to Y$ be a map. Then the following statements are equivalent.

(i) f is (1,2)*-g**-continuous.

(ii) the inverse image of each $\sigma_{1,2}$-closed set in Y is (1,2)*-g**-closed in X.

Proof. (i) ⇒ (ii) Let C be any $\sigma_{1,2}$-closed set in Y. Then Y\C is $\sigma_{1,2}$-open in Y. Since f is (1,2)*-g**-continuous, f^{-1}(Y\C) is (1,2)*-g**-open in X. Therefore X\f^{-1}(C) is (1,2)*-g**-open in X, which implies f^{-1}(C) is (1,2)*-g**-closed in X.

(ii) ⇒ (i) Let D be an $\sigma_{1,2}$-open set in Y. Then Y\D is $\sigma_{1,2}$-open in Y. This implies f^{-1}(Y\D) is (1,2)*-g**-closed in X, which implies X\f^{-1}(D) is (1,2)*-g**-closed in X. Therefore f^{-1}(U) is (1,2)*-g**-open in X. Hence f is (1,2)*-g**-continuous.

Theorem 4.6 If f: $X \to Y$ is a (1,2)*-g**-continuous map, then f($\tau_{1,2}$-g**-cl(A)) $\subseteq$ $\sigma_{1,2}$-cl(f(A)).

Proof. Since f(A) $\subseteq$ $\sigma_{1,2}$-cl(f(A)), implies A $\subseteq$ f^{-1}($\sigma_{1,2}$-cl(f(A))). Then $\sigma_{1,2}$-cl(f(A)) is a $\sigma_{1,2}$-closed set in Y and f is (1,2)*-g**-continuous map implies f^{-1}($\sigma_{1,2}$-cl(f(A))) is (1,2)*-g**-closed in X. Hence $\tau_{1,2}$-g**-cl(A) $\subseteq$ f^{-1}($\sigma_{1,2}$-cl(f(A))). Therefore f($\tau_{1,2}$-g**-cl(A)) $\subseteq$ $\sigma_{1,2}$-cl(f(A)).

5. Relation between (1,2)*-g**-continuous maps and (1,2)*-g**-irresolute maps

Definition 5.1. Let f: X $\rightarrow$ Y be a map. Then it is called (1,2)*-g**-irresolute if the inverse image of every (1,2)*-g**-open set of Y is (1,2)*-g**-open in X.

Remark 5.2. A map f: X $\rightarrow$ Y is a (1,2)*-g**-irresolute map if and only if the inverse image of every (1,2)*-g**-closed set in Y is (1,2)*-g**-closed in X.

Theorem 5.3. If f: X $\rightarrow$ Y is a (1,2)*-g**-irresolute, then f is (1,2)*-g**-continuous map.

Proof. Let F be an $\tau_{1,2}$-open set in X. Since f is (1,2)*-g**-irresolute map, implies $f^{-1}(F)$ is (1,2)*-g**-open in X. Hence f is (1,2)*-g**-continuous..

Example 5.4. The converse of Theorem 5.3 need not be true.

Let X = {a, b, c} = Y, τ_1 = {φ, X, {a, b}}, τ_2 = {φ, X, {b}, {c}, {b, c}}, σ_1 = {φ, Y, {a}} and σ_2 = {φ, Y, {b}}. Define f: X $\rightarrow$ Y by f(a) = c; f(b) = b; f(c) = a. Clearly f is (1,2)*-g**-continuous but it is not (1,2)*-g**-irresolute.

Theorem 5.5. Let f: X $\rightarrow$ Y be a (1,2)*-continuous map and Y is (1,2)*-g**-$T_{½}$-space then f is (1,2)*-g**-irresolute.

Proof. Let A be (1,2)*-g**-open set in Y. Since Y is (1,2)*-g**-$T_{½}$ space, implies A is an $\sigma_{1,2}$-open set in Y. Since f is (1,2)*-continuous, implies $f^{-1}(A)$ is (1,2)*-g**-open in X. Therefore f is (1,2)*-g**-irresolute map.

Theorem 5.6. Let f: X $\rightarrow$ Y be a (1,2)*-g**-irresolute map and g: Y $\rightarrow$ Z be a (1,2)*-g**-irresolute map. Then gof: X $\rightarrow$ Z is (1,2)*-g**-irresolute.

Proof. Let U be a (1,2)*-g**-open set in Z. Then $g^{-1}(U)$ is (1,2)*-g**-open in Y which implies $f^{-1}(g^{-1}(U))$ is (1,2)*-g**-open in X. Therefore $(gof)^{-1}(U)$ is (1,2)*-g**-open in X. Hence, gof is (1,2)*-g**-irresolute.

6. (1,2)*-g**-connected sets.

Definition 6.1. X is said to be (1,2)*-g**-connected if X cannot be written as disjoint union of two non-empty (1,2)*-g**-open sets.

Theorem 6.2. For a bitopological space X, the following statements are equivalent.

(i) X is (1,2)*-g**-connected.

(ii) The only subsets of X which are both (1,2)*-g**-open and (1,2)*-g**-closed are φ and X.

Proof. (i) ⇒ (ii) Let U be a (1,2)*-g**-open and (1,2)*-g**-closed subsets of X. Then X\U is both (1,2)*-g**-open and (1,2)*-g**-closed. Since X is the disjoint union of (1,2)*-g**-open set U and X\U, implies one of these must be empty, that is U = φ or X\U = φ.

(ii) ⇒ (i) Suppose that X = A ∪ B where A and B are disjoint non-empty (1,2)*-g**-open set of X. Then A(=X\B) is (1,2)*-g**-closed. Hence A is both (1,2)*-g**-open and (1,2)*-g**-closed subset of X. By assumption, A = φ or A = X. This implies X is (1,2)*-g**-connected.

Theorem 6.3. (i) If f: X → Y is a (1,2)*-g**-continuous surjection map and X is (1,2)*-g**-connected, then Y is (1,2)*-connected.

(ii) If f: X → Y is a (1,2)*-g**-irresolute surjection map and X is (1,2)*-g**-connected, then Y is (1,2)*-g**-connected.

Proof. (i) Suppose that Y is not (1,2)*-connected. Then Y = A ∪ B, where A and B are disjoint non-empty $\sigma_{1,2}$-open sets in Y. Since f is (1,2)*-g**-continuous and onto, X = f^{-1}(A) ∪ f^{-1}(B) where f^{-1}(A) and f^{-1}(B) are disjoint non-empty (1,2)*-g**-open sets which is a contradiction to our assumption that X is (1,2)*-g**-connected. Hence Y is (1,2)*-connected.

(ii) It follows from Definition 5.1

7. (1,2)*-g**-homeomorphisms

Definition 7.1. A map f: X → Y is said to be (1,2)*-g**-open map if f(U) is (1,2)*-g**-open in Y for every $\tau_{1,2}$-open set U in X.

Theorem 7.2. If f: X → Y is an (1,2)*-open map, then it is a (1,2)*-g**-open map.

Proof. Given f: X → Y is an (1,2)*-open map. Let G be any $\tau_{1,2}$-open set in X. Then f(G) is $\sigma_{1,2}$-open in Y. By Proposition 3.7, f(G) is (1,2)*-g**-open in Y. Hence f is a (1,2)*-g**-open map.

Example 7.3. The converse of Theorem 7.2 need not be true.

Let X = Y = {a, b, c}, τ_1 = {φ, X, {a}} and τ_2 = {φ, X, {b}}.

Let σ_1 = {φ, Y, {a}} and σ_2 = {φ, Y, {a, b}}. Then define f: X → Y as f(a) = a, f(b) = c and f(c) = b. Clearly f is (1,2)*-g**-open map but it is not an (1,2)*-open.

Definition 7.4. A map f: X $\to$ Y is said to be (1,2)*-g**-closed if f(U) is (1,2)*-g**-closed in Y for every $\tau_{1,2}$-closed set U in X.

Remark 7.5. If f: X $\to$ Y is a (1,2)*-closed map, then it is a (1,2)*-g**-closed and the converse need not be true.

Proof. Similar to the proof of Theorem 7.2.

Definition 7.6. A bijection map f: X $\to$ Y is called (1,2)*-g**-homeomorphism if f is both (1,2)*-g**-continuous and (1,2)*-g**-open.

Remark 7.7. Every (1,2)*-homeomorphism is a (1,2)*-g**-homeomorphism but not conversely.

Theorem 7.8. For any bijection map f: X $\to$ Y, the following statements are equivalent.

(i) Its inverse map f^{-1}: Y $\to$ X is (1,2)*-g**-continuous.

(ii) f is a (1,2)*-g**-open map.

(iii) f is a (1,2)*-g**-closed map.

Proof. (i) $\Rightarrow$ (ii) Let G be any $\tau_{1,2}$-open set in X. Since f^{-1} is (1,2)*-g**-continuous, the inverse image of G under f^{-1} namely f(G) is (1,2)*-g**-open in Y. Hence f is (1,2)*-g**-open map.

(ii) $\Rightarrow$ (iii) Let F be any $\tau_{1,2}$-closed set in X. Then X\F is $\tau_{1,2}$-open in X. Since f is (1,2)*-g**-open map, f(X\F) is (1,2)*-g**-open in Y. But f(X\F) = Y\f(F), implies f(F) is (1,2)*-g**-closed in Y. Therefore f is (1,2)*-g**-closed map.

(iii) $\Rightarrow$ (i) Let F be any $\tau_{1,2}$-closed set in X. Then the inverse image of F under f^{-1}, namely f(F) is (1,2)*-g**-closed in Y. Since f is a (1,2)*-g**-closed map, f^{-1} is (1,2)*-g**-continuous.

Theorem 7.9. Let f: X $\to$ Y be bijective and (1,2)*-g**-continuous map. Then the following statements are equivalent.

(i) f is (1,2)*-g**-open map.

(ii) f is (1,2)*-g**-homeomorphism.

(iii) f is (1,2)*-g**-closed map.

Proof. (i) ⇒ (ii) By assumption, f is bijective, (1,2)*-g**-continuous and (1,2)*-g**-open map. Then by definition, f is a (1,2)*-g**-homeomorphism.

(ii) ⇒ (iii) By assumption, f is (1,2)*-g**-open and bijective. By Theorem 7.8, f is (1,2)*-g**-closed map.

(iii) ⇒ (i) By assumption, f is (1,2)*-g**-closed and bijective. By Theorem 7.8, f is a (1,2)*-g**-open map.

References

1. **Abd El-Monsef, M.E., Thivagar, M.L. and Ravi, O.**,(2008) *Remarks on bitopological (1,2)*-Quotient mappings,* J. Egypt. Math. Soc. 16(1), 17.
2. **Balachandran, K., Sundaram, P., Maki, H,** (1991). *On generalized continuous maps in topological spaces,* Mem. Fac. Kochi Univ. Ser A., Math., 12, 5.
3. **Bhattacharrya, P., Lahiri, B.K.,** (1987). *Semi-generalized closed sets in a topology*, Indian J. Math., 29(3), 375.
4. **Dunham, W.,** (1977). $T_{½}$ *-spaces,* Kyungpook Math. J., 17, 161.
5. **Dunham, W.,** (1982). *A new closure operator for non* T_1 *topologies,* Kyungpook Math. J., 22, 55.
6. **Levine, N.,** (1970). *Generalized closed sets in topology, R*end. Circ. Math. Palermo., 19(2), 169.
7. **Levine, N.,** (1963). *Semi-open sets and semi-continuity in topological spaces,* Amer. Math. Monthly, 70, 36.
8. **Ravi, O. and Thivagar, M.L.,** (2004). *On stronger forms of (1,2)*-quotient mappings in bitopological spaces,* Internat. J. Math. Game Theory and Algebra, 14, No.6, 481.
9. **Ravi, O. and Thivagar, M.L.,** (2006). *A bitoplogical (1,2)*-semi-generalized continuous maps,* Bull. Malays. Math. Sci. Soc., (2)29(1), 79.
10. **Ravi, O. and Thivagar, M.L.,** *Remarks on extensions of (1,2)*-g-closed mappings,* (submitted).