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$f\tau^{\star}g^{\star}$ Semi-closed Sets in Fine-Topological Spaces

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ABSTRACT

Powar P. L. and Rajak K. have introduced fine-topological space which is a special case of generalized topological space. In this paper, we introduce a new class of sets called $f\tau^{\star}g^{\star}$ semi-closed sets and fine-generalized star semi-open sets in fine-topological spaces and study some of their properties.

KEYWORDS. – Fine-open sets, fine-closed sets, $f\tau^{\star}g^{\star}$ Semi-closed Sets.

1. Introduction

In 1970, Levine [3, 4] introduced the concept of generalized closed sets and semi open sets in topological spaces and studied most fundamental properties. Dunham [2] introduced the concept of closure operator cl^{*}, a new topology τ^{*} and studied some of their properties. Arya [1] introduced and investigated generalized semi closed sets in topological spaces. Pushpalatha et al. [10] introduced a new generalization of closed set in the weaker topological space (X, τ^{*}).

Powar P. L. and Rajak K. [9], have investigated a special case of generalized topological space called fine topological space. In this space, the authors have defined a new class of open sets namely fine-open sets which contains all α −open sets, β −open sets, semi-open sets, pre-open sets, regular open sets etc.. By using these fine-open sets they have defined fine-irresolute mappings which includes pre-continuous functions, semi-continuous function, α −continuouos function, β −continuous functions, α −irresolute functions, β −irresolute functions, etc (cf. [5]-[8]).

The aim of this paper is to introduce the concepts of $f\tau^{\star}g^{\star}s$ −closed sets in the fine topological space (X, τ, τ_f).

2. Preliminaries

Throughout this paper X and Y are topological spaces on which no separation axioms are assumed unless otherwise explicitly stated. For a subset A of a topological space X, int(A), $int^{\star}(A)$, cl(A), $cl^{\star}(A)$, $scl^{\star}(A)$ and A^c denote the interior, interior *, closure, closure *, semiclosure and complement of A respectively.

Let us recall the following definition which we shall require later.

Definition 2.1. [3] A subset A of a topological space (X, τ) is called generalized closed set (briey, g-closed) if $cl\ (A) \subseteq U$ whenever A_⊆U and U is open in X.

Remark 2.2. [2] For a subset A of a topological space (X, τ), the generalized closure of A denoted by cl*(A) is defined as the intersection of all g-closed sets containing A.

Definition 2.3. [3] Let (X, τ) be a topological space. Then the collection {G : cl*(G^c) = (G^c) is a topology on X and is denoted as τ* that is τ* ={G: cl*(G^c)=(G^c)}.

Remark 2.4. For a subset A of X

$$Cl^*(A) = \cap\{K : K \supset A; K^c \in \tau^*\},$$
$$int^*(A) = \cup\{G: G \subset A; G \in \tau^*\}$$

Definition 2.5. A subset A of a topological space X is said to be τ*-semi-closed-set (briefly τ*s-closed) if $int*(cl*(A)) \subseteq A$. The complement of a τ*s-closed set is called τ*-semi-open set (briey, τ*s-open).

Definition 2.6. For a subset A of a topological space (X, τ*), the generalized semi closure of A denoted as scl*(A) is defined as the intersection of all semi-closed sets in τ*containing A.

Definition 2.7 Let (X, τ) be a topological space we define

$\tau(A_\alpha) = \tau_\alpha$ (say) = {$G_\alpha(\neq X) : G_\alpha \cap A_\alpha = \phi$, for $A_\alpha \in \tau$ and $A_\alpha \neq \phi, X$, for some $\alpha \in J$, where J is the index set.}

Now, we define

$$\tau_f = \{\phi, X, \cup_{\{\alpha\in J\}}\{\tau_\alpha\}\}$$

The above collection τ_f of subsets of X is called the fine collection of subsets of X and (X, τ, τ_f) is said to be the fine space X generated by the topology τ on X (cf. [10]).

Definition 2.8 A subset U of a fine space X is said to be a fine-open set of X, if U belongs to the collection τ_f and the complement of every fine-open sets of X is called the fine-closed sets of X and we denote the collection by F_f (cf. [10]).

Definition 2.9 Let A be a subset of a fine space X, we say that a point x ∈ X is a fine limit point of A if every fine-open set of X containing x must contains at least one point of A other than x (cf. [10]).

Definition 2.10 Let A be the subset of a fine space X, the fine interior of A is defined as the union of all fine-open sets contained in the set A i.e. the largest fine-open set contained in the set A and is denoted by f_{Int} (cf. [10]).

Definition 2.11 Let A be the subset of a fine space X, the fine closure of A is defined as the intersection of all fine-closed sets containing the set A i.e. the smallest fine-closed set containing the set A and is denoted by f_{cl} (cf. [10]).

Definition 2.12 A function f : (X, τ, τ_f) → (Y, τ ′, τ_f') is called fine-irresolute (or f-irresolute) if $f^{-1}(V)$ is fine-open in X for every fine-open set V of Y (cf. [10]).

3. Generalized star semi-closed sets in topological spaces

In this section, we introduce the concept of $f\tau$*-generalized star-semi-closed sets in fine- topological spaces.

Definition 3.1. A subset A of a fine-topological space (X, τ, τ_f) is called fine-generalized closed set (briey, fg-closed) if $f_{cl}(A) \subseteq U$ whenever A⊆U and U is fine-open in X.

Definition 3.2 For a subset A of a fine - topological space (X,τ,τ_f), the fine-generalized closure of A denoted by f_{cl}*(A) is defined as the intersection of all fg-closed sets containing A.

Definition 3.3 Let (X, τ, τ_f) be a topological space. Then the collection {G : f_{cl}*(G^c) = (G^c) is a fine-topology on X and is denoted as τ_f* that is τ_f* ={G: f_{cl}*(G^c)=(G^c).

Definition 3.4 For a subset A of X

$$f^*{}_{cl}(A) = \cap\{K : K \supset A; K^c \in f\tau^*\},$$
$$f^*{}_{int}(A) = \cup\{G: G \subset A; G \in f\tau^*\}$$

Definition 3.5 A subset A of a fine-topological space X is said to be τ*-semi-closed-set (briefly τ*s-closed) if $f^*_{int}(f^*_{cl}(A)) \subseteq A$. The complement of a τ_f*s-closed set is called τ_f*-semi-open set (briefly,$f\ \tau$*s-open).

Example

Definition 3.6 For a subset A of a fine-topological space (X, τ, τ_f), the fine-generalized semi closure of A denoted as $f^*_{scl}(A)$ is defined as the intersection of all fine-semi-closed sets in τ_f*containing A.

Definition 3.7 A subset A of a fine-topological space X is said to be τ_f*-generalized star-semi-closed (briefly fτ*-g*s-closed) set if $f^*_{scl}(A) \subseteq G$ whenever $A \subseteq G$ and G is $f\tau$*-semi-open.

The complement of a $f\tau$*g*s-closed set is called as $f\tau$*g*s -open.

Example 3.8 Let X = {a, b, c} with the topology $\tau = \{\phi, X, \{b\}\}$, $\tau_f = \{\phi, X, \{b\}, \{a, b\}, \{b, c\}\}$. It can be easily check that $\tau_f^* = \{\phi, X, \{a\}, \{a, b\}, \{b\}\}$.

Remark 3.9 It can be easily check that τ_f^* is a topology on X.

Example 3.10 Let X = {a, b, c} with the topology $\tau = \{\phi, X, \{a,b\}\}$, $\tau_f = \{\phi, X, \{b\}, \{a\}, \{a,b\}, \{a,c\} \{b,c\}\}$. It can be easily check that $\tau_f^* = \{\phi, X, \{a\}, \{a,b\}, \{b\}, \{a,c\}\}$. Let A = {a} and B = {a, b}, it may be observe that $f_{cl}^*(A) = \{a,c\}$ and $f_{int}^*(B) = \{a,b\}$.

Theorem 3.11 Every fine-closed set in X is fτ*g*s -closed.

Proof. Let A be a closed set. Let $A \subseteq G$ and G is $f\tau$*g*s -semi-open. Since A is fine-closed, $f_{cl}(A) = A \subseteq G$. But $f_{scl}^*(A) \subseteq f_{cl}(A)$. Hence we have $f_{scl}^*(A) \subseteq G$ whenever $A \subseteq G$ and G is $f\tau$*-semi-open. Therefore, A is $f\tau$*g*s -closed.

Theorem 3.12 Every fτ*-closed set in X is fτ*g*s -closed.

Proof. Let A be a fτ*-closed set. Let $A \subseteq G$ and G is $f\tau$*-semi-open. Since A is fτ*- closed, $f_{cl}(A) = A \subseteq G$. But $f_{scl}^*(A) \subseteq G$. Hence we have $f_{scl}^*(A) \subseteq G$ whenever $A \subseteq G$ and G is fτ*-semi-open. Therefore A is fτ*g*s -closed.

Theorem 3.13 Every fτ*g-closed set in X is fτ*g*s -closed.

Proof. Let A be a fτ*g-closed set. Let $A \subseteq G$ and G is fτ*-semi-open. Since A is $f\tau$*g-closed, $f_{cl}(A) = A \subseteq G$. But $f_{scl}^*(A) \subseteq f_{cl}^*(A)$. Thus we have $f_{scl}^*(A) \subseteq G$ whenever $A \subseteq G$ and G is $f\tau$*g-semi-open. Therefore A is $f\tau$*g*s -closed.

The converse of the above Theorem need not be true as seen from the following example.

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