

# Fractional derivatives involving general classes of polynomials and product of two multivariable I-functions defined by Nambisan representable by Mellin-Barnes type contour integrals

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## ABSTRACT

In this document, first time, we established fractional derivative involving the product of two general classes of polynomials with the product of two multivariable I-functions defined by Nambisan et al [4] and generalized hypergeometric function. Second time, we established fractional derivative involving the product of two general classes of polynomials with the product of three multivariable I-functions defined by Nambisan et al [5] and generalized Lauricella function.

Keywords: General class of polynomial, fractional derivative, generalized Lauricella function, multivariable I-function, multivariable H-function, class of polynomials

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## 1. Introduction and preliminaries.

In this document, first time, we established fractional derivative involving the product of two general classes of polynomials with multivariable I-functions defined by Nambisan et al [5] and generalized Lauricella hypergeometric function. Second time, we established fractional derivative involving the product of two general classes of polynomials with three multivariable I-functions and generalized Lauricella function.

The I-function is defined and represented in the following manner.

$$I(z_1, z_2, \dots, z_r) = I_{p, q: p_1, q_1; \dots; p_r, q_r}^{0, n: m_1, n_1; \dots; m_r, n_r} \left( \begin{array}{c} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{array} \middle| \begin{array}{l} (a_j; \alpha_j^{(1)}, \dots, \alpha_j^{(r)}; A_j)_{1, p} : \\ \\ (b_j; \beta_j^{(1)}, \dots, \beta_j^{(r)}; B_j)_{1, q} : \end{array} \right. \\ \left. \begin{array}{l} (c_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{1, p_1}; \dots; (c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)})_{1, p_r} \\ (d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{1, q_1}; \dots; (d_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{1, q_r} \end{array} \right) \quad (1.1)$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \phi(s_1, \dots, s_r) \prod_{i=1}^r \theta_i(s_i) z_i^{s_i} ds_1 \dots ds_r \quad (1.2)$$

where  $\phi(s_1, \dots, s_r), \theta_i(s_i), i = 1, \dots, r$  are given by :

$$\phi(s_1, \dots, s_r) = \frac{\prod_{j=1}^n \Gamma^{A_j} \left(1 - a_j + \sum_{i=1}^r \alpha_j^{(i)} s_j\right)}{\prod_{j=n+1}^p \Gamma^{A_j} \left(a_j - \sum_{i=1}^r \alpha_j^{(i)} s_j\right) \prod_{j=1}^q \Gamma^{B_j} \left(1 - b_j + \sum_{i=1}^r \beta_j^{(i)} s_j\right)} \quad (1.3)$$

$$\theta_i(s_i) = \frac{\prod_{j=1}^{n_i} \Gamma^{C_j^{(i)}} \left(1 - c_j^{(i)} + \gamma_j^{(i)} s_i\right) \prod_{j=1}^{m_i} \Gamma^{D_j^{(i)}} \left(d_j^{(i)} - \delta_j^{(i)} s_i\right)}{\prod_{j=n_i+1}^{p_i} \Gamma^{C_j^{(i)}} \left(c_j^{(i)} - \gamma_j^{(i)} s_i\right) \prod_{j=m_i+1}^{q_i} \Gamma^{D_j^{(i)}} \left(1 - d_j^{(i)} - \delta_j^{(i)} s_i\right)} \quad (1.4)$$

For more details, see Nambisan et al [5].

Following the result of Braaksma [1] the I-function of r variables is analytic if :

$$U_i = \sum_{j=1}^p A_j \alpha_j^{(i)} - \sum_{j=1}^q B_j \beta_j^{(i)} + \sum_{j=1}^{p_i} C_j^{(i)} \gamma_j^{(i)} - \sum_{j=1}^{q_i} D_j^{(i)} \delta_j^{(i)} \leq 0, i = 1, \dots, r \quad (1.5)$$

The integral (2.1) converges absolutely if

$$|arg(z_k)| < \frac{1}{2} \Delta_k \pi, k = 1, \dots, r \text{ where}$$

$$\Delta_k = - \sum_{j=n+1}^p A_j \alpha_j^{(k)} - \sum_{j=1}^q B_j \beta_j^{(k)} + \sum_{j=1}^{m_k} D_j^{(k)} \delta_j^{(k)} - \sum_{j=m_k+1}^{q_k} D_j^{(k)} \delta_j^{(k)} + \sum_{j=1}^{n_k} C_j^{(k)} \gamma_j^{(k)} - \sum_{j=n_k+1}^{p_k} C_j^{(k)} \gamma_j^{(k)} > 0 \quad (1.6)$$

Consider the second multivariable I-function.

$$I(z'_1 z'_2, \dots, z'_s) = I_{p', q' : p'_1, q'_1; \dots; p'_s, q'_s}^{0, n' : m'_1, n'_1; \dots; m'_s, n'_s} \left( \begin{matrix} z'_1 \\ \vdots \\ z'_s \end{matrix} \middle| \begin{matrix} (a'_j; \alpha'_j{}^{(1)}, \dots, \alpha'_j{}^{(s)}; A'_j)_{1, p'} : \\ \\ (b'_j; \beta'_j{}^{(1)}, \dots, \beta'_j{}^{(s)}; B'_j)_{1, q'} : \end{matrix} \right.$$

$$\left. (c'_j{}^{(1)}, \gamma'_j{}^{(1)}; C'_j{}^{(1)})_{1, p'_1}; \dots; (c'_j{}^{(s)}, \gamma'_j{}^{(s)}; C'_j{}^{(s)})_{1, p'_s} \right)$$

$$(d'_j{}^{(1)}, \delta'_j{}^{(1)}; D'_j{}^{(1)})_{1, q'_1}; \dots; (d'_j{}^{(s)}, \delta'_j{}^{(s)}; D'_j{}^{(s)})_{1, q'_s} \quad (1.7)$$

$$= \frac{1}{(2\pi\omega)^s} \int_{L_1} \dots \int_{L_s} \psi(t_1, \dots, t_s) \prod_{i=1}^s \zeta_i(t_i) z_i^{t_i} dt_1 \dots dt_s \quad (1.8)$$

where  $\psi(t_1, \dots, t_s)$ ,  $\zeta_i(s_i)$ ,  $i = 1, \dots, s$  are given by :

$$\psi(t_1, \dots, t_s) = \frac{\prod_{j=1}^{n'} \Gamma^{A'_j} \left(1 - a'_j + \sum_{i=1}^s \alpha_j'^{(i)} t_j\right)}{\prod_{j=n'+1}^p \Gamma^{A'_j} \left(a'_j - \sum_{i=1}^s \alpha_j'^{(i)} t_j\right) \prod_{j=1}^{q'} \Gamma^{B'_j} \left(1 - b'_j + \sum_{i=1}^s \beta_j'^{(i)} t_j\right)} \quad (1.9)$$

$$\zeta_i(s_i) = \frac{\prod_{j=1}^{n'_i} \Gamma^{C_j'^{(i)}} \left(1 - c_j'^{(i)} + \gamma_j'^{(i)} t_i\right) \prod_{j=1}^{m'_i} \Gamma^{D_j'^{(i)}} \left(d_j'^{(i)} - \delta_j'^{(i)} t_i\right)}{\prod_{j=n'_i+1}^{p'_i} \Gamma^{C_j'^{(i)}} \left(c_j'^{(i)} - \gamma_j'^{(i)} t_i\right) \prod_{j=m'_i+1}^{q'_i} \Gamma^{D_j'^{(i)}} \left(1 - d_j'^{(i)} - \delta_j'^{(i)} t_i\right)} \quad (1.10)$$

For more details, see Nambisan et al [5].

Following the result of Braaksma [1] the I-function of r variables is analytic if :

$$U'_i = \sum_{j=1}^{p'_i} A'_j \alpha'_j{}^{(i)} - \sum_{j=1}^{q'_i} B'_j \beta'_j{}^{(i)} + \sum_{j=1}^{p'_i} C'_j{}^{(i)} \gamma'_j{}^{(i)} - \sum_{j=1}^{q'_i} D'_j{}^{(i)} \delta'_j{}^{(i)} \leq 0, i = 1, \dots, s \quad (1.11)$$

The integral (2.1) converges absolutely if

$$|arg(z'_k)| < \frac{1}{2} \Delta'_k \pi, k = 1, \dots, s \text{ where}$$

$$\Delta'_k = - \sum_{j=n'_k+1}^{p'_k} A'_j \alpha'_j{}^{(k)} - \sum_{j=1}^{q'_k} B'_j \beta'_j{}^{(k)} + \sum_{j=1}^{m'_k} D'_j{}^{(k)} \delta'_j{}^{(k)} - \sum_{j=m'_k+1}^{q'_k} D'_j{}^{(k)} \delta'_j{}^{(k)} + \sum_{j=1}^{n'_k} C'_j{}^{(k)} \gamma'_j{}^{(k)} - \sum_{j=n'_k+1}^{p'_k} C'_j{}^{(k)} \gamma'_j{}^{(k)} > 0 \quad (1.12)$$

Consider the third multivariable I-function.

$$I(z''_1, \dots, z''_u) = I_{p'', q''; p''_1, q''_1; \dots; p''_u, q''_u}^{0, n''; m''_1, n''_1; \dots; m''_u, n''_u} \left( \begin{matrix} z''_1 \\ \cdot \\ \cdot \\ \cdot \\ z''_u \end{matrix} \middle| \begin{matrix} (a''_j; \alpha''_j{}^{(1)}, \dots, \alpha''_j{}^{(u)}; A''_j)_{1, p''} : \\ \\ \\ (b''_j; \beta''_j{}^{(1)}, \dots, \beta''_j{}^{(u)}; B''_j)_{1, q''} : \end{matrix} \right.$$

$$\left. (c''_j{}^{(1)}, \gamma''_j{}^{(1)}; C''_j{}^{(1)})_{1, p''_1}; \dots; (c''_j{}^{(u)}, \gamma''_j{}^{(u)}; C''_j{}^{(u)})_{1, p''_u} \right)$$

$$(d''_j{}^{(1)}, \delta''_j{}^{(1)}; D''_j{}^{(1)})_{1, q''_1}; \dots; (d''_j{}^{(u)}, \delta''_j{}^{(u)}; D''_j{}^{(u)})_{1, q''_u} \quad (1.13)$$

$$= \frac{1}{(2\pi\omega)^u} \int_{L''_1} \dots \int_{L''_u} \psi(x_1, \dots, x_u) \prod_{i=1}^u \xi_i(x_i) z''_i{}^{x_i} dx_1 \dots dx_u \quad (1.14)$$

where  $\psi(x_1, \dots, x_u)$ ,  $\xi_i(x_i)$ ,  $i = 1, \dots, u$  are given by :

$$\psi(x_1, \dots, x_u) = \frac{\prod_{j=1}^{n''} \Gamma^{A''}_j (1 - a''_j + \sum_{i=1}^u \alpha''_j{}^{(i)} x_i)}{\prod_{j=n''+1}^{p''} \Gamma^{A''}_j (a''_j - \sum_{i=1}^u \alpha''_j{}^{(i)} x_i) \prod_{j=m''+1}^{q''} \Gamma^{B''}_j (1 - b''_j + \sum_{i=1}^u \beta''_j{}^{(i)} x_i)} \quad (1.15)$$

$$\xi_i(x_i) = \frac{\prod_{j=1}^{n''_i} \Gamma^{C''_j{}^{(i)}} (1 - c''_j{}^{(i)} + \gamma''_j{}^{(i)} x_i) \prod_{j=1}^{m''_i} \Gamma^{D''_j{}^{(i)}} (d''_j{}^{(i)} - \delta''_j{}^{(i)} x_i)}{\prod_{j=n''_i+1}^{p''_i} \Gamma^{C''_j{}^{(i)}} (c''_j{}^{(i)} - \gamma''_j{}^{(i)} x_i) \prod_{j=m''_i+1}^{q''_i} \Gamma^{D''_j{}^{(i)}} (1 - d''_j{}^{(i)} - \delta''_j{}^{(i)} x_i)} \quad (1.16)$$

For more details, see Nambisan et al [5].

Following the result of Braaksma [1] the I-function of r variables is analytic if :

$$U_i = \sum_{j=1}^{p''_i} A''_j \alpha''_j{}^{(i)} - \sum_{j=1}^{q''_i} B''_j \beta''_j{}^{(i)} + \sum_{j=1}^{p''_i} C''_j{}^{(i)} \gamma''_j{}^{(i)} - \sum_{j=1}^{q''_i} D''_j{}^{(i)} \delta''_j{}^{(i)} \leq 0, i = 1, \dots, u \quad (1.17)$$

The integral (2.1) converges absolutely if

$$\text{where } |arg(z_k'')| < \frac{1}{2} \Delta_k'' \pi, k = 1, \dots, u$$

$$\Delta_k' = - \sum_{j=n''+1}^{p''} A_j'' \alpha_j''(k) - \sum_{j=1}^{q''} B_j'' \beta_j''(k) + \sum_{j=1}^{m''} D_j'' \delta_j''(k) - \sum_{j=m_k''+1}^{q_k''} D_j'' \delta_j''(k) + \sum_{j=1}^{n_k''} C_j'' \gamma_j''(k) - \sum_{j=n_k''+1}^{p_k''} C_j'' \gamma_j''(k) > 0 \quad (1.18)$$

A general of Humbert polynomial is defined by the generating function [4,page 54, eq.(1.5)]

$$[C - ax t + b t^m (2x - 1)^d]^v = \sum_{n=0}^{\infty} \phi_n(x) t^n \quad (1.19)$$

where  $m, a, d \in \mathbb{N}$ , and other parameters are unrestricted in general.

The finite series representation for  $\phi_n(x)$  is also obtained there [4,page 55 31. eq(2.1)]

$$\phi_n(x) = \sum_{k=0}^{[n/m]} \frac{(-)^k c^{-v-n+(m-1)k} (v)_{n+(1-m)k} (ax)^{n-mk} [b(2x-1)^d]^k}{k!(n-mk)!} \quad (1.20)$$

$$= \sum_{k=0}^{[n/m]} \sum_{l=0}^{[dk]} \frac{(-dk)_l 2^l c^{-v-n+(m-1)k} (v)_{n+(1-m)k} (a)^{n-mk} b^k (-)^{(d+1)k} x^{n-mk+l}}{l!k!(n-mk)!} \quad (1.21)$$

The generalized polynomials defined by Srivastava [7], is given in the following manner :

$$S_{N_1', \dots, N_t'}^{M_1', \dots, M_t'} [y_1, \dots, y_t] = \sum_{K_1=0}^{[N_1'/M_1']} \dots \sum_{K_t=0}^{[N_t'/M_t']} \frac{(-N_1')_{M_1' K_1}}{K_1!} \dots \frac{(-N_t')_{M_t' K_t}}{K_t!}$$

$$A[N_1', K_1; \dots; N_t', K_t] y_1^{K_1} \dots y_t^{K_t} \quad (1.22)$$

Where  $M_1', \dots, M_s'$  are arbitrary positive integers and the coefficients  $A[N_1', K_1; \dots; N_t', K_t]$  are arbitrary constants, real or complex.

The Riemann-Liouville fractional derivative (or integral ) of order  $\mu$  is defined as follows [2 ,page49]

$$D_x^\mu f(x) = \begin{cases} \frac{1}{\Gamma(-\mu)} \int_0^x (x-t)^{-\mu-1} f(t) dt, Re(\mu) < 0 \\ \frac{d^m}{dx^m} [D_x^{\mu-m} \{f(x)\}], 0 \leq m, m \in \mathbb{N} \end{cases} \quad (1.23)$$

We have the following fractional derivative formula [3,page 67, eq. (4.4.4)] is also required :

$$D_x^\mu (x^\lambda) = \frac{\Gamma(\lambda + 1)}{\Gamma(\lambda - \mu + 1)} x^{\lambda - \mu}, \operatorname{Re}(\lambda) > -1 \quad (1.24)$$

The generalized Leibnitz formula for fractional calculus is required in the following form [6, page 317]

$$D_x^\alpha [f(x)g(x)] = \sum_{n=-\infty}^{\infty} a \binom{\alpha}{an + \epsilon} D_x^{\alpha - an - \epsilon} [f(x)] D_x^{an + \epsilon} [g(x)] \quad (1.25)$$

$0 \leq a \leq 1$ ;  $\alpha$  is a real or complex arbitrary number.

The following generalized Lauricella function in terms of multiple contour integrals is also required [8, page 253]

$$\begin{aligned} \text{We have } F_{Q; Q_1, \dots, Q_t}^{P; P_1, \dots, P_t}(-z_1, \dots, -z_t) &= \frac{\prod_{j=1}^Q \Gamma(B_j) \prod_{j=1}^{Q_1} \Gamma(g'_j) \cdots \prod_{j=1}^{Q_t} \Gamma(g_j^{(t)})}{\prod_{j=1}^P \Gamma(A_j) \prod_{j=1}^{P_1} \Gamma(e'_j) \cdots \prod_{j=1}^{P_t} \Gamma(e_j^{(t)})} \frac{1}{(2\pi\omega)^t} \int_{L_1} \cdots \int_{L_t} \\ &\frac{\prod_{j=1}^P \Gamma(A_j + s_1 + \cdots + s_t) \prod_{j=1}^{P_1} \Gamma(e'_j + s_1) \cdots \prod_{j=1}^{P_t} \Gamma(e_j^{(t)} + s_t)}{\prod_{j=1}^Q \Gamma(B_j + s_1 + \cdots + s_t) \prod_{j=1}^{Q_1} \Gamma(g'_j + s_1) \cdots \prod_{j=1}^{Q_t} \Gamma(g_j^{(t)} + s_t)} \Gamma(-s_1) \cdots \Gamma(-s_t) z_1^{s_1} \cdots z_t^{s_t} ds_1 \cdots ds_t \end{aligned} \quad (1.21)$$

where the contours are of Barnes type with indentations, if necessary, to ensure that the poles of  $\Gamma(A_j + s_1 + \cdots + s_t)$  are separated from those of  $\Gamma(-s_j)$ ,  $j = 1, \dots, t$ . The above result (1.21) can be easily established by an appeal to the calculus of residues by calculating the residues at the poles of  $\Gamma(-s_j)$ ,  $j = 1, \dots, t$

## 2. Required formula

The fractional derivative of multivariable I-function is as follows :

**Lemme**

$$\begin{aligned} D_z^\alpha [z^\lambda I(c_1 z^{\sigma_1}, \dots, c_r z^{\sigma_r})] &= z^{\lambda - \alpha} I_{p+1, q; p_1, q_1; \dots; p_r, q_r}^{0, n+1; m_1, n_1; \dots; m_r, n_r} \left( \begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \middle| \begin{matrix} (a_j; \alpha_j^{(1)}, \dots, \alpha_j^{(r)}; A_j)_{1,p}, \\ (b_j; \beta_j^{(1)}, \dots, \beta_j^{(r)}; B_j)_{1,q} \end{matrix} \right) \\ &(-\lambda; \sigma_1, \dots, \sigma_r; 1) : (c_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{1,p_1}; \dots; (c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)})_{1,p_r} \\ &(\alpha - \lambda; \sigma_1, \dots, \sigma_r; 1) : (d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{1,q_1}; \dots; (d_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{1,q_r} \end{aligned} \quad (2.1)$$

To prove the Lemme, express the multivariable I-function in Mellin-Barnes contour integral with the help of (1.2) and use the formula (1.19). Now interpreting the result thus obtained with the Mellin-barnes contour integral, we arrive at the desired result.

$$\text{Where : } \sigma_i > 0, i = 1, \dots, r; \operatorname{Re} \left[ \lambda + \sum_{i=1}^r \sigma_i \min_{1 \leq j \leq m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} + 1 \right] > 0$$

### 3. Main results

We will use these notations for the two following theorems:

$$X = m'_1, n'_1; \dots; m'_s, n'_s; m''_1, n''_1; \dots; m''_u, n''_u; 1, P_1; \dots; 1, P_t \quad (3.1)$$

$$Y = p'_1, q'_1; \dots; p'_s, q'_s; p''_1, q''_1; \dots; p''_u, q''_u; P_1, Q_1 + 1; \dots; P_t, Q_t + 1 \quad (3.2)$$

$$A = (a'_j; \alpha'_j{}^{(1)}, \dots, \alpha'_j{}^{(s)}, 0, \dots, 0, 0, \dots, 0; A_j)_{1,p'} \quad (3.3)$$

$$B = (b'_j; \beta'_j{}^{(1)}, \dots, \beta'_j{}^{(s)}, 0, \dots, 0, 0, \dots, 0; B'_j)_{1,q'} \quad (3.4)$$

$$A' = (a''_j; 0, \dots, 0, \alpha''_j{}^{(1)}, \dots, \alpha''_j{}^{(u)}, 0, \dots, 0; A''_j)_{1,p''} \quad (3.5)$$

$$B' = (b''_j; 0, \dots, 0, \beta''_j{}^{(1)}, \dots, \beta''_j{}^{(u)}, 0, \dots, 0; B''_j)_{1,q''} \quad (3.6)$$

$$\begin{aligned} C = & (c_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{1,p'_1}; \dots; (c_j^{(s)}, \gamma_j^{(s)}; C_j^{(s)})_{1,p'_s}; (c_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{1,p'_1}; \dots; \\ & (c_j^{(u)}, \gamma_j^{(u)}; C_j^{(u)})_{1,p'_u}; (1, 0; 1), (1 - e'_j, f'_j; 1)_{1,P_1}, \dots, (1, 0; 1), (1 - e_j^{(t)}, f_j^{(t)}; 1)_{1,P_t} \end{aligned} \quad (3.7)$$

$$\begin{aligned} D = & (d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{1,q'_1}; \dots; (d_j^{(s)}, \delta_j^{(s)}; D_j^{(s)})_{1,q'_s}; (d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{1,q'_1}; \dots; \\ & (d_j^{(u)}, \delta_j^{(u)}; D_j^{(u)})_{1,q''_u}; (0, 1; 1), (1 - g'_j, f'_j; 1)_{1,Q_1}, \dots, (0, 1; 1), (1 - eg_j^{(t)}, k_j^{(t)}; 1)_{1,Q_t} \end{aligned} \quad (3.8)$$

$$K_1 = [1 - A_j; 0, \dots, 0, 0, \dots, 0, a'_j, \dots, a_t^{(t)}; 1]_{1,P},$$

$$\begin{aligned} & [1 - l_i + \sigma_i + \sum_{j=1}^k \delta_i^{(j)} h_j; \lambda_i^{(1)}, \dots, \lambda_i^{(s)}, \lambda_i'^{(1)}, \dots, \lambda_i'^{(u)}, \mu'_i, \dots, \rho_i^{(t)}; 1]_{1,r'} \\ & (Mh - \mu - N - l - \sum_{j=1}^r vl_j - \sum_{j=1}^k \rho_j h_i; \mu_1, \dots, \mu_s, \mu'_1, \dots, \mu'_u, \lambda_1, \dots, \lambda_t; 1) \end{aligned} \quad (3.9)$$

$$K_2 = [1 - B_j; 0, \dots, 0, 0, \dots, 0, b'_j, \dots, b_t^{(t)}; 1]_{1,Q},$$

$$\begin{aligned} & [1 + \sigma_i + \sum_{j=1}^k \delta_i^{(j)} h_j; \lambda_i^{(1)}, \dots, \lambda_i^{(s)}, \lambda_i'^{(1)}, \dots, \lambda_i'^{(u)}, \mu'_i, \dots, \rho_i^{(t)}; 1]_{1,r'} \\ & (Mh + \alpha - \mu - N - l - \sum_{j=1}^r vl_j - \sum_{j=1}^k \rho_j h_i; \mu_1, \dots, \mu_s, \mu'_1, \dots, \mu'_u, \lambda_1, \dots, \lambda_t; 1) \end{aligned} \quad (3.10)$$

We have the following general formula

**Theorem 1**

$$D_x^a \left\{ x^\mu \prod_{i=1}^{r'} (x^v + \epsilon_i)^{\sigma_i} \phi_N(zx) S_{N_1, \dots, N_k}^{M_1, \dots, M_k} \left( \begin{array}{c} z_1 x^{\rho_1} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{\delta'_i} \\ \vdots \\ z_k x^{\rho_k} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{\delta_i^{(k)}} \end{array} \right) \right.$$

$$I \left( \begin{array}{c} Z_1 x^{\mu_1} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\lambda_i^{(1)}} \\ \vdots \\ Z_s x^{\mu_s} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\lambda_i^{(s)}} \end{array} \right) I \left( \begin{array}{c} Z'_1 x^{\mu'_1} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\lambda_i'^{(1)}} \\ \vdots \\ Z'_u x^{\mu'_u} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\lambda_i'^{(u)}} \end{array} \right)$$

$$F_{Q;Q_1,\cdots,Q_t}^{P;P_1,\cdots,P_t} \left[ -Z_1'' x^{\lambda_1} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\mu_i'}, \cdots, -Z_t'' x^{\lambda_t} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\mu_i^{(t)}} \right] \Bigg\}$$

$$= x^{\mu-a} \prod_{i=1}^{r'} \epsilon_i^{\sigma_i} \frac{\prod_{j=1}^Q \Gamma(B_j) \prod_{j=1}^{Q_1} \Gamma(g_j') \cdots \prod_{j=1}^{Q_t} \Gamma(g_j^{(t)})}{\prod_{j=1}^P \Gamma(A_j) \prod_{j=1}^{P_1} \Gamma(e_j') \cdots \prod_{j=1}^{P_t} \Gamma(e_j^{(t)})}$$

$$\sum_{h=0}^{[N/M]} \sum_{l=0}^{[dh]} \frac{(-dh)_l c^{-v'-N+(M-1)h} (v')_{N+(1-M)h} (a)^{N-Mh} b^h (-)^{(d+1)h} 2^l x^{N-Mh+1}}{l! h! (N-Mh)!}$$

$$\sum_{h_1=0}^{[N'_1/M'_1]} \cdots \sum_{h_k=0}^{[N'_k/M'_k]} A(h_1,\cdots,h_k) \prod_{j=1}^k \left[ \frac{(-N_j)_{M_j h_j}}{h_j!} (Z_j x^{\rho_j})^{h_j} \left( \epsilon_1^{\delta_1^{(j)}} \cdots \epsilon_{r'}^{\delta_{r'}^{(j)}} \right)^{h_j} \right]$$

$$\sum_{l_1,\cdots,l_{r'}=0}^{\infty} \prod_{i=1}^{r'} \left[ \frac{\left(-\frac{x^v}{\epsilon_i}\right)^{l_i}}{l_i!} \right]$$

$$I_{p'+p''+P+r'+1,q'+q''+Q+r'+1;V}^{0,n'+n''+P+r'+1;X} \left( \begin{array}{c} Z_1 x^{\mu_1} \prod_{i=1}^{r'} \epsilon_i^{-\lambda_i^{(1)}} \\ \vdots \\ Z_s x^{\mu_s} \prod_{i=1}^{r'} \epsilon_i^{-\lambda_i^{(s)}} \\ \vdots \\ Z'_1 x^{\mu'_1} \prod_{i=1}^{r'} \epsilon_i^{-\lambda_i'^{(1)}} \\ \vdots \\ Z'_u x^{\mu'_u} \prod_{i=1}^{r'} \epsilon_i^{-\lambda_i'^{(u)}} \\ Z''_1 x^{\lambda_1} \prod_{i=1}^{r'} \epsilon_i^{-\mu'_i} \\ \vdots \\ Z''_t x^{\lambda_t} \prod_{i=1}^{r'} \epsilon_i^{-\mu_i^{(t)}} \end{array} \middle| \begin{array}{l} A, K_1 : C \\ \\ B, K_2 : D \end{array} \right) \quad (3.11)$$

The quantities  $A, B, K_1, K_2, C$  and  $D$  are defined above, where the I-function on the right hand side is of  $(s + u + t)$  variables and  $F_{Q;Q_1,\dots,Q_t}^{P;P_1,\dots,P_t}(\cdot)$  is the generalized Lauricella function [9, page 253-254].

Provided that

- a)  $v, \mu, \sigma_i, \rho_j, \delta_i^{(j)}, \mu_l, \lambda_i^{(l)}, \mu'_n, \lambda_i^{(n)}, \lambda_m, \mu_i^{(m)} > 0; i = 1, \dots, r'; j = 1, \dots, k; l = 1, \dots, s; n = 1, \dots, u; m = 1, \dots, t$
- b)  $Re \left[ \mu + \sum_{i=1}^s \mu_i \min_{1 \leq j \leq m'_i} \frac{d_j'^{(i)}}{\delta_j'^{(i)}} + \sum_{i=1}^u \mu_i \min_{1 \leq j \leq m''_i} \frac{d_j''^{(i)}}{\delta_j''^{(i)}} \right] > 0$

Concerning the following theorem, the quantities  $A, B, C$  and  $D$  are similar

Let

$$K_3 = (-\mu; \sigma'_1, \dots, \sigma'_l; 1), K_4 = (-\alpha - em' - \epsilon - \mu; \sigma'_1, \dots, \sigma'_l; 1) \quad (3.12)$$

$$K_1 = [1 - A_j; 0, \dots, 0, 0, \dots, 0, a'_j, \dots, a_t^{(t)}; 1]_{1,P},$$

$$[1 + \sigma_i + \sum_{j=1}^k \delta_i^{(j)} h_j; \lambda_i^{(1)}, \dots, \lambda_i^{(s)}, \lambda_i'^{(1)}, \dots, \lambda_i'^{(u)}, \mu'_i, \dots, \rho_i^{(t)}; 1]_{1,r'}$$



$$(Mh - N - l - \sum_{j=1}^r vl_j - \sum_{j=1}^k \rho_j h_i; \mu_1, \dots, \mu_s, \mu'_1, \dots, \mu'_u, \lambda_1, \dots, \lambda_t; 1) \quad (3.13)$$

$$K_2 = [1 - B_j; 0, \dots, 0, 0, \dots, 0, b'_j, \dots, b_t^{(t)}; 1]_{1,Q},$$

$$[1 + \sigma_i + \sum_{j=1}^k \delta_i^{(j)} h_j; \lambda_i^{(1)}, \dots, \lambda_i^{(s)}, \lambda_i'^{(1)}, \dots, \lambda_i'^{(u)}, \mu'_i, \dots, \rho_i^{(t)}; 1]_{1,r'}$$

$$(Mh + em' + \epsilon - N - l - \sum_{j=1}^r vl_j - \sum_{j=1}^k \rho_j h_i; \mu_1, \dots, \mu_s, \mu'_1, \dots, \mu'_u, \lambda_1, \dots, \lambda_t; 1) \quad (3.14)$$

$$A_1 = (a_j; \alpha_j^{(1)}, \dots, \alpha_j^{(r)}, 0, \dots, 0; A_j)_{1,p} \quad (3.15)$$

$$B_1 = (b_j; \beta_j^{(1)}, \dots, \beta_j^{(r)}, 0, \dots, 0; B_j)_{1,q} \quad (3.16)$$

$$C_1 = (c_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{1,p_1}; \dots; (c_j^{(r)}, \gamma_j^{(r)}; C_j'^{(r)})_{1,p_r} \quad (3.17)$$

$$D_1 = (d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{1,q_1}; \dots; (d_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{1,q_r} \quad (3.18)$$

We have the following general formula

**Theorem 2**

$$D_x^a \left\{ x^\mu \prod_{i=1}^{r'} (x^v + \epsilon_i)^{\sigma_i} \phi_N(zx) S_{N_1, \dots, N_k}^{M_1, \dots, M_k} \begin{pmatrix} z_1 x^{\rho_1} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{\delta'_i} \\ \vdots \\ z_k x^{\rho_k} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{\delta_i^{(k)}} \end{pmatrix} \right\}$$

$$I \begin{pmatrix} Z_1 x^{\mu_1} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\lambda_i^{(1)}} \\ \vdots \\ Z_s x^{\mu_s} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\lambda_i^{(s)}} \end{pmatrix} I \begin{pmatrix} Z'_1 x^{\mu'_1} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\lambda_i'^{(1)}} \\ \vdots \\ Z'_u x^{\mu'_u} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\lambda_i'^{(u)}} \end{pmatrix}$$

$$I \left( \begin{array}{c} \omega x^{\alpha_1} \\ \vdots \\ \omega x^{\alpha_r} \end{array} \middle| \begin{array}{c} A; \mathfrak{A}_1; A_1 \\ \\ B; \mathfrak{B}_1; B_1 \end{array} \right) F_{Q; Q_1, \dots, Q_t}^{P; P_1, \dots, P_t} \left[ -Z_1'' x^{\lambda_1} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\mu'_i}, \dots, -Z_t'' x^{\lambda_t} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\mu_i^{(t)}} \right] \Bigg\}$$

$$=x^{\mu-a}\prod_{i=1}^{r'}\epsilon_i^{\sigma_i}\frac{\prod_{j=1}^Q\Gamma(B_j)\prod_{j=1}^{Q_1}\Gamma(g'_j)\cdots\prod_{j=1}^{Q_t}\Gamma(g_j^{(t)})}{\prod_{j=1}^P\Gamma(A_j)\prod_{j=1}^{P_1}\Gamma(e'_j)\cdots\prod_{j=1}^{P_t}\Gamma(e_j^{(t)})}$$

$$\sum_{h=0}^{[N/M]}\sum_{l=0}^{[dh]}\frac{(-dh)_lc^{-v'-N+(M-1)h}(v')_{N+(1-M)h}(a)^{N-Mh}b^h(-)^{(d+1)h}2^lx^{N-Mh+1}}{l!h!(N-Mh)!}\sum_{h_1=0}^{[N'_1/M'_1]}\cdots\sum_{h_k=0}^{[N'_k/M'_k]}$$

$$A(h_1,\cdots,h_k)\prod_{j=1}^k\left[\frac{(-N_j)_{M_jh_j}}{h_j!}(Z_jx^{\rho_j})^{h_j}\left(\epsilon_1^{\delta^{(j)}}\cdots\epsilon_{r'}^{\delta^{(j)}}\right)^{h_j}\right]\sum_{l_1,\cdots,l_{r'}=0}^{\infty}\prod_{i=1}^{r'}\left[\frac{\left(-\frac{x^v}{\epsilon_i}\right)^{l_i}}{l_i!}\right]$$

$$I_{p'+p''+P+r'+1,q'+q''+Q+r'+1;V}^{0,n'+n''+P+r'+1;X}\left(\begin{array}{c} Z_1x^{\mu_1}\prod_{i=1}^{r'}\epsilon_i^{-\lambda_i^{(1)}}\\ \cdot\\ \cdot\\ Z_sx^{\mu_s}\prod_{i=1}^{r'}\epsilon_i^{-\lambda_i^{(s)}}\\ \cdot\\ \cdot\\ Z'_1x^{\mu'_1}\prod_{i=1}^{r'}\epsilon_i^{-\lambda_i^{'(1)}}\\ \cdot\\ \cdot\\ Z'_ux^{\mu'_u}\prod_{i=1}^{r'}\epsilon_i^{-\lambda_i^{'(u)}}\\ Z''_1x^{\lambda_1}\prod_{i=1}^{r'}\epsilon_i^{-\mu'_i}\\ \cdot\\ \cdot\\ Z''_tx^{\lambda_t}\prod_{i=1}^{r'}\epsilon_i^{-\mu_i^{(t)}} \end{array}\right)\left|\begin{array}{l} \text{A , K}_5 : C\\ \\ \\ \\ \\ \text{B, K}_6 : D \end{array}\right.$$

$$I_{p+1,q;p_1,q_1;\cdots;p_r,q_r}^{0,n+1;m_1,n_1;\cdots;m_r,n_r}\left(\begin{array}{c|c} \text{w}_1x^{\alpha_1} & \text{A}',\text{K}_3:C'\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \text{w}_rx^{\alpha_s} & \text{B}',\text{K}_4:D' \end{array}\right)\tag{3.19}$$

Provided that

a)  $v, \mu, \sigma_i, \rho_j, \delta_i^{(j)}, \mu_l, \lambda_i^{(l)}, \mu'_n, \lambda_i^{(n)}, \lambda_m, \mu_i^{(m)} > 0; i = 1, \dots, r'; j = 1, \dots, k; l = 1, \dots, s; n = 1, \dots, u; m = 1, \dots, t$   
 $\lambda_p > 0; p = 1, \dots, r$

$$b) Re \left[ 1 + \sum_{i=1}^s \mu_i \min_{1 \leq j \leq m'_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} + \sum_{i=1}^u \mu_i \min_{1 \leq j \leq m''_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} \right] > 0 \text{ and}$$

$$Re \left[ \mu + 1 + \sum_{i=1}^r \alpha_i \min_{1 \leq j \leq m^{(i)}} \frac{d_j^{(i)}}{\delta_j^{(i)}} \right] > 0$$

To prove the theorem 1 ,we have

$$\begin{aligned} \text{L.H.S} &= D_x^a \left\{ x^\mu \prod_{i=1}^{r'} (x^v + \epsilon_i)^{\sigma_i} \phi_N(zx) S_{N_1, \dots, N_k}^{M_1, \dots, M_k} \begin{pmatrix} z_1 x^{\rho_1} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{\delta'_i} \\ \vdots \\ z_k x^{\rho_k} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{\delta_i^{(k)}} \end{pmatrix} \right. \\ &\quad I \begin{pmatrix} z_1 x^{\mu_1} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\lambda'_i} \\ \vdots \\ z_r x^{\mu_r} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\lambda_i^{(r)}} \end{pmatrix} I \begin{pmatrix} z'_1 x^{\mu'_1} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\lambda'_i{}^{(1)}} \\ \vdots \\ z'_u x^{\mu'_u} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\lambda'_i{}^{(u)}} \end{pmatrix} \\ &\quad \left. F_{Q; Q_1, \dots, Q_t}^{P; P_1, \dots, P_t} \left[ -Z'_1 x^{\lambda_1} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\mu'_i}, \dots, -Z'_t x^{\lambda_t} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\mu_i^{(t)}} \right] \right\} \end{aligned} \quad (3.20)$$

First, using the definitions of general class of polynomials  $\phi_n(\cdot)$ ,  $S_{N_1, \dots, N_t}^{M_1, \dots, M_t}[\cdot]$  given in (1.19) and (1.20) respectively and expressing the I-functions of Nambisan [5] of s variables and u-variables and generalized Lauricella function in Mellin-Barnes contour integral with the help of equations (1.8), (1.14) and (1.21) respectively, changing the order of summations and integrations (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process), we have :

$$\text{L.H.S} = x^{\mu-a} \prod_{i=1}^{r'} \epsilon_i^{\sigma_i} \frac{\prod_{j=1}^Q \Gamma(B_j) \prod_{j=1}^{Q_1} \Gamma(g'_j) \cdots \prod_{j=1}^{Q_t} \Gamma(g_j^{(t)})}{\prod_{j=1}^P \Gamma(A_j) \prod_{j=1}^{P_1} \Gamma(e'_j) \cdots \prod_{j=1}^{P_t} \Gamma(e_j^{(t)})}$$

$$\sum_{h=0}^{[N/M]} \sum_{l=0}^{[dh]} \frac{(-dh)_l x^{N-Mh+l} e^{-v'-N+(M-1)h} (v')_{N+(1-M)h} (a)^{N-Mh} b^h (-)^{(d+1)h} 2^l x^{N-Mh+1}}{l! h! (N-Mh)!}$$

$$\sum_{h_1=0}^{[N_1/M_1]} \cdots \sum_{h_k=0}^{[N_k/M_k]} A(h_1, \dots, h_k) \prod_{j=1}^k \left[ \frac{(-N'_j)_{M'_j h_j}}{h_j!} y_j^{h_j} \right] \frac{1}{(2\pi\omega)^{s+u+t}} \int_{L'_1} \cdots \int_{L'_s} \int_{L''_1} \cdots \int_{L''_u}$$

$$\int_{L_1} \cdots \int_{L_t} \phi(t_1, \dots, t_s) \prod_{i=1}^s \zeta_i(t_i) z_i'^{t_i} \psi(x_1, \dots, x_u) \prod_{i=1}^u \xi_i(x_i) z_i''^{x_i} \Gamma(-s_1) \cdots \Gamma(-s_t) z_1^{s_1} \cdots z_t^{s_t}$$

$$\frac{\prod_{j=1}^P \Gamma(A_j + s_1 + \cdots + s_t) \prod_{j_1}^{P_1} \Gamma(e'_j + s_1) \cdots \prod_{j_1}^{P_t} \Gamma(e_j^{(t)} + s_t)}{\prod_{j=1}^{Q_1} \Gamma(B_j + s_1 + \cdots + s_t) \prod_{j=1}^{Q_1} \Gamma(g'_j + s_1) \cdots \prod_{j=1}^{Q_t} \Gamma(g_j^{(t)} + s_t)}$$

$$D_x^\alpha \left\{ x^{\mu+N-Mh+l'+\sum_{j=1}^k \rho_j h_j + \sum_{j=1}^s \mu_j t_j + \sum_{j=1}^u \mu_u x_j + \sum_{j=1}^t \lambda_j s_j} \right.$$

$$\prod_{i=1}^{r'} \left[ (x^v + \epsilon_j)^{\sigma_j + \sum_{j=1}^k \delta_j h_j - \sum_{j=1}^s \lambda_i^{(j)} s_j - \sum_{j=1}^u \lambda_i'^{(j)} x_j - \sum_{j=1}^t \mu_i^{(j)} s_j} \right] \Big\} \, \mathrm{d}t_1 \cdots \mathrm{d}t_s \mathrm{d}x_1 \cdots \mathrm{d}x_u \, \mathrm{d}s_1 \cdots \mathrm{d}s_t \quad (3.21)$$

$$\text{Now use the binomial formula : } (x^v + a)^\lambda = a^\lambda \sum_{m=0}^{\infty} \binom{\lambda}{m} \left( \frac{x^v}{a} \right)^m ; \left| \frac{x^v}{a} \right| < 1 \quad (3.22)$$

twice and then the result (1.24) therein. On interpreting the contour integrals in the I-function of  $(s+u+t)$ -variables , we obtain the desired result.

To prove the theorem 2, we set  $f(x), g(x)$  in equation (1.20) as the following

$$f(x) = x^\mu I \left( \begin{matrix} w_1 x^{\alpha_1} \\ \vdots \\ \vdots \\ w_r x^{\alpha_r} \end{matrix} \middle| \begin{matrix} A':C' \\ \vdots \\ \vdots \\ B':D' \end{matrix} \right) \quad (3.23)$$

$$\text{and } g(x) = \left\{ x^\mu \prod_{i=1}^{r'} (x^v + \epsilon_i)^{\sigma_i} \phi_N(zx) S_{N_1, \dots, N_k}^{M_1, \dots, M_k} \left( \begin{matrix} z_1 x^{\rho_1} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{\delta_i'} \\ \vdots \\ \vdots \\ z_k x^{\rho_k} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{\delta_i^{(k)}} \end{matrix} \right) \right\}$$

$$I \left( \begin{matrix} Z_1 x^{\mu_1} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\lambda_i'} \\ \vdots \\ \vdots \\ Z_r x^{\mu_r} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\lambda_i^{(r)}} \end{matrix} \right) I \left( \begin{matrix} Z_1 x^{\mu_1'} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\lambda_i'^{(1)}} \\ \vdots \\ \vdots \\ Z_u x^{\mu_u'} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\lambda_i'^{(u)}} \end{matrix} \right)$$

$$F_{Q;Q_1,\cdots,Q_t}^{P;P_1,\cdots,P_t} \left[ -Z_1' x^{\lambda_1} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\mu_i'}, \cdots, -Z_t' x^{\lambda_t} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\mu_i^{(t)}} \right] \Big\} \quad (3.24)$$

$$\text{We have L.H.S} = \sum_{n'=-\infty}^{\infty} e\left(\frac{\alpha}{en'+\epsilon}\right) D_x^{\alpha-en'-\epsilon} [x^\mu I(w_1 x^{\alpha_1}, \dots, w_s x^{\alpha_r}) D_x^{an+\epsilon} \left\{ x^\mu \prod_{i=1}^{r'} (x^v + \epsilon_i)^{\sigma_i} \phi_N(zx) \right.$$

$$S_{N_1, \dots, N_k}^{M_1, \dots, M_k} \left( \begin{matrix} z_1 x^{\rho_1} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{\delta_i^{'}} \\ \cdot \cdot \cdot \\ z_k x^{\rho_k} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{\delta_i^{(k)}} \end{matrix} \right) F_{Q; Q_1, \dots, Q_t}^{P; P_1, \dots, P_t} \left[ -Z'_1 x^{\lambda_1} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\mu_i^{'}} , \dots , -Z'_t x^{\lambda_t} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\mu_i^{(t)}} \right] \Bigg\}$$

$$I \left( \begin{matrix} Z_1 x^{\mu_1} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\lambda_i^{'}} \\ \cdot \cdot \cdot \\ Z_r x^{\mu_r} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\lambda_i^{(r)}} \end{matrix} \right) I \left( \begin{matrix} Z'_1 x^{\mu'_1} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\lambda_i^{'(1)}} \\ \cdot \cdot \cdot \\ Z'_u x^{\mu'_u} \prod_{i=1}^{r'} (x^v + \epsilon_i)^{-\lambda_i^{'(u)}} \end{matrix} \right) \quad (3.25)$$

Now use the Lemme and the theorem 1, we obtain the theorem 2

#### Remarks :

If  $A'_j = B'_j = C_j^{(i)} = D_j^{(i)} = A''_j = B''_j = C_j^{''(i)} = D_j^{''(i)} = 1$ , the multivariable I-functions defined by Nambisan et al [5] reduce to multivariable H-functions defined by Srivastava and al [9], we obtain the similar relations.

By the following similar procedure, the results of this document can be extended to product of any finite number of multivariable I-functions defined by Nambisan et al [5].

#### 4. Conclusion

In this paper we have evaluated a generalized contour integral involving the multivariable I-functions defined by Nambisan et al [5], two classes of polynomials and the generalized hypergeometric function. The four formulae established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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