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CERTAIN TYPES OF NEUTROSOPHIC GRAPHS

[1]R.Dhavaseelan,[2]R.Vikramaprasad, *[3]V.Krishnaraj

[1,2,3]Department of Mathematics,

Sona College of Technology, Salem - 636 005,

Tamil Nadu, India.

Abstract

In this paper, the concept of strong neutrosophic graph is introduced. Some interesting properties of strong neutrosophic graphs are studied.

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1 Introduction

The notion graph theory was first introduced by Euler in 1736. In the history of mathematics, the solution given by Euler of the well known Königsberg bridge problem is considered to be the first theorem of graph theory. This has now become a subject generally regarded as a branch of combinatorics.The theory of graph is an extremely useful tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, operations research, optimization and computer science. On the other hand, fuzzy graph theory as a generalization of Eulers graph theory was first introduced by Rosenfeld [5] in 1975. In 1965, Zadeh [8] proposed the theory of fuzzy set theory which is applied in many real applications to handle uncertainty. Atanassov [2] added a new component (which determines the degree of non-membership) in the definition of fuzzy set. The concept of Neutrosophic set was introduced by F. Smarandache [6] which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. The notion of strong graphs and investigate some of their properties [1]. In this paper, the concept of strong neutrosophic graph is introduced. Some interesting properties of strong neutrosophic graphs are studied.

*Corresponding Author: krishna.raj1282@gmail.com, vikramprasad20@gmail.com, dhavaseelan.r@gmail.com

2 Preliminaries

Definition 2.1. [1] An intuitionistic fuzzy graph is of the form $G = \langle V, E \rangle$ where

1. $V = \{v_1, v_2, ..., v_n\}$ such that $\mu_1 : V \to [0, 1]$ and $\gamma_1 : V \to [0, 1]$ denote the degree of membership and nonmembership of the element $v_i \in V$ respectively, and

$$0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1 \tag{2.1}$$

2. $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0, 1]$ and $\gamma_2 : V \times V \to [0, 1]$ are such that

$$\mu_2(v_i, v_j) \leq min\{\mu_1(v_i), \mu_1(v_j)\} \tag{2.2}$$

$$\gamma_2(v_i, v_j) \leq max\{\gamma_1(v_i), \gamma_1(v_j)\} \tag{2.3}$$

and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$, $i, j = 1, 2, ..., n$.

3 Certain types of Neutrosophic Graphs

Definition 3.1. A Neutrosophic graph (NG) is of the form $G =< V,\ E >$ where

1. $V = \{v_1,\ v_2, v_3, ..., v_n\}$ such that $\mu_1 : V \to [0, 1]$, $\sigma_1 : V \to [0, 1]$ and $\gamma_1 : V \to [0, 1]$ denote the degree of membership, degree of indeterminacy and non-membership of the element $v_i \in V$, respectively, and $0 \leq \mu_1(v_i) + \sigma_1(v_i) + \gamma_1(v_i) \leq 3$ for every

$$v_i \in V,\ (i = 1, 2, \ldots, n)$$

2. $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0, 1]$, $\sigma_2 : V \times V \to [0, 1]$ and $\gamma_2 : V \times V \to [0, 1]$ are such that $\mu_2(v_i, v_j) \leq \min[\mu_1(v_i),\ \mu_1(v_j)]$, $\sigma_2(v_i, v_j) \leq \min[\sigma_1(v_i),\ \sigma_1(v_j)]$ and $\gamma_2(v_i,\ v_j) \leq \min[\gamma_1(v_i),\ \gamma_1(v_j)]$ and $0 \leq\ \mu_2(v_i, v_j) +\ \sigma_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 3$ for every

$$(v_i,\ v_j) \in E(i,\ j = 1, 2, 3, ..., n).$$

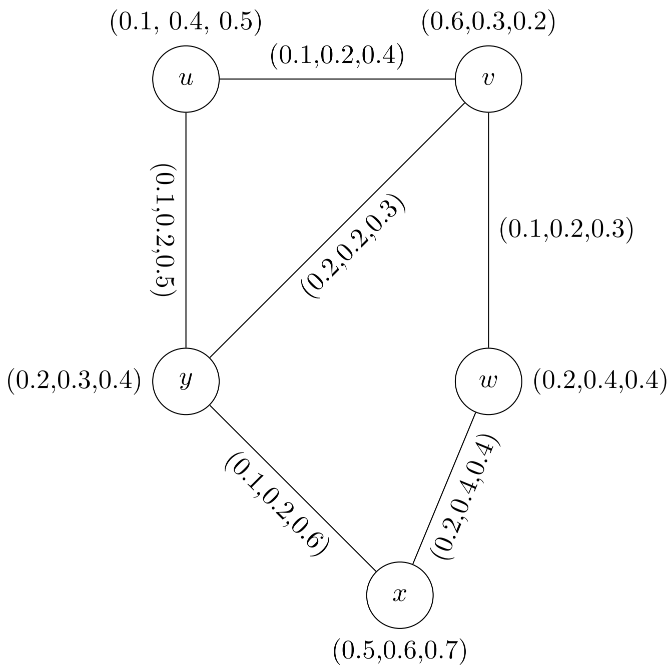

Figure 1: G : Neutrosophic Graph

Definition 3.2. An neutrosophic graph $G = \langle V, E\rangle$ is said to be complete neutrosophic graph if $\mu_{2ij} = min(\mu_{1i}, \mu_{1j})$, $\sigma_{2ij} = min(\sigma_{1i}, \sigma_{1j})$ and $\gamma_{2ij} = max(\gamma_{1i}, \gamma_{1j})$, for every $v_i, v_j \in V$.

Definition 3.3. Let $G = \langle V, E\rangle$ be a neutrosophic graph. Then the degree of a vertex v is defined by $d(v) = (d_\mu(v), d_\gamma(v))$ where $d_\mu(v) = \sum_{u\neq v} \mu_2(u, v)$,$d_\sigma(v) = \sum_{u\neq v} \sigma_2(u, v)$ and $d_\gamma(v) = \sum_{u\neq v} \gamma_2(u, v)$

Definition 3.4. A neutrosophic graph $G =< V, E >$ and G^* is called Strong Neutrosophic graph.

$$\mu_2\left(v_i,\ v_j\right) = \min\ \left[\mu_1\left(v_i\right),\ \mu_1\left(v_j\right)\right]$$

$$\sigma_2\left(v_i,\ v_j\right) = \min\ \left[\sigma_1\left(v_i\right),\ \sigma_1\left(v_j\right)\right]$$

$$\gamma_2\left(v_i,\ v_j\right) = \max\ \left[\gamma_1\left(v_i\right),\ \gamma_1\left(v_j\right)\right]$$

for all $(v_i,\ v_j) \in E$

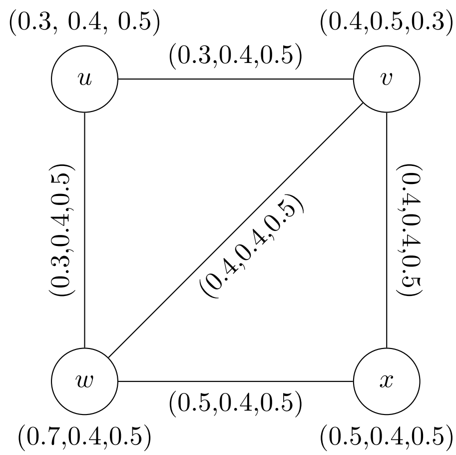

Figure 2: G : Strong Neutrosophic Graph

Definition 3.5. The complement of a strong neutrosophic graph $G\ on\ G^*$ a Strong Neutrosophic Graph $\overline{G}\ on\ G^*$ where

1. $\overline{V} = V$

2. $\overline{\mu_1}(v_i) = \mu_1(v_i)$, $\overline{\sigma_1}(v_i) = \sigma_1(v_i)$, $\overline{\gamma_1}(v_i) = \gamma_1(v_i)$, $v_i \in V$

3.
$$\overline{\mu_2}(v_i,\ v_j) = \begin{cases} 0 & if \quad \mu_2(v_i,\ v_j) > 0 \\ \min\ [\mu_1(v_i),\ \mu_1(v_j)] & if \quad \mu_2(v_i,\ v_j) = 0 \end{cases}$$

$$\overline{\gamma_2}(v_i,\ v_j) = \begin{cases} 0 & if \quad \gamma_2(v_i,\ v_j) > 0 \\ \max\ [\gamma_1(v_i),\ \gamma_1(v_j)] & if \quad \gamma_2(v_i,\ v_j) = 0 \end{cases}$$

$$\overline{\sigma_2}(v_i,\ v_j) = \begin{cases} 0 & if \quad \sigma_2(v_i,\ v_j) > 0 \\ \min\ [\sigma_1(v_i),\ \sigma_1(v_j)] & if \quad \sigma_2(v_i,\ v_j) = 0 \end{cases}$$

Remark 3.1. If $G =< V, E >$ is a neutrosophic graph on G^*. Then from above definition, it follow that $\overline{\overline{G}}$ is given by the neutrosophic graph $\overline{\overline{G}} =< \overline{\overline{V}},\ \overline{\overline{E}} >$ on G^* where $\overline{\overline{V}} = V$ and $\overline{\overline{\mu_2}}(v_i,\ v_j) = \min\ [\mu_1(v_i),\ \mu_1(v_j)]$, $\overline{\overline{\sigma_2}}(v_i,\ v_j) = \min\ [\sigma_1(v_i),\ \sigma_1(v_j)]$, $\overline{\overline{\gamma_2}}(v_i,\ v_j) = \max\ [\gamma_1(v_i),\ \gamma_1(v_j)]$. for all $(v_i,\ v_j) \in E$

Thus $\overline{\overline{\mu_2}} = \mu_2$, $\overline{\overline{\sigma_2}} = \sigma_2$, $and\ \overline{\overline{\gamma_2}} = \gamma_2$ on V, where $E = (\mu_2\ ,\ \sigma_2\ ,\ \gamma_2)$ is the strong neutrosophic relation on V. For any neutrosophic graph G, $\overline{G}$ is strong neutrosophic graph and $G \subseteq \overline{\overline{G}}$.

Proposition 3.1. $G = \overline{\overline{G}}$ if and only if G is a strong neutrosophic graph.

Proof. It is obvious. □

Definition 3.6. A strong neutrosophic graph G is called self complementary if $G \approx \overline{G}$.

Example 3.1. Consider a graph $G^* = (V,\ E)$ such that $V = \{a, b, c, d\}$, $E = \{ab,\ ac,\ bc,\ cd\}$. Consider a strong neutrosophic graph G;

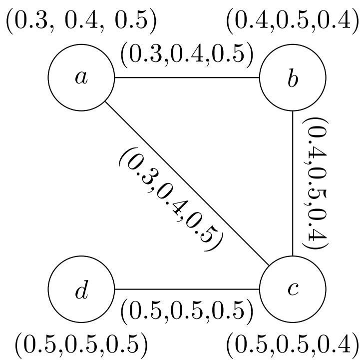

Figure 3: G

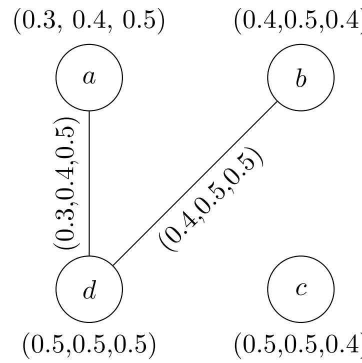

Figure 4: $\overline{G}$

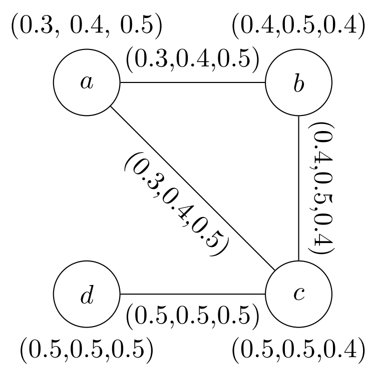

Figure 5: $\overline{\overline{G}}$

Clearly, $G = \overline{\overline{G}}$. Hence G is self complementary.

Proposition 3.2. Let G be a strong neutrosophic graph. If $\mu_2(v_i,\ v_j) = \min\ [\mu_1(v_i),\ \mu_1(v_j)]$, $\sigma_2(v_i,\ v_j) = \min\ [\sigma_1(v_i),\ \sigma_1(v_j)]$, $\gamma_2(v_i,\ v_j) = \max\ [\gamma_1(v_i),\ \gamma_1(v_j)]$ for all $v_i,\ v_j \in V$. Then G is self complementary.

Proof. Let G be a strong neutrosophic graph such that

$$\mu_2(v_i,\ v_j) = \min\ [\mu_1(v_i),\ \mu_1(v_j)]$$

$$\sigma_2(v_i,\ v_j) = \min\ [\sigma_1(v_i),\ \sigma_1(v_j)]$$

$$\gamma_2(v_i,\ v_j) = \max\ [\gamma_1(v_i),\ \gamma_1(v_j)]$$

for all $v_i,\ v_j \in V$. Then $G \simeq\ \overline{G}$ under the identity map $I : V \to V$.Hence G is self complementary. □

Proposition 3.3. Let G be a self complementary strong neutrosophic graphic. Then

$$\sum_{v_i \neq v_j} \mu_2(v_i,\ v_j) = \sum_{v_i \neq v_j} \min\ [\mu_1(v_i),\ \mu_1(v_j)]$$

$$\sum_{v_i \neq v_j} \sigma_2(v_i,\ v_j) = \sum_{v_i \neq v_j} \min\ [\sigma_1(v_i),\ \sigma_1(v_j)]$$

$$\sum_{v_i \neq v_j} \gamma_2(v_i,\ v_j) = \sum_{v_i \neq v_j} \max\ [\gamma_1(v_i),\ \gamma_1(v_j)]$$

Proposition 3.4. Let G_1 and G_2 be strong neutrosophic graphics. $G_1 \approx G_2$.

Proof. Assume that G_1 and G_2 are isomorphic, there exist a bijective map $f :_{V_1} \to_{V_2}$ satisfying $\mu_{V_1}(v_i) = \mu_{V_2}(f(v_i))$, $\sigma_{V_1}(v_i) = \sigma_{V_2}(f(v_i))$,$\gamma_{V_1}(v_i) = \gamma_{V_2}(f(v_i))$ for all $v_i \in_{V_1}$.
$\mu_{E_1}(v_i, v_j) = \mu_{E_2}(f(v_i),\ f(v_j))$,$\sigma_{E_1}(v_i, v_j) = \sigma_{E_2}(f(v_i), f(v_j))$,$\gamma_{E_1}(v_i, v_j) = \gamma_{E_2}(f(v_i),\ f(v_j))$ for all $(v_i,\ v_j) \in_{E_1}$

By definition of complement, we have

$$\begin{aligned}\overline{\mu_{E_1}}(v_i, v_j) &= \min[\mu_{V_1}(v_i), \mu_{V_1}(v_j)] \\ &= \min[\mu_{V_2}(f(v_i)), \mu_{V_2}(f(v_j))] \\ &= \overline{\mu_{E_2}}(f(v_i), f(v_j)),\end{aligned}$$

$$\begin{aligned}\overline{\sigma_{E_1}}(v_i,\ v_j) &= \min\ \left[\sigma_{V_1}(v_i),\ \sigma_{V_1}(v_j)\right] \\ &= \min[\sigma_{V_2}(f(v_i))\ ,\ \sigma_{V_2}(f(v_j))] \\ &= \overline{\sigma_{E_2}}(f(v_i),\ f(v_j)),\end{aligned}$$

$$\begin{aligned}\overline{\gamma_{E_1}}(v_i,\ v_j) &= \max\left[\gamma_{V_1}(v_i),\ \gamma_{V_1}(v_j)\right] \\ &= \max[\gamma_{V_2}(f(v_i))\ ,\ \gamma_{V_2}(f(v_j))] \\ &= \overline{\gamma_{E_2}}(f(v_i),\ f(v_j)).\end{aligned}$$

For all $(v_i,\ v_j) \in E_1$. Hence $\overline{G_1} \approx \overline{G_2}$. The converse is straight forward. □

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